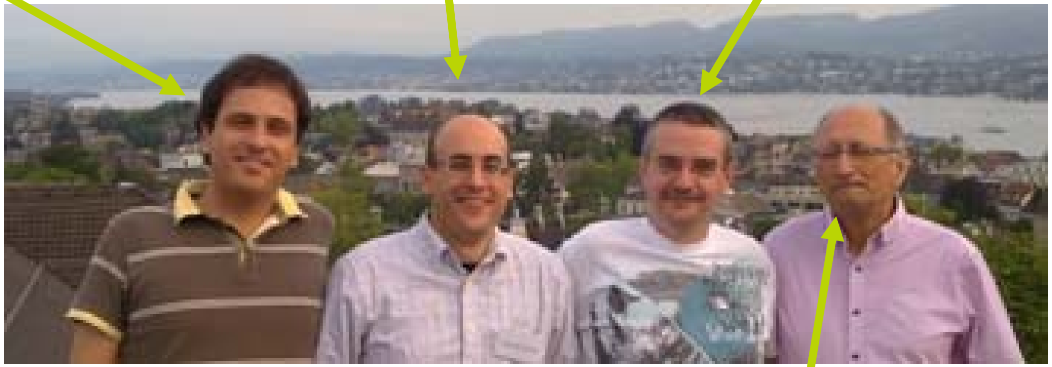
Social and strategic imitation: the way to consensus

DANIELE VILONE, JOSÉ J. RAMASCO, *ANGEL SÁNCHEZ*, (*GISC-UC3M*)



MAXI SAN MIGUEL

*IFISC

CSIC Universitat de les Illes Balears

Decision making:

Adaptation of new technologies
Consensus problems

Making decisions can be a hard task, especially when facing *a priori* equivalent choices...

Interplay of STRATEGIC and SOCIAL factors

Interactions: i) Mechanism → **IMITATION**

ii) Network of interactions

Social imitation **Strategic behavior**

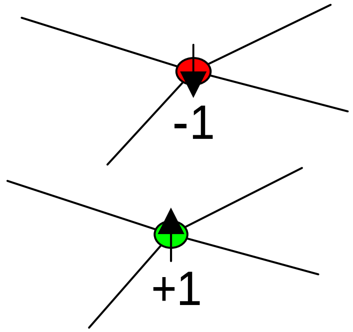
Fast and Slow Thinking

D. Kahneman

VOTER MODEL **COORDINATION GAME**

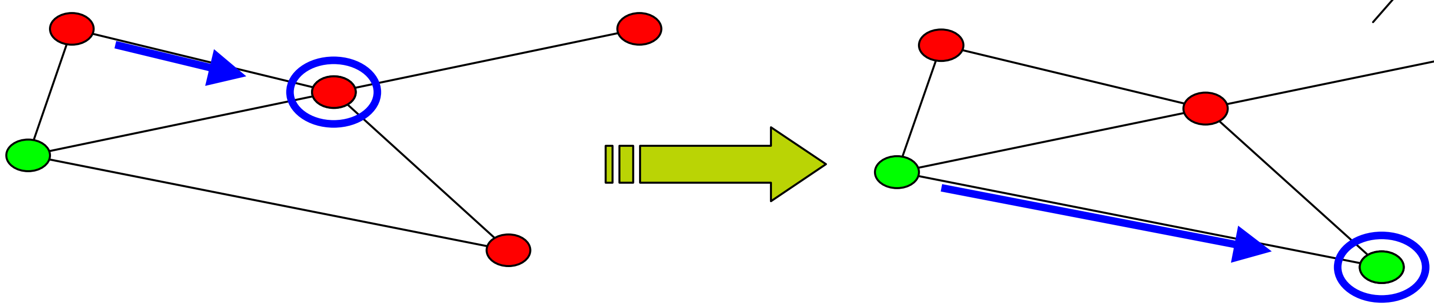
Voter Model (VM)

Ann. Probability (1975)



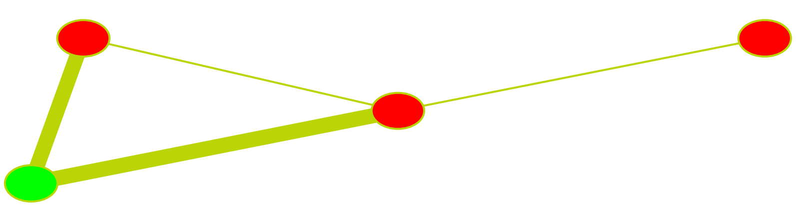
"Voters" located in the nodes of a network have "opinions" σ_i=1 or σ_i=-1.

