Optimal Power Control Game Algorithm for Cognitive Radio Networks with multiple interference temperature limits

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Abstract—Based on interference temperature model, the problem of secondary spectrum sharing can be formulated as a power optimization problem at physical layer. In this paper, we consider decentralized cognitive radio networks, and we especially focus on the spectrum sharing scenario where multiple measurement points are located in the licensed system. Game theory is used to investigate the distributed power control for providing the maximum throughput in cognitive radio networks. There are two aspects that should be considered in the design of the payoff function for each player, one is counteracting negative externalities in cognitive radio networks, and the other is satisfying all the interference temperature limits from measurement points. A tax-based power control game algorithm is introduced to implement power allocation optimization in a distributed mode, and guarantees the convergence to globally optimal power allocations.

Keywords- cognitive radio, game theory, power control, interference temperature

I. INTRODUCTION

The concept of Cognitive Radio(CR) [1] is first introduced by J. Mitola to achieve a more efficient spectrum usage by using spectrum opportunities in time, frequency and space which is not fully used by a licensed system (primary system), but without disturbing the primary system. The methods that realize the spectrum sharing or system coexisting between primary system and cognitive radio network can be classified as two types, one is based on precise spectrum sensing, and the other is based on interference temperature model. The interference temperature is introduced into CR by Federal Communications Commission (FCC) as a metric for the measurement of interference for “quantifying and managing interference” [2]. In order to prevent the negative impact to the licensed users, the Interference Temperature Limit (ITL) is proposed to indicate the allowed worst RF environment. Interference temperature allows unlicensed users to transmit in licensed bands with carefully adjusted power, provided that unlicensed users’ transmission does not raise the interference temperature over the ITLs. Hence, interference temperature model is an important secondary spectrum sharing method which can realize underlying system coexisting between primary system and secondary cognitive system.

Based on the interference temperature model, the problem of efficient secondary spectrum sharing can be formulated as a power control problem at the physical layer. In this paper, we consider a decentralized CR network where secondary users compete for a short-term data service through secondary spectrum sharing. We focus on maximizing the total CR network throughput, subject to ITLs from Measurement Points (MPs) which are located in primary system.

Related work on secondary spectrum sharing based on interference temperature has appeared in [3] and [4]. In [3], auction mechanisms that allocate the received power for secondary users under the constraint of interference temperature are studied, but Pareto optimal solution can be achieved only when the manager know all the users’ utility functions, which is unpractical for decentralized cognitive radio network. In [4], the quality of service from secondary users is also considered besides the interference temperature limit from primary system, and a potential game is proposed to deal with the spectrum access problem. However, the research in [3] and [4] studies only single measurement point scenario. In practical applications there should exist many MPs in primary system to detect the interference from cognitive radio networks.

The contributions of this paper are described as follows. We investigate the secondary spectrum sharing scenario where multiple measurement points exist in primary system; A tax-based power control game algorithm was introduced to solve the power allocation optimization problem in a decentralized mode, and the tax-based power control game considers not only counteracting the negative externalities in secondary networks but also satisfying the ITLs from the primary system. We prove that the proposed algorithm will always converge to the globally optimal solution if the parameters are appropriately set.

The rest of this paper is organized as follows. In section II, the system model of secondary spectrum sharing was addressed. Section III formulates the power allocation optimization problem. In section IV, a tax-based power control
game algorithm is proposed. Finally, the simulation is provided to verify the performance of proposed algorithm in Section V.

II. SYSTEM MODEL

Consider a distributed secondary network with n logical transmitter-receiver pairs and m MPs. The system model is shown in Fig. 1.

Denote each transmitter-receiver pair by a single index \( i(i=1,\cdots,n) \), which we refer to as a user. \( p_i \) is the transmit power, \( h_{ij} \) is the channel gain from user \( i \)'s transmitter to user \( j \)'s receiver, and \( \sigma^2 \) is the background noise power that is assumed to be the same for all users. Denote each measurement point by a index \( k(k=1,\cdots,m) \), \( g_{ik} \) is the channel gain from user \( i \)'s transmitter to the \( k \)th measurement point.

For each user \( i \), the received SINR is given by

\[
\gamma_i = \frac{p_i h_{ij}}{\sigma^2 + \sum_{j \neq i} p_j h_{ji}}, \quad i=1,\cdots,n
\]  

The maximum interference power tolerance at the \( k \)th measurement point can be calculated as \( Z_k=BT_k \), where \( B \) is Boltzmann’s constant, \( T_k \) is the interference temperature limit which is a pre-defined threshold and is proposed to indicate the allowed worst RF environment.

