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M E X I C O

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Abstract

We study the aggregate effects of a social security reform in a large overlapping generations model where markets are incomplete and households face uninsurable idiosyncratic income shocks. We depart from the previous literature by assuming that, because of lack of commitment in the credit market, the borrowing constraint in the unique asset is endogenously determined by the agents' incentives to default on previous debts. We find that a model with exogenous borrowing constraints overestimates the positive effect of reforming social security on the capital stock and the saving rate, compared to our model with endogenous borrowing limit. The reason is that, in the latter, the size of precautionary savings is smaller because after the reform the incentives to default on previous debts are lower and consequently households face more relaxed borrowing limits. Adding retirement accounts to the basic model does not change these conclusions, although the quantitative importance of endogenizing borrowing constraints is reduced.

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1 Introduction

The reform of social security is at the center of the political debate in most countries, at least for two reasons: First, the aging of the population is rising concern about the sustainability of public social security systems over the next decades. Second, in some countries the gross private saving rate has shown a downward trend over the last decades. For example,in the U.S. economy the private saving rate as a percent of GDP decreased from 20.9 in 1980 to 12.7 in 2000. This trend, and its possible negative consequences for economic growth, has exacerbated the debate about the desirability of reducing the generosity of public pension systems as a tool to promote savings.

In this paper, we focus on the second issue, abstracting completely from the first one. The question we address is whether the lack of commitment in the credit market is important to understand the effects of reforming social security on aggregate savings. Our answer is that when borrowing constraints are endogenous and idiosyncratic income risk is important, the magnitude of the increase in the capital to output ratio and the saving rate due to a social security reform is reduced.

Recent theoretical studies have found that reforming social security, eliminating the pay-as-you-go (PAYG) public pension system, is an important policy tool to foster capital accumulation and savings. The seminal example of this tradition is the deterministic model by Auerbach and Kotlikoff (1987). More recently, there are some papers that have addressed the aggregate effects of social security in model economies with idiosyncratic income risk. The work of Imrohoroglu et al. (1995), Conesa and Krueger (1999) and Storesletten et al. (1999) are the typical examples. Perhaps closer to the spirit of our work is the study by Fuster (1999). She finds that introducing altruism in a large overlapping generations model substantially reduces the effect of eliminating social security on the capital stock.

One common feature of these studies is that individuals face exogenous borrowing constraints (in fact, no borrowing restrictions) and consequently have not accounted for the possible effect that a change in the social security regime may have on the incentives to default on previous debts. Without commitment, financial intermediaries take into account an individual rationality constraint from the borrower's side, namely, that the value of paying back the debt is no less than the value of defaulting and accepting the legal punishment (normally associated to some level of exclusion from the credit market in the future). Hence, changes in the economic environment like a social security reform are likely to affect the relative value of default and the endogenous borrowing limit associated to it. In an environment with idiosyncratic income risk, the reform could then have an impact on the size of precautionary savings.

To study this issue, we build a large overlapping generations model where households face uninsurable idiosyncratic income shocks. Agents live for a maximum number of periods, and retire at an exogenous age. In addition to idiosyncratic income risk, agents face a life cycle profile of mean earnings, and mortality risk after retirement. All agents are ex-ante equal. Markets are incomplete by assumption, since we only allow for one asset (later, we introduce a second asset, retirement accounts). These features of the model are standard in the social security literature and correspond closely to the assumptions in Imrohoroglu et al. (1995).

We depart from this literature by assuming that, because of lack of commitment, financial intermediaries are only willing to lend to a worker the maximum amount of resources that satisfies the rationality condition of no default for *all* possible values of the income shock tomorrow. Consequently, individuals face state-specific borrowing limits and there is no default in equilibrium. This assumption has been used by Zhang (1997) and Fernandez Villaverde and Krueger (2002). Notice that in the incomplete markets literature we are somewhere in between two well-established theories. On one hand, there are papers which exogenously assume the number of assets (usually one) and the borrowing constraint for each asset (in most cases, no borrowing). This is the framework studied by Huggett (1993) and Aiyagari (1994), in which most of the social security literature is based on. On the other hand, the seminal paper by Kehoe and Levine (1993), followed by Alvarez and Jermann (2000), assumes a full set of contingent assets but with an endogenous borrowing limit in each, arising from the lack of commitment from the borrower´s side. We do restrict the set of assets, as in the first framework, but endogenize the borrowing limit, as in the second.[1]

We start from the steady state of our economy in a PAYG social security system with a generous replacement rate. Each worker pays a social security tax proportional to her labor income when working, and collect benefits after retirement. We assume that the benefits are the same for all retirees, independently of the amount contributed to the social security fund. Hence, social security plays an insurance role against bad income shocks during the working life. Still, because of the type of market incompleteness assumed, workers have to self-insure against short run negative income shocks through precautionary savings. The experiment is to compare this steady state to one in which the replacement rate is zero, so workers have also to build their own savings for retirement.

We calibrate the model to U.S. data, solve for the stationary equilibria, and find that when the borrowing constraints are exogenous eliminating social security has important effects on the capital stock and the saving rate. In contrast, when the borrowing constraints are endogenous, the magnitude of the increase in the saving rate is reduced (although not by a huge amount) and there is a debt boom. The reason is that with the elimination of social security individuals have less incentives to default on debts. The relaxation of borrowing constraints associated to this fact

[1] The formal properties of our framework have not been fully developed. We do not claim that the resulting optimal debt contract is optimal in any meaningful sense. Moreover, we are aware of a third alternative, which is a framework in which interest rates are type specific, maximize banks ex-ante profits, and there is default in equilibrium, as in Chatterjee et al. (2004). Adding this default option is an interesting issue for future research.

implies that the agents need to save less for precautionary reasons. This effect goes in the opposite direction to the traditional increase in savings associated with the reform of public pensions.[2]

Our result goes in the direction of the empirical literature, which finds that actual experiences of social security reform around the world have had a small impact on saving rates. The first privatization experience, Chile in 1981, has been used as a textbook example of a successful reform. However, the aggregate effects on the saving rate show an increase of at most 10% in twenty years, of which is hard to disentangle how much comes from corporate savings instead of households savings.[3] Using Chilean household level data, Coronado (2002) finds that the saving rate increase by about 7% for rich families, while Butelmann and Gallego (2000) find an increase in debt for low income households. It is still early to assess the results of the reform in other Latin American countries, as Mexico and Peru, which reformed their systems in the 90s, but so far there is no evidence of increases in their saving rates while consumer debt has significantly increased. A "rich" country which has been reforming its social security system since 1986 is the U.K. Disney et al. (2003) find no trend in household aggregate savings rate after the reform, while Granvick and Mallick (2002) finds that savings for retirement substitute other types of household savings, with no increase in the aggregate saving rate.

One of the multiple problems in comparing the predictions of the model with actual reforms is that these reforms do not look like the theoretical experiment of eliminating the PAYG social security system. In all the cases the government ensures a minimum, less generous pension to all workers. Moreover, the reduction of the social security replacement rate comes together with the creation of new financial instruments as tax favored retirement accounts to promote savings when young. In the second part of this paper, we extend the basic model by introducing a second asset (retirement accounts) and experiment with a more realistic social security reform. Again, the model with endogenous borrowing constraints features a smaller increase in the saving rate after the reform than the model with exogenous borrowing limits, and also a larger debt boom. The differences between the two models, however, are now very small, mostly because in our model retirement accounts are protected against creditor claims upon default, as it is in most legislations. Even if our explanation goes in the right direction, there is still a long way to go in order to understand why social security reforms do

[2]A previous attempt to incorporate endogenous borrowing constraints in social security models is the work by Andolfatto and Gervais (2001). They use a three-period overlapping generations model with no income risk to assess the role played by endogenous borrowing constraints in shaping the aggregate effects of social security. Their findings indicate that it is not quantitatively important to model these constraints endogenously. The reason is that their model does not account for the change in the magnitude of precautionary savings associated with the relaxation of borrowing limits after the elimination of social security. In contrast, in our paper we account for this additional effect and find that is important for the question at hand.