A randomly chosen voter takes the opinion of one of its neighbors (node update).



Qs?: When and how one of the two absorbing states (consensus) is reached? Effect of network of interactions?

<u>Order Parameter:</u> Average interface density



$$n_A \equiv \rho = \frac{1}{2N\langle k\rangle}\left(\sum_{i=1}^{N}\sum_{j\in\nu(i)}(1-\sigma_i\sigma_j)\right)$$

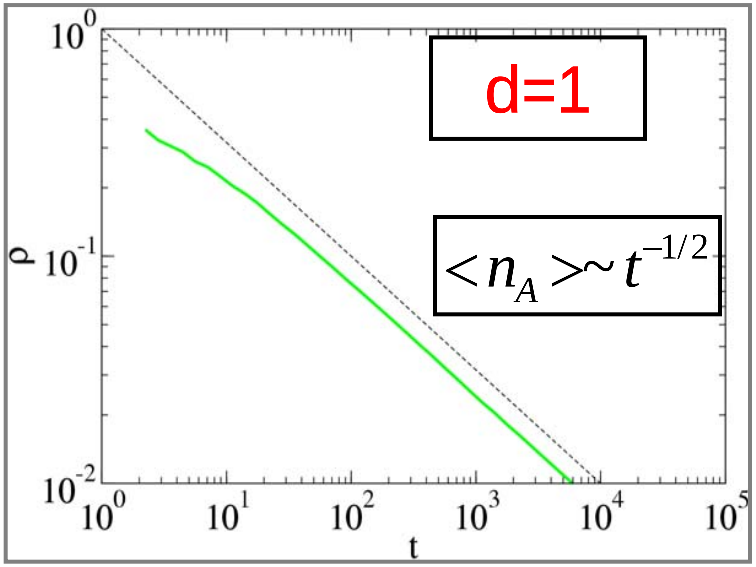
n_A=0 in absorbing state

Interface: a link connecting nodes with different states.

Voter Model (VM)

