INSTITUTE FOR RISK AND RELIABILITY

Leibniz Universität Hannover

Risk analysis of infinite slope failure using advanced Bayesian networks

Longxue He, Antonio Topa Gomes, Matteo Broggi, Michael Beer

16.07.2018

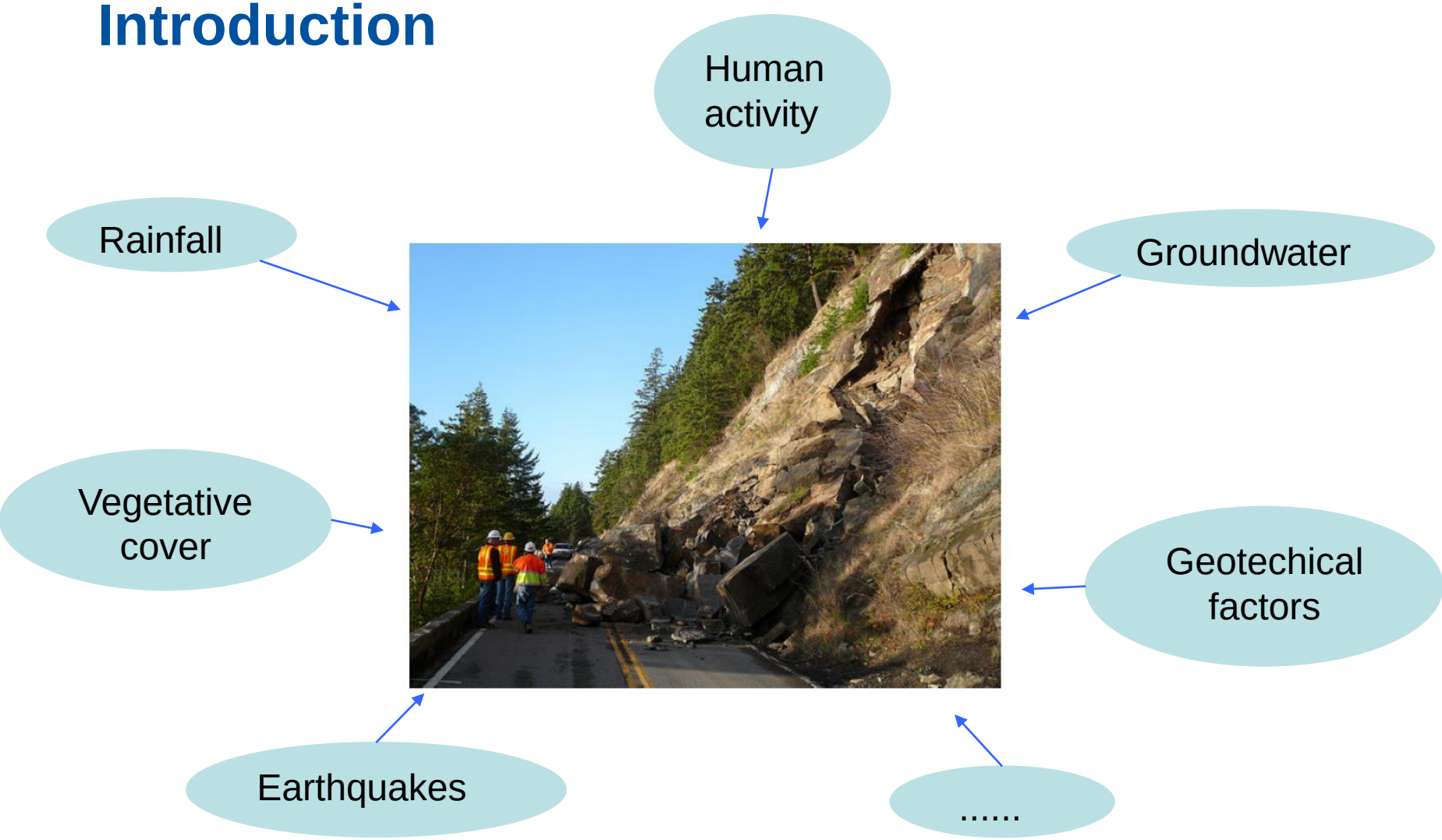
REC 2018

CHINA SCHOLARSHIP COUNCIL

Outline

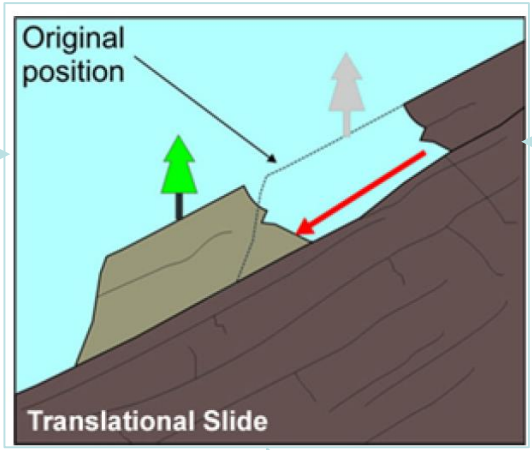
- Introduction
- Methodology
- The advanced BNs model of the infinite slope
- Example
- Result
- Conclusion

Introduction



Introduction

Which factor is critical or less important to the slope stability?



How the effect of the key factors on the stability of a slope?

What is the probability of slope failure?

Methodology: Bayesian Networks

Bayesian Networks (BNs)

