Asynchronous Signal Detection in Frequency-Selective Non-Gaussian Channels

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Abstract—We present a signal detection algorithm for digital amplitude-phase modulated signals in frequency-selective fading channels with non-Gaussian noise. We consider an asynchronous scenario in which the timing (symbol transition epochs) is unknown and a symbol rate offset is present due to clock drift. A Gibbs sampling-based algorithm is proposed to estimate the unknown parameters and signal detection is performed using a maximum-likelihood procedure. The additive noise is modeled by a Gaussian mixture distribution, a well-known model for man-made and natural noise. Numerical results are presented to characterize the performance of the proposed algorithm.

I. INTRODUCTION

Signal detection or spectrum sensing can be defined as the process of determining the presence or absence of a signal from a set of noisy observations. This process has many applications in wireless communications, including autonomous multi-mode radios [1] and electronic warfare and surveillance systems [2]. In most scenarios of interest, the difficulty in performing signal detection is due primarily to the fact that the detectors operate with incomplete or no knowledge of the fading experienced by the signal and the distribution of the noise added in the channel.

In this paper, we present a signal detection algorithm for digital amplitude-phase modulated signals transmitted over a frequency-selective fading channel with additive white non-Gaussian noise. We propose a detector capable of operating without prior knowledge of the fading, noise distribution parameters, and symbol timing (symbol transition epochs). In addition, we assume that error in the internal clocks of the transmitter and receiver lead to clock drift and, as a result, uncertainty in the symbol interval exists.

It is important to note that signal processing algorithms designed for optimal performance in Gaussian noise typically perform significantly worse when non-Gaussian noise is present [3]. This is due, in part, to “the lack of robustness of linear and quadratic type signal processing procedures to many types of non-Gaussian statistical behavior” [4], [5]. Our assumption that the additive noise is non-Gaussian is motivated by studies which have shown that most radio channels experience both man-made and natural noise, and that the combined noise is impulsive [3], [6]. Although different detectors have been developed for the case when the additive noise is non-Gaussian – see, for example, [7] and references there in – a detector has not yet been proposed for frequency-selective non-Gaussian channels in the absence of timing synchronization.

The rest of the paper is organized as follows. In Section II, we introduce the system and the noise models used in this paper. Section III briefly describes likelihood-based signal detection. A signal detection algorithm for the case in which the receiver is aware of the symbol timing interval but is not aware of the delay introduced by the wireless channel, i.e., the symbol timing offset, is presented in Section IV. In Section V, the signal detection algorithm is extended to the case when both the symbol timing interval and the delay introduced by the channel are unknown. Numerical results for the two scenarios are presented in the corresponding sections. Finally conclusions are presented in Section VI.

II. SYSTEM MODEL

We assume that the data conveyed in the transmitted signal is mapped onto a known digital amplitude-phase modulated constellation denoted by \( S \). The low-pass equivalent of the transmitted signal is

\[
s(t) = \sum_{m=-\infty}^{\infty} s_m g(t - mT),
\]

where \( T \) is the symbol interval and \( g(t) \) is a real-valued pulse. The random variables \( \{s_m\}_{m=-\infty}^{\infty}, s_m \in S \), denote the modulated symbols and are assumed to be independent and uniformly distributed among all constellation points in the set \( S \). Without loss of generality, the energy of \( g(t) \) and the average energy of \( S \) \( (E[|s_m|^2]) \) are normalized to unity.

The channel is assumed to be slowly varying and, in particular, constant during the observation interval. By using a conventional tapped delay line channel model with tap spacing equal to \( T \) [8], the low-pass equivalent of the received signal is given by

\[
r(t) = \sum_{m=-\infty}^{\infty} \sum_{l=0}^{L-1} h_l s_m g(t - mT - lT - \tau) + w(t),
\]

where \( h_l \) are the complex channel coefficients for each of the \( L \) resolvable paths, \( \tau \in [0, T) \) is an unknown time delay and \( w(t) \) is the low-pass equivalent of the noise added in the channel. Using (2), the signal at the output of a matched filter, when sampled at \( t = kT \), is given by

\[
y_k = \{r(t) * g(t)\}_{t=kT} = \sum_{m=-\infty}^{\infty} \sum_{l=0}^{L-1} h_l s_{k-l+m} p(-mT - \tau) + w_k,
\]