[3]See Coronado (2002) for a review of the problems of backing up saving rates using macro data in Chile.

not lead to the huge increases in savings predicted by current models.

The paper is organized as follows. Section 2 describes the main features of our economy with only one asset and defines a stationary equilibrium for it. Section 3 explains the calibration procedure and show some features of our benchmark quantitative model. In Section 4, we perform the experiment of a social security reform which eliminates social security benefits, comparing the effects on the saving rates with fixed (exogenous) borrowing constraints and endogenous borrowing limits. Section 5 extends the model adding a second asset, retirement accounts, and analyzes a more realistic social security reform. Finally, we conclude.

2 The Environment: One-Asset Economy

Our economy is in many aspects similar from the economy described in Imrohoroglu et al. (1995). We work in a large overlapping generations setup, with mortality risk and individual income risk. We also preclude for contingent markets, including markets for annuities. Individuals save and borrow through only one asset, negative amounts of which corresponds to unsecured debt. We depart from the previous literature, however, by introducing limited commitment in the credit market. Individuals can only commit to pay back their previous debt if it is optimal for them to do so, compared to the option of default. In particular, we assume as in Zhang (1997) that financial intermediaries are only willing to lend to a worker the maximum amount of resources that satisfies the rationality condition of no default for all possible values of the income shock tomorrow. Consequently, there is no default in equilibrium, and individuals face state-specific, endogenously determined, borrowing constraints.

The main features of the model are the following. The economy is populated by individuals that live for a maximum of I periods. Agents with age $i \in \{1, ..., I_r - 1\}$ are workers and provide $\eta e(i)$ efficiency units of labor to the market, where e is an exogenously given efficiency profile and η is a stochastic shock. The transitions across states of nature follow the stationary probability matrix π. Individuals with age $i \geq I_r$ are retired, collect social security benefits (TR_{ss}) and face a probability of surviving to the next period $s(i)$ with $s(I) = 0$. Each agent which dies is replaced by a newborn with age $i = 1$, employed. and with no assets. Total assets of dead agents (liquid plus retirement accounts) are seized by the government.

Workers and retirees decide how much to consume (c) and save/borrow (a) to maximize their lifetime utility. The one period utility function is of the class of CRRA, with risk-aversion coefficient σ and discount factor β. In order to determine a worker's endogenous borrowing constraint, we have to compare the continuation value of paying back a loan of a given size, or defaulting. The latter implies a complete discharge of the debt but also a punishment: defaulters are permanently excluded form the credit market and can only save through a storage technology which yields no interest income. [4] We assume that retirees cannot borrow.

Workers pay a social security tax proportional to their labor income (τ_{ss}). The proceedings are used to finance social security benefits, computed as a replacement rate times the average wage in the economy. The social security system is self-financed. The government also collects labor income taxes (τ), capital income taxes (τ_k) and unintended bequests from all agents and spends it in unproductive government consumption (G) keeping each period a balanced budget. Finally, there is a technology to produce the only good in the economy (Y) using labor (L) and capital (K), described by a Cobb-Douglas production function.

We consider only a stationary equilibrium, in which all prices and aggregate quantities remain constant over time. This allows us to write the model in a simple recursive language.

2.1 Consumer's Problem

To characterize consumer´s problem, we first have to describe the continuation value for agents that defaulted in their previous debts and consequently are excluded from the borrowing market and from the possibility of earning capital income on accumulated liquid assets. This value is necessary to compute the endogenous borrowing limit in the problem faced by consumers in equilibrium.

Value of default:

Let´s start with a retired individual in her last period of life ($i = I$). This agent will consume all its income, given by the common social security transfer plus any remaining assets. Hence her utility is given by:

$$v_r^*(a, I) = \frac{[TR_{ss} + a]^{1-\sigma}}{1-\sigma}$$

Going backwards, a retired individual of age $i = \{I_r, ..., I-1\}$ faces the following problem:

$$v_r^*(a, i) = \max_{\{c,a'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta s(i) \, v_r^*(a', i+1) \right\}$$

$$\begin{aligned} s.t. \quad c + a' &= TR_{ss} + a \\ a' &\geq 0 \end{aligned}$$

[4] This punishment is weaker than the one in Zhang (1997) and Kehoe and Levine (1993). In these papers, defaulters revert to autarky, this is, they cannot borrow nor save in their remaining periods of life. Such a harsh punishment would certainly work in our favor, although it might not be realistic. We also explored the possibility of precluding defaulters only to borrow, allowing them to earn interest income. It turned out that this punishment was too weak to sustain a positive amount of debt in equilibrium, a result already suggested by Bulow and Rogoff (1989).

Notice that this individual cannot borrow and does not earn interest income, as a punishment from past behavior. Now let´s move to workers which defaulted in their previous debts and are in their last period of working life ($i = I_r - 1$). Their problem is described by:

$$v_e^*(a, I_{r-1}, \eta) = \max_{\{c,a'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta v_r^*(a', I_r) \right\}$$

$$\begin{aligned} s.t. \quad c + a' &= (1-\tau)(1-\tau_{ss})\eta w e(I_r - 1) + a \\ a' &\geq 0 \end{aligned}$$

with an additional state variable η representing their current income shock. The problem is similar for workers at ages $i = \{1, ..., I_r - 2\}$:

$$v_e^*(a, i, \eta) = \max_{\{c,a'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \sum_{\eta' \in E} \pi(\eta, \eta') v_e^*(a', i+1, \eta') \right\}$$

$$\begin{aligned} s.t. \quad c + a' &= (1-\tau)(1-\tau_{ss})\eta w e(i) + a \\ a' &\geq 0 \end{aligned}$$

with the probabilities $\pi(\eta, \eta')$ used to compute expectations about next period´s income.

Consumer´s Problem with Endogenous Borrowing Constraints:

We are ready now to described the problem faced by consumers in equilibrium. Again, we start with a retired agent in her last period of life ($i = I$):

$$v_r(a, I) = \frac{[TR_{ss} + (1 + r(1-\tau_k))a +]^{1-\sigma}}{1-\sigma}$$

and at ages $i = \{I_r, ..., I-1\}$:

$$v_r(a, i) = \max_{\{c,a'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta s(i) v_r(a', i+1) \right\}$$

$$\begin{aligned} s.t. \quad c + a' &= TR_{ss} + (1 + r(1-\tau_k))a \\ a' &\geq 0 \end{aligned}$$

