ROBOT CALIBRATION USING LEAST-SQUARES AND POLAR-DECOMPOSITION FILTERING

Gregory Ioannides[1], Jorge Angeles[1], Randall Flanagan[2]

and David Ostry[2]

McGill University, Montréal, Canada.

Abstract

This paper reports the experimental results of a novel method to calibrate geometric errors of multi-axis robotic manipulators. The method proposed by the authors is based on a least-square estimation of the rotation matrix of a rigid body in three-dimensional Cartesian space. The error is filtered by imposing the orthogonality constraint on the rotation matrix, using the polar-decomposition theorem. The axis of rotation of the rigid body, then, is computed from the linear invariants of the rotation matrix. Finally, the wrist of the Yaskawa Motoman Robot was calibrated. The measurements of the Cartesian coordinates of points were performed using a computer vision system and LED markers on a rigid body grasped by the end-effector.

1. INTRODUCTION

It is well known that the current accuracy of robots is outside acceptable tolerance for certain manufacturing applications such as high precision assembly. Different authors have tried to give an estimate of the error involved [10],[12],[15] the need for calibrating robotic manipulators and compensating for the errors thus becoming evident. There are two kinds of errors, namely, geometric and non-geometric. The latter are caused by backlash, structure flexibility, etc. The former exist due to imprecise manufacturing of the

[1]Dept. of Mech. Eng., McGill Univ., 3480 University St., Montréal,Québec, H3A 2A7, Canada.
[2]Dept. of Psych., McGill Univ.,1205 Penfield Ave., Montréal, Québec, H3A 1B1, Canada.

robot links and joints. However, because the lengths can be measured easily and accurately, it is assumed that the error lies mostly in the joints and their alignment, which, in turn, causes kinematic errors.

Even though in the teach-mode programming of robotic trajectories the accurate knowledge of the parameters defining the robot architecture is not required, the off-line programming of those trajectories definitely requires the accurate knowledge of these parameters. The measurement of the aforementioned parameters is referred to as *robot geometric calibration* or, simply, *robot calibration*. Due to the unavoidable manufacturing errors mentioned above, the nominal values of the parameters defining the robot kinematic structure, e.g., the Hartenberg-Denavit parameters of the robot at hand, will be in error. Thus, axes which are nominally parallel, perpendicular or intersecting, will fail to be so in reality. This means that the positioning accuracy will be affected, unless these errors are compensated. Work in connection with robot calibration has been extensive, as reported in the literature [7],[12],[15],[9],[1]. However, as pointed out by Hayati in [7], the presence of two neighboring axes which are ideally or nearly parallel introduces serious numerical instabilities. In this paper, a method is introduced for robot calibration based on a least-square estimation of the axis of rotation of a moving rigid body, that is independent of the relative orientation of consecutive axes.

The paper is organized as follows: We begin with a short account of the problem of estimating the orientation of rigid bodies from the measurements of the Cartesian coordinates of a set of points in two finitely-separated configurations of the body. Next, we refer briefly to the least-square/polar-decomposition technique adopted for data processing, which allows us to obtain acceptable estimates of the orientation matrix. We end up with a description of the experimental procedure and discuss the results obtained.

2. THEORETICAL BACKGROUND

The determination of the orientation matrix of a rigid body from perfect measurements of the Cartesian coordinates of any three non-collinear points of the body has been the subject of extensive research [11], [3]. However, actual coordinate measurements are bound to contain noise, which can be filtered if redundant measurements, i.e., measurements of the coordinates of more than three non-collinear points, are taken. From these, an estimate of the attitude of the body, with respect to a reference configuration, is made by resorting to a simple least-square fit. However, since no constraints are imposed on the orthogonality of the estimated orientation matrix, the estimate will most likely fail to be orthogonal. A method is proposed by Higham in [4] for computing the most likely orthogonal matrix from the non-orthogonal estimate based on the *Polar-Decomposition Theorem*. Once an orthogonal estimate of the matrix defining the attitude of the body is available, the direction of the axis of the associated rotation, from the reference configuration, is determined using the concept of *vector* of a 3×3 matrix [2]. The position vector of a point of the axis of rotation is then computed using the method proposed Angeles in [3], thereby completing the calibration of one joint. The same procedure is repeated for all joints.

