Seamless Channel Transition for the Modified BroadCatch Broadcasting Scheme

Chih-Jen Wu¹, Yu-Wei Chen², and Yue-Li Wang³

Abstract

In this paper, we present a scheme that possess seamless channel transition property and is suitable for heterogeneous receivers simultaneously. We first modify the BroadCatch (MBC) scheme [28][29] by allowing an arbitrary number of base channels defined in BroadCatch. We then propose a seamless channel transition enhancement on top of MBC so that service provider can be capable of adjusting the channel allocation policy to make the most benefit out of the available bandwidth. MBC is not only adaptive to receivers with heterogeneous bandwidth capacities but also with zero heterogeneous scalability when channel transition is performed. With zero heterogeneous scalability, new clients can benefit from newly added server channels without paying any extra cost. As compared with BroadCatch and Fast schemes, our results show that MBC greatly reduces startup latencies for low-end clients while slightly sacrifices those for high-end clients. Maximum buffering requirement is greatly reduced as well.

Keywords: Video-on-Demand (VOD), BroadCatch, Seamless channel transition, Heterogeneous scalability.

¹Chih-Jen Wu is currently working toward the Ph. D. degree at the Department of Information Management, National Taiwan University of Science and Technology, No. 43, Sec. 4, Keelung Rd., Taipei, 106, Taiwan, Republic of China. E-mail address: D9409102@mail.ntust.edu.tw.
²Yu-Wei Chen is with the Graduate Institute of Information and Logistics Management, National Taipei University of Technology, No. 1, Sec. 3, Chung-hsiao E. Rd., Taipei, 10608 Taiwan, E-mail address: ywchen@ntut.edu.tw.
³Corresponding author: Yue-Li Wang is with the Department of of Information Management, National Taiwan University of Science and Technology, No. 43, Sec. 4, Keelung Rd., Taipei, 106, Taiwan, Republic of China. E-mail address: ylwang@cs.ntust.edu.tw. This work is partially supported by NSC-97-2221-E260-007-MY3.
1 Introduction

For popular CBR-encoded videos, broadcasting schemes [1]-[16] are the most efficient approach for utilizing bandwidth. Some efficient schemes for broadcasting VBR-encoded videos are proposed as well. For example, Saparilla et al. [17] proposed a series of multiplexing schemes based on smoothing, server buffering, and client prefetching to achieve nearly 100% link utilization with negligible packet loss. In addition, Zhao et al. [18] proposed an efficient scheme for broadcasting a “non-linear” video in which a user can dynamically select which branch of the video s/he wants to follow. However, throughout this paper, we focus on the schemes for broadcasting linear CBR-encoded videos.

Current broadcasting schemes typically work by first dividing a video into a series of segments, and then repeatedly transmitting them on a few specified channels in a certain manner. Users then download segments from all or some of the channels and store segments in their local buffers during watching the video. Under the broadcasting environment, the requirement of server bandwidth is independent of the number of users on the system. However, users have to wait for some amount of time before watching a video. The waiting time is called startup latency which is usually the duration of transmitting a segment. That is the reason why it is called the near VOD system.

Although broadcasting schemes have the advantage of reducing server bandwidth, they suffer from the problem of insensitivity to the popularity of videos. However, the popularity of a video usually changes with time, social events, and so on. Given a set of popular videos and limited server bandwidth, it is desirable for the service provider to be capable of adjusting bandwidth allocated to a video according to its popularity seamlessly, so as to make the most benefit on the available bandwidth. It is called as positive channel transition (PCT for short) if more bandwidth is allocated to a video, while it is called as negative channel transition (NCT for short) if less bandwidth is allocated to a video.

Recently, several channel transition protocols have been proposed for different broadcasting schemes, such as the Fast scheme [3], the Staircase scheme [4], the Bespoke scheme [5], the Enhanced Mirrored-Pyramid scheme [6], the Skyscraper scheme [7] and the Generalized Fibonacci
Broadcasting scheme [8]. For ease of explanation, we call a segment scheduled before and after channel transition the “old” and “new” segments, respectively. These channel transition protocols can be roughly classified into two types: Content-Relation Approach [19]-[21] builds the relationship of old and new segments among different channels; whereas Segment-Placement Approach [22]-[25] carefully arranges the placement of old and new segments transmitted on the same channel without overlap.