Notice that the only difference with the default values is that retirees now earn some interest income for their assets. A worker in her last period of working life ($i = I_r - 1$) faces the problem

$$v_e\left(a, I_{r-1}, \eta\right) = \max_{\{c,a'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta v_r\left(a', I_r\right) \right\}$$

$$\begin{aligned} s.t. \quad c + a' &= (1-\tau)(1-\tau_{ss})\,\eta w e\left(I_r - 1\right) + \left(1 + r\left(1-\tau_k\right)\right) a \\ a' &\geq 0 \end{aligned}$$

again similar to the default value except for the interest income. The interesting problem is the one faced by worker of age $i = \{1, ..., I_r - 2\}$:

$$v_e\left(a, i, \eta\right) = \max_{\{c,a'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \sum_{\eta' \in E} \pi(\eta, \eta') v_e\left(a', i+1, \eta'\right) \right\}$$

$$\begin{aligned} s.t. \quad c + a' &= (1-\tau)(1-\tau_{ss})\,\eta w e\left(i\right) + \left(1 + r\left(1-\tau_k\right)\right) a \\ a' &\geq -b(i) \end{aligned}$$

where the endogenous borrowing constraint $-b(i)$ satisfies the individual rationality constraint:

$$\min\left\{-b(i) \,|\, v_e\left(-b(i), i+1, \eta'\right) \geq v_e^*\left(0, i+1, \eta'\right)\right\} \qquad \forall \eta' \in E$$

Workers are able to borrow only that amount of resources for which the value of paying back this debt is no less than the value of defaulting next period for all possible realizations of the income shock. This gives rise to an endogenous, age-specific, borrowing constraint. Solving each of the previous problems, we obtain optimal decision rules for consumption and next period assets for workers $g_e^c\left(a, i, \eta\right)$, $g_e^a\left(a, i, \eta\right)$ and retirees $g_r^c\left(a, i\right)$, $g_r^a\left(a, i\right)$.

2.2 Firm's Problem

Firms in this economy have access to a Cobb-Douglas production function that converts capital and labor into (net) output:

$$Y = K^{\alpha} L^{1-\alpha} - \delta K$$

From firm's maximization problem we obtain the marginal conditions

$$w = (1-\alpha) K^{\alpha} L^{-\alpha} \qquad r = \alpha K^{\alpha-1} L^{1-\alpha} - \delta$$

2.3 Aggregate Distribution and Law of Motion

Let $\mu_s(a,i,\eta)$ be the mass of consumers (agents) in situation $s \in \{e,r\}$, with age $i \in \{1,..,I\}$, asset holdings $a \in A$ and shock realization $\eta \in E$. Define also the marginal distributions

$$\mu_s(i) = \sum_{\eta \in E} \int_A d\mu_s(a,i,\eta)$$

and the partial sum of asset holdings

$$A_s(i) = \sum_{\eta \in E} \int_A a \; d\mu_s(a,i,\eta)$$

The distribution μ represents the aggregate state variable, that we normalize as a probability measure

$$\sum_{\eta \in E} \sum_{s \in \{e,,r\}} \sum_{i=1}^{I} \int_A d\mu_s(a,i,\eta) = 1$$

The law of motion of the aggregate state variable is given by $\mu' = \Gamma(\mu)$, and can be defined using the optimal policy rules. In our stationary equilibrium, μ is a fixed point of the operator Γ.

2.4 Government Budget

The government in our model faces two separate budgets, one for the social security system and another for current expenditures. We assume that the two budgets are balanced each period.

Social Security System:

Social security benefits are computed applying a replacement rate θ over the average earnings per worker in the economy:

$$TR_{ss} = \theta \frac{\sum_{\eta \in E} \sum_{i=1}^{I_r - 1} \int_A w\eta e(i) \mu_e(a,i,\eta)}{\sum_{i=1}^{I_{r-1}} \mu_e(i)}$$

This transfer is assumed to be the same for all retirees. An interesting extension is to allow for some dependence of the transfer on each individual´s previous wages, as in Storesletten et al. (1999). For simplicity, we abstract from this feature.

Since the social security system is assumed to be balanced, the total amount of transfers to retirees has to be equal to the social security tax collected from all workers:

$$TR_{ss} \sum_{i=I_r}^{I} \mu_r(i) = \tau_{ss} w \sum_{\eta \in E} \sum_{i=1}^{I_r - 1} \eta e(i) \int_A \mu_e(a,i,\eta).$$

Government Expenditures:

The government also levies labor and capital income taxes and unintended bequests to finance an exogenous level of government consumption. This part of the budget is also assumed to be balanced, hence

$$G = \tau (1 - \tau_{ss}) w \sum_{\eta \in E} \sum_{i=1}^{I_r - 1} \eta e(i) \int_A \mu_e (a, i, \eta) + \tau_k r \sum_{s \in \{e,r\}} \sum_{i=2}^{I} A_s (i)$$

$$+ \sum_{i=I_r}^{I} (1 - s(i)) \int_A [g_r^a (a, i)] \, d\mu_r (a, i)$$

2.5 Definition of a Stationary Equilibrium

A Stationary Equilibrium is a set of value functions, optimal decision rules, price functions, a set of government policies, aggregate law of motion Γ, and invariant distribution μ such that:

1. Given prices, taxes, transfers, and aggregate law of motion, consumers optimize
2. Price functions satisfy marginal conditions from firm's problem
3. Government budget constraints are satisfied
4. Markets clear

$$Y = K^{\alpha} L^{1-\alpha} - \delta K = C + K' - K + G$$

where aggregate capital and labor is the sum of asset holdings and the supply of labor efficiency units of all individuals in the economy respectively

$$K = \sum_{s \in \{e,r\}} \sum_{i=1}^{I} A_s (i)$$

$$L = \sum_{i=1}^{I_r - 1} \sum_{\eta \in E} \int_A \eta e (i) \, d\mu_e (a, i, \eta)$$

and aggregate consumption is

$$C = \sum_{s \in \{e,r\}} \sum_{\eta \in E} \sum_{i=1}^{I} \left[\int_A g_s^c (a, i, \eta) \, d\mu_s (a, i, \eta) \right].$$

5. The invariant distribution satisfies $\mu = \Gamma (\mu)$.

3 A Quantitative Benchmark Economy

We calibrate the model above to reproduce some features of the U.S. economy during the eighties. We chose that period since retirement accounts (IRA's and 401(k)´s) where still not available, and therefore the social security system was closer to the pay-as-you-go system described in the model. Moreover, it had been in such a system for a long time, so presumably the old generations had time to adjust their asset holdings. We will talk later about the role played by retirement accounts and how do they change the default incentives in our model.

3.1 Demographics and Employment Process

Each model period is taken to be 5 years. Individuals enter the economy at 20 and may live until age 85 (model period 13). At each point in time individuals supply $e(i)\,\eta$ units of labor. The exogenous profile of age specific efficiency units $e(i)$ is taken from Hansen (1993) and adjusted to our demographic setup through interpolation. The results are presented in Table 1.

For the stochastic idiosyncratic labor shocks we use a discretized version of the process estimated by Storesletten et al. (1999)[5], with five states:

$$\eta \in E = \left[\begin{array}{ccccc} 0.4234, & 0.6507, & 1.000, & 1.5368, & 2.3618 \end{array}\right]$$

transition matrix:

$$\pi = \begin{bmatrix} 0.3684 & 0.3561 & 0.1989 & 0.0637 & 0.0129 \\ 0.2425 & 0.3165 & 0.2657 & 0.1321 & 0.0432 \\ 0.1177 & 0.2313 & 0.3021 & 0.2313 & 0.1177 \\ 0.0432 & 0.1321 & 0.2657 & 0.3165 & 0.2425 \\ 0.0129 & 0.0637 & 0.1989 & 0.3561 & 0.3684 \end{bmatrix}$$

and invariant distribution

$$\Pi = \left[\begin{array}{ccccc} 0.1509, & 0.2217, & 0.2548, & 0.2217, & 0.1509 \end{array}\right]$$

Finally, we assume that individuals survive with probability one until the age of 65 (the retirement age) and after that age individuals may die according to an

[5]In their Table 1, Storesletten, Telmer and Yaron (1999) estimate a first order autoregressive process for individual income with large persistence ($\rho = 0.935$). This is the process that we discretize using five states. Other studies have also relied on these estimates. For example, Conesa and Krueger (1999) and Fernandez Villaverde and Krueger (2002) use a two-state and three-state discretization of the same process, respectively. In Storesletten et al. (2001) the authors discuss that the true process might be close to a unit root. This issue is relevant for our analysis, although we don´t think the results would change by much.