In order to guarantee non-intrusion of the primary system, the total received power at measurement point \( k \) must satisfy:

\[
\sum_{i=1}^{n} p_i g_{ik} \leq Z_k, \quad k=1,2,\cdots,m
\]  

III. PROBLEM FORMULATION

In this paper, we formulate the power allocation problem to a constrained total network utility maximization problem. Secondary user \( i \)'s QoS is characterized by a utility \( u_i \), which is defined as \( u_i = \log \gamma_i \) when only high SINR scenario is considered. Therefore, the total system utility can be calculated as \( \sum u_i (p) \). Optimality here is to maximize the total secondary system utility with the interference temperature limits from all the MPs. The constrained power control problem can be mathematically described as

\[
\max \sum_{i=1}^{n} u_i (p)
\]

subject to:

\[
\sum_{i=1}^{n} p_i g_{ik} \leq Z_k, \quad k=1,2,\cdots,m \quad (a)
\]

\[
p_i^{\min} \leq p_i \leq p_i^{\max}, \quad i=1,2,\cdots,n \quad (b)
\]

Where the optimization variable is the transmit power vector \( p=(p_1, p_2, \cdots, p_n) \). The constraint (a) reflects the fact that the aggregated received interference power at the \( k \)th measurement point is bounded by the interference threshold \( Z_k \) which should not be overstepped. The constraint (b) is regulatory or system limitations on transmit powers. Note that constraints (a) and (b) respectively denote a set of constraints.

Literature [5] shows that through logarithmic change of variables, problem (P1) can be transformed to a strictly convex optimization problem which has a unique optimal solution and can be solved by geometric programming method. Although the geometric programming method has shown to be effective in certain applications, it has limitation due to the requirement of collecting global information at a central point of computation. In a decentralized cognitive radio network, decentralized implementations are preferred.

In the following, we explore the distributed implementation of optimal power allocation for problem (P1). In section IV, we propose a tax-based game theoretic method to deal with the power allocation optimization which is played in an asynchronous update mode.

IV. TAX-BASED POWER CONTROL GAME ALGORITHM

Game theory is founded on the hypothesis that players act rationally, in the sense that each player has a payoff function that it tries to optimize. In a power control game, each link is modeled as a player with an aim of maximizing its own payoff function. Hence the design of payoff function is crucial for a game to converge to a desire Nash equilibrium.

In this section, we aim at seeking the distributed power control algorithm to realize the maximum throughput of a secondary network using game theory. Hence the desire Nash equilibrium is the maximum throughput of the cognitive radio network. To achieving the above target two aspects should be considered when designing the payoff function for each player. The first one is how to embody the social network benefit. The second one is how to meet the constraint (a) in (P1). Then we give our solution about the design of payoff function.

Denote \( p_i=(p_{i1}, p_{i2}, \cdots, p_{in}) \), we define the payoff function of use \( i \) as

\[
s_i (p_{i1}, p_{-i}) = u_i (p_{i1}, p_{-i}) - p_i \sum_{j=1, j \neq i}^{n} v_j - p_i \sum_{k=1}^{m} \mu_k, \forall i
\]

Where \( u_i = \log \gamma_i \), \( v_j \) and \( \mu_k \) are positive factors, \( v_j \) is the tax factor which represents user \( j \)'s unit decrease in utility with
respect to per unit increase in \( p_i \), and \( \mu_k \) is the penalty factor which reflects the penalty degree to the payoff of user \( i \) when the ITL at the \( k \)th measurement point is overstepped.

Then the power control game can be expressed as

\[
\begin{align*}
\max_{p_i} & \quad s_i(p_i, p_{-i}), \forall i \\
\text{subject to:} & \quad p_{i_{\text{min}}} \leq p_i \leq p_{i_{\text{max}}}
\end{align*}
\]  

(4)

In the following we will interpret the theory foundation based on which our payoff function are proposed and give specific definitions of the two kinds of factors in (3).

