LOW-COMPLEX ICI REDUCTION METHOD BY APPLYING FRANKS WINDOW COEFFICIENTS IN LINEAR TIME-VARYING CHANNEL

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ABSTRACT

In this paper, we consider the ICI effect induced by time-varying channel. A simple and low-complex method is proposed to suppress ICI by windowing the received signal using Franks window coefficients in time domain and combining the extended signal interval which was not interfered by multipath to the corresponding information part of the cyclic-prefix (cp) based OFDM system. The ICI are suppressed by equivalently reducing the channel time-variation according to the useful extended signal interval. If the channel time-variation is linear in one OFDM symbol and the useful extended signal interval period as long as the information part period, the equivalent channel time-variation of information part will turn into a constant which means no ICI occurs. And then, a N-point fast Fourier-transform (FFT) and one-tap equalizer is used for de-modulation and channel equalization. The complexity of the proposed method is only \( N \log N \). The analysis of Franks window coefficients for equivalent channel mitigation in linear time-varying channel are investigated and simulation results are provided to validate the performance of the proposed method.

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a powerful transmission scheme for improving bandwidth efficiency and mitigating frequency-selective fading channel by dividing a serial high-rate data stream into several parallel low-rate data stream. A cyclic-prefix (cp) is used as guard interval which is inserted before each transmitted data block to prevent inter-symbol interference (ISI). If the length of guard interval is larger than maximum channel delay, then ISI can be completely eliminated and one-tap equalizer can be used in frequency domain to compensate channel impact in time-invariant multipath channel. However, the drawback of OFDM systems is very sensitive to orthogonality among sub-carriers which might be destroyed by oscillator frequency offset or Doppler spread. The loss of orthogonality among sub-carriers results in inter-carrier interference (ICI) which degrades the performance of OFDM systems and error floor occurs.

There are several ICI cancellation methods that have been proposed. Some of the methods[1,2] suppress ICI by using equalizer. The complexity of those methods are much higher than conventional one-tap equalizer and they need complicated channel estimation method[2] to estimate the channel time-variations. Moreover, the pilot symbol for channel estimation also suffers ICI and debase the accuracy of channel estimation. Another proposed methods[3-5] suppress ICI by utilizing the extended signal interval. In [3], the author consider for the impairment of a constant frequency offset, like receiver oscillator mismatch, and proposed a method that uses Nyquist window in time domain to exploit the ISI-free part of guard interval and demodulate the signal by applying a 2N-point fast Fourier-transform (FFT). The reception performance of several window functions for the constant frequency offset are also investigated in [4]. The results shows that the Franks window function outperforms than other window functions. However, the consideration of constant frequency offset is not suitable for time varying channel analysis. In [5], Chang consider the channel time-variation as linear varying within one OFDM symbol. Chang’s proposed method utilizes extended signal interval in frequency domain and ICI can be complete removed when extended signal interval as long as information part interval. Nevertheless, the complexity of the method is quite huge for the requirement of N sets of N-point FFT.

In this paper, we consider the ICI phenomenon induced by time-varying channel and suppress the ICI by the mitigation of equivalent channel time-variation. A simple and low-complex method is proposed by using Franks window coefficients and combining the extended signal interval to the corresponding information part in time domain. And then, a N-point FFT and one-tap equalizer is used for demodulation and channel equalization. The ICI can be complete removed if the extended signal interval as long as information part in linear time-varying channel.

The remainder of this paper is organized as follows: Section II describes the system signal model and analyzes the ICI effect in time-varying channel. The proposed method for ICI reduction by applying Franks window coefficients in linear time-varying channel is described in section III. The simulation results are given in section IV and the conclusion is drawn in section V.

II. SIGNAL MODEL OF ICI EFFECT

A discrete-time baseband OFDM signal in time domain during one symbol interval can be expressed as

\[
x'_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{j2\pi mn/N}, \quad 0 \leq n \leq N - 1.
\]  

where \( n \) is integer represents discrete-time index, \( N \) is the duration of symbol interval, \( X_m \) is the transmitted symbol modulated on the \( m \)th sub-carrier. The cp is inserted before \( x'_n \) as guard interval to prevent ISI by periodic extend the original interval of \( x'_n \). The OFDM symbol with cp insertion can be expressed by
where \( N_g \) is the duration of guard interval. In this study, we assume the maximum channel delay is smaller than guard interval which means no ISI occurs. The received baseband signal over multipath time-varying channel after removal of the guard interval is given as

\[
y_n = \sum_{l=0}^{L-1} h_{nl}^{(l)} x_{n-l} + z_n \quad 0 \le n \le N - 1.
\]

where \( L \) is the number of paths of the channel, \( h_{nl}^{(l)} \) represents the complex channel fading of the \( l \)th path at the \( n \)th snapshot, \( x_{n-l} \) is the corresponding delay time of the \( l \)th path and \( z_n \) is the complex additive white Gaussian noise (AWGN).

