Mapping Link SNRs of Wireless Mesh Networks onto an Indoor Testbed

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Overview

• Introduction
• Methodology
• Results
• Summary
Mapping Real-World onto Testbed

Real-World Mesh Network

ORBIT Radio Testbed

What equivalence do we seek?

Answer: Equivalent SINR matrices!
Link Signal-to-Interference-Plus-Noise Ratio, SINR

• Assume $N$ nodes, each with receiver noise power $N_0$

• Path gain from Node $k$ to Node $j$ is $g_{kj}$ in power ratio, or $G_{kj} = 10\log g_{kj}$ in dB

• Link SINR from Node $i$ to Node $j$:

$$SINR_{ij} = \frac{P_i g_{ij}}{\sum_{k \neq i} P_k g_{kj} + N_0} = \frac{\gamma_{ij}}{\sum_{k \neq i} \gamma_{kj} + 1}$$

where $\gamma = P g_{ij} / N_0$ (link SNR)

• Conclusion: If $\{\gamma_{ij}\}$ is identical for the grid and real-world environments, $\{SINR_{ij}\}$ is also identical for both.
Link Gain Is A Major Element

- **Link SNR**
  - Fundamental to system level emulation
  - Proportional to link gain: $\gamma = \frac{P_g}{N_0}$

- **Link Gain**
  - Characterized by distance ($d$) and propagation parameters, e.g., ($\alpha$, $\sigma$)
    - $\alpha$: path loss exponent
    - $\sigma$: standard deviation of dB shadow fading

- **Link Gain Model**
  \[
  G = 10 \log g = [A - 10 \alpha \log d] + 10 \log s
  \]
  - Bracketed term: deterministic, $d$-based median path gain
  - $s$: spatially random shadow fading, usually log-normal
  - $S = 10 \log s = \sigma u$, where $u \sim N(0, 1)$
## Differences Between “Real World” and ORBIT Testbed

<table>
<thead>
<tr>
<th></th>
<th>Real World</th>
<th>ORBIT Testbed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Loss Exponent, $\alpha$</td>
<td>$&gt; 2$</td>
<td>2 (assumed)</td>
</tr>
<tr>
<td>Shadow Fading, $S$</td>
<td>Present, and following Gaussian distribution</td>
<td>Absent (assumed)</td>
</tr>
<tr>
<td>Network Dimensions</td>
<td>Arbitrary</td>
<td>20 m by 20 m</td>
</tr>
</tbody>
</table>
Link Gain Matrix: The Starting Point of Efficient SNR Mapping

Real-world Network Topology
\((\alpha > 2, S \neq 0)\)

Testbed Replication of Real-world
\((\alpha = 2, S = 0)\)
Outline

• Introduction
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Two Mapping Approaches

Forward Mapping

Reverse Mapping
Forward Mapping — Attractive But Naïve

Feasible positions for C
= Intersection of rings
NO intersection → NO feasible “C”

User A

30 m

20 m

User B

30 m

User C

Range of C decided by A

Range of C decided by B

α > 2, S ≠ 0

α = 2, S = 0
Reverse Mapping : \( N=3 \)

- Nodes A, B and C can be mapped exactly from the grid to the real world based on their link gains \( \{ G_{ij} \} \)
Reverse Mapping: $N>3$

- Given a grid link gain matrix, we create real-world mappings such that the link gain differences between the two scenarios approximate the effects of shadow fading.

\[ \alpha=2, \ S=0 \]

\[ \alpha \neq 2, \ S \neq 0 \]
SNR Mapping Procedure

Step 1: Create Node Configuration on the Grid

- Choose a configuration with desired features, e.g.,
  -- two clusters of nodes
  -- random uniform spatial array of nodes
- Compute $N \times N$ matrix $G$ of inter-node path gains, e.g.,
  $$G_{ij} = 10\log\left(\frac{4\pi d_{ij}/\lambda}{\lambda}\right)^2$$  (line-of-sight path gain)
  where $\lambda$ = wave length, $d_{ij}$ = distance between nodes $i$ and $j$
- Path gains may not be simple line-of-sight values; alternative is to obtain $G$ via measurements and store the measured matrix (grid calibration)
SNR Mapping Procedure (Cont.)