A. Negative Externalities, Pigovian Tax and definition of \( v_j \)

The concept of negative externalities comes from microeconomics [6]. There are externalities when the actions of one producer directly affect the payoff of the other producers. Further, if the actions decrease the payoff of the others, the externalities are negative with the term as negative externalities. Due to the negative externalities the social benefit is greatly reduced. Pigovian tax is considered one efficient means of counteracting the negative effect of externalities for achieving better social benefit. The basic idea of Pigovian tax is to add a tax function which reflects interference level to other users in (3) is a penalty function which penalizes the violation of the constraint (a) in (P1) by reducing the payoff of user \( i \). In this paper, we define \( \mu_k \) as

\[
\mu_k = \lambda_k g_{\alpha}
\]  

(11)

B. Lagrange Multipliers and definition of \( \mu_k \)

We relax the constraint (a) in (P1) using Lagrange relaxation method. The Lagrange function of problem (P1) is

\[
L(p, \lambda) = \sum_{i=1}^{n} u_i(p) - \sum_{k=1}^{m} \lambda_k (\sum_{i=1}^{n} p_i g_{\alpha} - Z_k)
\]  

(6)

Then problem (P1) is equivalent to

\[
\max_{p} L(p, \lambda^*)
\]

(7)

subject to:

\[
p_{i_{\text{min}}} \leq p_i \leq p_{i_{\text{max}}}, \quad i=1,2,\ldots,n
\]

(8)

where \( \lambda^*=(\lambda_1^*, \lambda_2^*, \ldots, \lambda_m^*) \) is Lagrange multiplier vector corresponding to constraint (a) in (P1), and we use the subgradient method to search it. Let \( w \) be a time (iteration) counter and \( N_k(w) \) be the aggregate received power at the \( k \)th measurement point at time \( w \). Let

\[
N_k(w) = \sum_{i=1}^{n} p_i(w) g_{\alpha}(w), \quad k=1,\ldots,m
\]  

(9)

Then the variables \( \lambda_k(w+1) \) is updated via subgradient with step sizes \( l_k \)

\[
\lambda_k(w+1) = \max \left\{ 0, \lambda_k(w) + l_k(w) (N_k(w) - Z_k) \right\}
\]

Where step sizes \( l_k \) are defined as

\[
l_k(w) = \frac{\alpha}{Z_k \sqrt{w}}, \quad k=1,\ldots,m
\]  

(10)

Step sizes in (10) obey nonsummable diminishing step size rules, and \( \alpha \) is a positive parameter.

The expression \( p_i \sum_{j=1}^{n} \mu_k \) in (3) is a penalty function which penalizes the violation of the constraint (a) in (P1) by reducing the payoff of user \( i \). In this paper, we define \( \mu_k \) as

\[
BR(p, \mu) = \arg \max_{p_i} s_i(p_i, p_{-i}), \forall i
\]  

(12)

Each active link \( i \) solves \( \max_{p_i} s_i(p_i, p_{-i}) \) by choosing appropriate \( p_i \). The partial derivative of \( s_i(p_i, p_{-i}) \) with respect to \( p_i \) is

\[
v_j = \frac{\partial u_i}{\partial p_i}, \quad j=1,\ldots,n, \quad j \neq i
\]  

(5)
\[
\frac{\partial s_i}{\partial p_i} = \frac{\partial u_i}{\partial p_i} + \sum_{j \neq i}^n \nu_j + \sum_{k \neq i}^n \mu_k \tag{13}
\]

Let the above equation in (13) equal to zero, we obtain the best response function

\[
BR_i(p_{-i}) = \left[ \frac{1}{\sum_{j \neq i}^n \pi_j h_{ij} + \sum_{k \neq i}^n \lambda_k g_{ik}} \right]^{\pi_i^{\max}} \tag{14}
\]

Note that the denotation \([X]^+\) means \(\max\{a, \min\{b, X\}\}\).

For distributed implementation, it is necessary to obtain information \(\nu_j\) from every other link \(j\). The large amount of information exchange need to be performed between links, thus will greatly decrease the performance of distributed power control. In [7], a method of decreasing information exchange will greatly decrease the performance of distributed power control. In [8], the power control game (4) has at least one pure Nash Equilibrium.

Proof: The action set \(a_i = [p_i^{\min}, p_i^{\max}]\) of player \(i\) is a nonempty compact convex set. \(s\) is continuous in \(p\), and \(s_i\) is strictly concave in \(p_i\) with fixed \(p_{-i}\). According to Proposition 20.3 in [8], the power control game (4) has at least one pure Nash Equilibrium. \(\square\)

Theorem 2: TPCG algorithm always converges to the globally optimal solution of the network throughput optimization problem (P1), provided that step sizes \(l_k\) are appropriately chosen.

Proof: Supposing the Nash equilibrium of game (4) is \(p^*\), then there exist unique Lagrange multipliers, \(\Psi_i, \zeta_i\), such that

\[
\frac{\partial \psi_i(p^*)}{\partial p_i} = \Psi_i - \zeta_i, \forall i \tag{18}
\]

where \(\psi_i(p^*) - p_i^{\max} = 0, \psi_i \geq 0\)

\[
\zeta_i(p_i^{\max} - p_i^*) = 0, \zeta_i \geq 0 \tag{19}
\]

The left part of equation in (18) can be expressed as

\[
\frac{\partial \psi_i(p^*)}{\partial p_i} = \frac{\partial u_i(p^*)}{\partial p_i} + \sum_{j \neq i}^n \frac{\partial u_j(p^*)}{\partial p_i} - \sum_{k \neq i}^n \lambda_k g_{ik} \tag{20}
\]

By (18), (19) and (20), it shows that \(p^*\) also satisfies the KKT necessary conditions of problem (7).