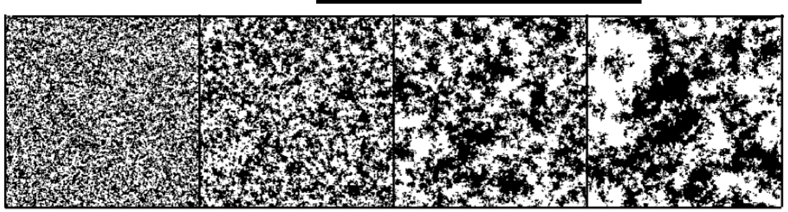
Regular Networks

Ordering: Unbounded growth of domains of absorbing states



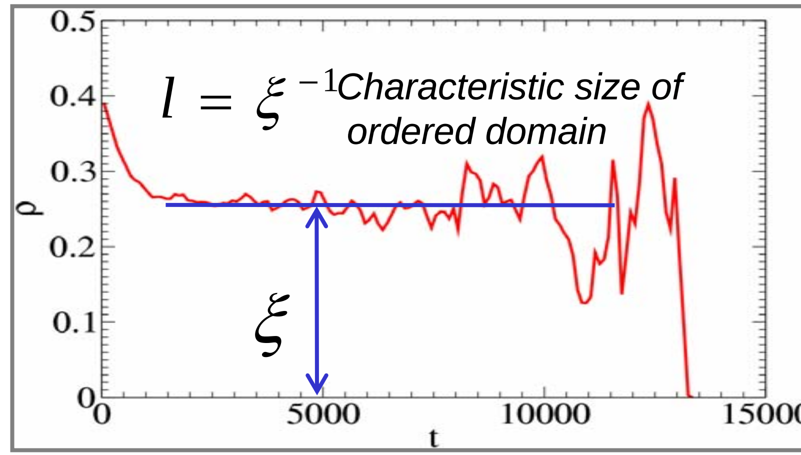
d=2

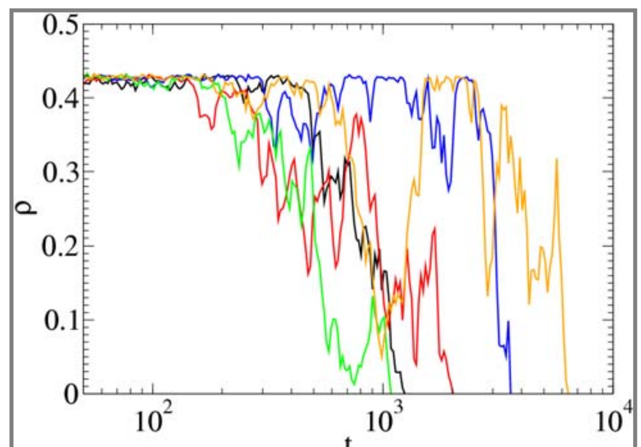
$< n_A > \sim (\ln t)^{-1}$



Complex Networks (+ fully connected)

No Ordering: Dynamical Metastability





$< n_A > \sim e^{-t/\tau}$

Coordination games (CG)

- Individuals choose between two strategies. Their pay off is larger when they choose the same strategy than the other player (coordination)

- The fully symmetrical coordination game is described by the payoff matrix

	L	R
L	1,1	0,0
R	0,0	1,1

Two degenerate Nash Equilibria in pure strategies: Coordination in ++ (LL) or in - - (RR)

Coordination game dynamics of N interacting agents

-Agents located at the nodes of a network play the coordination game with all their neighbors and aggregate a pay-off.

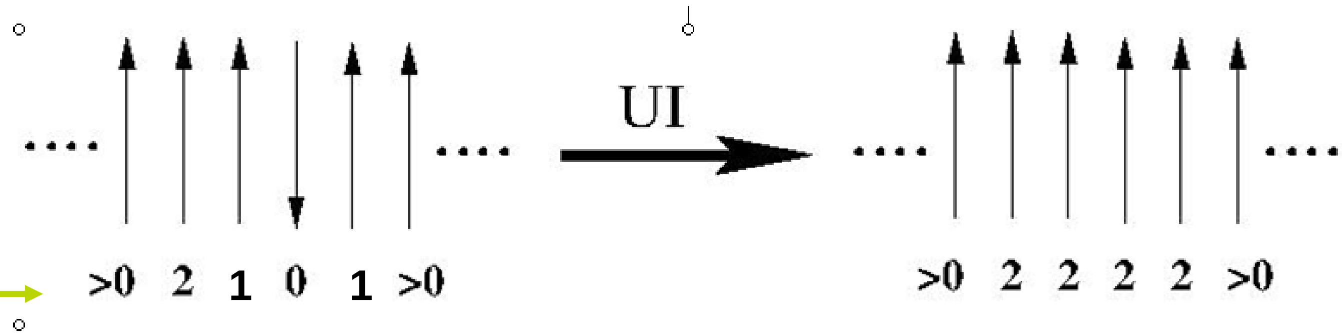
-**STRATEGIC** dynamical rule of **UNCONDITIONAL IMITATION (UI)**: at the end of each round, individuals imitate the strategy of their neighbour with largest pay off

-FULLY CONNECTED NETWORK: Coordination (consensus) in one time step

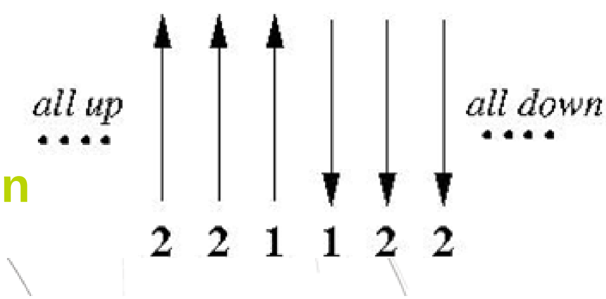
CG: Unconditional Imitation in Regular Networks