Table 1: Demographics in the Benchmark Economy

Period	Age	e	s
1	20-24	0.36	1
2	25-29	0.46	1
3	30-34	0.53	1
4	35-39	0.59	1
5	40-44	0.63	1
6	45-49	0.64	1
7	50-54	0.64	1
8	55-59	0.61	1
9	60-64	0.56	1
10	65-69	0	0.54
11	70-74	0	0.52
12	75-79	0	0.44
13	80-84	0	0

age specific mortality probabilities profile taken from the U.S. Bureau of the Census, International Data Base. These probabilities have been slightly adjusted so as to reproduce the U.S. dependency ratio (+65 over 20-64) of 21%, as shown in Table 1.

3.2 Government and Social Security taxes

The role of the government in our model economy is twofold. The first is to run a pay-as-you-go social security system collecting taxes on labor earnings and paying pensions to retirees. In the initial steady state (or the benchmark economy), we set the social security replacement rate to 44%. Later, our experiment will be to reduce this rate to zero. The associated social security tax rate that balances the pension budget is 9.4%, a realistic number in the U.S. once Medicare and disability insurance is taken away.

With respect to current expenditures, we set exogenously unproductive government expenditures to be 20% of GDP, as in the U.S. data. We also set a capital income tax of 35%, also consistent with the U.S. legislation. Given these numbers, the labor income tax rate is set endogenously to balance the government budget. This procedure implies a labor income tax of 17%, again a reasonable approximation for the U.S. tax system.

Table 2: Calibration of the Benchmark Economy

Parameters		**Calibration**		
		Targets		Results
α	0.360	rK/Y	0.360	0.360
β	0.994	K/Y	2.500	2.475
δ	0.085	I/Y	0.210	0.211
σ	2.100	Debt/Y	0.028	0.028

3.3 Other Parameters

The remaining parameters are simultaneously solved to match a set of targets for the U.S. economy. In particular, the value of the capital share in output α, the risk aversion parameter σ, the discount parameter β, and the depreciation rate δ are simultaneously calibrated to match a capital share in income of 0.36, a capital output ratio of 2.5, an investment rate of 21% and a debt-to-output ratio of 0.028 which corresponds to the ratio of revolving (unsecured) debt over GDP between 1980-1989 in the U.S. (Federal Reserve Statistical Release). The results of the calibration procedure (at a yearly frequency) are presented in Table 2

3.4 Assets, Debt, and Borrowing Limits in the Benchmark Economy

The model economy has been calibrated to match certain key ratios of the US economy. Hence it reproduces by construction G/Y, I/Y and a realistic capital-output ratio. The average age profile of asset holdings of the initial steady state is also consistent with the empirical evidence. As seen in Figure 1, on average young individuals start accumulating wealth until they reach the age of 60-64 and after they exhaust those assets until they die. Notice also that young agents have on average negative assets, so that the aggregate debt-to-output ratio is non-zero. We do obtain some borrowing in equilibrium.

In our model, borrowing limits depends on the incentives to default on debts. Notice also from Figure1 that young individuals face more relaxed borrowing limits, since for these agents the cost of doing default is higher. The intuition is that agents defaulting early in life lose access to (compounded) interest income in all their savings, compared to an old worker whose stock of savings is already built.

As in most models with incomplete markets, in our economy workers accumulate assets for two reasons: life cycle considerations (including retirement) and precautionary reasons, due to the positive probability of a long sequence of bad shocks that would make individuals hit the borrowing constraint and start to reduce current consumption. As we will see, in a world with endogenous borrowing constraints, the

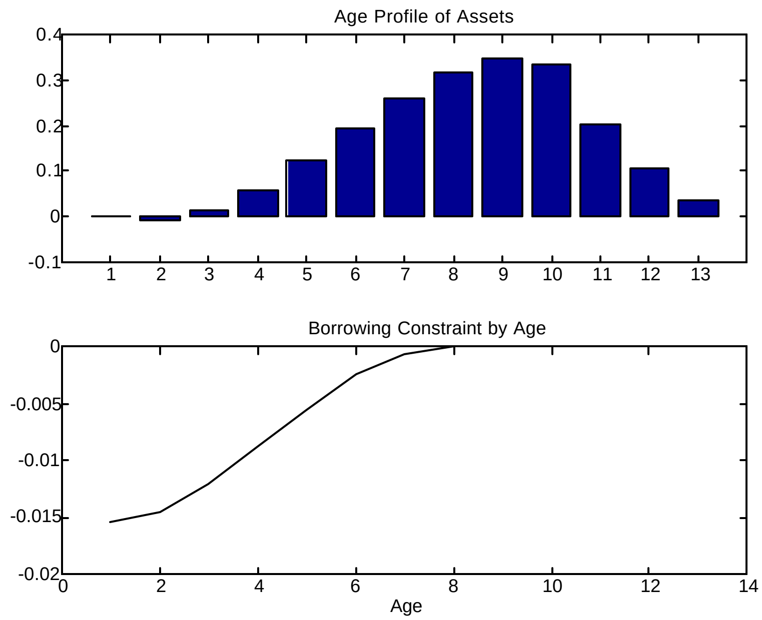

Figure 1: Age Profile of Assets and Borrowing Constraints

effects on precautionary savings are key to understand the aggregate effects of changes in policy, as a social security reform.

4 Effects of Eliminating Social Security

We now study the effects of a social security reform changing the replacement rate from 44% to 0%. We will perform this experiment in two cases. In the first one, we keep fixed the (endogenous) borrowing constraint of the benchmark economy, consequently the change in the social security regime does not change the age specific borrowing limits. We want to think of this case as the one analyzed in the previous literature of social security (Imrohoroglu et al. (1995), Conesa and Krueger (1999), Fuster (1999), etc.). In contrast, in the second case (labelled Endogenous Constraints) borrowing constraints are allowed to adjust to the variation of default incentives induced by the elimination of social security. The difference between the results in the two cases will give us an idea of the quantitative importance of endogenous borrowing constraints in analyzing social security reform.

4.1 Fixed Borrowing Constraints

The results of this experiment can be seen in the top panel of Table 3. Notice that, as in previous quantitative studies, the elimination of the social security system increases

Table 3: Aggregate Effects of Eliminating Social Security

		Fixed Constraints	
	Initial S.State	Final S.State	Percent Change
τ_{ss}	0.09	0.00	–
τ	0.17	0.15	–
K/Y	2.48	3.59	45.01
I/Y	0.21	0.31	45.01
Debt/Y	0.03	0.03	14.86
		Endogenous Constraints	
	Initial S.State	Final S.State	Percent Change
τ_{ss}	0.09	0.00	–
τ	0.17	0.15	–
K/Y	2.48	3.42	38.25
I/Y	0.21	0.29	38.25
Debt/Y	0.03	0.12	334.31

the capital-output ratio and the saving rate. The magnitude of this change, about 45%, is in line with other related studies. For example, Imrohoroglu et al (1995) report that changing the U.S. social security replacement rate from 40% to zero increases the capital-output ratio by 42% in the model economy with zero population growth and certain lifetimes, which is the closest environment to our paper. The increase in the capital stock is due to several factors. First, individuals save more to finance consumption during the retirement period and to insure against income risk since the social security benefits have been remove. And second, a PAYG pension system partially substitutes for missing annuity markets. Consequently, without these public pension benefits individuals have to save more in order to insure themselves against the risk of living more than expected. Hence, the aggregate saving rate and the capital-output ratio increases and the interest rate falls, increasing debt moderately.