Three broadcasting schemes, HeRO [26], CAR [27] and BroadCatch [28][29], concerning with the heterogeneity issue have been proposed. These three schemes allow users to choose among a range of bandwidths to download a video at the cost of their startup latency, not the video quality. Besides, a new criterion, heterogeneous scalability, is introduced in [28]. It is defined as the amount of bandwidth that a client has to add on average to benefit from a newly added server channel. Besides, Guo et al. [30] proposed a patching-based scheme that can be applied to provide services to heterogeneous clients as well. The method [30] also proposed a channel allocation scheme, called dynamic multiplexing with channel merging, to assign the numbers of regular and patching channels dynamically. Recently, Wu et al. [31] explored the heterogeneous behavior of the Fast scheme [3] which is originally designed for homogeneous users.

To our best knowledge, only the Fast scheme and BroadCatch scheme can be applied in heterogeneous environment and possess seamless channel transition property at the same time. In fact, seamless channel transition for the BroadCatch scheme is accomplished effortlessly by increasing or decreasing the number of the so-called catching channels [28]. However, heterogeneous scalabilities of these two schemes are both larger than zero, which implies that new clients must pay some extra cost to benefit from the conducting of channel transition. This motivates our work to design a scheme that not only is suitable for heterogeneous environment, but also exhibits seamless channel transition property with zero heterogeneous scalability when channel transition is performed.

Given $\alpha$ server channels, we modify the BroadCatch scheme by allowing an arbitrary number (ranging from 2 to $\alpha$ ) of base channels defined in BroadCatch. The modified BroadCatch scheme is denoted by MBC. Seamless channel transition on MBC is then performed by
dynamically increasing or decreasing the number of base channels. When channel transition is conducted, the heterogeneous scalability of the proposed MBC scheme remains zero, which implies that new clients can benefit from newly added server channels without paying extra cost. As compared with BroadCatch and Fast schemes, our results show that MBC greatly reduces startup latencies for low-end clients while slightly sacrifices those for high-end clients. Maximum buffering requirement is greatly reduced as well.

The rest of the paper is organized as follows. In Section 2, we review the BroadCatch scheme and introduce its channel transition strategy. In Section 3, we first present the MBC scheme. Then, the seamless channel transition protocols based on it is proposed. After that, we analyze its heterogeneous scalability. In Section 4, we compare performances of MBC with those of BroadCatch and Fast schemes. Finally, some concluding remarks are given in Section 5.

2 Overview of the BroadCatch Scheme

Suppose that \(\alpha\) channels \(C_1, C_2, \ldots, C_\alpha\) are allocated to deliver a video of length \(L\). All channels are of bandwidth equal to the video consumption rate at normal playback, say \(b\) Mbps. We first review the BroadCatch scheme in Subsection 2.1, and then review the seamless channel transition protocols on BroadCatch by adding/decreasing catching channels in Subsection 2.2.

2.1 Review of the BroadCatch Scheme

The BroadCatch scheme [29] enables users with variant bandwidth capabilities from \(b\) to \((\alpha - 2) \cdot b\) Mbps to download a video. With \(\alpha\) channels, the number of divided segments is \(2^{\alpha-1}\). These \(\alpha\) channels are synchronously divided as an infinite set of time slots, where each time slot is used to deliver one segment. Channels \(C_1\) and \(C_2\) broadcast the whole video from time slots 0 and \(2^{\alpha-2}\), respectively, and are referred to as the base channels. Channel \(C_i\), \(3 \leq i \leq \alpha\), broadcasts \(2^{\alpha-i+1}\) segments \(\{S_1, \ldots, S_{2^{\alpha-i}}\}\) from time slot \(2^{\alpha-i}\), whereas channel \(C_\alpha\) only broadcasts segment \(S_1\) as indicated in [29]. These \(\alpha - 2\) channels are referred to as the catching channels. Figure 1 shows the channel design strategy of the BroadCatch scheme with six channels.
Figure 1. Channel design strategy of the BroadCatch scheme with six channels.