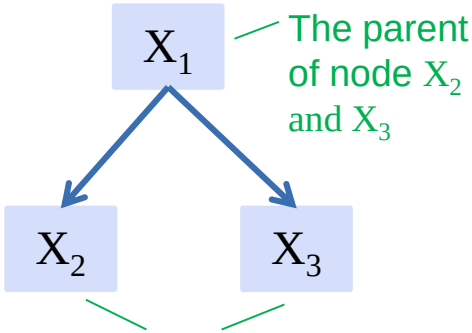
- Qualitative part
  - Directed acyclic graph (DAG)
  - Nodes : random variables
  - Edges : direct cause influences
- Quantitative part*
  - The joint probability of a simple BN,

$$P(X_1, X_2, X_3) = P(X_1)P(X_2)P(X_3|X_1, X_2)$$

  - Calculation with evidence e: X_2=T,

$$P(X_1, X_3|e) = \frac{P(X_1, e, X_3)}{P(e)} = \frac{P(X_1)P(e)P(X_3|X_1, e)}{P(e)}$$

X_1	$P(X_1)$
T	0.6
F	0.4



X_1	X_2	$P(X_2\|X_1)$
T	T	0.2
T	F	0.8
F	T	0.75
F	F	0.25

X_1	X_3	$P(X_3\|X_1)$
T	T	0.8
T	F	0.2
F	T	0.1
F	F	0.9

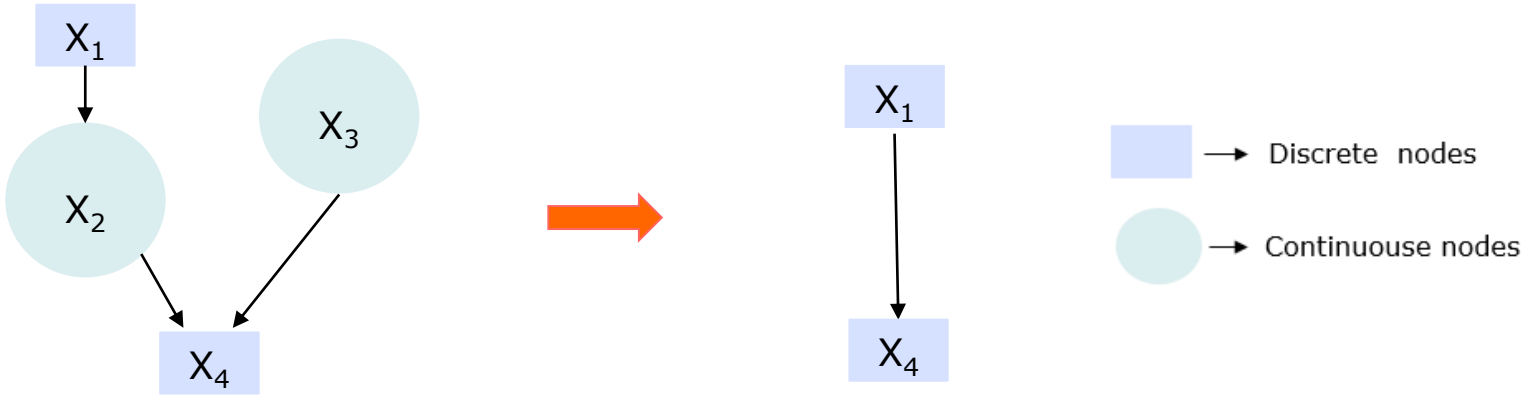
*Pearl, J. Probabilistic Reasoning in Intelligent Systems. Morgan Kaufmann Publishers, San Mateo, California, 1988