Studies have shown that these coordinates can be used to determine the pose—position and orientation—of a rigid body based on only three non-collinear points [3]. The work reported here is based on redundant measurements, i.e., the coordinates of $m > 3$ points are measured. Theoretically, by equating the coordinates of those points in a displaced configuration with a linear transformation of the coordinates of the same points in a reference configuration, an overdetermined system of linear algebraic equations is formed, whose least-square approximation produces the orthogonal rotation matrix involved in the coordinate transformation. However, due to unavoidable measurement and roundoff errors, rather than working with the position vectors themselves, we define a set of m vectors stemming from a common point, the centroid of the set, and pointing to each point of the set. We call the ith vector of this set $\mathbf{r}_i$, for $i = 1, \ldots, m$. Moreover the corresponding set defines, in a finitely separated configuration, a set of corresponding vectors $\mathbf{r}'_i$, for $i = 1, \ldots, m$.

Now, let the rotation carrying the rigid body from its reference to its current configuration be represented by matrix $\mathbf{Q}$. Hence, a set of m 3D vector equations linear in $\mathbf{Q}$ can be written that relate each vector $\mathbf{r}'_i$ with its counterpart $\mathbf{r}_i$. Now the entries of $\mathbf{Q}$,

q_{ij}, are to be determined. In this way, a system of m 3D vector equations that are linear in the nine unknowns q_{ij} is derived. These equations can be rewritten in a standard form by grouping the nine unknowns into the 9-dimensional vector $\mathbf{q}$, matrix $\mathbf{R}$ and vector $\mathbf{r}$ which are defined correspondingly. Clearly, $\mathbf{R}$ and $\mathbf{r}$ are known, for they are determined entirely by the components of the sets $\{\mathbf{r}_j\}_1^m$ and $\{\mathbf{r}'_j\}_1^m$, respectively. One then has:

$$\mathbf{R}\mathbf{q} = \mathbf{r} \tag{1}$$

where $\mathbf{R}$ is a $3m \times 9$ matrix and $\mathbf{r}$ is a $3m$-dimensional vector. Since we assumed redundant measurements, $m > 3$ and hence, $3m > 9$, which renders the system of eq.(1) overdetermined. Now, let matrix $\mathbf{Q}_k$ represent the rotation carrying the rigid body from its reference to its kth current configuration, for $k = 1, \ldots, n$. This matrix contains nine unknowns and is now rewritten as the 9-dimensional vector $\mathbf{q}_k$. Since coordinate measurements of m points are made, a set of $3m$-dimensional vectors $\mathbf{v}'_k$, for $k = 1, \ldots, n$, is then formed as follows:

$$\mathbf{v}'_k \equiv [\,\mathbf{r}'^T_{1k} \quad \mathbf{r}'^T_{2k} \quad \ldots \quad \mathbf{r}'^T_{mk}\,]^T \tag{2}$$

where $\mathbf{r}'_{jk}$, for $j = 1, \ldots, m$ and $k = 1, \ldots, n$, denote vectors $\mathbf{r}'_j$ associated with point P'_j for the kth current configuration. Furthermore, a set of n $3m \times 9$ matrices $\mathbf{R}_k$, for $k = 1, \ldots, n$, is defined for every measurement, eq.(1) thus being written n times as:

$$\mathbf{R}_k\mathbf{q}_k = \mathbf{r}_k, \quad k = 1, \ldots, n \tag{3}$$

In this way, the matrix and the right-hand side of the algebraic system of eq.(3) are now available. The least-square approximation of this system, $\tilde{\mathbf{q}}_k$, can be expressed symbolically via the generalized inverse of $\mathbf{R}_k$, $\mathbf{R}_k{}^I$, as follows:

$$\tilde{\mathbf{q}}_k = \mathbf{R}_k{}^I\mathbf{v}'_k, \quad \mathbf{R}_k \equiv (\mathbf{R}_k{}^T\mathbf{R}_k)^{-1}\mathbf{R}_k{}^T \tag{4}$$

