Infinitesimal Perturbation Analysis Based Optimization for a Manufacturing-Remanufacturing System

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Abstract

In this paper, a manufacturing/ remanufacturing system composed by two parallel machines, a manufacturing buffer, a recovery buffer and a customer who demands a constant quantity of product. To describe the system, a stochastic fluid model is adopted and which take into account returned products and remanufacturing products. The objective of this paper is to evaluate the optimal manufacturing buffer. This optimal level allows minimizing the total cost which is the sum of inventory and lost sales costs. Infinitesimal perturbation analysis is used for optimization of the manufacturing/ remanufacturing system. The trajectories of the buffer level are studied and the infinitesimal perturbation analysis estimates are evaluated. These estimates are shown to be unbiased and then they are implemented in an optimization algorithm which determines the optimal buffers levels in the presence of returned products and remanufacturing products.

1. Introduction

In recent years, research on supply chain management has been paying attention on the recovery processes of end of life products for recycling or remanufacturing (Rubio and Corominas [1]). However, a new research area called has appeared by focusing on the management of products once they are no longer desired or can no longer be used by the consumers. Indeed, the reverse logistics has become a matter of strategic importance and which is supposed as an element that companies must consider in decision-making processes concerning the design and development of their supply chains (Flapper et al [2]). However, in the literature we find many of the studies published on reverse logistics and which focus on aspects of production planning and inventory management. Van Der Laan et al [3] considered a production planning and inventory control in systems where manufacturing and remanufacturing operations occur simultaneously. The authors presented a methodology to analyze a push control strategy (in which all returned products are remanufactured as early as possible) and a pull control strategy (in which all returned products are remanufactured as late as is convenient). Indeed, the authors compared the traditional systems without remanufacturing to push and to pull controlled systems with remanufacturing, then they concluded that the efficient planning and control in these systems tends to be more complex than in traditional systems without remanufacturing. Zhou et al [4] studied a hybrid system with both manufacturing and remanufacturing. The authors used an inventory control strategy in the manufacturing loop which is an automatic pipeline, inventory and order based production control system and in the remanufacturing loop they employ a Kanban policy to represent a typical pull system and to control the remanufacturing process. Indeed, the authors analyzed the dynamic performance of the system which has implications on total costs in terms of inventory holding, capacity utilization and customer service failures. Klausner and Hendrickson [5] used a continuous flow model for modeling a remanufacturing system and to take into account the returned products.

In this paper a stochastic fluid model is adopted to describe the system and to take into account returned products and remanufacturing products. Indeed, stochastic fluid models (Cassandras et al. [6], [7], Markou and Panayiotou [8]) provide an alternative modeling technique to queuing theory with applications including communication networks and manufacturing systems. Furthermore, stochastic fluid models are simple to study; they make the performance analysis efficient without the need to track part by part and hence allowing focusing on important events such as machine failures and buffer full/empty. Another reason of the choice of this model, that stochastic fluid model allows us to use an interesting method for optimizing our system; this method called infinitesimal perturbation analysis (IPA).