Limitation of traditional Bayesian networks

- Two main drawbacks of traditional BNs :
  - The restriction to the use of only discrete nodes
    - Limitation: Hybrid BNs
      - continuous nodes and discrete nodes
  - The input should be precise
    - Limitation: imprecise nodes
      - Interval variables
      - Discrete variables with bounds
      - Imprecise probability
      - …

→ **Methodology: advanced BNs**

Enhanced Bayesian Networks(eBNs)



- Calculation of removing continuous nodes,
  - The joint distribution for this network,

$$P(X_1, X_2, X_3, X_4) = P(X_1)P(X_4|X_2, X_3)f(X_3)f(X_2|X_1)$$

  - The joint probability over the discrete variables,

$$P(X_1, X_4) = \iint_{X_2,X_3} P(X_1)P(X_4|X_2, X_3)f(X_3)f(X_2|X_1)dX_2dX_3$$

  - After further rearranged,

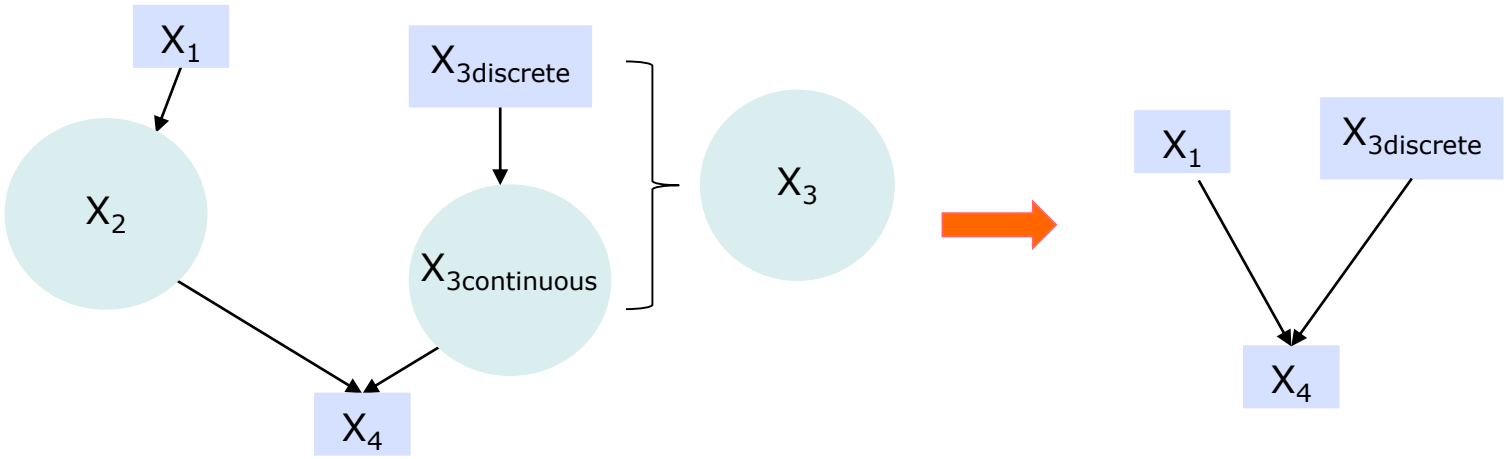
$$P(X_1, X_4) = P(X_1) \iint_{\Omega_{X_4}(X_2,X_3)} f(X_3)f(X_2|X_1)dX_2dX_3$$ → SRMs

Evidence on Continuous nodes

Discrete nodes

Continuous nodes

- A discretization approach for eBNs,



- For each sub-domains, the PMF of node $X_{3discrete}$ with i states,

$$P(X^{i}_{3\,discrete}) = F_{X_3}\left[\overline{x_{3i}}\right] - F_{X_3}\left[\underline{x_{3i}}\right]$$ (Conditional CDF $F_{X_3}[x_3]$)