The least-square estimation of vector $\mathbf{q}_k$, $\tilde{\mathbf{q}}_k$, associated with that system, is most efficiently computed using Householder reflections [5]. However, the foregoing least-square approximation provides only a first estimate, $\tilde{\mathbf{Q}}_k$, of the orthogonal matrix $\mathbf{Q}_k$. Since the measurements are noisy, the estimated rotation matrix does include some error. This error is filtered by imposing the orthogonality condition on the rotation matrix, which is done with the aid of the Polar-Decomposition Theorem (PDT) [6]. According to the PDT, a $n \times n$ matrix can be decomposed in the form :

$$\bar{\mathbf{Q}} = \mathbf{O}\mathbf{U} \tag{5}$$

where $\mathbf{O}$ and $\mathbf{U}$ are both $n \times n$ matrices, the former being orthogonal, the latter, at least positive semidefinite. The polar decomposition theorem states that, if the original matrix is non singular, then matrix $\mathbf{U}$ is necessarily positive definite and both $\mathbf{O}$ and $\mathbf{U}$ are unique. Otherwise, $\mathbf{U}$ is only positive semidefinite and not unique. In our case, if neither measurement nor roundoff noise were present, $\mathbf{O}$ would be $\mathbf{Q}$ and $\mathbf{U}$ would be the identity matrix, $\mathbf{1}$. Since we are acknowledging the presence of errors, $\mathbf{O}$ is not $\mathbf{Q}$, but merely an estimate of it, denoted here by $\hat{\mathbf{Q}}$, and $\mathbf{U}$ is not the identity, but $\mathbf{1} + \mathbf{E}$, where $\mathbf{E}$ is the estimation error. Note that $\mathbf{E}$ provides us with an estimate of the error involved, and hence, allows us to decide whether the estimate is acceptable or not.

The algorithm proposed by Higham (1986), aimed at computing the factors of the said decomposition was utilized on all $\bar{\mathbf{Q}}_k$. Hence, the error was filtered out from the initial estimation of each rotation matrix and the filtered estimates $\hat{\mathbf{Q}}_k$ were obtained. In this experiment, the components of the error matrix $\mathbf{E}$ had no more than 1% error, which was considered acceptable. Subsequently, all information about the axes was derived from the $\hat{\mathbf{Q}}_k$ matrices, as described next.

2. COMPUTATION OF AXIS OF ROTATION FROM ORTHOGONAL MATRIX COMPONENTS

Once the estimate $\hat{\mathbf{Q}}_k$ of the orientation matrix is available, the direction of the axis of rotation is determined from the *vector* [2] of that estimate, denoted by $\text{vect}(\hat{\mathbf{Q}}_k)$. In fact, if $\hat{\mathbf{e}}_k$ denotes the estimate of the unit vector parallel to the axis of rotation and $\hat{\phi}_k$

the estimate of the associated angle of rotation, then,

$$\text{vect}(\hat{\mathbf{Q}}_k) = \hat{\mathbf{e}}_k \sin\hat{\phi}_k \tag{6}$$

where the angle of rotation is estimated from eq.(6) and the following:

$$\text{tr}(\hat{\mathbf{Q}}_k) = 1 + 2\cos\hat{\phi}_k \tag{7}$$

Note that each measurement provides an estimate $\hat{\mathbf{e}}_k$ of the same unit vector $\mathbf{e}$ parallel to the axis under calibration. Thus, the best estimate of this vector is chosen as the normalized mean of the said estimates, i.e., for the n measurements associated with the same axis,

$$\hat{\mathbf{e}} = \frac{\mathbf{f}}{\|\mathbf{f}\|}, \quad \mathbf{f} \equiv \frac{1}{n}\sum_{1}^{n}\hat{\mathbf{e}}_k \tag{8}$$

in which $\|\cdot\|$ denotes the Euclidean norm of its vector argument. In order to completely determine the axis under calibration, all that remains is to estimate the location of one of its points. This can be, for example, the point closest to the origin, of the position vector $\mathbf{s}_0$. This vector is calculated based on the relations derived in [3], which lead to the following overdetermined linear algebraic system:

$$\mathbf{A}\mathbf{s}_0 = \mathbf{b} \tag{9}$$

where $\mathbf{A}$ is a $(3m+1)\times 3$ matrix and $\mathbf{b}$ is a $(3m+1)$-dimensional vector, both of which are given below:

$$\mathbf{A} \equiv \begin{bmatrix} \hat{\mathbf{Q}}_1 - \mathbf{1} \\ \hat{\mathbf{Q}}_2 - \mathbf{1} \\ \vdots \\ \hat{\mathbf{Q}}_n - \mathbf{1} \\ \hat{\mathbf{e}}^T \end{bmatrix}, \quad \mathbf{b} \equiv \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_n \\ 0 \end{bmatrix} \tag{10a}$$

with $\mathbf{b}_k$, for $k = 1,\ldots,n$, defined as follows:

$$\mathbf{b}_k \equiv \hat{\mathbf{Q}}_k\mathbf{c} - \mathbf{c}' + \left[(\mathbf{c}-\mathbf{c}')^T\hat{\mathbf{e}}\right]\hat{\mathbf{e}}^T \tag{10b}$$

thereby completely determining the direction and the position of the axis under calibration. This procedure was applied to the calibration of the wrist axes of an industrial robot, as described in the following sections.

3. SOFTWARE TESTING

Before any experiment was performed the method was tested using the software developed for this purpose. The problem was formulated first in a constrained least-squares fashion only to find that the orthogonal rotation matrices that were obtained were unacceptable. The results of the constrained problem were far from those of the unconstrained problem. The conclusion is that the constraints were too strong and caused the solution to zero-in on an orthogonal matrix which was very different from the expected one. Hence, the unconstrained least-squares formulation was utilized.

The algorithm was written in the C programming language and it takes as input the position vectors of a number of points assumed to be on the rigid body and the same vectors after being multiplied by a specified rotation matrix. The objective of the tests was to figure out the amount of error that could be filtered out of the actual and presumably noisy measurements. For that reason, random noise was added to the input data of the program. The results showed that the least-squares computations alone performed on input contaminated with noise up to 10% yielded the required rotation matrix with an accuracy of 10^{-3}. This figure improved considerably when the polar-decomposition algorithm was used and decreased the error to 10^{-8}.

After the tests proved that it was feasible to obtain the desired robot axis with reasonable accuracy, the experiment was set up.

4. EXPERIMENTAL PROCEDURE

The objective of the experimental part of the method was to determine the pose of each link of the robot. This could be accomplished by taking measurements of the coordinates of a certain number of points on each link before and after the manipulator underwent a programmed motion. Knowing these coordinates, the matrix representing the rotation of the specific link could be evaluated. The axis of rotation of the particular link could then be computed from the linear invariants of the rotation matrix. However, because of the architecture of the robot, i.e., the limited space and the curved surfaces, it was not possible to attach a sufficient amount of markers on each link.

In order to facilitate the collection of the data, a bracket, shown in Fig. 1, was manufactured and all the light-emitting diode (LED) markers were attached to the bracket. Consequently the bracket was rigidly attached to the end-effector. This configuration allowed for better measurements since the LEDs were concentrated in one area and were not spread over the length of a link. It also has the advantage that the markers can be positioned in a configuration allowing for maximum robustness in the measurements. Another advantage is that the whole process of taking measurements was sped up by a considerable amount of time since, if the bracket is ready from the onset, all that needs to be done is to connect the bracket to the end-effector.

Even though the markers were not any more on individual links, the calibration was done now by considering motions of one joint at a time, while the others were kept locked. Hence, the locked links were considered as a single rigid body undergoing a rotation about a fixed axis, which was the robot axis to be identified.

4.1 Equipment

After researching the possibilities of equipment to be used for data acquisition, the system that is next described was chosen over laser methods and traveling microscopes because of its simplicity, suitability and availability.

The system that was used is the WATSMART (WATerloo Spatial Motion Analysis and Recording Technique) vision system [13]. It is comprised of two high resolution infrared cameras, one pre-surveyed calibration frame, several (LED) markers, along with the appropriate controllers for the cameras and the marker strobers. Finally, a computer