The goal of this paper is to develop an Infinitesimal Perturbation Analysis (IPA) for optimization of the manufacturing/ remanufacturing system. Infinitesimal perturbation analysis, (Ward et al. [9], Yao and Cassandras [10], Cassandras et al. [11]) is an approach for sensitivity analysis. Indeed, IPA is a technique
which allows obtaining sample path derivatives of a random variable with respect to some parameters of interest. The most important advantage of IPA method is that the simulation based on IPA allows reducing the simulation time comparing to a classical simulation method. This advantage is explained by the fact that the optimization algorithm based on IPA computes at every step the gradient estimates which corresponds to the new value of a parameter of interest (for example: buffer level). The IPA technique was applied initially to queuing networks (Wardi et al. [12], Egerstedt and Wardi [13], Manitz [14]). Ho and Cao [15] derived sensitivity information of the throughput of the system with respect to various parameters. This information can then be used for the optimization of queuing networks. However, the queuing networks become increasingly difficult to handle for actual systems, especially for telecommunication and computer networks with enormous traffic volumes. Subsequently IPA has been applied to stochastic flow models, where much of the action is today (Wardi and Melamed [16], Tan and Gershwin [17]). Panayiotou and Cassandras [18] determined the optimal capacities (or hedging points) of the finished goods and work-in-process buffers to minimize a cost function. Using a stochastic fluid model, they estimated the gradient of the cost function with respect to these hedging points for a two stages and single product. For determining the optimal buffer levels Turki et al. [19] adopted a stochastic fluid model to a manufacturing system composed by two machines in serial configuration, two intermediate buffers and two buffers of finished products, the authors applied the IPA method on the buffer levels and determined the gradient estimates. The authors proved that these estimates are unbiased. Indeed, the unbiasedness is the principal condition for making the application of IPA useful in practice, since it enables the use of the sample IPA derivative in control and optimization methods that employ stochastic gradient-based techniques. Then, these estimates could be used in stochastic approximation algorithms. In this paper, the IPA estimates are determined and then used in an optimization algorithm for determining the optimal buffer level.

However, the main contribution of this paper is to apply the IPA method on the stochastic fluid model which describe a manufacturing/ remanufacturing system and take into account of the returned products, then we derive gradient estimators and we show them their unbiasedness in order to use them in optimization algorithm for determining the optimal buffer level.

The paper is organized as follows. The stochastic fluid model with the problem formulation is presented in section 2. The IPA approach is presented in section 3. In section 4, The IPA estimates are determined, the unbiasedness is proved and numerical results are presented. Finally, the last section concludes the paper and gives some perspectives to our work.

2. Model and explanations

The studied manufacturing/ remanufacturing system consists of two parallel machines which are subject to random failures and repairs denoted $M_1$ and $M_2$ for manufacturing and remanufacturing, respectively. Both machines are producing the same type of product. We take into account in our system the production activity in forward direction and reverse logistics (i.e. activity of the remanufacturing of the used products). We consider customers who demand a quantity of products per unit time denoted $d$ and which is supposed known and constant. This demand is satisfied from a manufacturing buffer $B_1$ which is filled up by the machines $M_1$ and $M_2$. Another buffer $B_2$ is available for the stock keeping of the returned products ahead of the remanufacturing process. These returned products will be then remanufactured by the machine $M_2$ and then stored in the buffer $B_2$ with the manufactured products. We denote $R$ the return rate of the products (i.e. the number of the returned products per unit time)

![Figure 1. Manufacturing-remanufacturing system.](image)

We assume that the machine $M_1$ is never starved. The machine $M_1$ is either up or down. The state of the machine at time $t$, denoted $\alpha(t)$, is given by:

$$\alpha(t) = \begin{cases} 1 & \text{machine } M_1 \text{ is up} \\ 0 & \text{machine } M_1 \text{ is down} \end{cases}$$ (1)

The state of the machine $M_2$ at time $t$, denoted $\beta(t)$, is given by:

$$\beta(t) = \begin{cases} 1 & \text{machine } M_2 \text{ is up} \\ 0 & \text{machine } M_2 \text{ is down} \end{cases}$$ (2)

When the machine is up, the production rate of $M$, denoted by $u_i(t)$, could take a value between 0 and its maximum rate $U_i$, i.e., $0 \leq u_i(t) \leq U_i$. When the machine is down $u_i(t)$=0. The times to failure and times to repair are exponentially distributed with rate $\lambda_i$ and $\mu_i$. For the machine $M_2$ we have the same state,
with \( u_2(t) \) is the production rate, \( U_2 \) is the maximal production rate, \( x_2 \) and \( y_2 \) are The times to failure and times to repair of the machine \( M_2 \). The failure/repair process is an independent random process. It does not depend on the system parameters.