4.2 Endogenous Borrowing Constraints

In the second case we perform the same experiment as before, i.e. eliminating the PAYG pension system. In contrast to the previous case, borrowing limits adjust to the change in individual incentives to default on previous debts. The results can be compared in Table 3. As before, the removal of the pension gives incentives to individuals to save more in order to finance consumption when retired. This mechanism increases the saving rate in the economy.

The key point, however, is that in this case the removal of social security benefits

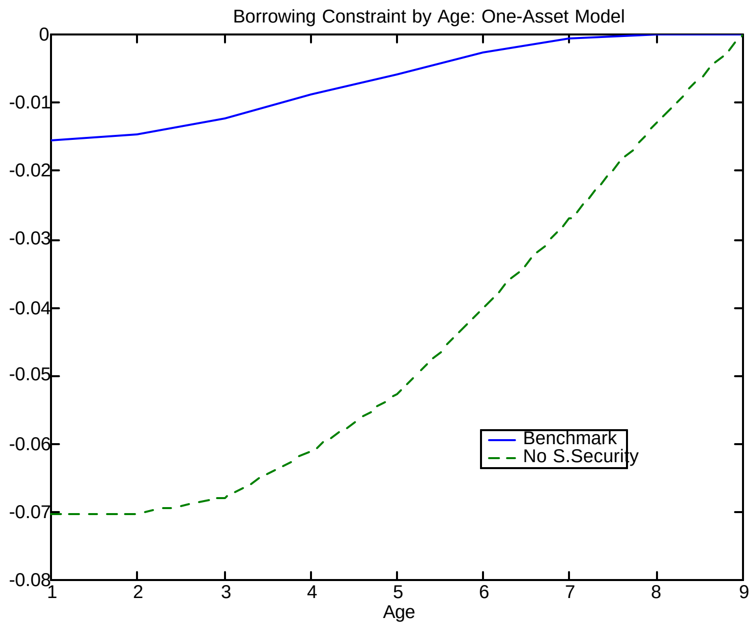

Figure 2: Changes in Age Profile of Borrowing Constraints

also relaxes the borrowing limits agents faced by agents, as seen in Figure 2. The intuition is that without social security individuals have to save on their own for retirement. Then losing interest income on their savings upon default is more costly and incentives to default are lower. The lower incentives to default translates into more relaxed borrowing limits, in particular for young agents which still have to build most of their savings for retirement.

More relaxed borrowing constraints translate into less precautionary savings. Agents save less for precautionary reasons in good states, since they can enter more easily into debt in bad states. This effect goes in the opposite direction to the conventional increase in savings due to the need of self-financing for retirement. Therefore, in contrast to an economy with exogenous borrowing constraints, the aggregate saving rate and the capital-output ratio increases less with the removal of social security. In particular, in our economy the capital-output ratio and the saving rate increases by 38% compared to the 45% increase with fixed borrowing limits. Consequently, *the introduction of endogenous borrowing constraints significantly reduce the increase in the saving rate due to the elimination of social security.* Moreover, the social security reform is followed by a debt boom, since young agents benefit from more relaxed borrowing limits to smooth consumption during life cycle.

Table 4: Effects of Eliminating Social Security without Income Risk

		Fixed Constraints	
	Initial S.State	Final S.State	Percent Change
τ_{ss}	0.09	0.00	–
τ	0.22	0.18	–
K/Y	2.44	3.18	30.35
I/Y	0.21	0.27	30.35
Debt/Y	0.00	0.00	–
		Endogenous Constraints	
	Initial S.State	Final S.State	Percent Change
τ_{ss}	0.09	0.00	–
τ	0.22	0.18	–
K/Y	2.44	3.18	30.35
I/Y	0.21	0.27	30.35
Debt/Y	0.00	0.00	–

4.3 The Role of Idiosyncratic Income Risk

To further understand the importance of idiosyncratic income uncertainty when analyzing the effects social security on the incentives to default on debts, we performed the same set of experiments in a model without income risk. For this, we set the value of the stochastic shock to be always equal to its expected value. The rest of the parameters for the benchmark remain the same. This experiment is important to compare our previous results with the ones obtained by Andolfatto and Gervais (2001), namely that in an OLG models without income risk there are no quantitative differences between modelling borrowing constraints exogenously or endogenously to analyze the effects of social security.

With the elimination of income risk, individuals do not have to self-insure against bad shocks. Consequently, the initial steady state displays a lower capital-output ratio, as observed in Table 4. With a lower capital stock the interest rate and the cost of borrowing are higher, yielding a virtually zero debt-to-output ratio. After the reform, the increase in the capital stock is now smaller because the elimination of social security does not increase precautionary savings by much. This reflects the fact that, without income risk, the PAYG social security system plays no insurance role against lifetime earnings differences. In addition, the differences between the fixed vs. endogenous constraints are almost zero: in our calibrated model, the elimination of social security increases the saving rate by 30% in both cases.

The role of endogenous borrowing constraints in our model depends crucially on the fact that, with income risk, more relaxed borrowing constraints give individuals less incentives to save for precautionary reasons. Abstracting from income risk shuts down the main channel of our model and does not give a fair chance for endogenous borrowing limits to matter.

5 Introducing Retirement Accounts

The social security reform analyzed in the previous section allows us to compare our results with most of the current literature. However, this type of reform is a theoretical extreme which does not correspond to the observed attempts to reform social security in some Latin American and European countries. In all these cases, the generosity of the government managed PAYG system is significantly reduced, but some form of retirement accounts are created instead to promote private savings for retirement. There are three reasons why we should consider retirement accounts as a different asset in our analysis: First, because they are less liquid, since early withdrawal is either prohibited or subject to high penalties. Second, because they receive an special tax treatment. Finally, and more importantly for our purposes, because retirement accounts cannot confiscated by creditors upon default.

We extend the model to add retirement accounts, and perform the following experiment. Staring from the same benchmark economy as before (with 44% social security replacement rate and no retirement accounts), we now compare this steady state to one without social security (zero replacement rate) but with the possibility to save using retirement accounts. We believe this is a more realistic description of a feasible, politically implementable, social security reform.

5.1 A New Environment with Two Assets

The final steady state of the experiment is described by a model with two assets. Individual savings can now take the form of liquid assets (a) or retirement accounts (d). We model retirement accounts following the general rules that characterize the functioning of the most popular retirement accounts (401(k) and IRAs) in the U.S., which are summarized in Appendix A. Other countries (Chile, Mexico, UK, etc.) have introduced similar financial instruments, differing in the limits to contributions, tax treatment, and penalties for early withdrawal.