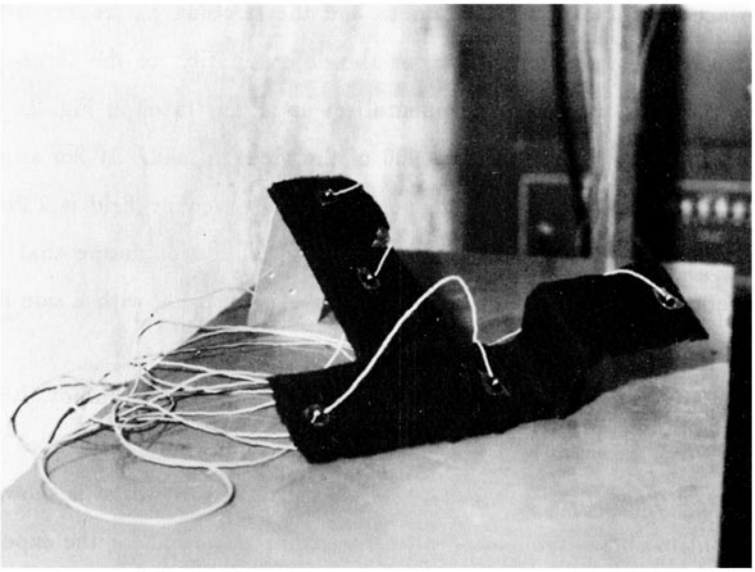

Fig.1: The T-bracket.

Fig.2: Experimental set up.

with an interface board to drive the camera and the strobing controllers is included in the system, as well as special software for the determination of the three-dimensional coordinates of the markers. The experimental set up is illustrated in Fig. 2.

The cameras have a resolution of 1:4000 of the viewing field. At 2m away from the cameras, where the measurements were performed, the viewing field is 1.2m wide and hence the accuracy of the cameras is 0.3mm. The calibration frame that serves as a permanent reference object is in the shape of a cube, see Fig. 3, with a side length of 21 inches. The manufactured accuracy of the frame is 0.1mm.

The robot that was calibrated is the Yaskawa Motoman AID 810 Robot. It is a six-axis manipulator designed for welding applications. The second and third axes are parallel and the wrist is supplied with an offset. A representation of the wrist is shown in Fig. 4. The AID32v controller was used to produce the required motion for the experiment and the programming language was RAIL[3].

4.2 Data Acquisition

The procedure followed to acquire the data is described next. As already mentioned the LED markers were attached to a T-shaped bracket and a part of it was bent so as to produce a three-dimensional body rather than a two-dimensional one—this has the effect of a better depth estimation. It was proved to be important to cover the bracket with a black cloth, made of colortron foam, in order to reduce reflections from the area in the proximity of the LED markers, the reason being that the cameras take the position of the marker to be the centroid of the area with the strongest light emission. The markers are strobed one at a time and hence there is no correspondence problem to solve.

Before the actual monitoring of the three-dimensional structure commenced, calibration of the vision system was needed. This was done by placing the calibration cube in the middle of the range of motion of the robot and viewing it with the cameras. The provided software, knowing the geometry of the calibration cube, calculates the regression

[3] Acronym for Robot Automatix Incorporated Language

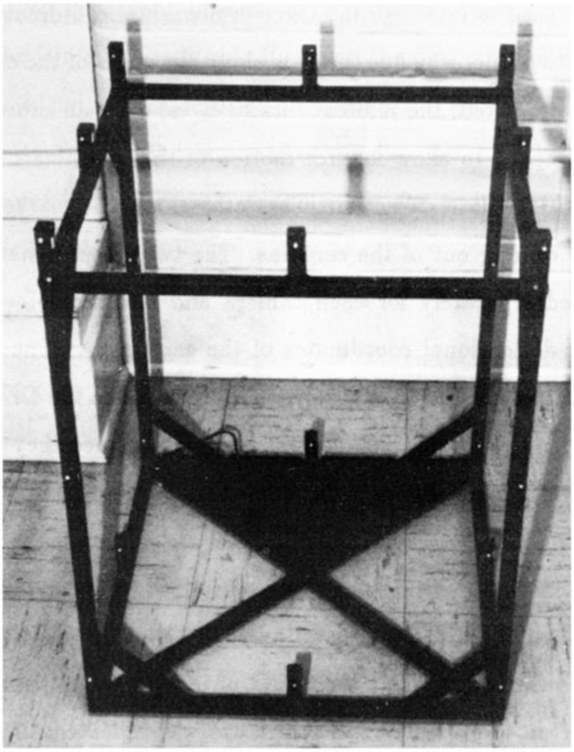

Fig.3: The calibration cube.