Furthermore, we assume in this paper that:

- The maximal production rate of the machine \( M_1 \) permits to satisfy the demand, i.e. \( U_1 > d \). This assumption allows avoiding having always the manufacturing buffer empty.
- If the demand is unsatisfied, the demand is lost with a corresponding cost (lost sales cost).
- The maximal production rate of the machine \( M_2 \) is upper to the return rate, i.e. \( U_2 > R \). This assumption allows avoiding having always the recovery buffer very full.
- The remanufactured products can be considered meeting the same quality level as the new products so that both type of products can be distributed like new.

We denote by \( x_1(t) \) and \( x_2(t) \) the buffer levels respectively for \( B_1 \) and \( B_2 \).

We consider the case of infinite capacity for \( B_1 \) and \( B_2 \), where the dynamics of the system are given by the following equations:

\[
\frac{dx_1(t)}{dt} = u_1(t) + u_2(t) - d \tag{3}
\]

\[
\frac{dx_2(t)}{dt} = R - u_2(t) \tag{4}
\]

Remark 1: In the first step of the approach for determining the optimal buffer level, we suppose that the capacity of buffer \( B_1 \) is infinite, and then the optimal buffer level will be determined according to the hedging point.

The chosen control policy is a hedging point policy (Mokou and Porter [20]) which ensures that the part does not exceed a given number of products, denoted by \( h \). Furthermore, the hedging point has been proved to be the optimal policy for a one-product manufacturing system (Akella and Kummar [21]).

Indeed, according to this policy, the production rate is equal to the demand rate when the buffer \( x_1(t) \) become full (i.e. \( x_1(t) = h \)). In this case we assume in our system that the machines \( M_1 \) and \( M_2 \) share the production. In other word: each machine produces half of the production (i.e. \( u_1(t) = d/2 \) and \( u_2(t) = d/2 \)). Therefore the control policy is defined as follows:

For the machine \( M_1 \):

\[
u_1(t) = \begin{cases} U_1 & \text{if } \alpha(t) = 1 \text{ and } x_1(t) < h \\ d/2 & \text{if } \alpha(t) = 1 \text{ and } x_1(t) = h \\ 0 & \text{if } \alpha(t) = 0 \text{ or } x_1(t) > h \end{cases} \tag{5}
\]

For the machine \( M_2 \):

When the buffer \( B_2 \) is empty, the machine \( M_2 \) is supplied directly by the returned products.

\[
u_2(t) = \begin{cases} U_2 & \text{if } \beta(t) = 1, x_2(t) > 0 \text{ and } x_1(t) < h \\ d/2 & \text{if } \beta(t) = 1, x_2(t) > 0 \text{ and } x_1(t) = h \\ R & \text{if } \beta(t) = 1, x_2(t) = 0 \text{ and } x_1(t) \leq h \\ 0 & \text{if } \beta(t) = 0 \text{ or } x_1(t) > h \end{cases} \tag{6}
\]

The number of unsatisfied demands (lost) per unit time is denoted by \( D^o(t) \), and depends on the customer demand and the buffer level. Indeed, when the customer orders a demand \( d \) and the buffer is empty, the demand will not be satisfied at all and will be lost. Therefore, \( D^o(t) \) is null if the buffer level is positive and is equal to the demand if the buffer is empty. The number of unsatisfied demands per unit time is defined as follows:

\[
D^o(t) = \begin{cases} 0 & \text{if } x_1(t) > 0 \\ d & \text{if } x_1(t) = 0 \end{cases} \tag{7}
\]

The number of unsatisfied demands at time \( t \) denoted by \( L(t) \) is given by:

\[
\frac{dL(t)}{dt} = D^o(t) = d \quad \text{if } x_1(t) = 0 \\
L(t) = 0 \quad \text{if } x_1(t) > 0 \tag{8}
\]

Remark 2: we defined \( L(t) \), because we will need it for writing the cost function.