In our second version of the model, only workers accumulate retirement accounts, choosing a percent of their labor income as their deposit in each period. We impose minimum ($\Delta_{\min}$) and maximum ($\Delta_{\max}$) contributions, as percent of individuals labor income. This deposit is deducted from the current labor income tax. Retirement accounts do not pay capital income taxes. Workers can withdraw from retirement accounts, but paying a penalty (τ_{pen}) proportional to the amount withdrew plus the deferred income tax. Retired agents can withdraw from the retirement account without paying the penalty, only the deferred income tax. There is a minimum amount required to withdraw depending on age ($\varpi(i)$). Finally, and this is a key point, retirement accounts are protected from creditors claims upon a bankruptcy declaration. This means that an individual can maintain the full property of funds invested in retirement accounts upon default.[6]

The new model features an interesting portfolio choice. Agents have to decide how much to save in each type of asset. The liquid asset is more suitable for precautionary reasons, since it can be used at no cost in the case of bad shocks. On the other hand, retirement accounts are better instruments to save for retirement, because of the tax incentives, but are less suitable to smooth consumption during worker's life, since early withdrawals are heavily penalized. This differentiated role for the two assets allows us to pin down the optimal portfolio choice.

Notice also that borrowing constraints are going to depend now not only on age, but on the stock of retirement accounts. Everything else equal, we should expect agents with more retirement accounts to face tighter borrowing limits for the liquid asset. The reason is that, since retirement accounts cannot be confiscated upon default, these agents savings for retirement are more protected and therefore the punishment for default is less costly for them.

5.2 Effects of Social Security Reform

Suppose that an economy starts in an initial steady state corresponding to our Benchmark economy, with a social security replacement rate of 44% and no retirement accounts. The following reform is introduced: (i) the replacement rate is reduced to 10%, and (ii) a system of retirement accounts is created. We set the minimum and maximum contributions to retirement accounts to 4% and 10% of worker's labor income, respectively. We also choose a penalty for early withdrawal of 50%, and follow the same rules for minimum withdrawals after retirement as in the U.S. legislation.[7]

In Table 5, we analyze the steady state after the reform to the initial steady state. As before, we perform one experiment with fixed borrowing constraints, and another with endogenous borrowing limits. In both cases, the social security tax necessary

[6] The formalization of an equilibrium for this new environment is presented in Appendix B. The resulting model is close to Imrohoroglu et al (1998), except that we abstract from the unemployment state (in which early withdrawals from retirement accounts are allowed) and, of course, we use endogenous borrowing constraints instead of a no borrowing restriction.

[7] The experiment does not intend to capture a particular experience of social security reform, neither the current U.S. system. The parameters are chosen as to represent the *average* reform, just as an illustrative example. For example, government in all countries ensure a minimum pension, captured by our positive replacement rate. Neither the U.S. nor the U.K. have minimum contributions to retirement accounts, but in Chile this is 10% of worker's wages and in Mexico, around 6%. The maximum amount of tax deductible contributions is a difficult number to obtain, since legislations set absolute instead of relative caps. Finally, in Chile and Mexico early withdrawals are prohibited (except for unemployment or disability), while in the U.S. they are allowed with a 10% penalty. See Piñera (1996) for a description of the Chilean system, Soliz (2003) for Mexico, and Disney et al. (2003) for the U.K.

Table 5: Social Security Reform with Retirement Accounts

		Fixed Borrowing Constraints	
	Initial S.State	Final S.State	Percent Change
τ_{ss}	0.09	0.02	-7.33
τ	0.17	0.19	1.55
K/Y	2.48	3.42	38.92
I/Y	0.21	0.29	38.92
Debt/Y	0.03	0.04	28.11
RA/K	0.00	0.54	–
		Endogenous Borrowing Constraints	
	Initial S.State	Final S.State	Percent Change
τ_{ss}	0.09	0.02	-7.33
τ	0.17	0.18	1.40
K/Y	2.48	3.38	36.29
I/Y	0.21	0.29	36.29
Debt/Y	0.03	0.05	79.46
RA/K	0.00	0.52	–

to balance the PAYG part of the system is reduced from 9% to 2%, while the labor income tax increases slightly from 17% to 18-19% to pay for tax deductions. The reform increases workers disposable income, as intended.

The main results of this experiment in the model economy with retirement accounts are qualitatively similar to the ones obtained in the previous model economy with one asset. That is, when borrowing constraints are allowed to change as a function of the different incentives to default, eliminating social security translates into a less pronounced increase in the saving rate and a debt boom. However, in this case, the differences between the model with exogenous and endogenous borrowing are less important. Endogenizing borrowing constraints, the capital-output ratio and the saving rate increases by 36% compared to the 39% increase with fixed borrowing limits

We expected to obtain this result for two reasons: First, with a positive replacement rate the government provides for some insurance, therefore reducing the role of precautionary savings. Second, retirement accounts are protected from creditors claims upon default, so replacing the PAYG system with private savings through this instrument is not going to have an important impact in the borrowing limits.

Still, since retirement accounts and liquid assets are imperfect substitutes, we do observe in Figure 3 that borrowing constraints are less tight after the reform. Agents

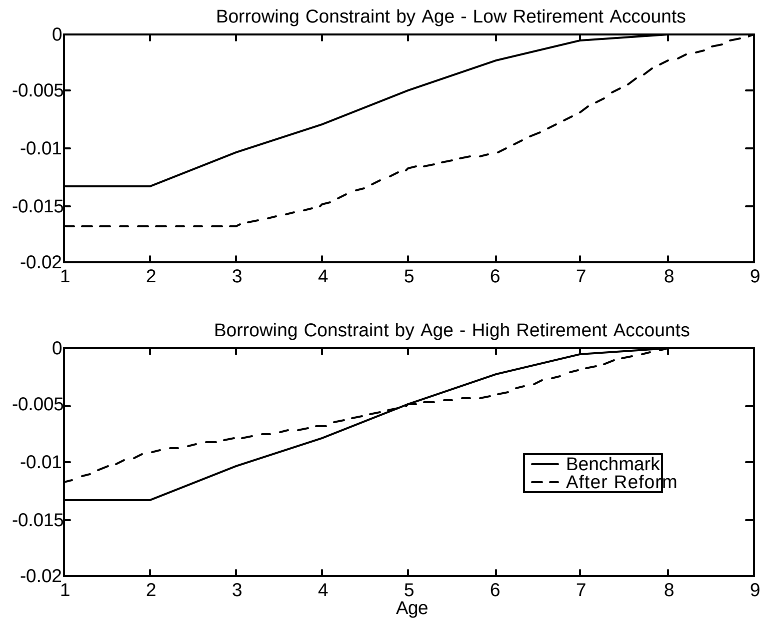

Figure 3: Changes in Borrowing Limits after Reform

which default on their debts can build savings for retirement without any punishment through retirement accounts, but still lose the interest income on their savings through liquid assets, which are more suitable to protect them from bad income shocks in the future when borrowing is precluded. More relaxed borrowing constraints translate again into more debt and less precautionary savings, but not to the same extent as in the model with only one asset.

Finally, notice that the optimal portfolio of households between liquid assets and retirement accounts is similar (around half each) in the two setups. With endogenous borrowing constraints one would expect individuals, facing more relaxed constraints so they do not need to save much for precautionary reasons via the liquid asset, to tilt their portfolio towards retirement accounts. However, if anything we observe the opposite. The reason, as shown in Figure 3, is that accumulating retirement accounts actually tightens worker's borrowing constraints, for a given social security system. *Excessive savings in retirement accounts protects workers too much in the case of default, making financial intermediaries less willing to lend them in equilibrium* and offsetting the initial effect of the reduction of the social security replacement rate.