We introduce the reception strategy [29], for self-contained purpose, as follows. A client always downloads segment $S_i$ first, then tunes to another channel to download the rest of the video. Suppose that a client starts downloading segment $S_i$ broadcast on channel $C_i$, if $i$ is either 1 or 2, the client downloads all segments from the base channel $C_i$. If $3 \leq i \leq \alpha - 1$ the client continues its downloading process until segment $2^{i-1}$ on $C_i$ has been downloaded. While the downloading proceeds, the client looks for the earliest occurrence of $S_{2^{i-1}+1}$ on one of the upper channel $C_m$, $m < i$, and catches up to $C_m$, then downloads segments $S_{2^{i-1}+1}, S_{2^{i-1}+2}$, and so forth, until finishing downloading the last segment $S_{2^{i-1}+1}$ on $C_m$. The above process repeats until the client finally reaches to one of the base channels and downloads the rest of the video.

When a client gets into the system to download a video, the server informs the client of the total number of channels and the beginning of the first broadcast on base channel $C_1$. Using these two parameters, the client can precisely construct an image of a segment placement as the example in Figure 1. According to the reception strategy, it is easy to compute the receiving schedule (and thus the required bandwidth) for clients arriving at a specific time slot. Let $T_j$ denote time slot $j$ for $j \geq 0$. Take a client who starts downloading $S_i$ at $T_{10}$ from channel $C_5$ in Figure 1 as an example. The client downloads segments $\{S_1, S_2\}$, $\{S_3, \ldots, S_{16}\}$, and $\{S_{17}, \ldots, S_{32}\}$ from channels $C_5$, $C_3$, and $C_1$ during $T_{10} \sim T_{11}$, $T_{10} \sim T_{23}$, and $T_{16} \sim T_{31}$, respectively. The bandwidth required by this client at $T_{10}$ is thus $2 \cdot b$. With the same reasoning, for clients starting to download $S_i$ at $T_{13}$, their required bandwidth will be $3 \cdot b$ Mbps. Obviously, bandwidth requirements are different for clients arriving
at different time. For clients without enough bandwidth, they must wait for the earliest occurrence of segment $S_i$ that fits their bandwidth capabilities before starting the downloading process.

Here, we review the definition of heterogeneous scalability [29] as follows.

**Definition 1** Given any integer $\alpha \geq 2$, the heterogeneous scalability is the difference of average worst client bandwidth requirements associated with $\alpha$ and $\alpha - 1$ server channels.

The heterogeneous scalability of the BroadCatch scheme is at most $b/4$ Mbps on average.

### 2.2 Seamless Channel Transition for BroadCatch

The basic idea of seamless channel transition protocols in the BroadCatch scheme [28] is to dynamically increase or decrease the number of catching channels. Since each catching channel broadcasts a half of the video data that delivered on its previous channel, adding a new channel or releasing an old one works thus naturally. To perform PCT, it just needs to broadcast half of the video portion of the previous channel on the newly added channel; whereas it just needs to drop the last channel to perform NCT. Note that the whole process can be achieved without disturbing the delivery of other channels.

### 3 Seamless Channel Transition for the MBC scheme

We present the MBC scheme in Subsection 3.1. Then, we propose the seamless channel transition protocols for MBC in Subsection 3.2. In Subsection 3.3, we discuss heterogeneous scalability of MBC. Besides, given a set of popular videos, we present a channel allocation policy which allows the service provider to be capable of determining the number of channels assigned to each video according to its popularity so as to make the most benefit from the broadcasting service in Subsection 3.4.