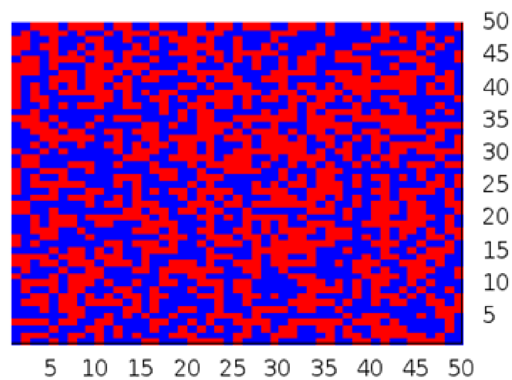
Example
of
UI Evolution

Pay off

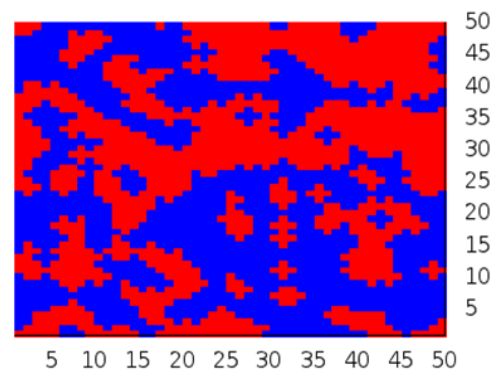


d=1
Frozen
Configuration





t=0
Random
configuration



Final
Frozen
configuration

Unconditional imitation of the strategy of the neighbor with largest payoff typically leads to frozen configurations in topologies in which social imitation (voter dynamics) leads to a consensus

	FULLY CONNECTED NETWORK	REGULAR LATTICE (d=1,2)	RANDOM NETWORK
VOTER MODEL (q=1) Social imitation	DYNAMICAL DISORDER	ORDERING TO CONSENSUS	DYNAMICAL DISORDER
COORDINATION GAME (q=0) Strategic imitation	COORDINATION	FROZEN DISORDER	FROZEN DISORDER

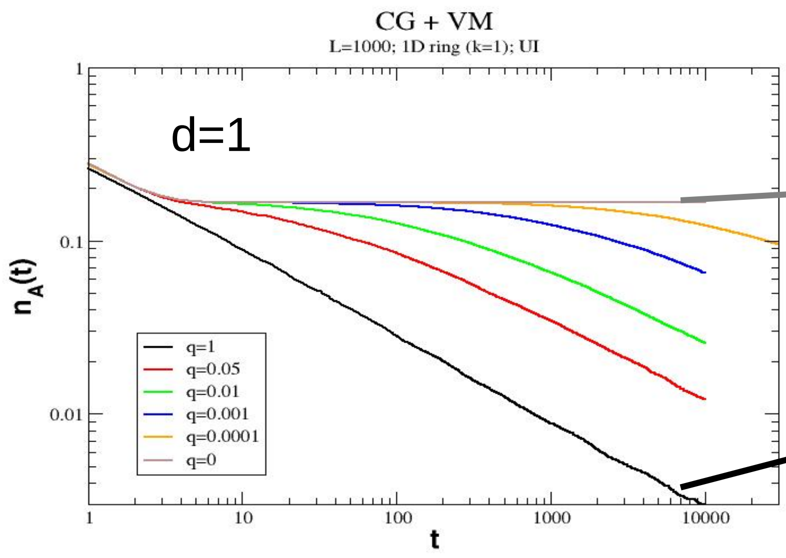
D. Vilone et al. Scientific Reports 2, 686 (2012)

- System of N interacting agents, characterized by a binary variable state (+-,RL,↑↓,CD) which can be seen as both “opinion” or strategy

- At each elementary time step, an agent is picked up at random: with probability *q* she will update her state by Social Imitation (VM), and with probability *1-q* by Strategic Imitation (UI in CG): q=1 VM, q=0 CG

- Now not only topology is important, but also the social-strategic interplay parameter *q*.

Regular Networks



UI Coordination Game (frozen)

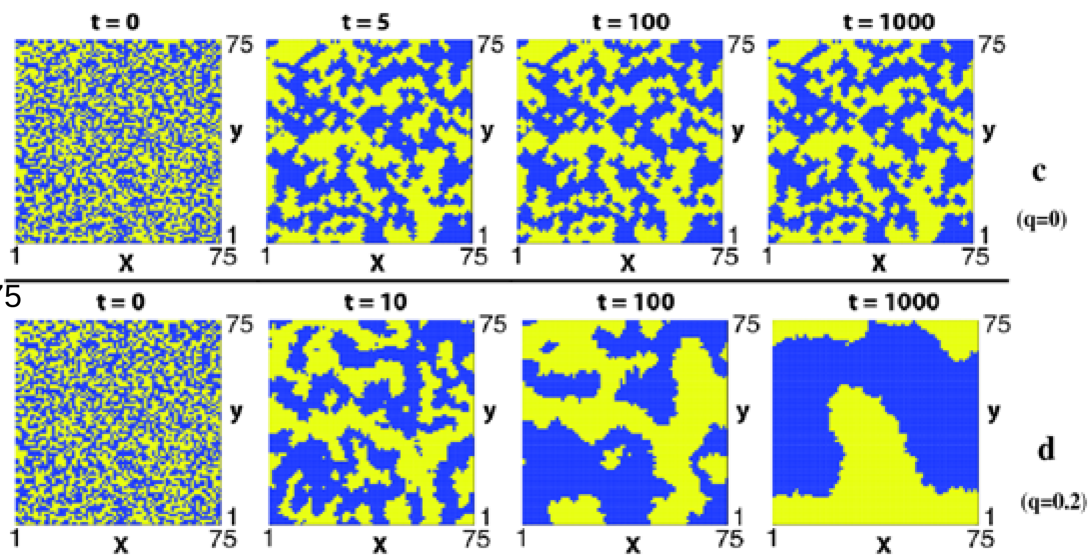
Voter Model (ordering)