6 Conclusions

We have shown that when borrowing constraints are endogenous and idiosyncratic income risk is important, the magnitude of the increase in the capital to output ratio and the saving rate due to a social security reform is reduced. The reason is that with the elimination of social security individuals have less incentives to default on debts. The relaxation of borrowing constraints associated to this fact implies that the agents need to save less for precautionary reasons, and this effect goes in the opposite direction to the traditional increase in savings associated with the reform of public pensions.

From a quantitative perspective, the effects are significant but not large, especially if we allow for retirement accounts protected from creditors upon default. Still, our mechanism goes in the direction of the empirical literature, which finds that current models overestimate the impact of social security reforms on the saving rate.

There are two important directions for future research that we plan to pursue. One is to move to a credit contracting framework in which interest rates are type specific and there is default in equilibrium, as in Chatterjee et al. (2004). The effect of a social security reform on default rates is an interesting unexplored issue. Second, it seems relevant to add a third asset to the model (as durables or housing) which could be used as collateral for loans, in the direction of Fernandez Villaverde and Krueger (2002). Then changes in the social security system might have an additional impact on borrowing limits through the price of such an asset.

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A The Basics of Retirement Accounts: The 401(k) and IRAs

The first appendix describes briefly the current legislation on the two most important types of retirement accounts in the U.S.: 401(k) and IRAs. We focus on the features of each plan that are relevant for our model, as the limits to contributions, tax treatment, and penalties for early withdrawal.

401(k): Employer-sponsored retirement accounts

A 401(k) plan is a retirement plan set up by an employer. It is a simple way to build up retirement savings and get significant tax benefits while an individual is working. When a worker joins a 401(k), he agrees to contribute part of his salary to the 401(k) account. The money contributed is deducted from the paycheck before income taxes are taken out, although social security taxes are levied over the gross wage before deducting any contribution to a retirement account. Consequently, a worker does not pay income taxes on contributions and any interest it earn until the money is withdrawn from the account. Some employers offer a match, meaning that for every dollar one contributes up to a certain amount, your employer will also make a contribution (10 cents, 50 cents, a dollar - it depends on your employer).

- *Minimum Contributions:* The existence of a minimum contribution to a 401k depends on the rules of the particular plan. There is no federally imposed minimum contribution to a 401(k), but many plans require participants to contribute at least 1-2 percent of their salary in order to offset administration costs.

- *Maximum Contributions:* Each company decides the contribution limits for its own plan, within the IRS guidelines of a maximum individual contribution limit of 20% (in 2001) of annual earnings or \$10,500, and \$35,000 maximum combined employer and employee contribution limit (through the matching contributions), and the 25 percent of pay limitation.

- *Withdrawals before 59.5 years old:* If one is younger than 59.5, the amount withdrawn in a year subject to income tax at your current income tax rate and a 10% early withdrawal penalty unless one of the following conditions apply: A. If you become totally disabled. B. If you die, and your beneficiary collects the money. C. If you are in debt for medical expenses that exceed 7.5 percent of your adjusted gross income. D. If you are required by court order to give the money to your divorced spouse, a child, or a dependent. E. If you are separated from service (through permanent layoff, termination, quitting or taking early retirement) in the year you turn 55, or later. F. If you are separated from service and you have set up a payment schedule to withdraw money in substantially equal amounts over the course of your life expectancy. (Once you begin taking this kind of distribution you are required to continue for 5 years or until you reach age 59.5, whichever is longer). If you do this, you would pay ordinary income tax on the distributions but no 10 percent penalty. However, this type of distribution may or may not be available to you under the terms of a particular 401(k) plan, and for this reason we abstract from modelling this feature.

- *Withdrawals after retirement:* To make sure that most of the retirement benefits are paid to you during your lifetime, rather than to your beneficiaries after your death, the payments that you receive from a retirement plan must begin no later than your required beginning date (which is the retirement age or 70 and 1/2 if you are not retired). The payments each year cannot be less than the minimum required distribution. The minimum required distribution is computed by dividing your account balance at retirement by the expected lifetime at that age. If the actual distributions in any year are less than this minimum you are subject to an additional tax, which equals 50% of the part of the minimum distribution that was not distributed (this applies to a qualifies employee plan like the 401ks).

- *Changing Jobs and Unemployment:* Workers who just changed jobs or get unemployed can rollover their previous retirement plans (e.g. 401(k)) to IRAs to continue to benefit from tax-deferred growth. Moreover, there is no penalty on the rollover amount from the previous retirement plan to an IRA. This also means that a unemployed individual cannot benefit from an employer-sponsored retirement account (401(k)) but can continue to invest in an individual retirement account which is also a tax-deferred investment vehicle.

Traditional IRAs

An Individual Retirement Arrangement (IRA), commonly called an Individual Retirement Account, is a personal retirement savings plan available to anyone who receives taxable compensation during the year. For IRA contribution purposes, compensation includes wages, salaries, fees, tips, bonuses, commissions, taxable alimony, and separate maintenance payments.

- *Age Limit:* Under some conditions, even a child can have an IRA. However, you can no longer contribute money into your Traditional IRA when you have reached 70.5.

- *Setting up:* An IRA should be opened with a bank trust or custodial company.

- *Contribution Limit:* There is no lower limit for contribution. There is an upper limit of $2,000 a year for the total amount in the IRAs under the same person's name. Moreover, if a person's income is under $2000 in a particular year, the maximum contribution for that year is 100% of his/her work income. (excluding rental income, interest income, etc.). For a couple, the maximum total is $4000 with $2000 each. If the joint income is below $4000, the maximum amount of contribution is 100% of their joint work income. You cannot contribute money into your traditional IRA when you have reached 70.5,

- *Tax Treatment:* 1.Contributions to a traditional IRA is income tax deferred. The interest earned in your traditional IRA is also income tax deferred. 2.Interest earned in a traditional IRA is subject to income tax when the amount is withdrawn.

- *Withdrawal:* Withdrawing money from a traditional IRA is subject to the same rules as the employer-sponsored retirement accounts (401(k)): If you are younger than 59.5, the amount withdrawn this year is subject to income tax at your current income tax rate and a 10% early withdrawal penalty unless you die or become disabled. If you do not start withdrawing some required minimum amount from your IRA after you have reached 70.5, you will be subjected to a 50% penalty.

- Workers who just changed jobs can rollover their previous retirement plans (e.g. 401(k)) to IRAs to continue to benefit from tax-deferred growth. Moreover, there is no penalty on the rollover amount from the previous retirement plan to an IRA.

- Individuals with a retirement plan at work (401k) can still have an IRA provided that their modified adjusted gross income satisfies one of the following conditions (in 2000): 1) between $52000 and $62000 for a married couple filing jointly, 2) between $32000 and $42000 for a single individual or head of household, or 3) between $0 and $10000 for a married person filing separately.

Bankruptcy

Finally, because both programs are personal investment programs for your retirement, it is protected by pension (ERISA) laws, which means that the benefits may not be used as security for loans outside the program. This includes the additional protection of the funds from garnishment or attachment by creditors.

B The Model with Two-Assets

This appendix formalizes the descripion of the Two-Assets economy. For compactness, we only present those equations and problems which differ to the corresponding expressions in the One-Asset economy. We keep referring to a as the liquid asset, and introduce the notation d for retiremnt accounts. We start with the consumer´s value functions (default values and endogenous borrowing constraints) and then state the definition of equilibrium for this economy.