### 3.1 The Modified BroadCatch Scheme
Let MBC(α, β), 1 ≤ β < α, denote the modified BroadCatch scheme where α is the number of channels and β is the number of base channels. The first β channels will be referred to as base channels and others are referred to as catching channels. We first define three functions \( N_{\alpha,\beta}, P^i_{\alpha,\beta}, \) and \( O^i_{\alpha,\beta} \) to represent the total number of partitioned segments, the number of different segments broadcast on channel \( C_i \), and the offset, i.e., the number of time slots, of the starting broadcast on \( C_i \) relative to \( C_1 \), respectively, as follows:

\[
N_{\alpha,\beta} = 2^{\alpha-\beta} \cdot \beta,
\]

\[
P^i_{\alpha,\beta} = \begin{cases} 
2^{\alpha-\beta} \cdot \beta & 1 \leq i \leq \beta \\
2^{\alpha-\beta+i+1} & \beta+1 \leq i \leq \alpha-1 \\
1 & i = \alpha,
\end{cases}
\]

and

\[
O^i_{\alpha,\beta} = \begin{cases} 
2^{\alpha-\beta} \cdot (i-1) & 1 \leq i \leq \beta \\
2^{\alpha-\beta+i} & \beta+1 \leq i \leq \alpha.
\end{cases}
\]

When context is clear, we drop the subscripts \( \alpha, \beta \) on \( N, P^i \) and \( O^i \). For example, \( N_{6,3} = 2^{6-3} \cdot 3 = 24, \quad P^4_{6,3} = 2^{6-4+1} = 8, \quad \) and \( O^4_{6,3} = 2^{6-4} = 4 \) (see Figure 2). Server transmission rules are now stated below.

**Server Transmission Rules**

1) Partition the video into \( N \) equal segments. The length of each segment is \( l = L/N \).

2) Let each channel have a series of length \( l \) time slots so that each segment can be transmitted within one time slot. The initial time of time slot \( x \) is denoted by \( T_x \) for \( x \geq 0 \).

3) At time \( O^i \cdot l \) for \( 1 \leq i \leq \alpha \), the following transmission is taken on channel \( C_i \). If \( 1 \leq i \leq \alpha - 1 \), channel \( C_i \) broadcasts segments \( S_1, S_2, \ldots, S_{\mu_i} \) periodically while channel \( C_\alpha \) only broadcasts segment \( S_1 \) once every two time slots.

**End of Server Transmission Rules**
Figure 2 shows the segment placement of MBC(6,3) as an example. It is easy to verify that MBC(α,2) is the BroadCatch scheme with α channels. The reception strategy is the same as that of BroadCatch. Take a client who starts downloading S₁ at T₆ from channel C₅ in Figure 2 as an example. The client downloads segments {S₁,S₂}, {S₃,…,S₈}, and {S₉,…,S₂₄} from channels C₅, C₄, and C₁ during T₆ ~ T₇, T₆ ~ T₁₁, and T₈ ~ T₂₃, respectively.

3.2 The Channel Transition Strategy for the MBC Scheme

3.2.1 Preliminaries

The basic concept of our proposed channel transition protocol is to arrange the placement of old and new segments on the same channel without overlap so that channel transition can be conducted seamlessly. Thus, in this subsection, we deduce the time of ceasing transmission old segments on each channel once the channel transition is conducted. We first define some terms.

Definition 2 The transmitting pattern of channel Cᵢ, denoted by TPᵢ in MBC(α,β) is defined as $S₁ \circ S₂ \circ \cdots \circ S_\beta$, where $\circ$ is the concatenation operator.

Definition 3 The index of the segment that occurs on channel Cᵢ, 1 ≤ i ≤ α, at time slot p can be obtained by the following function:
\[
I_{a,\beta}(i, p) = \begin{cases} 
((p - O')) \mod P' + 1, & \text{if } 1 \leq i \leq \alpha - 1 \text{ and } O' \leq p \\
1, & \text{if } i = \alpha \text{ and } p \text{ is odd} \\
0, & \text{otherwise},
\end{cases}
\]

where \( I_{a,\beta}(i, p) = 0 \) means that time slot \( p \) is idle, i.e., no segment is transmitted at time slot \( p \).