Fig 1. Receiver structure of the proposed method

A periodically extended OFDM signal in transmitter side with a guard interval appended before the extended signal can be given as

\[
x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{j2\pi \frac{m}{N} n}, \quad -N_g - N_e \le n \le N - 1
\]

where \( N_e \in \{0,1,\cdots,N-1\} \) represents the duration of extension interval. Then, the received signal in time domain over time-varying multipath channel at receiver side after removal of the guard interval is expressed as

\[
r_n = \sum_{l=0}^{L-1} h_{nl}^{(l)} x_{n-l} + z_n \quad -N_e \le n \le N - 1
\]

where \( h_{nl}^{(l)} \) represents the complex channel gain of the \( l \)th path at the time instant \( n=0 \). \( a_l \) is the slope of the \( l \)th path channel time-variation. Substituting (8) into (7), then

\[
r_n = \sum_{l=0}^{L-1} (c_l + a_l \cdot n) x_{n-l} + z_n \quad -N_e \le n \le N - 1
\]

From equation (4) and (5), it is obvious that ICI occurs when \( H_{m,m'} \neq 0 \) for \( m' \neq m \) if \( h_{nl}^{(l)} \) is not a constant during 0 \( \le n \le N - 1 \). Nevertheless, if \( h_{nl}^{(l)} \) is a constant during 0 \( \le n \le N - 1 \) then there will be no ICI terms.

### III. ICI REDUCTION BY APPLYING FRANKS WINDOW COEFFICIENTS IN LINEAR TIME-VARYING CHANNEL

The proposed method uses franks window coefficients to mitigate channel time variation of the received OFDM signal in time domain and combines the extended signal to the corresponding information part. The receiver structure of the proposed method is illustrated in Fig. 1. and weighting coefficients is investigated and derived as follows.

\[
y_n = w_n \cdot r_n = \sum_{l=0}^{L-1} (c_l + a_l \cdot n) x_{n-l} + w_n \cdot z_n
\]

where \( -N_e \le n \le N - 1 \). Because of cyclic extension of OFDM signal in transmitter side we can combine the
extended signal to the corresponding information part since
\[ x_{n-N-e^{(i)}} = x_{n-e^{(i)}} \]
where \( x_{n-N} \) represents the signal of cyclic extension interval and \( x_n \) represents the corresponding signal of information parts for \( N - N_e \leq n \leq N - 1 \). After combing, the combined signal is expressed as
\[
y_n' = \begin{cases} 
y_n , & 0 \leq n \leq N - N_e - 1 \\
y_n + y_{n-N} , & N - N_e - 1 < n \leq N - 1 
\end{cases}
(11)
\]
The equation (11) can be further expressed by
\[
y_n' = \begin{cases} 
w_n r_n , & 0 \leq n \leq (1-\alpha)N - 1 \\
w_n r_n + w_{n-N} r_{n-N} , & (1-\alpha)N - 1 < n \leq N - 1 
\end{cases}
(12)
\] where \( \alpha = \frac{N_e}{N} \) represents the extended signal ratio. Substituting (9) into (12), we will get
\[
y_n' = \left\{ \frac{1}{N} - \frac{(1-\alpha)}{N} \right\}
\sum_{l=0}^{N-1} \left[ w_n c_l + a_l \cdot n \cdot x_{n-e^{(i)}} + w_n z_n \right], \quad (1-\alpha)N \leq n \leq (1-\alpha)N - 1
(13)
\]
For the convenience of analysis, we shift the time index letting \( \tilde{y}_n' = y_n' \) where \( n' = n - \frac{(1-\alpha)}{2}N , \quad 0 \leq n \leq N - 1 \).