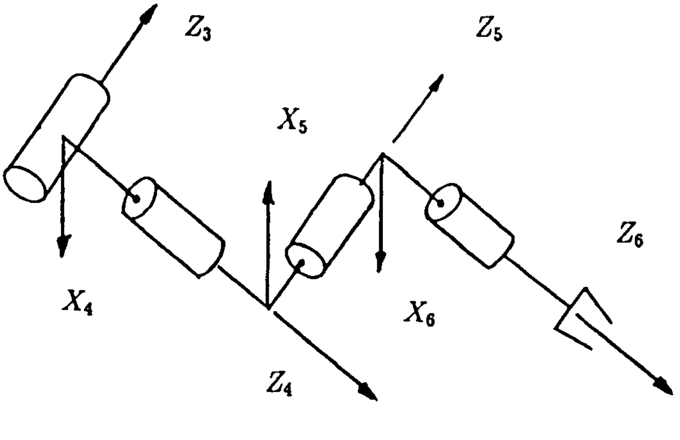

Fig.4: The wrist representation.

equations that are used to establish the three-dimensional coordinates of the markers in a reference coordinate frame, which actually is along the edges of the calibration cube. Once this procedure is completed, the reference frame is valid for all subsequent measurements and the cube is removed to allow for free motion of the robotic arm.

The next step is the data collection. The cameras see the markers on a plane perpendicular to an axis coming out of the cameras. The two-dimensional coordinates of those points are recorded separately for each camera and later combined by the software to produce the three-dimensional coordinates of the each point. The method used for this purpose is called the *direct linear transformation method* or the DLT method [14].

In this experiment, five infrared markers were used, since they are enough to render the linear algebraic system overdetermined. A small routine was written in RAIL to produce the joint-by-joint motion needed. Before any motion occurred the angle between the camera line of vision and the LEDs was optimized. The first joint of the robot arm moved in the joint coordinate space by an angle of 10°. The coordinates of the landmark points were subsequently recorded. This kind of motion was repeated six times, thereby obtaining data for an arc of 60°. The procedure was repeated for the remaining joints, always taking into account the orientation of the end-effector with respect to the cameras. Special care was taken to secure that the data had the minimum of noise, mainly due to the high reflectivity of the infrared light of the LED markers. This was achieved by covering every object in the room with black colortron foam cloth.

After the measurements were performed the data were examined using signal processing algorithms to make sure that they did not conform to any set pattern. For this purpose, a Butterworth filter in different frequencies was utilized. Since fifty measurements were taken for each position, an averaging was performed to screen out the random noise that might have affected the measurements. Having now obtained the set of coordinates of the marked points in different positions, the axes of the robot were calculated as explained in previous sections.

The experiment was repeated a second time to verify and improve the accuracy of the results. This time 12 points were marked so as to get a better least-square estimation, since that is the most critical part of the calculations. In order to increase the accuracy, the cameras were moved in closer, the number of samples was increased to 200 and the angle of rotation of the joints was reduced. However, the new plate that was manufactured to fit the 12 LEDs was flat, which resulted in loosing the good depth estimation of the cameras and therefore getting a bad estimate of the axes from the least-squares solution. We are currently manufacturing a new structure for the end-effector in the shape of a cube to eliminate this problem.

5. CALIBRATION OF THE YASKAWA MOTOMAN AID 810 ROBOT

The method described in this paper was applied on the Yaskawa Motoman Robot. The nominal Hartenberg-Denavit parameters of the robot are presented in Table 1.

Since the axis of rotation of each link is known, the angles α_i are found from the dot product of the unit vectors representing the ith and $(i+1)$st axes. The length a_i

Table 1: Nominal H-D Parameters of the manipulator

$Joint_i$	a_i (mm)	b_i (mm)	α_i (deg)
1	0	785	90°
2	670	0	0°
3	0	0	90°
4	0	950	90°
5	0	90.0	90°
6	0	0	0°

is calculated by taking the projection on the common perpendicular of two neighboring axes of the vector joining the two known points on the same axes. The length b_i is computed as the distance between two points on the ith axis. These points are defined as the intersections of the ith axis and the two common perpendiculars between the ith and $(i-1)$st as well as the ith and $(i+1)$st axes. The results are listed in Table 2.