Furthermore, through logarithmic change of variables, the objective function of problem (7) can be transformed to a strictly concave function. Since the constraint sets are convex, the problem (7) is a convex optimization problem which has a unique globally optimal solution. Hence there exists only one solution that meets KKT necessary conditions. Thus we conclude that \(p^*\) is the globally optimal solution of problem (7).

D. Nash Equilibrium and Globally Optimal Solution

In the following, we will analyze the convergence of game (4), and prove that the proposed TPCG algorithm will always converge to the globally optimal solution if step sizes \(l_k\) are appropriately chosen.

Theorem 1: the power control game (4) always converges.

Proof: The action set \(a_i = [p_i^{\min}, p_i^{\max}]\) of player \(i\) is a nonempty compact convex set. \(s\) is continuous in \(p\), and \(s_i\) is strictly concave in \(p_i\) with fixed \(p_{-i}\). According to Proposition 20.3 in [8], the power control game (4) has at least one pure Nash Equilibrium. \(\square\)

Algorithm 1: Tax-based Power Control Game (TPCG) Algorithm

1) Initialization: \(w = 0, p_i^{\min} \leq p_i(0) \leq p_i^{\max}, \lambda_i(0) > 0, \lambda_i(0) = 0 \)

2) Power Update: iteratively update power \(p_i\) as follows:

\[
p_i(w + 1) = \left[ \frac{1}{\sum_{j \neq i}^n \pi_j h_{ij} + \sum_{k \neq i}^n \lambda_k g_{ik}} \right]^{\pi_i^{\max}} \tag{20}
\]

3) \(\lambda_k\) Update via Subgradient with Step size \(l_k\)

\[
\lambda_k(w + 1) = \max \{0, \lambda_k(w) + l_k(w)\{N_i(w) - Z_i\}\} \tag{20}
\]

Where \(l_k(w) = \lambda / (Z_i \sqrt{w})\)

4) \(\lambda_i\) Update and Broadcast

\[
\pi_i(w + 1) = 1 / (\sigma^2 + I_i(w + 1)) \tag{20}
\]

Where \(I_i(w + 1) = \sum_{j \neq i}^n p_i(w + 1)h_{ij}\)
V. SIMULATION RESULTS

In this section, we present some numerical results to illustrate the performance of the proposed power allocation algorithm. We consider a decentralized cognitive radio network with 5 transmitter-receiver pairs which are randomly distributed over a 500m × 500m square area. There are three MPs which are located about 1000 meters away from the cognitive radio nodes. We assume that the moving of cognitive radio nodes is slower than the convergence time for the proposed algorithms and the MPs are fixed. Fig. 2 shows the spectrum sharing scenario, where we use dashed lines to connect the transmitting node to its intended receiving node and square markers to denote the MPs.

We then illustrate the convergence properties of the proposed power control algorithm for the proposed spectrum sharing scenario in Fig. 2. For simplification, we define that all cognitive radio users have the same maximum transmit power, i.e. $p_t^{\text{max}}=1W$. The channel gains are simplified as $h_{ij}=d_{ij}^{-4}$ and $g_{ik}=d_{ik}^{-3}$. The background noise is defined as $\sigma^2=10^{-4}$. The interference thresholds at the three MPs are set as $Z_1=0.3\sigma$, $Z_2=0.2\sigma$, and $Z_3=0.1\sigma$. Fig. 3 shows that the power control algorithm converges to a pure strategy Nash equilibrium. Fig. 4 shows that the received interference power 1, 2 and 3 at the three MPs converge below the corresponding interference thresholds. Hence our TPCG algorithm is efficient to realize the maximum throughput and satisfies all the ITLs.

VI. CONCLUSION

In this work, we have investigated the issue about spectrum sharing between a decentralized CR network and a primary system. Based on interference temperature model, we have designed a power control algorithm to implement the spectrum sharing and underlying system coexisting. Especially we focused on the scenario with multiple measurement points. The proposed TPCG algorithm used a tax-based game model to realize the efficient power control in a distributed mode, which resulted in the maximum throughput for the CR network and satisfied all the interference temperature limits as well.

REFERENCES