Value of default:

a) Last period of life ($i = I$):

$$v_r^*(a,d,I) = \frac{[TR_{ss} + a + (1-\tau)(1+r)\,d]^{1-\sigma}}{1-\sigma}$$

b) At age $i = \{I_r, ..., I-1\}$:

$$v_r^*(a,d,i) = \max_{\{c,a',d'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta s(i)\, v_r^*(a',d',i+1) \right\}$$

$$\begin{array}{rrcl} s.t. & c + a' + d' & = & TR_{ss} + a + (1+r)d - \tau \Delta d \\ & \Delta d & = & (1+r)\,d - d' \\ & a' & \geq & 0 \\ & \Delta d & \in & [\varpi(i)(1+r)\,d, (1+r)\,d] \end{array}$$

c) Last period of working life ($i = I_r - 1$):

$$v_e^*(a,d,I_{r-1},\eta) = \max_{\{c,a',d'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta v_r^*(a',d',I_r) \right\}$$

$$\begin{array}{rrcl} s.t. & c + a' + d' & = & (1-\tau)(1-\tau_{ss})\,\eta we(I_r - 1) + a + (1+r)d \\ & & & +\tau\Delta d - \tau_{pen}(I_r - 1)\max\{-\Delta d, 0\} \\ & \Delta d & = & d' - (1+r)\,d \\ & a' & \geq & 0 \\ & \Delta d & \in & [-(1+r)\,d, 0] \quad \text{or} \quad \dfrac{\Delta d}{\eta we(I_r - 1)} \in [\Delta_{\min}, \Delta_{\max}] \end{array}$$

d) At ages $i = \{1, ..., I_r - 2\}$:

$$v_e^*(a, d, i, \eta) = \max_{\{c, a', d'\}} \left\{ u(c) + \beta \sum_{\eta' \in E} \pi(\eta, \eta') v_e^*(a', d', i+1, \eta') \right\}$$

$$\begin{aligned} s.t. \quad c + a' + d' &= (1-\tau)(1-\tau_{ss}) \eta we(i) + a + (1+r)d \\ & \quad + \tau \Delta d - \tau_{pen}(i) \max\{-\Delta d, 0\} \\ \Delta d &= d' - (1+r)d \\ a' &\geq 0 \\ \Delta d &\in [-(1+r)d, 0] \quad \text{or} \quad \frac{\Delta d}{\eta we(I_r - 1)} \in [\Delta_{\min}, \Delta_{\max}] \end{aligned}$$

Consumer´s Problem with Endogenous Borrowing Constraints:

a) Last period of life ($i = I$):

$$v_r(a, d, I) = \frac{[TR_{ss} + (1 + r(1-\tau_k))a + (1-\tau)(1+r)d]^{1-\sigma}}{1-\sigma}$$

b) At ages $i = \{I_r, ..., I-1\}$:

$$v_r(a, d, i) = \max_{\{c, a', d'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta s(i) v_r(a', d', i+1) \right\}$$

$$\begin{aligned} s.t. \quad c + a' + d' &= TR_{ss} + (1 + r(1-\tau_k))a + (1+r)d - \tau \Delta d \\ \Delta d &= (1+r)d - d' \\ a' &\geq 0 \\ \Delta d &\in [\varpi(i)(1+r)d, (1+r)d] \end{aligned}$$

c) Last period of working life ($i = I_r - 1$):

$$v_e(a, d, I_{r-1}, \eta) = \max_{\{c, a', d'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta v_r(a', d', I_r) \right\}$$

$$\begin{aligned} s.t. \quad c + a' + d' &= 1 - \tau)(1-\tau_{ss}) \eta we(I_r - 1) + (1 + r(1-\tau_k))a + (1+r)d \\ & \quad + \tau \Delta d - \tau_{pen}(I_r - 1) \max\{-\Delta d, 0\} \\ \Delta d &= d' - (1+r)d \\ a' &\geq 0 \\ \Delta d &\in [-(1+r)d, 0] \quad \text{or} \quad \frac{\Delta d}{\eta we(I_r - 1)} \in [\Delta_{\min}, \Delta_{\max}] \end{aligned}$$

d) At ages $i = \{1, ..., I_r - 2\}$:

$$v_e(a, d, i, \eta) = \max_{\{c, a', d'\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \sum_{\eta' \in E} \pi(\eta, \eta') v_e(a', d', i+1, \eta') \right\}$$

$$\begin{array}{lrcl} s.t. & c + a' + d' & = & (1-\tau)(1-\tau_{ss})\eta we(i) + (1 + r(1-\tau_k))a + (1+r)d \\ & & & +\tau\Delta d - \tau_{pen}(i)\max\{-\Delta d, 0\} \\ & \Delta d & = & d' - (1+r)d \\ & a' & \geq & -b(d', i) \\ & \Delta d & \in & [-(1+r)d, 0] \quad \text{or} \quad \dfrac{\Delta d}{\eta we(I_r - 1)} \in [\Delta_{\min}, \Delta_{\max}] \end{array}$$

where the endogenous borrowing constraint $-b(d', i)$ satisfies

$$v_e(-b(d', i), d', i+1, \eta') \geq v_e^*(0, d', i+1, \eta') \qquad \forall \eta' \in E$$

Definition of a Stationary Equilibrium:

A Stationary Equilibrium is a set of value functions, optimal decision rules, price functions, a set of government policies, aggregate law of motion Γ, invariant distribution μ, marginal distributions

$$\mu_s(i) = \sum_{\eta \in E} \int_{A \times D} d\mu_s(a, d, i, \eta)$$

and partial sums

$$A_s(i) = \sum_{\eta \in E} \int_{A \times D} a \, d\mu_s(a, d, i, \eta)$$

$$D_s(i) = \sum_{\eta \in E} \int_{A \times D} d \, d\mu_s(a, d, i, \eta)$$

such that:

1. Given prices, taxes, transfers, and aggregate law of motion, consumers optimize

2. Price functions satisfy marginal conditions from firm's problem

3. Government budget constraints are satisfied

$$TR_{ss}\sum_{i=I_r}^{I}\mu_r\left(i\right)=\tau_{ss}w\sum_{\eta\in E}\sum_{i=1}^{I_r-1}\eta e(i)\int_{A\times D}\mu_e\left(a,d,i,\eta\right).$$

and

$$G=\tau\left(1-\tau_{ss}\right)w\sum_{\eta\in E}\sum_{i=1}^{I_r-1}\eta e(i)\int_{A\times D}\mu_e\left(a,d,i,\eta\right)+\tau_k r\sum_{s\in\{e,r\}}\sum_{i=2}^{I}A_s\left(i\right)$$

$$+\tau\sum_{s\in\{e,r\}}\sum_{i=1}^{I}\left[\left(1+r\right)D_s\left(i\right)-\sum_{\eta\in E}\int_{A\times D}g_s^d\left(a,d,i,\eta\right)d\mu_s\left(a,d,i,\eta\right)\right]$$

$$+\sum_{i=I_r}^{I}\left(1-s\left(i\right)\right)\int_{A\times D}\left[g_r^a\left(a,d,i\right)+g_r^d\left(a,d,i\right)\right]d\mu_r\left(a,d,i\right)$$

$$+\sum_{i=1}^{I_r-1}\tau_{pen}(i)\int_{A\times D}\max\left[\left(1+r\right)d-g_e^d\left(a,d,i,\eta\right),0\right]d\mu_e\left(a,d,i,\eta\right)$$

4. Markets clear

$$Y=K^{\alpha}L^{1-\alpha}-\delta K=C+K'-K+G$$

with:

$$K=\sum_{s\in\{e,r\}}\sum_{i=1}^{I}\left[A_s\left(i\right)+D_s\left(i\right)\right]$$

5. The invariant distribution satisfies $\mu=\Gamma\left(\mu\right)$.