Similarly, subscripts \( \alpha, \beta \) will be dropped on \( I(i, p) \) when the context is clear. For example, Table 1 below lists \( TP_i, i = 1, 2, \ldots, 6 \), in MBC(6,3). The index of the segment transmitted on \( C_4 \) at time slot 8, i.e., \( I_{6,3}(4, 8) \), is equal to \( ((p - O')) \mod P' + 1 = ((8 - 4) \mod 8) + 1 = 4 + 1 = 5 \), namely segment \( S_5 \).

Table 1. Transmitting pattern of each channel in MBC(6,3)

<table>
<thead>
<tr>
<th>Channel</th>
<th>( TP_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>( S_1 \circ S_2 \circ S_3 \circ \cdots \circ S_{24} )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( S_1 \circ S_2 \circ S_3 \circ \cdots \circ S_{24} )</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>( S_1 \circ S_2 \circ S_3 \circ \cdots \circ S_{24} )</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>( S_1 \circ S_2 \circ S_3 \circ \cdots \circ S_6 )</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>( S_1 \circ S_2 \circ S_3 \circ S_4 )</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>( S_1 )</td>
</tr>
</tbody>
</table>

Let \( C_e, 1 \leq e \leq \beta \), be the base channel which finishes transmitting \( TP_e \) earliest among all base channels after a new channel is available. Let \( T_\delta \) be its finishing transmission time for transmitting \( TP_e \). Let \( T^{'}_\delta \) be the finishing transmission time of channel \( C_i, 1 \leq i \leq \alpha \), \( i \neq e \), for transmitting the remaining old segments in \( TP_i \) after \( T_\delta \). As we shall show in the next two subsections, channel transition will be conducted after \( T_\delta \). Lemma 4 shows the formula for computing \( T^{'}_\delta, 1 \leq i \leq \alpha, i \neq e \).

**Lemma 4**

\[
T^{'}_\delta = \begin{cases} 
T_\delta + O'((i - e) \mod \beta) + 1 \cdot l, & \text{if } 1 \leq i \leq \beta, \text{ and } i \neq e \\
T_\delta + O' \cdot l, & \text{if } \beta + 1 \leq i \leq \alpha - 1 \\
T_\delta, & \text{if } i = \alpha.
\end{cases}
\]
Proof. See Appendix A.

According to Lemma 4, the base channel $C_i$, $1 \leq i \leq \beta$, can be relabelled as channel $C_{(i-c) \mod \beta+1}$. Without loss of generality, hereafter, we assume that $C_e = C_1$. This means that any channel transition is performed after channel $C_i$ has finished transmitting $TP_i$. We now define some terms and functions that will be used for describing the proposed channel transition protocol.

**Definition 5** A channel sequence $x_1x_2\cdots x_{\alpha-1}$ with respect to $\alpha$ and $\beta$ is called a B-sequence if

$$x_i = \begin{cases} C_{a-i+1} & \text{if } 1 \leq i \leq \alpha - \beta \\ C_{i-a+\beta+1} & \text{if } \alpha - \beta + 1 \leq i \leq \alpha - 1. \end{cases}$$

Equivalently, a B-sequence $x_1x_2\cdots x_{\alpha-1}$ with respect to $\alpha$ and $\beta$ is the sequence $C_aC_{\alpha-1}\cdots C_{\beta+1}C_2C_3\cdots C_{\beta}$.

**Definition 6** For $x_i$ and $x_j$ in a B-sequence with $1 \leq i < j \leq \alpha - 1$, channel $x_i$ starts the transmission of its first transmission pattern earlier than channel $x_j$ transmitting its first transmission pattern. Accordingly, channel $x_i$ finishes transmitting its last transmission pattern of old segments earlier than channel $x_j$ transmitting its last transmission pattern after channel transition is conducted.