A small component of social imitation (non-rational behavior) makes the agreement (consensus) possible

q=0
CG

d=2
N=75x75

q=0.2
CG + VM



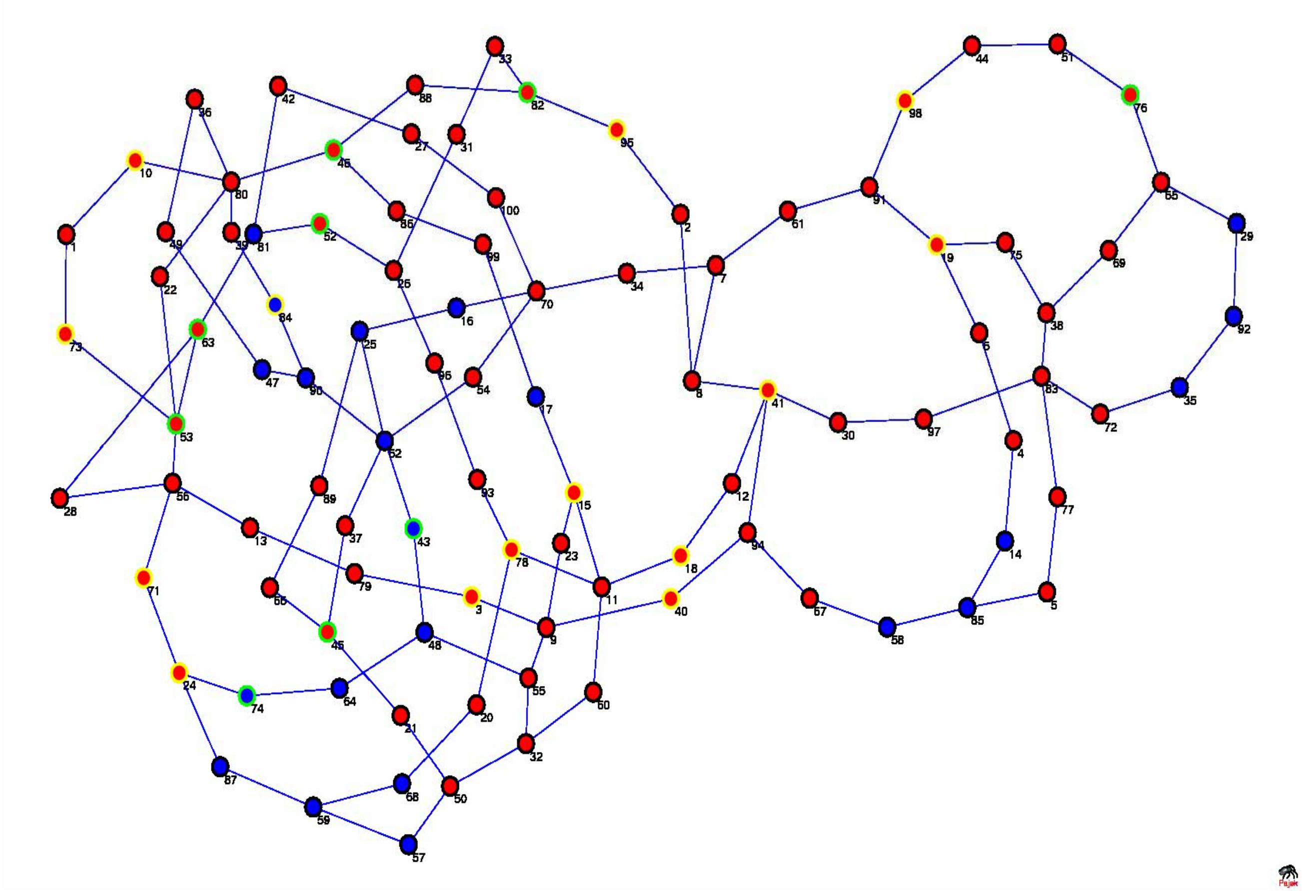
Fully Connected Nets:

VM: Disorder
CG: Coordination
A small component of strategic imitation makes the agreement possible

Social (VM) + Strategic Imitation (CG)

Random Networks

ER network, N=100, <k>=3, *q*=0.4

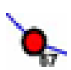


VM:
Dynamical Disorder

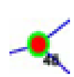
CG:
Frozen Disorder

**VM+CG (q=0.4):
Reaching agreement**

 R

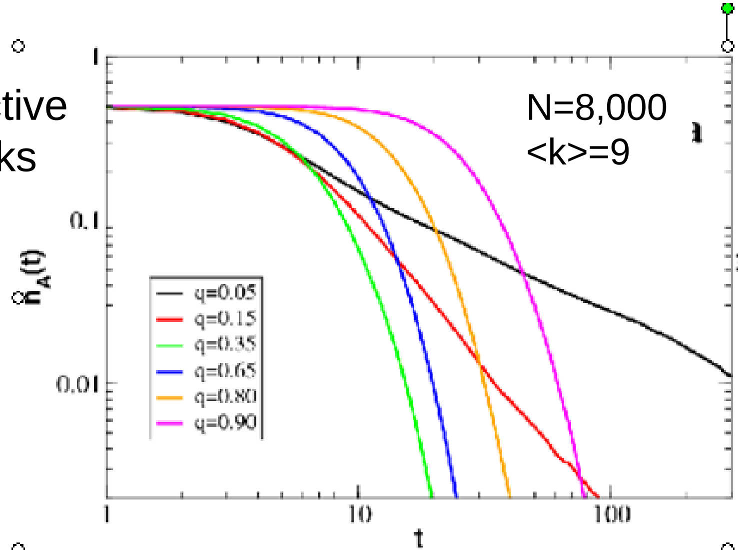
 L

 L, last updated by CG

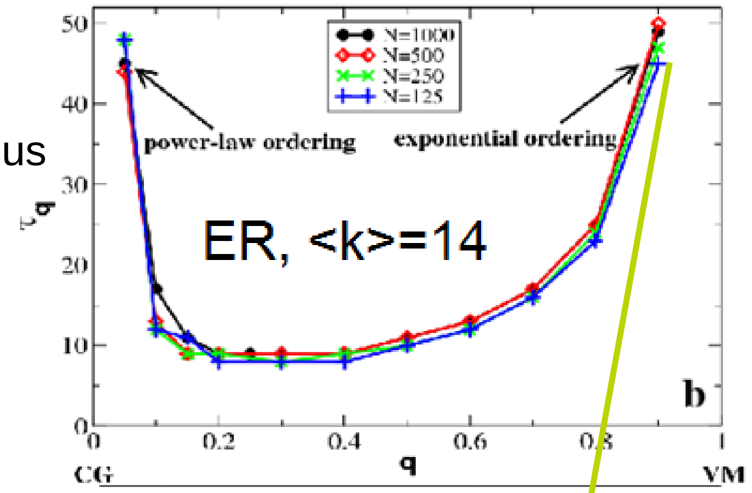
 L, last updated by VM

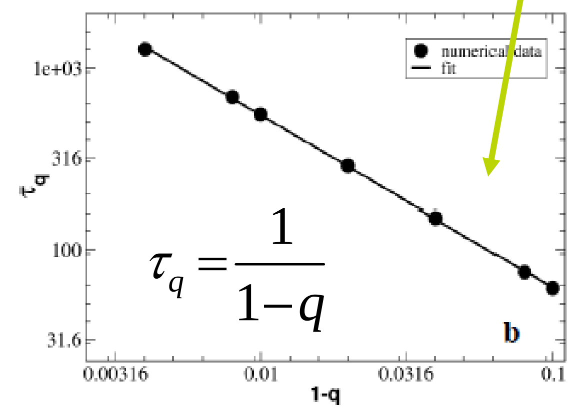
ER Random Networks

Active links



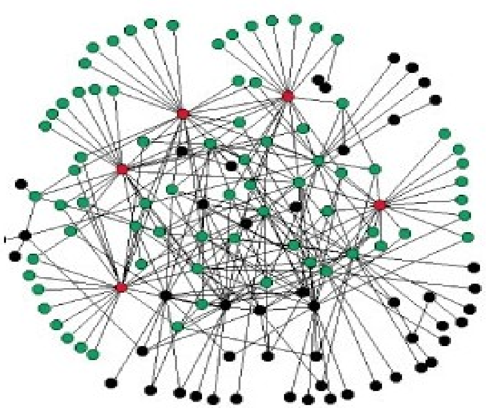
Time to consensus