For the return rate, an assumption is made that it is proportional to the customer demand rate. We denote by \( r (0 < r < 1) \) the percentage of sales (demand \( d \)) which are returned for remanufacturing. Indeed, the customer returns rate may be as high as 15% of sales in the coming years, and in sectors such as catalogue sales and e-commerce it could reach as much as 35% (Rubio and Corominas [1]). Then we can write:

\[
R = r \cdot d \tag{9}
\]
The possible events at every time $t$ are: machine $M_1$ and $M_2$ failure i.e. $u_1(t)=0$ and $u_2(t)=0$ (PM1 and PM2), machine repair (RM1 and RM2), buffer full i.e. $x_i(t) = h$, (SS) and buffer empty i.e. $x_i(t) = 0$ (SV).

The cost function $C(t)$, at time $t$, which is composed by the inventory cost and the lost sale cost, is given by:

$$C(t) = cs_1x_1(t) + cs_2x_2(t) + cs^-L(t)$$

(10)

Where:

- $cs_1$ and $cs_2$: are the units inventory cost respectively for $B_1$ and $B_2$;
- $cs^-$: unit lost sale cost.

The expected average cost, denoted by $J(h)$ depending on $h$ is given by:

$$J(h) = \lim_{T \to \infty} \frac{1}{T} E \left[ \int_{0}^{T} C(t) dt \right]$$

(11)

$\forall t \in [0,T]$ with $T$ the total simulation time.

In the following section, we will investigate the sample path trajectories of $x_i(t)$. This study will allow us to find the IPA estimates and prove that these estimates are unbiased.

3. IPA approach

In this section, we apply the IPA method to the stochastic fluid model. The IPA is an approach for computing a sample path derivative with respect to an input parameter. Indeed, this method is intended to estimate gradients of performances metric with respect to some parameters of interest, for example, the optimal buffer level $h$ of $B_1$. It consists on observing and analyzing two sample paths, one is the nominal sample path (for example $x_i(t)$), and the other is the perturbed sample path (for example $x'_i(t)$) (see Figure. 2). We assume that the optimal buffer level is increased by a perturbation, denoted by $\eta$. In this work, we consider $\eta > 0$ and we study the resulting changes in the cost function using geometric arguments (similar results could be easily obtained for $\eta < 0$). The optimal buffer level of the perturbed sample path ($x'_i(t)$) is $h + \eta$.

The following assumptions are considered:

- The perturbation of $h$ is $\eta$.
- The maximal production and unit costs are the same for both sample paths.

- The same distribution of random variables (time to failure, time to repair) is used for both sample paths.

In the following – (Figure. 2), we give an example of simple paths for $x_i(t)$ and $x'_i(t)$.

The following notations are used:

- $x'_i(t)$: The manufacturing buffer ($B_1$) level for the perturbed path;
- $x'_2(t)$: The recovery buffer ($B_2$) level for the perturbed path;
- $u'_1(t)$: The production rate of the machine $M_1$ at time $t$ for the perturbed path;
- $u'_2(t)$: The production rate of the machine $M_2$ at time $t$ for the perturbed path;
- $D'^0(t)$: The number of unsatisfied demands per unit time for the perturbed trajectory.

- $t_{ij}^p$: $j^{th}$ instant for which $B_i$ on the nominal path becomes full and causing a lag between the perturbed path and the nominal path;
- $t_{ij}^n$: $j^{th}$ instant for which $B_i$ on the disturbed path becomes full.
- $t_{ij}^e$: $j^{th}$ instant for which $B_i$ on the perturbed path becomes empty and for which the perturbed and the nominal path merge;
- $t_{ij}^l$: is the last instant between $t_{ij}^e$ and $t_{ij}^n$ for which the manufacturing buffer becomes empty on the nominal path;
- $t_{ij}^m$: $j^{th}$ instant which causes a lag between the perturbed path and the nominal path on the recovery buffer level $B_2$;
- $t_{ij}^r$: $j^{th}$ instant for which $B_2$ on the perturbed path becomes empty and for which the perturbed and the nominal path merge.

![Figure 2. Manufacturing buffer level for the perturbed and nominal paths.](image-url)
We denote by $T$ the total simulation time, we consider the trajectories of $x_j(t)$ and $x_j^p(t)$ over $t \in [0,T]$. However, the interval $[0,T]$ is divided on two alternating periods: the first when $t \in [t_{j},t_{j+1}]$ and the other when $t \in [t_{j},t_{j+1}]$. Then we can analyze the perturbed and the nominal paths using these intervals. Indeed, the study of the trajectories is generalized in the the following lemmas and theorems.