Consider the situation when $k$ channels are added to perform PCT. Consequently, there are two B-sequences, say $X$ and $Y$, before and after, respectively, channel transition. Assume that $X = x_1x_2\cdots x_{\alpha-1}$ and $Y = y_1y_2\cdots y_{\alpha+k-1}$. Note that by Definition 6, for $x_i$ and $x_j$ with $1 \leq i < j \leq \alpha - 1$, channel $x_i$ finishes transmitting its last transmission pattern of old segments earlier than channel $x_j$ finishes transmitting the old segments in its last transmission pattern, whereas for $y_i$ and $y_j$ with $1 \leq i < j \leq \alpha + k - 1$, channel $y_i$ starts its transmission of its first new transmission pattern earlier than channel $y_j$ transmitting its first new transmission pattern. The situation of NCT is similar. Instead, channel indexes of old and new B-sequences are with respect to $\alpha$ and $\beta$ and
\( \alpha - k \) and \( \beta - k \), respectively. The remaining question is to accommodate the channels before and after PCT (or NCT) so that old and new segments would not overlap each other.

Let \( C_1, C_2, \cdots, C_a \) be the channels used before PCT and \( A_1, A_2, \cdots, A_k \) be the \( k \) available channels for PCT. Furthermore, \( B\)-sequence \( X = x_1 x_2 \cdots x_{a-1} \) is with respect to \( \alpha \) and \( \beta \). After PCT (respectively, NCT), let \( C_1, C_2, \cdots, C_{a+k} \) (respectively, \( C_1, C_2, \cdots, C_{a-k} \)) be the resulting channels. To accommodate the channels before and after PCT (or NCT), we define four new channel sequences: \( E\)-sequence \( e_1 e_2 \cdots e_{a+k} \), \( F\)-sequence \( f_1 f_2 \cdots f_{a-k} \), \( P\)-sequence \( p_1 p_2 \cdots p_{a+k} \), and \( N\)-sequence \( n_1 n_2 \cdots n_{a-k} \) as follows.

\[
e_i = \begin{cases} C_i & \text{if } i = 1 \\ A_{i-1} & \text{if } 2 \leq i \leq k+1 \\ x_{i-k-1} & \text{if } k+2 \leq i \leq \alpha + k. \end{cases}
\]

\[
f_i = \begin{cases} C_i & \text{if } i = 1 \\ x_{i-1} & \text{if } 2 \leq i \leq \alpha - k. \end{cases}
\]

\[
p_i = \begin{cases} C'_i & \text{if } i = 1 \\ C'_{a+k-i+2} & \text{if } 2 \leq i \leq \alpha - \beta + 1 \\ C'_{i-a+\beta} & \text{if } \alpha - \beta + 2 \leq i \leq \alpha + k. \end{cases}
\]

\[
n_i = \begin{cases} C'_i & \text{if } i = 1 \\ C'_{a-k-i+2} & \text{if } 2 \leq i \leq \alpha - \beta + 1 \\ C'_{i-a+\beta} & \text{if } \alpha - \beta + 2 \leq i \leq \alpha - k. \end{cases}
\]

The mapping function \( P(e_i) = p_i \) for PCT (respectively, \( N(f_i) = n_i \) for NCT) ensures that old and new segments do not overlap each other. Table 2 is used to illustrate the channel accommodation for PCT from MBC(6,3) to MBC(10,7). Similarly, Table 3 is used to illustrate the channel accommodation for NCT from MBC(10,7) to MBC(6,3).

**Table 2. PCT from MBC(6,3) to MBC(10,7)**

<table>
<thead>
<tr>
<th>E-sequence</th>
<th>( C_1 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_4 )</th>
<th>( C_6 )</th>
<th>( C_1 )</th>
<th>( C_4 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-sequence</td>
<td>( C'_1 )</td>
<td>( C'_{10} )</td>
<td>( C_9 )</td>
<td>( C_8 )</td>
<td>( C_7 )</td>
<td>( C_6 )</td>
<td>( C_4 )</td>
<td>( C_5 )</td>
<td>( C_3 )</td>
</tr>
</tbody>
</table>

**Table 3. NCT from MBC(10,7) to MBC(6,3)**

<table>
<thead>
<tr>
<th>F-sequence</th>
<th>( C_1 )</th>
<th>( C_{10} )</th>
<th>( C_9 )</th>
<th>( C_8 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-sequence</td>
<td>( C'_1 )</td>
<td>( C_6 )</td>
<td>( C_5 )</td>
<td>( C_4 )</td>
<td>( C_2 )</td>
<td>( C_3 )</td>
</tr>
</tbody>
</table>
The detailed PCT and NCT protocols will be discussed in the following two subsections, respectively.