Credal Networks

- CNs computation
  - The joint probability of a simple CN with bounds can be obtained as:

$$P\left(\underline{\overline{X}}_1, X_2, \underline{\overline{X}}_3\right) = P(X_2)P\left(X_2\middle|\underline{\overline{X}}_1\right)P(\underline{\overline{X}}_3|\underline{\overline{X}}_1)$$

  - Each combination of input parameters with bounds

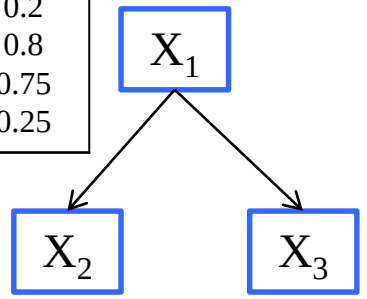
$$P(\overline{X}_3) = \max\left(\sum_{\underline{\overline{X}}_1, X_2} P(X_2)P\left(X_2\middle|\underline{\overline{X}}_1\right)P(\overline{X}_3|\underline{\overline{X}}_1)\right)$$

Complex network (include many nodes)

Combinatorial explosion with exact inference

X_1	$P(X_1)$
T	[0.68, 0.69]
F	[0.31, 0.32]

X_1	X_2	$P(X_2\|X_1)$
T	T	0.2
T	F	0.8
F	T	0.75
F	F	0.25



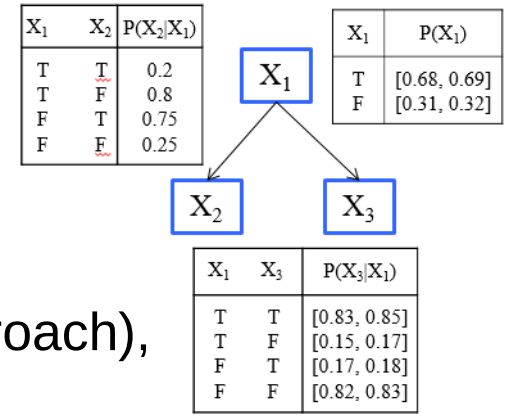
X_1	X_3	$P(X_3\|X_1)$
T	T	[0.83, 0.85]
T	F	[0.15, 0.17]
F	T	[0.17, 0.18]
F	F	[0.82, 0.83]

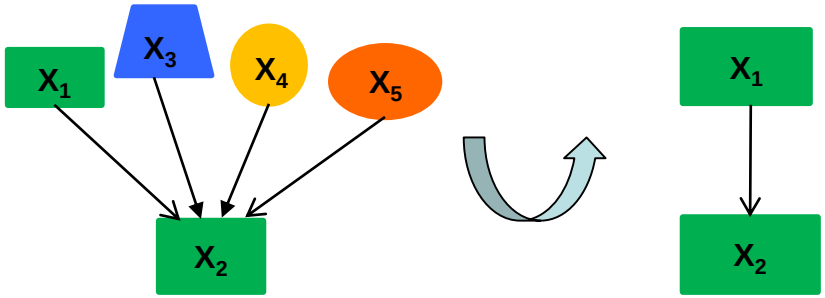
Removing the unnecessary nodes with SRMs

- Low-cost computational methods*
  - Inner bounds of the query probability

$$P(\bar{X}_1) = \max\left(\sum_{X_2,\underline{X}_3} P(\bar{X}_1, X_2, \underline{X}_3)\right)$$

- Reduction of a simple CN (similar with eBNs approach),





Notes: node X_2 should be characterized by a domain in the outcome space of its parents

Notes:

Variable Type	Node Definition	CPD	Graph
Discrete	Discrete	Discrete Probabilities	Rectangle
Interval	bounded	Interval or Probability Intervals	Ellipse
Random	Probabilistic	Probability distribution	Circle
Hybrid	Hybrid	Parametric P-boxes	Trapezoid

* S. Tolo, E. Patelli, and M. Beer, An open toolbox for the reduction, inference computation and sensitivity analysis of credal networks, Advances in Engineering Software 115, 126 (2018).