The asterisks indicate parameters that could not be calculated. The maximum error in axis location occurred between axes 5 and 6. The maximum offset error at joint 4 and

Table 2: Calibrated H-D Parameters of the manipulator

$Joint_i$	a_i (mm)	b_i (mm)	α_i (deg)
3	*	*	89.6°
4	5.9	962.1	93.7°
5	9.3	97.3	93.2°

the maximum error in the angular parameters occurred between axes 4 and 5.

6. CONCLUSIONS

The experimental aspects of a new robot calibration method were presented. The method is independent of the robot architecture. It is based on a least-square and polar-decomposition filtering of the errors in the measurement of the Cartesian coordinates of more than three non-collinear landmark points of a rigid link. Limitations in the instrument accuracy allowed the calibration of the wrist only. However, our results are promising and hence, further investigation of the method is recommended.

7. ACKNOWLEDGEMENTS

This research was possible under NSERC (Natural Sciences and Engineering Research Council of Canada) Research Grant No. A4532, and FCAR (Fonds pour la formation de chercheurs et l' aide à la recherche, du Québec) Grant No. 88AS2517.

References

[1] An, C.H., Atkeson. C.G. and Hollerbach, J.M. 1988, *"Model-Based Control of a Robot Manipulator"*, The MIT Press, ambridge, Massachusetts.

[2] Angeles, J., 1985., "On the Numerical Solution of the Inverse Kinematics Problem", *The International Journal of Robotics Research,* Vol. 4, No. 2, Summer 1985.

[3] Angeles, J., 1986, "Automatic Computation of the Screw Parameters of Rigid-Body Motions. Part I: Finitely-Separated Positions", *ASME Journal of Dynamic Systems,Measurement and Control.* Vol. 108, March 1986.

[4] Angeles, J., and Ioannides, G. 1989., "Calibration of a Robot Wrist with an Offset Axis", *ASME Winter Annual Meeting.* (To be presented.)

[5] Golub, G., and Van Loan. C., 1983. *Matrix Computations*, The Johns Hopkins University Press, Baltimore, MD.

[6] Halmos, P. R., 1974. *Finite-Dimensional Vector Spaces*, Springer-Verlag, New York.

[7] Hayati, S .A." Robot Arm Geometric Link Calibration.", *Proceedings of the 22nd IEEE Conference on Decision and Control,* pp.1477-pp.1483, December 1983.

[8] Higham, N. J., 1986, "Computing the Polar Decomposition - with Applications", *SIAM J. Sci. Stat. Comput.*, Vol. 7, No. 4, October.

[9] Kirchner H. O. K., Gurumoorthy, B., and Prinz, F. B., 1987, "A Perturbation Approach to Robot Calibration", *The International Journal of Robotics Research,* Vol. 6, No. 4, Winter.

[10] Kumar, A. and Waldron, K.J., 1981. "Numerical Plotting of Positioning Accuracy of Manipulators."*Mechanism Machine Theory* Vol. 16, No 4, pp. 361-368.

[11] Laub, A. J. and Schifflet, G. R. 1983., "A Linear Algebra Approach to the Analysis of Rigid-Body Displacement from Initial and Final Position Data", *ASME Journal of Dynamic Measurements and Control* , Vol. 105, June.

[12] Mooring, B. W., 1983, (August 7-11,Chicago)." The Effect of Joint Axis Misalignment on Robot Positioning Accuracy.",*Proc. 1983 ASME Int. Conf. on Computers in Enginnering.* New York: ASME, pp. 151-155.

[13] Nothern Digital Inc., 1986, "The WATerloo Spatial Motion Analysis and Recording Technique", *Technical Description*, August 1986.

[14] Shapiro, R., 1978. "Direct Linear Transformation Method for Three-Dimensional Cinematography", *The Research Quarterly*, Vol. 49, No. 2, pp. 197-205.

[15] Veitschegger, W. K. and Wu, C. H., 1986, "Robot Accuracy Analysis Based on Kinematics", *IEEE Journal of Robotics and Automation,* Vol. RA-2, No. 3, Fall.

[16] Veitschegger, W. K. and Wu, C. H., 1987, "A Method for Calibrating and Compensating Robot Kinematic Errors", *IEEE International Conference on Robotics and Automation,* Vol. 1, pp. 39-44.