From equation (13), the weighted and combined received signal in time domain after shifting time index can be expressed as
\[
\tilde{y}_n' = \frac{1}{N} \sum_{l=0}^{N-1} \left[ w_n c_l + a_l \cdot n \cdot x_{n-e^{(i)}} + (1-\alpha)N \right], \quad (1-\alpha)N \leq n' \leq (1-\alpha)N - 1
\]
\[
\tilde{y}_n = \left\{ \frac{1}{N} - \frac{(1-\alpha)}{N} \right\}
\sum_{l=0}^{N-1} \left[ w_n c_l + a_l \cdot n \cdot x_{n-e^{(i)}} + (1-\alpha)N \right], \quad (1-\alpha)N \leq n' \leq (1-\alpha)N - 1
(14)
\]
According to (14), the channel impulse response of the weighted and combined signal can be separated into two parts.
\[
\tilde{h}_n' = \left\{ \frac{1}{N} - \frac{(1-\alpha)}{N} \right\}
\sum_{l=0}^{N-1} \left[ w_n c_l + a_l \cdot n \cdot x_{n-e^{(i)}} \right], \quad (1-\alpha)N \leq n' \leq (1-\alpha)N - 1
(15a)
\]
\[
\tilde{h}_n = \left\{ \frac{1}{N} - \frac{(1-\alpha)}{N} \right\}
\sum_{l=0}^{N-1} \left[ w_n c_l + a_l \cdot n \cdot x_{n-e^{(i)}} \right], \quad (1-\alpha)N \leq n' \leq (1-\alpha)N - 1
(15b)
\]
where \( a_l \) is the variation slope of the \( l \)th path, \( c_l' \) is considered as static term of the \( l \)th path channel impulse response and which is equal to the average channel time variation of (15a) for \( w_{n-e^{(i)}} = 1 \). In (15a), the channel time variation can not be mitigated just by a single coefficient so we let \( w_{n-e^{(i)}} = 1 \). In (15b), the channel time variation can be mitigated by letting \( (w_{n'-N} + w_{n'-N} \cdot (n'-N)) = 0 \). After solving these two equation
\[
w_{n'-N} = \begin{cases} 
1 , & \frac{(1-\alpha)}{2}N - 1 < n' \leq \frac{(1+\alpha)}{2}N - 1 \\
0 , & \text{otherwise}
\end{cases}
(16)
\]
we can get the weighting coefficients as
\[
\begin{cases} 
\frac{n'}{N} , & \frac{(1-\alpha)}{2}N - 1 < n' \leq \frac{(1+\alpha)}{2}N - 1 \\
1 , & \text{otherwise}
\end{cases}
(17)
\]
For symmetrical expression, we further write (17) as
\[
w_{n'-N} = \frac{n'}{N} , \quad \frac{(1-\alpha)}{2}N - 1 < n' \leq \frac{(1+\alpha)}{2}N - 1
(18)
\]
and (18) can be expressed as
\[
w_{n} = \frac{1 - \frac{n}{N}}{N} , \quad \frac{(1-\alpha)}{2}N - 1 < n \leq \frac{(1+\alpha)}{2}N - 1
(19)
\]
Finally, the weighting coefficients for linear time-varying channel mitigation are derived and shown similar to Franks window given in [4] as
\[
p_f(t) = \begin{cases} 
1 , & 0 \leq |t| \leq \frac{T_n(I-\alpha)}{2} \\
1 - \frac{|t|}{T_n(I-\alpha)} , & \frac{T_n(I-\alpha)}{2} \leq |t| < \frac{T_n(I+\alpha)}{2} \\
0 , & \text{otherwise}
\end{cases}
(20)
\]
where \( p_f(t) \) represents the continuous-time Franks window function, \( T_n \) is the duration of information part signal. The window shape of Franks window function is shown in Fig. 2 for different \( \alpha \) values. When \( \alpha = 1 - \frac{1}{\nu} \), the Franks window equivalent to a triangular window.
The equivalent mitigation of channel time variation for linear time-varying channel by Franks window coefficients is shown. The channel time variation of information part corresponding to extended signal can be equivalent mitigated by Franks window coefficients after combination in linear time-varying channel case.