Step 2: Replicate Gain Matrix in Real World

- **Sub-Step A: Initialization**
  - Compress grid layout based on the difference in path loss exponents, i.e., attempt to scale all distances via \( d \rightarrow d^{2/\alpha} \)
  - Can be done precisely for \( N = 3 \), assuming \( \alpha \geq 2 \)
  - For \( N > 3 \), \( (G_{ij})_{rw} \rightarrow (G_{ij})_{grid} + \Delta_{ij} \)

**Idea!** Note that

\[
(G_{ij})_{grid} = (G_{ij})_{rw} - \Delta_{ij}
\]

where \( (G_{ij})_{rw} \) is the **distance-based median path gain** for real-world link \( ij \).

If the set of mapping errors \( \{\Delta_{ij}\} \) can be "shaped" to have Gaussian statistics, in particular,

\[
\{\Delta_{ij}\} \rightarrow N(0, \sigma^2)
\]

then the set of grid gains can be said to correspond to a set of real-world gains, with distance based medians plus Gaussian shadow fading.
SNR Mapping Procedure (Cont.)

Step 2: Replicate Gain Matrix in Real World (Cont.)

- **Sub-Step B: Constrained Optimization**
  -- Successively adjust node positions to control the statistics of $\{\Delta_{i,j}\}$
  -- Goal: Make the $N(N-1)/2$ values of $\Delta_{i,j}$ Gaussian, $N(0, \sigma^2)$
  -- Method: If $\mu_n = n^{th}$ moment of the set $\{\Delta_{i,j}\}$, drive the process towards

$$
\begin{align*}
\text{excess kurtosis} & \quad \kappa = \frac{\mu_3}{\mu_2^{3/2}} \quad \rightarrow \quad 0 \\
\text{skewness} & \quad \xi = \frac{\mu_4}{\mu_2^2} - 3 \quad \rightarrow \quad 0 \\
\text{s.t.} & \quad \begin{cases} 
\mu_1 = 0 \\
\mu_2 \leq \sigma^2
\end{cases}
\end{align*}
$$
SNR Mapping Procedure (Cont.)

Step 3: Real-World Sizing (Scaling)
- Scale all $x$- and $y$- coordinates by a common factor, $\tau$, to achieve desired network area size, while preserving shape of the node layout.
- All inter-node path gains will decrease by a common value of $10\overline{\alpha} \log \tau$

Step 4: Real-World Transmit and Noise Powers
- Assign transmit powers, $\{\overline{P}_i\}$ and receiver noise power, $\overline{N}_0$, where $\overline{\text{overbar}}$ denotes real-world quantities.
- Then, $\overline{\text{SINR}}_{ij} = \overline{P}_i + (G_{ij})_{\text{grid}} - 10\overline{\alpha} \log \tau - \overline{N}_0$

Step 5: Grid Node Powers
- Set grid transmit powers $\{P_i\}$:
  $$P_i = \overline{P}_i - 10\overline{\alpha} \log \tau + N_0 - \overline{N}_0$$
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• Introduction
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Architecture of Catalogue

Population

Environment

Model Parameters

Topology

N-Node Grid Configuration

Indoor Scenario

Outdoor Scenario

\( \alpha = 5 \)

\( \sigma = 8 \)

\( \alpha = 3 \)

\( \sigma = 8 \)

\( \alpha = 4 \)

\( \sigma = 5 \)

\( \alpha = 3 \)

\( \sigma = 5 \)

\( \alpha = 4 \)

\( \sigma = 8 \)

\( \alpha = 5 \)

\( \sigma = 8 \)

Cluster

Flat Mesh

Cluster

Flat Mesh

...
Experiment 1: Three Indoor Clusters with $N=30$ ($\alpha=4$, $\sigma=5$ dB)