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for \( k = 1, \ldots, K \). If the unknown time delay \( \tau \) was known (that is, if it was perfectly estimated) and the matched filter output was sampled at \( t = kT + \tau \), we would have

\[
y_k = \{r(t) * g(t)\} \big|_{t=kT+\tau} = \sum_{l=0}^{L-1} h_l s_{k-l} + w_k, \tag{4}
\]

for \( k = 1, \ldots, K \). In (3), \( p(t) = g(t) * g(t) \) is a Nyquist pulse (for example, raised-cosine).

The samples of \( w(t) \) are assumed to be independent and identically distributed (i.i.d) with probability density function (pdf) given by the \( N \)-term Gaussian mixture

\[
p(w_k) = \frac{\prod_{n=1}^{K} \sum_{n=1}^{N} \frac{\lambda_n}{\sigma_n^2} \exp \left( -\frac{|w_k|^2}{\sigma_n^2} \right)}{\sum_{n=1}^{N} \frac{\lambda_n}{\sigma_n^2} \exp \left( -\frac{|w_k|^2}{\sigma_n^2} \right)} \tag{5}
\]

where \( \lambda_n \) is the probability that \( w_k \) (kth sample of \( w(t) \)) is from the \( n \)th term in the pdf, with \( \sum_{n=1}^{N} \lambda_n = 1 \). This model closely approximates Middleton’s canonical Class-A interference model [3, 9] and other symmetric pdfs [6].

In the analysis that follows, the symbol interval \( T \), the time delay \( \tau \), the channel coefficients \( \{h_l\}_{l=0}^{L-1} \), and the noise distribution parameters \( \{\lambda_n\}_{n=1}^{N}, \{\sigma_n\}_{n=1}^{N} \) are modeled as deterministic unknowns. The pulse shape \( g(t) \) is assumed to be known [7]. Also, the following notations are used: \( h = \{h_0, \ldots, h_{L-1}\}^T \), \( \sigma^2 = \{\sigma_1^2, \ldots, \sigma_N^2\} \), and \( \lambda = \{\lambda_1, \ldots, \lambda_N\} \). Also, \( |.|^T \) denotes the transpose.

### III. Maximum-Likelihood Signal Detection

Maximum-likelihood signal detection is a binary hypothesis (\( \mathcal{H}_0 \) versus \( \mathcal{H}_1 \)) testing problem in which the hypothesis that maximizes the (log-)likelihood of the received signal is chosen [10]. Assuming that all channel and noise distribution parameters are known, the maximum-likelihood decision rule is given in (6) where \( |S| \) is the order of the constellation, \( \{y_k\}_{k=1}^{K} \) are the samples taken at the optimal sampling times as shown in (4). Further, in the derivation of (6), the received samples are assumed to be independent\(^1\)

\[ D = \frac{\prod_{k=1}^{K} \sum_{|y_k|^2=\lambda_n} \exp \left( -\frac{|y_k|^2}{\sigma_n^2} \right)}{\prod_{k=1}^{K} \sum_{|y_k|^2} \exp \left( -\frac{|y_k|^2}{\sigma_n^2} \right)} \tag{6} \]

for \( k = 1, \ldots, K \). If the unknown time delay \( \tau \) was known (that is, if it was perfectly estimated) and the matched filter output was sampled at \( t = kT + \tau \), we would have

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y_k = \{r(t) * g(t)\} \big|_{t=kT+\tau} = \sum_{l=0}^{L-1} h_l s_{k-l} + w_k, \tag{4}
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It can be seen in (6) that the detector requires knowledge of \( h, \sigma^2 \) and \( \lambda \). In addition, the unknown symbol interval \( T \) and the time offset \( \tau \) were assumed to be known in the derivation of (6). We rely on a composite hypothesis testing procedure [10] in which the unknown channel and noise distribution parameters are estimated in a pre-processing stage before likelihood-based signal detection. We use a Gibbs sampling-based numerical Bayesian technique to estimate these unknowns. In the Bayesian framework, the unknown parameters are treated as random variables and estimates are obtained by using the unknowns’ posterior pdf [11]. For more details on Gibbs sampling please refer to [11].