**Theorem 1** shows that the manufacturing buffer level of the perturbed path is equal to nominal path plus the perturbation when $t \in [t_{j},t_{j+1}]$ and the nominal path and of the perturbed path are equal when $t \in [t_{j},t_{j+1}]$.

**Theorem 1:** If $\eta > 0$, $x_j^p(0) = x_j(0)$ we have:

- If $t \in [t_{j},t_{j+1}]$, then $x_j^p(t) = x_j(t) + \eta$.

- If $t \in [t_{j},t_{j+1}]$, then $x_j^p(t) = x_j(t)$.

The proof of this theorem is similar to the proof of *Theorem 1* and *Theorem 2* in [22].

**Theorem 2** shows that the recovery buffer level of the perturbed path is equal to nominal path minus the perturbation when $t \in [t_{j},t_{j+1}]$ and the nominal path and of the perturbed path are equal when $t \in [t_{j},t_{j+1}]$.

**Theorem 2:** If $\eta > 0$, $x_j^p(0) = x_j(0)$ we have:

- $t \in [t_{j},t_{j+1}]$, then $x_j^p(t) = x_j(t) - \eta$.

- $t \in [t_{j},t_{j+1}]$, then $x_j^p(t) = x_j(t)$.

**Proof:** The perturbation on the buffer level $B_j$ is caused by the perturbation of the buffer level $B_i$ when $t \in [t_{j},t_{j+1}]$ and if the machine $M_j$ is up (i.e. $\beta(t) = 1$). We assume that we have $t' \in [t_{j},t_{j+1}]$ with $t' < t$ and at time $t$ the machine becomes up. We assume also that we have $x_j^p(t') = x_j(t')$ (just for assuming that we have not a lag between the perturbed and nominal paths at time $t'$) and $x_j(t) > 0$.

Then when $t \in [t_{j},t_{j+1}]$, we have $x_j^p(t) = x_j(t) - \eta$.

Q.E.D.

**Theorem 3** shows that the number of the unsatisfied demand on the perturbed path is equal to nominal path minus the perturbation when $t \in [t_{j},t_{j+1}]$ and the nominal path and of the perturbed path are equal when $t \in [t_{j},t_{j+1}]$.

**Theorem 3:** If $\eta > 0$, $x_j^p(0) = x_j(0)$ we have:

- $t \in [t_{j},t_{j+1}]$, then $L^p(t) = L(t) - \eta$.

- $t \in [t_{j},t_{j+1}]$, then $L^p(t) = L(t)$.

**Proof:** When $t \in [0,T] \backslash [t_{j},t_{j+1}]$ the demands are satisfied for the two types of trajectories, then we have $L^p(t) = L(t)$ when $t \in [t_{j},t_{j+1}]$ we have, the number of satisfied products is equal to the number of products in the buffer and then we have $x_j^p(t) - x_j(t) = \eta$. Contrariwise, the difference between the number of unsatisfied products for the perturbed trajectory and that for the nominal trajectory is equal to the opposite for the case of satisfied products, thus $L^p(t) - L(t) = -\eta$.

Q.E.D.
In what follows, we will determine the IPA estimates and will prove their unbiasedness.

4. IPA for optimization

In this section, we will use the results of the trajectories study for determining the IPA estimates; these estimates will be implemented in an optimization algorithm, which allows us to determine the value of $h$. Indeed, the values of IPA estimates allow orienting quickly the algorithm to the optimal value of $h$. Therefore, the advantage of our optimization algorithm based on IPA compared with an exhaustive optimization algorithm is that it takes smaller computational time (simulation time).