If \( \alpha = \frac{N-1}{N} \) then \( \tilde{h}^{(i)}_n = c_i \) for \( 0 \leq n \leq N-1 \), means the channel gain is a constant over the duration of information part. Thus equation (5) becomes

\[
H_{m,m'} = \frac{1}{N} \sum_{l=0}^{N-1} e^{-j2\pi \frac{m}{N}l} \sum_{n=0}^{N-1} h^{(i)}_n e^{-j2\pi \frac{m-m'}{N}n} = \frac{1}{N} \sum_{l=0}^{N-1} c_i e^{-j2\pi \frac{m}{N}l} \sum_{n=0}^{N-1} e^{-j2\pi \frac{m-m'}{N}n} (21)
\]

so there will be no ICI terms. The ICI can be complete removed if \( \alpha = \frac{N-1}{N} \) for linear time-varying channel. When \( \alpha \) decreased, the time variation of channel can be partially mitigated according to the extended signal and ICI can be reduced by relieving the channel time variation.

**IV. SIMULATION RESULTS**

The uncoded BER simulation results are given in this section. We consider an OFDM system with 64 subcarriers and 16-QAM modulation are used in time-varying multipath Rayleigh fading channel. The duration of information part is \( N = 64 \), the extension interval is \( N_e = \alpha \cdot N \) and the guard interval is \( N_g = 16 \). The propagation channel are assumed with 6 paths, and the power decay of the channel model and the delay time of each path is sample-by-sample as shown in table 1. Each channel tap is generated by Jake’s model[6] with normalized Doppler frequency \( f_d \cdot N \) where \( T = N \cdot T_s \) represents the duration of information part and \( T_s \) represents the sample time. The channel impulse response is assumed ideal known and a one-tap equalizer is used to compensate multipath fading.

**TABLE 1**

<table>
<thead>
<tr>
<th>Power</th>
<th>Delay</th>
<th>Tap1</th>
<th>Tap2</th>
<th>Tap3</th>
<th>Tap4</th>
<th>Tap5</th>
<th>Tap6</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3</td>
<td>0</td>
<td>-2</td>
<td>-6</td>
<td>-8</td>
<td>-10</td>
<td>dB</td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 2. Franks window shape for different \( \alpha \).](image2)

In Fig. 3, equivalent time variant channel mitigation within one OFDM symbol.

![Fig. 3. Equivalent time variant channel mitigation within one OFDM symbol.](image3)

![Fig. 4. BER performance comparison for extension ratio \( \alpha = 1 - \frac{1}{N} \) and \( f_d \cdot N = 0.085 \).](image4)
Although the performance of our method are the same as Franks windowing with 2N-FFT and Chang’s method when $\alpha = 1 - \frac{1}{N}$ and the BER curve are overlapping in Figure. 4, the proposed method is much simpler than those two methods. As shown in Fig. 4, the performance of Raised cosine windowing with 2N-FFT is the worst scheme compared to other methods.

Fig. 5. BER performance comparison for extension ratio $\alpha = 0.5$ and $f_{d,\text{norm}} = 0.085$

Fig. 5. shows the BER performance comparison for extended signal ratio $\alpha = 0.5$ and the normalized Doppler frequency $f_{d,\text{norm}} = 0.085$. When $\alpha$ decreased, the proposed method outperforms Chang’s method. As shown in Fig. 5, the proposed method improved the performance about 1.5 dB in Eb/No at the BER = $10^{-2}$ compared to Chang’s method. Although the performance of our method and Franks windowing with 2N-FFT method are the same in Figure. 5, the proposed method only needs N points FFT.

Fig. 6. shows the BER performance when extended signal ratio reduced to $\alpha = 0.25$ which is the parameter that most of the OFDM systems used and the normalized Doppler frequency $f_{d,\text{norm}} = 0.085$. Again, the proposed method outperforms Chang’s method.

V. CONCLUSIONS

A low-complex ICI reduction method by windowing the received signal with the extended signal interval using Franks window in time domain and combining the extended signal to the corresponding information part signal was proposed in this paper. We proved that ICI can be complete removed when $\alpha = 1 - \frac{1}{N}$ in linear time varying channel. As the simulation results shown, the proposed method can improve the performance to achieve BER = $10^{-2}$ before coding when $\alpha = 0.25$ and $f_{d,\text{norm}} = 0.085$ while Chang’s method can not achieve this criterion. Compared to Chang’s method, the performance of the proposed method outperforms Chang’s and the complexity is less than Chang’s method, $N \log N + N e^2$. Compared to the method of Franks windowing with 2N-FFT, the performance are the same. However, our method reduce the complexity that only needs N points FFT.

REFERENCES