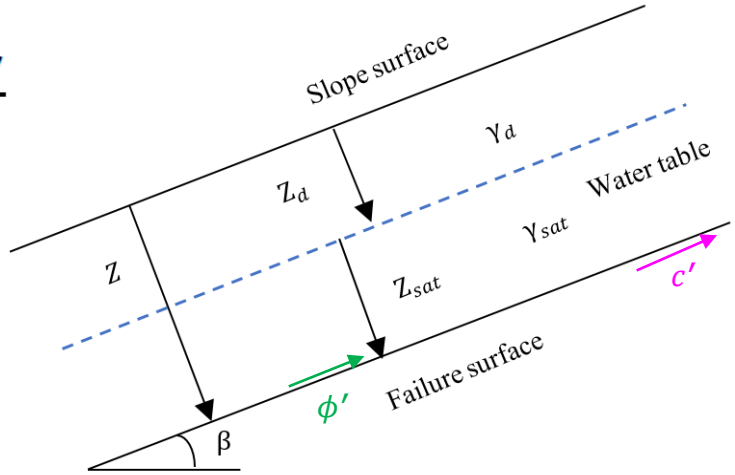
Outline

- Introduction
- Methodology
- **The advanced BN model of the infinite slope**
- Example
- Result
- Conclusion

Formulation of an infinite slope failure

- Factors of safety,

$$FOS = \frac{c\prime + (\gamma_d Z_d + \gamma_{sat} Z_{sat} - \gamma_w Z_{sat}) * cos\,\beta * tan\,\phi\prime}{(\gamma_d Z_d + \gamma_{sat} Z_{sat}) * sin\,\beta}$$



c'-- cohesion ;

ϕ'-- friction angle;

β --slope inclination;

γ_w -- the unit weight of water, 9.81kN/m^3.

Z -- the total thickness of the soil;

Z_d , Z_{sat} – unsaturated, saturated soil thickness.

γ_d, γ_{sat} -- the dry and saturated unit weight of the material .

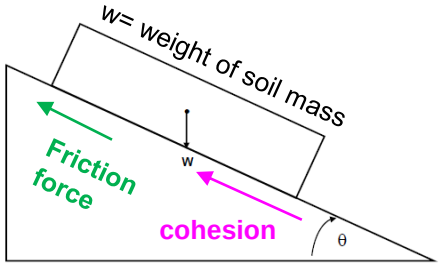
  - A slope is safe: *FOS*≥1, or unsafe if *FOS*<1.

- The failure model of the slope:

$$G(X) = FOS - 1$$

The BN model of Slope Stability

- Resist Force:
  - c'
  - ϕ'