- Ordering for any $q \neq 0,1$
- Fast ordering close to *q*=1, VM
  Slow ordering close to *q*=0, CG
- Crossover regime from slow ordering (q<q*) to fast ordering (q>q*)
- **Pure strategic or social imitation leaves the system disordered, but any amount of mixing of them allows to reach total consensus.**
  Optimum mixture for q=q* with a smallest time to reach consensus τ_q

Scale Free Networks

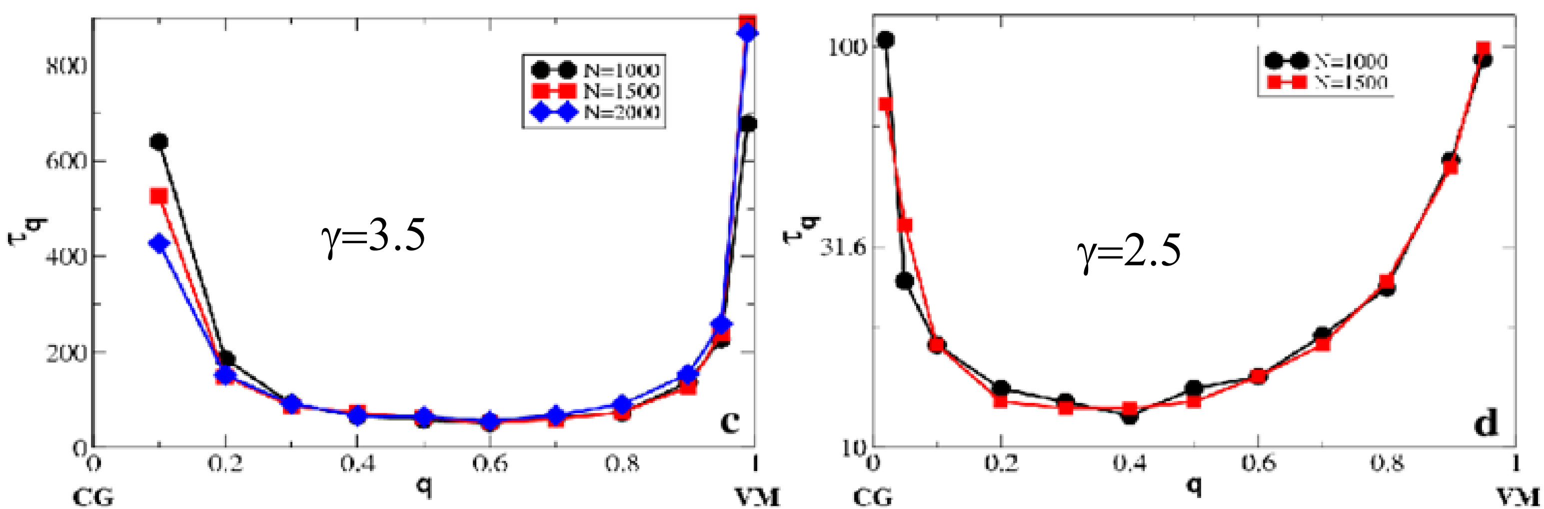


Power law for the degree distribution

$P(k) \sim k^{-\gamma}$

Preferential attachment

Importance of hubs



Again, interplay ($q \neq 0,1$) of social and strategic imitation needed to reach agreement

