G N Ramachandran's Contributions to Medical Imaging

Substance From Shadows

Anil Kumar and Ashok Ajoy

G N Ramachandran made several remarkable contributions to biophysics and structural biology over the course of his celebrated career. In a pair of seminal papers in 1971, he also laid the foundation for the technique of 'Projection Reconstruction' that has come to revolutionize medical imaging. It is now routinely employed in modern CT (Computed Tomography) and MRI (Magnetic Resonance Imaging) scans. In this article, we review the salient features of this technique, that allows the 'reconstruction of an object from its shadows'.


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1. Introduction

G N Ramachandran (GNR) made seminal contributions to the field of medical imaging by publishing two papers in 1971 on 'Projection Reconstruction'. In the first paper published in *Proceedings of the Indian Academy of Sciences* [1], he suggests the use of Fourier domain for image reconstruction. The paper is titled 'Reconstruction of substance from shadow: 1. Mathematical theory with application to three-dimensional radiography and electron micrography'. In the second paper (published jointly with A V Lakshminarayanan), in the *Proceedings of National Academy of Sciences*, USA [2], he suggests (instead of Fourier domain) the use of convolutions in the spatial dimension.

Projection reconstruction is at the heart of obtaining three-dimensional (3D) images of objects from their two-dimensional (2D) projections. In human radiology, the best example is that of Computed Tomography scan

There is so much mathematical elegance underlying medical imaging. Think about it the next time you go in for a CT or MRI scan.

(CT-scan). Normal X-rays revolutionized human radiology in the beginning of the last century by providing 2D images of human anatomy. These were highly successful and continue to be so. However, these images lack depth information. The projection reconstruction algorithm of G N Ramachandran [1,2] and others [3–5] revolutionized human radiology by enabling the construction of 3D images of human anatomy from these 2D projections and storing the 3D images thus obtained in a computer. This became possible because of two major inventions in 1960s. First was the availability of small desktop computers and the second was an algorithm for Fast Fourier Transform (FFT) by Cooley and Tukey in 1965 [6].

2. Projection Reconstruction

In a set of remarkable papers [1,2], which ushered in a new era in the field of computed tomography, Ramachandran outlined the 'convolution back projection' technique for reconstructing an object from its projections efficiently and with low error. In this section, we review the salient features of this method.

The aim is to reconstruct an object "from its shadows", i.e., take many projections of the object with the X-ray source at different angles θ, and from all these projections reconstruct the object. While the totality of the object information is contained in the projections, it is non-trivial to extract since each projection is composed from many object points. For simplicity, let us consider a 2D object (*Figure* 1), and the projections are then 1D.

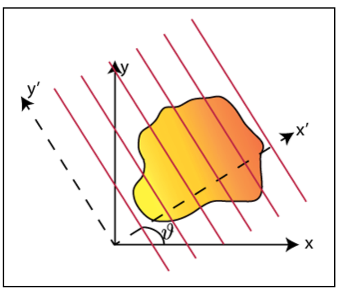

Figure 1. Schematic representation of projection reconstruction. One obtains projections of the object with the source at different angles θ. The aim is to reconstruct the shape of the object from these projections.

The projection is defined as just being the sum of intensities after transmission through the object along the line parallel to the incident beam. Mathematically, this is just the line integral – if the 2D object is described by the function $f(x, y)$ then, for instance, the projection along the y-axis is

$$g(x) = \int f(x, y) dy \,. \quad (1)$$

Projections are just akin to shadows. Hence, the reconstruction aims to obtain "substance from its shadows".

Similarly, one can define $g(r; \theta)$ as being the projection due to a beam incident at angle θ. In this notation, the LHS of (1) would be $g(r; \frac{\pi}{2})$.