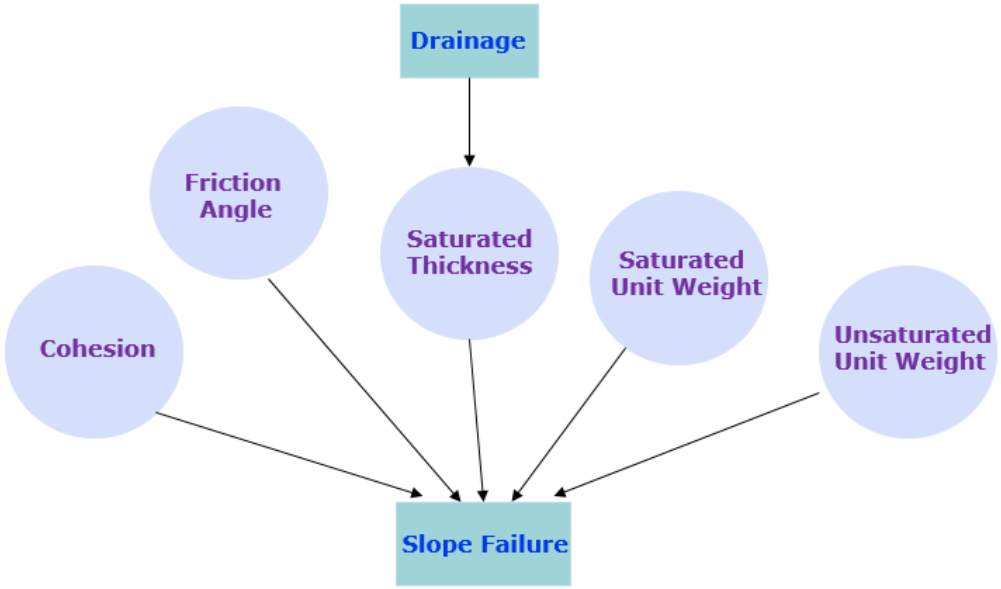


Conceptual model of the slope

- Soil water content
  - γ_d
  - γ_{sat}
  - Z_{sat}

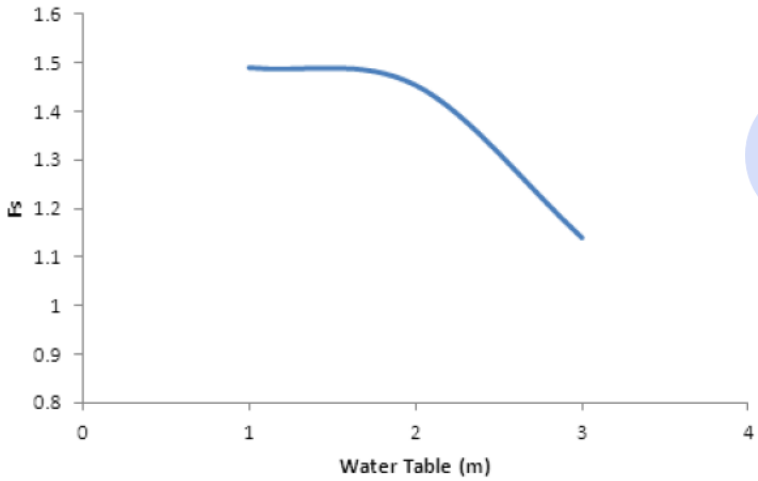
(The vertical stress = the product of its thickness and saturated or dry unit weight)