\(^1\)To avoid this assumption, independent samples can be taken at intervals of \( kLT \) for the purpose of evaluating the maximum likelihood decision rule.

### IV. Scenario 1: \( T \) known, \( \tau \)-unknown

In this section, we discuss the proposed signal detection algorithm for the case in which the receiver is aware of the symbol timing \( T \) but is asynchronous with the received signal due the unknown symbol timing offset \( \tau \). In this scenario, since \( T \) is known, the estimation of the unknown parameters and the signal detection is done after matched filtering. The received signal after matched filtering and sampling at symbol intervals \( \{kT\}_{k=1}^{K} \) is given by (3).

#### A. Estimation Using Gibbs Sampling

The prior and the full conditional posterior distributions (FCPD) used for the unknown parameter vector \( \{h, \sigma^2, \lambda, \tau, x\} \), where \( x = [x_{-(M+L-2)}, \ldots, x_{K+M}] \) is the estimated symbol sequence, are discussed here. Using (3), the received signal vector \( y \) can be written as \( y = Xh + w \), where \( X \) is defined as

\[ X = \sum_{m=-M+1}^{M} p(-mT - \tau)X_m \]

where,

\[ [X_m]_{K \times L} = \begin{pmatrix} x_{1+m} & \cdots & x_{1+m-(L-1)} \\ \vdots & \ddots & \vdots \\ x_{K+m} & \cdots & x_{K+m-(L-1)} \end{pmatrix}, \]

and the pulse shape \( p(t) \) is taken to be zero for \( |t| > MT \). Given that \( w \) has a Gaussian mixture distribution, using (5), the likelihood function is given by

\[
p(y|h, \sigma^2, \lambda, \tau, x) = \prod_{k=1}^{K} \sum_{n=1}^{N} \lambda_n N((Xh)_k, \sigma_n^2), \tag{7}
\]

where \( N((Xh)_k, \sigma_n^2) \) represents a complex Gaussian pdf with mean \( (Xh)_k \) and variance \( \sigma_n^2 \), and \( (Xh)_k \) is the \( k \)th term in the \( \mathbf{X} \times 1 \) vector \( \mathbf{X}h \).

The prior pdfs [11] used for the unknown parameters are given in Table I. An indicator variable \( I_k \) is introduced to represent the mixture component to which \( w_k \) belongs. It should be noted that \( P(I_k = n|\lambda) = \lambda_n \). The parameters of the prior pdfs of \( h, \sigma^2, \lambda \) and \( \tau \) are chosen such that they are non-informative [11]. The FCPDs within a proportionality constant are shown in Table I. It is seen that the posterior distribution of the timing offset does not have a closed form expression. A greedy-Gibbs sampling-based algorithm [12] is used to estimate \( \tau \).

#### B. Results

Using the FCPDs, the proposed Gibbs sampling algorithm is executed by iteratively sampling for \( I, h, \sigma^2, \lambda, \tau, \) and \( x \) under hypothesis \( \mathcal{H}_1 \) and for \( I, \sigma^2, \) and \( \lambda \) under hypothesis \( \mathcal{H}_0 \) [11]. After the algorithm converges, the means of the resulting estimates are taken as the MMSE estimates of the
unknown parameters and maximum-likelihood signal detection is performed using (6).

In the results that follow, the modulation scheme is assumed to be BPSK. The complex channel gains \( \{h_l\}_{l=0}^{L-1} \) are assumed to be independent zero-mean Gaussian random variables with \( \sum_{l=0}^{L-1} E[|h_l|^2] \) equal to 1. The multipath intensity profile is assumed to be exponential with \( \beta \). The pulse \( p(t) \) is taken to be a root raised-cosine with 65\% excess bandwidth and is truncated to have a time duration of \( 2T \). The number of terms in the Gaussian mixture distribution is taken to be \( N = 2 \) [4], [13] where the first and second terms of the mixture represent the thermal noise component (with variance \( \sigma_1^2 \) and proportion \( \lambda_1 = 0.8 \)) and the impulsive noise component (with variance \( \sigma_2^2 \gg \sigma_1^2 \) and proportion \( \lambda_2 = 1 - \lambda_1 = 0.2 \)), respectively. The ratio of the variances \( \sigma_2^2/\sigma_1^2 \) is set to 100.