Coordination games (CG): Pay-off and Risk Dominance

Pay-off matrix (*b*>0)

$$\begin{array}{ccc} & L & R \\ L & 1,1 & 0,-b \\ R & -b,0 & 2,2 \end{array}$$

RR equilibrium: pay-off dominant

b<1, RR risk dominant equilibrium

b>1, LL risk dominant equilibrium

-FULLY CONNECTED NETWORK:

b<1 Coordination in RR equilibrium

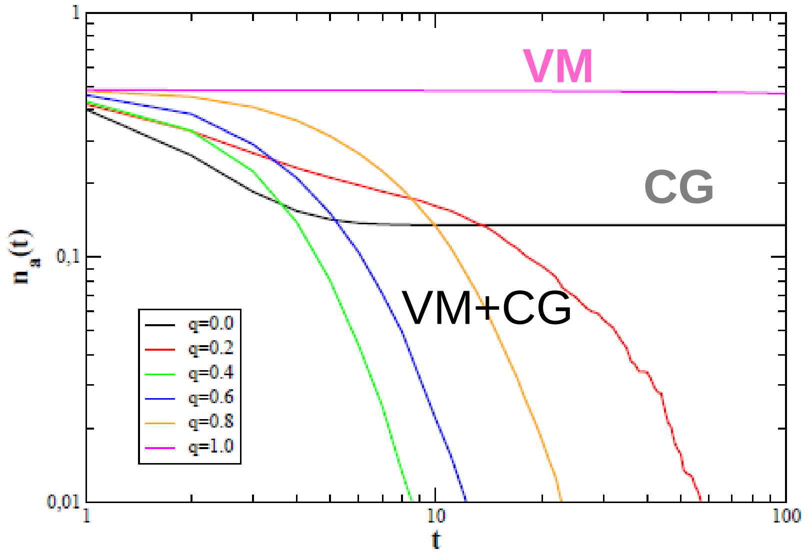
b>1 Coordination in LL equilibrium

-COMPLEX NETWORKS:

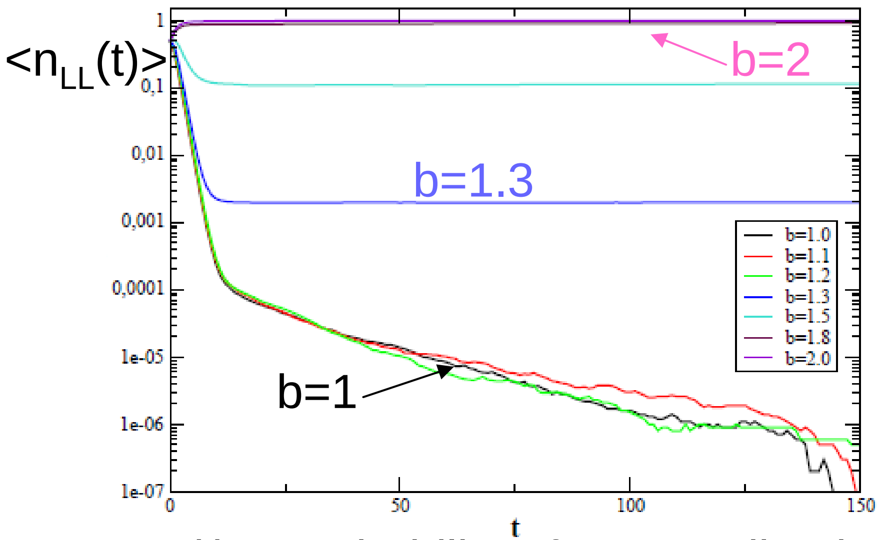
Unconditional imitation leads to frozen disordered configurations

Pay-off and Risk Dominance in ER Random Networks

N=10,000, <k>=30, b=1.5 (LL rd)



N=10,000, <k>=9, q=0.1



Interplay ($q \neq 0,1$) of social and strategic imitation needed to reach agreement

$\langle n_{LL}(t) \rangle$~probability of LL coordination

b>1, LL risk dominant equilibrium

$$q \neq 0,1; \quad 1 < b < b^*(q); \quad \langle n_{LL}(t \to \infty) \rangle = 0$$
$$(b^*(q) \approx 1.26)$$

Coordination in RR: non-risk dominant equilibrium

- **Social imitation makes coordination possible, AND promotes pay-off dominance (rationality?) against risk dominance**

Question Addressed:

Competition of strategic ("the self") and socially motivated decisions

Illustrative minimal model:

Social imitation (Voter Model) +
Strategic imitation (Coordination Game)

Take home message:

- Social consensus only reached by combining social and strategic imitation. Optimum mixing q*
- Social imitation promotes pay-off dominance (rationality?) against risk dominance