3. Fourier Approach to Reconstruction

There is a simple procedure for reconstructing the function $f(x, y)$, shown in the following equations:

$$F(R; \theta) = \int g(r; \theta) \exp[i2\pi r R] dr \,,$$

$$f(r, \phi) = \int \int F(R; \theta) \exp[-i2\pi r R \cos(\phi - \theta)] R dR d\theta \,. \quad (2)$$

The first equation is just the Fourier transform (FT) of the projection obtained for different values of the incident beam θ. When this is substituted in the second equation, which is a 2D FT, one can reconstruct the real space object $f(r, \phi)$ (which is the same as $f(x, y)$ above, but in polar coordinates).

The above equations (2) are hard to calculate in practice, because it requires a 1D FT followed by a 2D FT to reconstruct the function $f(x, y)$. Instead, it was the insight of Ramachandran and Laksminarayanan that this could be simplified substantially. (See *Box* 1.)

It was Ramachandran's key insight that a transformation could reduce tremedously the computational power required for reconstruction.

4. Magnetic Resonance Imaging (MRI)

G N Ramachandran's contributions to Magnetic Resonance Imaging (MRI) are also well documented. In the

Box 1. Convolution Back Projection

Consider that one can rewrite (2) by doing the integral over R first to define a new function $g'(r;\theta)$.

$$g'(r;\theta) = \int |R| F(R;\theta) \exp[-i2\pi rR] dR \,. \tag{i}$$

The LHS of (2) can now be written as

$$f(r,\phi) = \int g'[r\cos(\phi-\theta);\theta] d\theta \,. \tag{ii}$$

Defining $q(r)$ as the 1D FT of $|R|$ by the equation $|R| = \int q(r)\exp[i2\pi rR]dr$, one recognizes from (ii) that,

$$\{\mathrm{FT}[g'(r;\theta)]\} = \{\mathrm{FT}[g(r;\theta)\} \times \{FT[q(r)]\} \,. \tag{iii}$$

This allows one to express (i) as the convolution,

$$g'(r;\theta) = \int g(r-r_1;\theta) q(r_1) \,. \tag{iv}$$

If $r = na$, i.e., the projection is obtained at discrete points given by the integral index n, then the convolution above can be expressed as the converging series,

$$g'(na;\theta) = \frac{1}{4a} g(na;\theta) - \frac{1}{n^2} \sum_{p \in \mathrm{odd}} \frac{1}{p^2} g[(n+p)a;\theta] \,. \tag{v}$$

Here we have used the formal result that the FT of $|R|$ is $\frac{1}{p^2}$ away from $p = 0$.

The procedure for reconstruction is now eminently simple: from the projections $g(na;\theta)$, calculate $g'(na;\theta)$ and substitute in (ii) to obtain the function describing the object $f(r,\phi)$. This forms a remarkable transformation of the original 2D FT in (2). It just consists of single integrals (or sums) that can be computed very efficiently – about an order of magnitude faster than the 2D FT.

This transformation has come to be known as the 'convolution back projection' technique and is now widely used in all types of tomography, especially in all commercial CT scanners. It is indeed a testament to the creative ingenuity of Ramachandran and his coworkers.

very first paper on MRI by Paul Lauterbur in *Nature* in 1973 [7] (for which Lauterbur shared the 2003 Nobel Prize in Physiology/Medicine[1]), he mentions six papers on projection reconstruction including the paper [2] by

[1] See Kavita Dorai, Magnetic Resonance Imaging: Window to a Watery World, *Resonance*, Vol.9, No.5, pp.19–32, May 2004.

G N Ramachandran and Lakshminarayanan. Lauterbur's idea on MRI is briefly described below.

In MRI, the object (in Lauterbur's case two capillaries filled with water) is placed in a homogeneous magnetic field of an NMR spectrometer. One then applies a linear gradient perpendicular to the two tubes. This makes the magnetic field at the the two tubes slightly different (proportional to the distance between them). The NMR spectrum is thus a projection of the 2D object into a 1D spectrum. The process is repeated with several different projections and the image reconstructed using the projection reconstruction algorithm of Ramachandran [2] (see *Figure* 2). The principle of MRI is thus to convert space information into frequency information using linear gradients.