(a) Grid Configuration

(b) Real World Configuration

(c) $\sqrt{\mu_2}=4.99$ dB, $\kappa=1.9E-4$, $\xi=-1.5E-4$
Experiment 2: Two Outdoor Clusters with $N=30$ ($\alpha=4$, $\sigma=8$ dB)

(a) Grid Configuration

(b) Counterpart Real World Configuration

(c) $\mu_2 = 7.97$ dB, $\kappa = 2 \times 10^{-5}$, $\xi = 6 \times 10^{-6}$
Experiment 3: Outdoor Flat Mesh with $N=50$

($\alpha=4$, $\sigma=8$ dB)

(a) Grid Configuration

(b) Counterpart Real World Configuration

(c) $\sqrt{\mu_2} = 8$ dB, $\kappa = 1E-5$, $\xi = 7.8E-6$
Experiment 4: Multiplicity

One grid configuration can correspond to multiple real-world scenarios.
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Summary

• A new procedure for testbed / real-world mapping of SINR
• Reverse mapping (testbed-to-real-world) is the key
• Shadow fadings closely emulate desired Gaussian statistics
• Efficient mapping of 50+ nodes and various topologies
• Both indoor and outdoor environments
• One grid configuration can produce multiple real-world configurations, permitting choice of “preferred” mappings
• Permits “catalogue” of mappings for different populations, environments, propagation parameters, node topologies
• Next steps: -- Spatial variations of shadow fading
  -- Emulating terminal (node) mobility
Questions ?
Demo for Mobility

-- Creating Choreography of Nodes

• 2 Clusters
• 30 Nodes in Total
• Path loss exponent $\alpha = 3$
• Standard deviation of shadowing $\sigma = 8$ dB
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping

Testbed X, m

Testbed Y, m

Realworld X, m

Realworld Y, m
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

Testbed X, m

Testbed Y, m

(b) Realworld Mapping

Realworld X, m

Realworld Y, m
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping

Testbed X, m

Testbed Y, m

Realworld X, m

Realworld Y, m
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping
(a) Testbed Configuration

(b) Realworld Mapping
Mobility of One Node in Ad Hoc Network

\( (\alpha=3, \sigma=8) \)
Definition and Properties of Skewness

- Skewness is the degree of asymmetry of a distribution, defined as a normalized form of the third central moment of a distribution.
- If the distribution has a “longer tail” less than the maximum, the function has negative skewness. Otherwise, it has positive skewness.
- The skewness for a normal distribution is zero, regardless of the values of its parameters.

Both PDFs have the same expectation and variance. The one on the left is positively skewed. The one on the right is negatively skewed.
Definition and Properties of Kurtosis

- Kurtosis is the degree of peakedness of a distribution, defined as a normalized form of the fourth central moment of a distribution.
- A high kurtosis distribution has a sharper "peak" and fatter "tails", while a low kurtosis distribution has a more rounded peak with wider "shoulders".
- Distributions with zero kurtosis are called mesokurtic. The most prominent example of a mesokurtic distribution is the normal distribution, regardless of the values of its parameters.

The PDF on the right has higher kurtosis than the PDF on the left. It is more peaked at the center, and it has fatter tails.
Jarque-Bera test

From Wikipedia, the free encyclopedia

In statistics, the Jarque-Bera test is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. The test statistic $JB$ is defined as

$$JB = L/6 \times (\xi^2 + \kappa^2/4)$$

where $\xi$ is the skewness, $\kappa$ is the excess kurtosis, and $L$ is the number of observations. The statistic has a chi-squared distribution with two degrees of freedom and can be used to test the null hypothesis that the data are from a normal distribution. Samples from a normal distribution have an expected skewness of 0 and an expected kurtosis of 0. As the equation shows, any deviation from this increases the JB statistic.

References

Algorithms for Nonlinearly Constrained Optimization

• A technical but nontrivial issue
• Choice is not unique and depends on the structure of a problem, e.g. interior-point method, stochastic gradient method
• MATLAB routine: FMINCON

References