### 3.2.2 Positive Channel Transition Protocol

Suppose that PCT is conducted by adding $k$ channels where $k \geq 1$ is an integer. Channel transition is performed after channel $C_i$ has finished transmission of $TP_i$ at time $T_\delta$. We describe the PCT protocol below.

**PCT Protocol:**

1) Allocate $k$ extra channels to deliver the video. The total number of available channels becomes $\alpha + k$ in which $\beta + k$ and $\alpha - \beta$ channels are base channels and catching channels, respectively.

2) At time $T_\delta$, divide the video into $N_{\alpha+k,\beta+k}$ new segments such that each new segment has length $l = L/((\beta + k) \cdot 2^{2-\beta})$.

3) Use function $P$ to accommodate channels for PCT. Let $C'_i$ denote the $i$th new channel.

4) At time $T_\delta + O_{\alpha+k,\beta+k} \cdot l$ for $1 \leq i \leq \alpha + k$, the following actions are taken:
   4.1) Adjust a time slot to be the time for broadcasting a length $l$ segment.
   4.2) Channel $C'_i$, $1 \leq i \leq \alpha + k - 1$, begins to broadcast $P'_{\alpha+k,\beta+k}$ new segments $S'_j$, $1 \leq j \leq P'_{\alpha+k,\beta+k}$, repeatedly while channel $C_{\alpha+k}$ only broadcasts segment $S'_i$ once every two time slots.

*End of PCT Protocol*

As an example, the mapping of old and new channels when transiting from MBC(4,2) to MBC(5,3) is listed in Table 4 below. To give a clear picture, Figure 3 depicts the segment placement when transiting from MBC(4,2) to MBC(5,3). As shown in Figure 3, PCT is conducted at time $T_\delta$. All requests made before or at time $T_\delta - l$ are served with old segments. For clients
arriving after $T_\delta - 1$, they start to receive new segments. Theorem 7 proves the proposed PCT for MBC is seamless.

Table 4. PCT from MBC(4,2) to MBC(5,3)

<table>
<thead>
<tr>
<th>E-sequence</th>
<th>$C_1$</th>
<th>$A_1$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-sequence</td>
<td>$C_1$</td>
<td>$C_1$</td>
<td>$C_2$</td>
<td>$C_1$</td>
</tr>
</tbody>
</table>

Figure 3. Seamless channel transition from MBC(4,2) to MBC(5,3).

**Theorem 7** Our proposed positive channel transition protocol for the MBC scheme is able to seamlessly increase the number of channels allocated to a video.

**Proof.** See Appendix B.

**3.2.3 Negative Channel Transition Protocol**

The behavior of NCT is similar to that of PCT. Assume that NCT is conducted by releasing $k$ channels, where $k$ is an integer satisfying $1 \leq k \leq \beta - 1$. Channel transition is performed after channel $C_i$ has finished transmission of one TP at time $T_\delta$. We now state the proposed NCT protocol.
**NCT Protocol:**

1) The total number of channels becomes $\alpha - k$ in which $\beta - k$ channels are base channels and $\alpha - \beta$ channels are catching channels.

2) At time $T_\beta$, divide the video into $N_{\alpha-k,\beta-k}$ new segments such that each new segment has length $l = L/((\beta - k) \cdot 2^{a-\beta})$.

3) Use function $N$ to accommodate channels for NCT. Let $C_i$ denote the $i$th new channel.

4) At time $T_\beta + O'_{a-k,\beta-k} \cdot l'$ for $1 \leq i \leq \alpha - \beta$, the following actions are taken:
   
   4.1) Adjust a time slot to be the time for broadcasting a length $l'$ segment.