We define the received SNR as the ratio of the average signal power and the thermal noise power. In simulation there exists 20\% error in the symbol timing, i.e., \( \tau = 0.2T \).

Fig. 1 shows the convergence of the estimates of the unknown parameters for one particular run of the Gibbs sampling algorithm. It is seen that the estimates converge in about 30 iterations. As a conservative estimate and to cover all different ranges of SNR, we allow the algorithm to run for 200 iterations with a burn-in period of 100 iterations. The signal detection performance is shown in Fig. 2 where we observe that the probability of detection improves with SNR.

In the analysis that follows, we assume that the receiver has knowledge of the nominal value of the symbol interval denoted by \( \bar{T} \). The received signal after oversampling is given by

\[
\tilde{x}_k = \sum_{m=-\infty}^{\infty} \sum_{l=0}^{L-1} h_l s_m g(kT_s - mT - lT - \tau) + w_k, \tag{8}
\]

where \( T_s \) is the sampling period chosen such that \( w_k \) are i.i.d. Let \( \tau = \{\tau_1, \tau_2, \ldots, \tau_K\}^T \). We next discuss the Gibbs sampling-based estimation procedure in such an asynchronous scenario.

A. Estimation via Gibbs sampling

The unknown parameter vector is \( \{h, \sigma^2, \lambda, T, \tau, x\} \). By following the steps in Section IV, the FCPDs for \( h, \sigma^2, \lambda, x \) can be obtained using the received signal in (8). In order to model our uncertainty in the symbol interval we define the prior distribution of \( T \) as \( \mathcal{N}(\bar{T}, \bar{T}^2) \) where \( \bar{T} \) is the
To estimate uncertainty in the clock drift. We use the Metropolis-Hastings algorithm, a variant of the Gibbs sampling algorithm, to obtain estimates for unknown parameters in the data-aided scenario. The algorithm is shown in Fig. 4. Because an incorrect estimate of $T$ leads to large timing offsets (especially towards the end of the sequence), estimates of $T$ do not tend to represent the original sequence. Thus, we observe in Fig. 4 that the Gibbs sampling algorithm does not converge. Hence, we propose a data-aided estimation algorithm where the algorithm exploits the presence of training symbols in standard wireless communications systems such as LTE. Thus the unknown parameter vector is now $\{h, \sigma^2, \lambda, T, \tau\}$.

B. Results

Figs. 5-7 show the convergence of estimates of various unknown parameters in the data-aided scenario. The algorithm is allowed to run for 500 iterations in this scenario due to the unknown symbol interval $T$. The estimates obtained after the burn-in period (250 iterations) are used to evaluate the MMSE estimates of the unknown parameters. Fig. 8 shows the performance of the signal detection algorithm performed on 50 unknown symbols following the training sequence. Again
in this case where the symbol interval $T$ is unknown, the proposed signal detection algorithm improves with increasing SNR. The improved performance with respect to that observed in Section IV is due to more accurate estimation of the unknown parameters through the use of a training sequence.

VI. CONCLUSION

We proposed an asynchronous signal detection algorithm for digital amplitude-phase modulated signals transmitted over frequency selective fading channels with non-Gaussian noise. We consider two asynchronous scenarios, 1) the symbol interval is known but the symbol timing offset is unknown and 2) both the symbol interval and the symbol timing offset are unknown. While convergence of the algorithm was not an issue in the first scenario, the second scenario required knowledge of the data to achieve good convergence. In systems where a training sequence is known, the proposed algorithm can exploit this knowledge to estimate the unknown parameters, even with an uncertain symbol interval. The presented results demonstrate that the proposed estimation stage enables reliable signal detection in an asynchronous environment.

REFERENCES