The above protocol of Lauterbur was tremendously improved by the use of Fourier Transform NMR, as suggested by Richard Ernst in 1966. The projection from one plane could be recorded by a Continuous-Wave[2] (CW) method, as was done by Lauterbur, or by a much faster method using FT-NMR. This itself would lead to some improvement in signal-to-noise ratio. However, the major breakthrough was obtained using 2D FT-NMR again by Ernst, who obtained the 1991 Chemistry Nobel Prize for one and two dimensional FT-NMR. (A *Resonance* special issue of Nov 2015 explains FT-NMR in

[2] For CW versus FT-NMR, see *Resonance,* Vol.20, No.11, p.995, 2015.

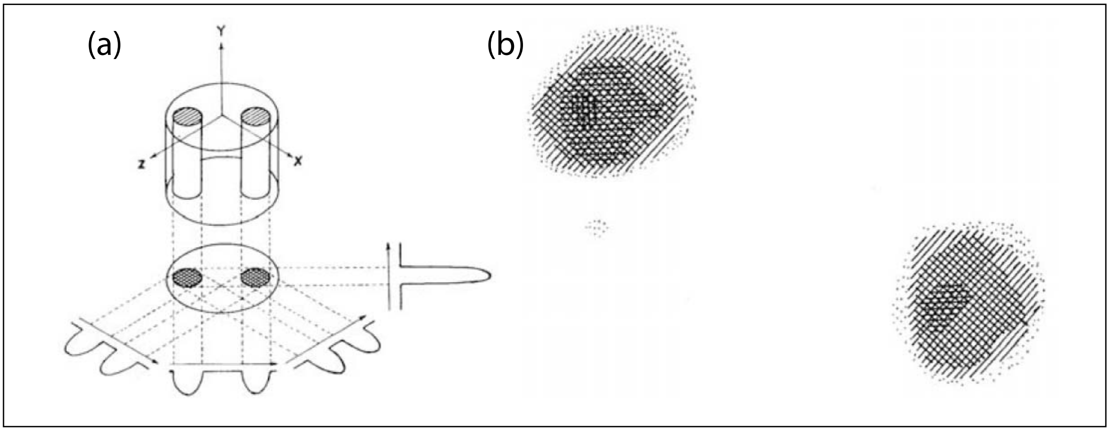

Figure 2. (**a**) 2D projections of two tubes of water for gradients perpendicular to them. (**b**) The first MRI images [7] of the two tubes obtained by Lauterbur using projection reconstruction protocol of Ramachandran and others.

more detail [8,9].) It was suggested that the magnetization of a 2D plane be excited by a radiofrequency (RF) pulse and the resulting induced signal be collected as a function of two time variables t_1 and t_2 under two orthogonal gradients [10]. The 2D plane is chosen by a frequency selective pulse under the influence of the third linear gradient perpendicular to the plane. The data $s(t_1, t_2)$ thus collected is subjected to a 2D FT yielding $F(f_1, f_2)$ which contains the 2D image. This protocol yields the whole 2D image much faster. The data under the t_1 period is collected in a single excitation by the use of 'Echo-Planar-Imaging' (EPI) as suggested by Peter Mansfield [11] (for which he shared the 2003 Nobel Prize with Paul Lauterbur), a technique that has been described in detail in [8].

5. Summary

[3] See *Resonance*, Vol.6, No.10, 2001.

G N Ramachandran[3] is highly regarded for his path-breaking research in the area of protein structure determination protocols, first for obtaining the 3D structure of collagen by single crystal X-ray crystallography (1954), and later for suggesting the famous 'Ramachandran phi/psi plots' for allowed conformations of amino acids in a protein (1963). He also made seminal contributions towards medical imaging (1971), which is the subject of this article. It is noteworthy that his work rekindled interest in the early work of J Radon [12], which anticipated features of the mathematics of projection reconstruction, although obviously not the application. Both CT-scan and MRI have benefited enormously from G N Ramachandran's 1971 papers and he can be rightly credited with having a deep impact on the 'Medical Imaging' field.

Acknowledgements

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Suggested Reading

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