4.1. Determination of the IPA estimates

The average cost of the nominal path is given by:

$$J(h) = \lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T C(t) \, dt \right]$$

(12)

The average cost of the perturbed path is given by:

$$J^\eta(h + \eta) = \lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T C^\eta(t) \, dt \right]$$

With

$$C^\eta(t) = cs_1 x_1^\eta(t) + cs_2 x_2^\eta(t) + cs^- L^\eta(t)$$

(14)

The sampled estimation for the expected average cost of the nominal path is given by:

$$J^\eta_a(h) = \frac{1}{T} \mathbb{E} \left[ \int_0^T C(t) \, dt \right]$$

(15)

The sampled estimation for the expected average cost of the perturbed path is given by:

$$J^\eta_a(h + \eta) = \frac{1}{T} \mathbb{E} \left[ \int_0^T C^\eta(t) \, dt \right]$$

(16)

We determine the IPA estimates of the cost function by computing the difference between the perturbed average cost and the nominal average cost.

The difference between the perturbed expected average cost and the nominal expected average cost is given by:

$$J^\eta_a(h + \eta) - J^\eta_a(h) = \frac{1}{T} \mathbb{E} \left[ \int_0^T (C^\eta(t) - C(t)) \, dt \right]$$

$$J^\eta_a(h) - J^\eta_a(h) = \frac{1}{T} \mathbb{E} \left[ \int_0^T \left( cs_1 (x_1^\eta(t) - x_1(t)) + cs_2 (x_2^\eta(t) - x_2(t)) \right) \, dt \right] + \mathbb{E} \left[ \int_0^T cs^- (L^\eta(t) - L(t)) \, dt \right]$$

We assume that in the interval $[0,T]$ we have $m$ intervals $[t^\eta_{ij}, t^\eta_{ij+1}]$, $n$ intervals $[t^\delta_{ij}, t^\delta_{ij+1}]$, $p$ intervals $t \in [t^\eta_{pi}, t^\eta_{pi+1}]$ and $q$ intervals $t \in [t^\eta_{qi}, t^\eta_{qi+1}]$ then we have:

- For the trajectories of $x_1(t)$ and $x_2^\eta(t)$ the interval $[0,T]$ is divided on two sums of periods: $T_1^\eta$ is the sum of periods when $t \in [t^\eta_{ij}, t^\eta_{ij+1}]$ (when $x_1^\eta(t) = x_1(t) + \eta$) i.e. $T_1^\eta = \sum_{j=1}^{j=m} (t^\eta_{ij+1} - t^\eta_{ij})$ and $T_1^\eta$ is the sum of periods when $t \in [t^\delta_{ij}, t^\delta_{ij+1}]$ (when $x_1^\eta(t) = x_1(t)$) i.e. $T_1^\delta = \sum_{j=1}^{j=n} (t^\delta_{ij+1} - t^\delta_{ij})$. Thus, according to the theorems 1 we have:

$$\int_0^T cs_1 (x_1^\eta(t) - x_1(t)) \, dt = cs_1 T_1^\eta \eta$$

- For the trajectories of $x_2(t)$ and $x_2^\eta(t)$ the interval $[0,T]$ is divided on two sums of periods: $T_2^\eta$ is the sum of periods when $t \in [t^\eta_{pi}, t^\eta_{pi+1}]$ and $T_2^\eta$ is the sum of periods when $t \in [t^\eta_{qi}, t^\eta_{qi+1}]$.

Thus, according to the theorems 2 we have:

$$\int_0^T cs_2^\eta (x_2^\eta(t) - x_2(t)) \, dt = -(cs_2^\eta T_2^\eta \eta)$$

- For the trajectories of $L(t)$ and $L^\eta(t)$ the interval $[0,T]$ is divided on two sums of periods: $T_3^\eta$ is the sum of periods when $t \in [t^\eta_{ij}, t^\eta_{ij+1}]$ and $T_3^\eta$ is the sum of periods when $t \in [0,T]$.