The BN of an infinite slope

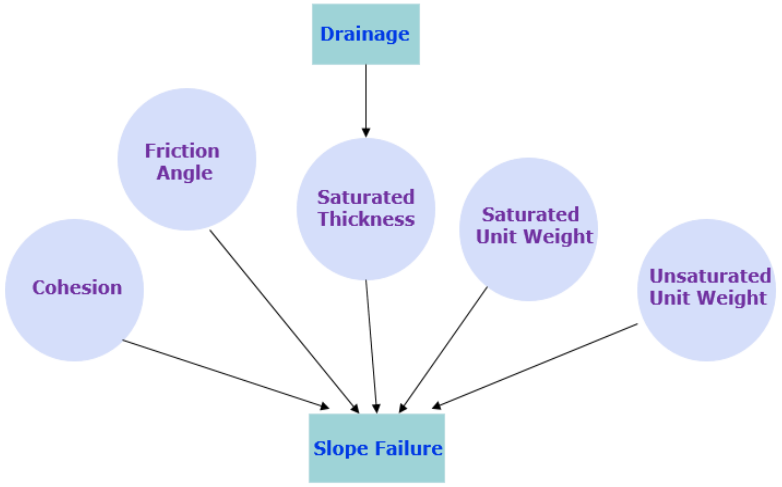


The BN model of Slope Stability

- Water table:
  - Saturated Thickness



The effect of water table on the slope*

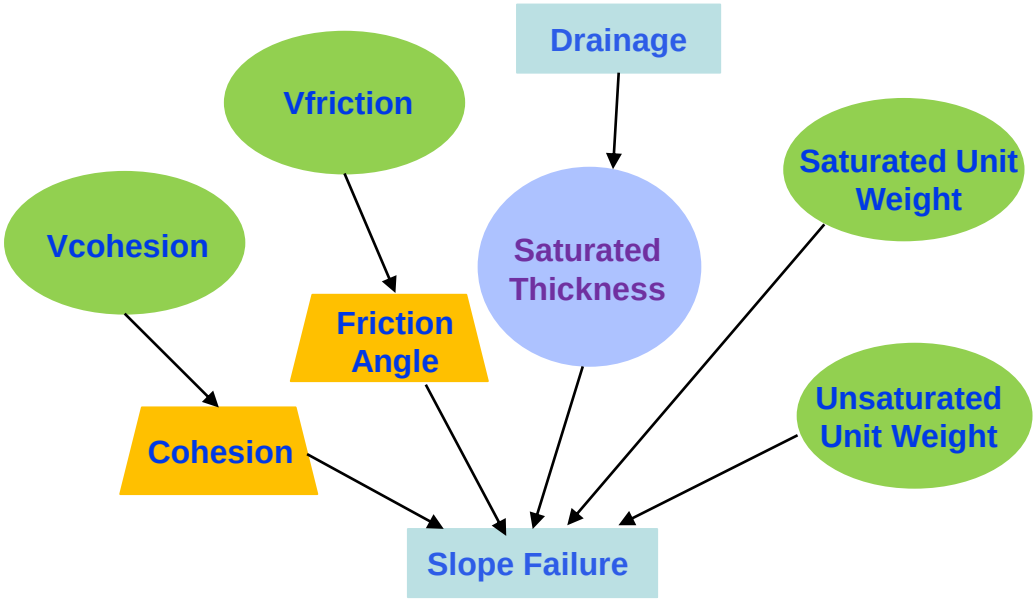


- Drainage influence

*Khanmohammadi, S and V. Hosseinitoudeshki. The effect of water level on the stability of slopes. Journal of Novel Applied Sciences, 2014

The CN model of Slope Stability

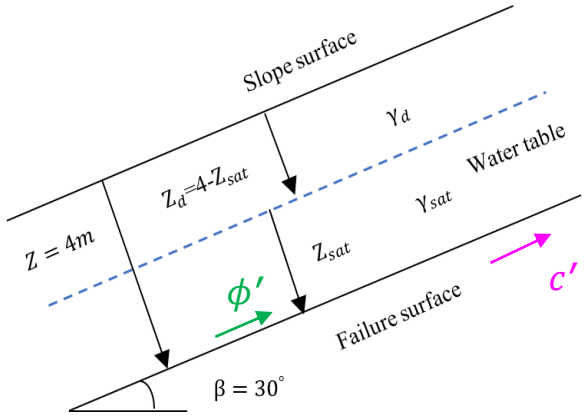
- Imprecise nodes
  - Interval variables: γ_d, γ_{sat} , Z_{sat} and the average of c' and ϕ'
  - P-boxes: c' and ϕ',



Example 1

- The shape of the failure surface
  - Translational slip :

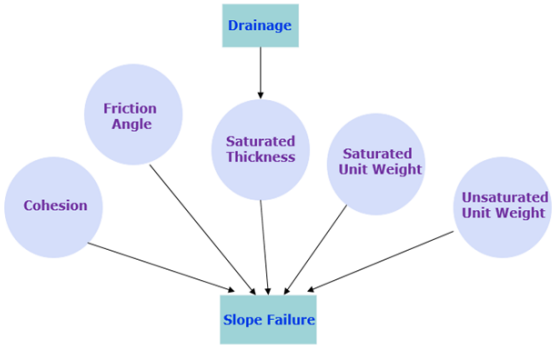
plane and roughly parallel to the slope surface.



- Input parameters of the unsaturated soil,

Parameters	Variable type	CPD*
Cohesion [kPa]	Continuous	logN(22,10)
Friction angle [°]	Continuous	N(35,3)
Unsaturated unit weight [kN/m^3]	Continuous	N(17, 0.4)
Saturated unit weight [kN/m^3]	Continuous	N(19, 0.5)
Saturated thickness of soil [m]	Continuous	U(0, 4) or 0
Drainage (D)	Discrete	[0.5, 0.5]
Slope failure(SF)	Discrete	[P_f,P_s]

*N, logN, represents normal and lognormal distribution with mean and standard deviation, respectively. U represent uniform distribution with lower and upper bound.



Example implementation

- The prior probabilities of node SF,

$$P(SF) = P(D) \int_{\dots\dots} \int_{\Omega_{SF}(\gamma_d, \gamma_{sat}, Z_{sat}, c', \varphi')} f(\gamma_d) f(\phi') f(c') f(\gamma_{sat}) f(Z_{sat}|D)\, d\gamma_d d\phi' dc' d\gamma_{sat} \quad (8)$$

- The outcome space $\Omega_{SF}(\gamma_d, \gamma_{sat}, Z_{sat}, c', \phi')$:

$$P(SF) = \begin{cases} P_f, & FOS - 1 < 0 \\ P_s, & FOS - 1 > 0 \end{cases} \quad (9)$$

  - P_f – the failure probility of node SF;
  - P_s – the safe Failure probility of node SF

Results of example 1

- The failure probability,

State	No evidence	Drainage	No Drainage
$P(FS)$	0.0274	$< 10^{-7}$	0.0513

  - The influence of water table in the soil.
  - Drainage problems for the Decision makers

Results of example 1

- The comparison with narrowing the observation in continuous nodes
  - Slope failure probability updated with new information,

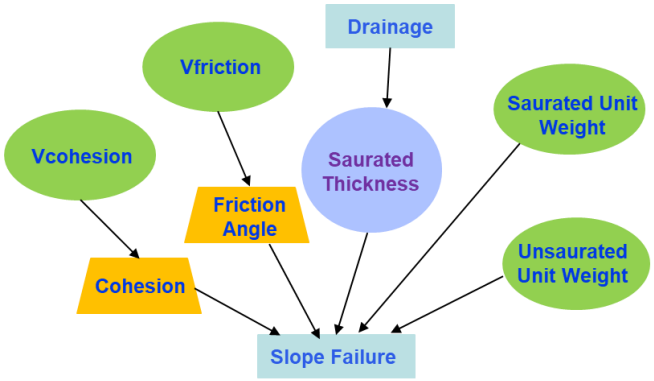
Nodes	Cohesion	Friction Angle	Unsaturated Unit Weight	Saturated Unit Weight
Evidence	$0 \leq c \leq 100$	$25 \leq \phi \leq 45$	$16 \leq \gamma_d \leq 19$	$18 \leq \gamma_{sat} \leq 21$
$P(FS)$	0.0255	0.0267	0.0259	0.0256

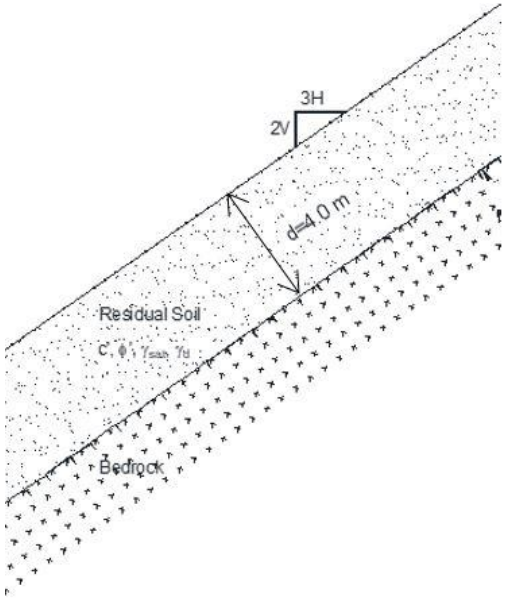
  - Slope failure probability updated with further information,

Nodes	Cohesion	Friction Angle	Unsaturated Unit Weight	Saturated Unit Weight
Evidence	$10 \leq c \leq 90$	$30 \leq \phi \leq 40$	$16.5 \leq \gamma_d \leq 18.5$	$18.5 \leq \gamma_{sat} \leq 20.5$
$P(FS)$	0.0181	0.0237	0.0262	0.0253

Example 2

- Slope in residual soil from Porto
  - Input parameters of the residual soil,





Parameters	Precise information	Imprecise information min	Imprecise information max	P-boxes*
Cohesion [kPa]	logN(20,4)	0	70	logN(μ_c,4), $\mu_c \in [16, 22]$
Friction angle [°]	N(37,1.85)	25	47	N(μ_f,1.85), $\mu_f \in [36, 38.5]$
Unsaturated unit weight [kN/m^3]	N(18.5,0.51)	17	20	[17, 20]
Saturated unit weight [kN/m^3]	N(20,0.6)	18	22	[18, 22]

$*\mu$ indicates the mean of the distributions.

Results of example 2

- The comparison with three different input information,

Different information	Precise	Imprecision	Parametric p-box
$P(FS)$	0.0247	[0, 1]	[0, 0.0711]

- The influence of Drainage,

Different information	Precise	Imprecision	Parametric p-box
$P(FS\|D = true)$	0	[0, 1]	0
$P(FS\|D = false)$	0.0514	[0, 1]	[0, 0.1531]

Conclusion

- Information updating of the advanced BNs is effective for the real-time decision making.
- The results of CNs can be further accurate by use of P-boxed.
- Advanced BNs are effective and feasible approach to assess the risk for the slope subject to drainage state.

Acknowledgements: This research is supported by the Chinese Scholarship Council.

Thank you for your attention