Thus, according to the theorems 3 we have:
Then we have
\[
J_\alpha(h + \eta) - J_\alpha(h) = \frac{1}{T} \mathbb{E} \left[ cs_1, T_1^\eta - cs_2, T_2^\eta - cs_3, T_3^\eta \right] \eta
\]

Then, the gradient estimates of the cost function are given by:
\[
\frac{\partial J_\alpha(h)}{\partial h} = \frac{1}{T} \mathbb{E} \left[ cs_1, T_1^\eta - cs_2, T_2^\eta - cs_3, T_3^\eta \right]
\]

For making these estimates useful in practice, the unbiasedness should be proved.

**Theorem 4**: The gradient estimates of the average cost are unbiased.

The proof of this theorem is similar to the proof of theorem 12 in [22].

Remark 3: The fact that the estimates are statistically unbiased, mean that the estimated value equal to the real value.

In the followings, numerical results are presented to show the impact of the percentage of the returned products \( r \) on the value of the optimal buffer level \( h \).

### 4.2. Numerical results

In this part we use the IPA estimates in an optimization algorithm, which allows us to determine the values of \( h \).

The following parameters are used for the simulation:
- \( U_1 = 6 \) products /time unit;
- \( U_2 = 3 \) products /time unit;
- \( d=4 \) products /time unit;
- The total simulation time is equal to \( T=10E+07 \) time units;
- The times to failure or repair are given by exponential distribution, the mean time between failures MTBF is equal to 2.7 and the mean time to repair MTTR is equal to 1;
- The unit inventory cost \( cs_1 \) is equal to 2 monetary units;
- The unit lost sales cost \( cs_2 \) is equal to 50 monetary units;
- The unit inventory cost \( cs_2 \) is equal to 2 monetary units.

Such as the cost function depends of return rate of the products \( R = r \cdot d \), the value of \( h \) which minimize this cost function will certainly also depend on the percentage of the returned products \( r \).

Simulation results are presented in the following table to show the impact of the percentage of the returned products \( r \) on the value of the optimal buffer level \( h \).

<table>
<thead>
<tr>
<th>( r )</th>
<th>( R )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.4</td>
<td>11.281</td>
</tr>
<tr>
<td>20%</td>
<td>0.8</td>
<td>9.882</td>
</tr>
<tr>
<td>30%</td>
<td>1.2</td>
<td>6.203</td>
</tr>
<tr>
<td>40%</td>
<td>1.6</td>
<td>4.494</td>
</tr>
<tr>
<td>50%</td>
<td>2</td>
<td>3.998</td>
</tr>
<tr>
<td>60%</td>
<td>2.4</td>
<td>2.791</td>
</tr>
<tr>
<td>70%</td>
<td>2.8</td>
<td>2.013</td>
</tr>
</tbody>
</table>

**Table. Impact of the percentage of the returned products on the value of \( h \).**

We see that more the percentage of the returned products increases; more the value of \( h \) decreases. Indeed, when the number of returned products increases, the buffer \( B_2 \) fills up and then the machine \( M_2 \) can fill more the buffer \( B_1 \) (i.e. \( u_2(t) > R \) ). However, if the machine \( M_2 \) fills more the buffer \( B_1 \) the customers demand is more satisfied. Therefore, the unsatisfied demand decreases and normally the lost sales cost decreases, thus the optimal buffer level which minimizes the total cost decreases.

### 5. Conclusion

In this paper, a manufacturing/remanufacturing system composed by two parallel machines, a manufacturing buffer, a recovery buffer and a customer who demands a constant quantity of product is considered. A stochastic fluid model is adopted to describe the system and to take into account returned products and remanufacturing products. The times to failure and times to repair are random variables with exponential distribution. The buffer levels trajectories is studied and analyzed. The infinitesimal perturbation analysis estimates are determined and shown to be unbiased. These estimates are then implemented in an optimization algorithm for determining the optimal levels value of the manufacturing buffer. The impact of the percentage of the returned products on the value of the optimal buffer level \( h \) is studied. Indeed, more the percentage of the returned products increases; more the value of \( h \) decreases.
For future research, we will take into account the delivery time between the manufacturing and the customer. Also, we will consider a more complex model with random customer demand and random delivery time.

References


Research, 2013.