   4.2) Channel $C_i$, $1 \leq i \leq \alpha - k - 1$, begins to broadcast $P'_{a-k,\beta-k}$ new segments $S'_j$, $1 \leq j \leq P'_{a-k,\beta-k}$, repeatedly while channel $C'_{a-k}$ only broadcasts segment $S'_1$ once every two time slots.

5) Release every channel $C_i$, $\beta - k + 1 \leq i \leq \beta$, after all old segments in its last $TP_i$ are transmitted. Actually, by Lemma 4, the released time is at $T_\beta + O' \cdot l$.

**End of NCT Protocol**

Table 5 and Figure 4 illustrate an NCT example when transiting from MBC(5,3) to MBC(4,2). Again, what remains now is that we have to prove that the proposed NCT protocol is seamless as well.

**Table 5. NCT from MBC(5,3) to MBC(4,2)**

<table>
<thead>
<tr>
<th>F-sequence</th>
<th>$C_1$</th>
<th>$C_4$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-sequence</td>
<td>$C'_1$</td>
<td>$C'_4$</td>
<td>$C'_2$</td>
</tr>
</tbody>
</table>
Figure 4. Seamless channel transition from MBC(5,3) to MBC(4,2).

**Theorem 8** Our proposed negative channel transition protocol for the MBC scheme is able to seamlessly decrease the number of channels allocated to a video.

**Proof.** See Appendix B

In the NCT protocol of the MBC method, the number of released channels must be less than that of base channels, i.e., $k < \beta$, since the rear portion of a video is only broadcast on base channels. However, a combination of the NCT protocols of the MBC and BroadCatch schemes can be applied to meet the need of removing more channels than the number of base channels. That is, in the situation that $k \geq \beta$, we can first perform the NCT protocol of the MBC scheme to remove $\beta - 1$ base channels, and then perform the NCT protocol of the BroadCatch scheme to remove the last $k - \beta + 1$ catching channels.

### 3.3 Heterogeneous Scalability

Recall that heterogeneous scalability is defined as the amount of bandwidth that a client has to add on average to benefit from a newly added server channel. In this subsection, we shall show
that the heterogeneous scalability of MBC is zero when PCT is conducted. Again, this implies that clients can benefit from conducting of PCT without paying extra cost. Toward that end, we define some terms first.

**Definition 9** Let $TS(C_i,s,n)$ be the collection of time slots $\{s,s+1,\ldots,s+n-1\}$ of channel $C_i$ in MBC$(\alpha,\beta)$. A cluster $CL_j$ for an integer $j \geq 1$, is defined as

$$CL_j = \{TS(C_i, O^t + m \cdot N, N) | r = ((j-1) \mod \beta) + 1 \text{ and } m = \lfloor (j-1)/\beta \rfloor \} \cup \{TS(C_i, O^s + (j-1) \cdot 2^{a-\beta}, 2^{a-\beta}) | 1 \leq s \leq \alpha \}.$$

As an example, $CL_1$ which is the first cluster of MBC(6,3) is the set $\{TS(C_1,0,24), TS(C_4,4,8), TS(C_5,2,8), TS(C_6,1,8)\}$ shown as the shaded region in Figure 2. Lemma 10 proves that the transmitting patterns of any two clusters are the same.

**Lemma 10** The transmitting patterns of any two clusters $CL_j$ and $CL_w$ of MBC$(\alpha,\beta)$ are identical, for $j \neq w$, $j, w \geq 1$.

**Proof.** It is trivial that base channels of cluster $CL_j$ and $CL_w$ broadcast the same transmitting patterns by Definition 3 and Definition 9. Consider the catching channels $C_i, \beta + 1 \leq i \leq \alpha - 1$, of cluster $CL_1$. Recall that at time $O^t \cdot l$, $C_i$ starts to broadcast segments $S_1, S_2, \ldots, S_p$, periodically. The segment $S_p, 1 \leq p \leq P'$, is thus broadcast at time slot $O^t \cdot 1 + p$, of channel $C_i$ in which the time slot $O^t \cdot 1 + p$ is in $TS(C_i, O^t, 2^{a-\beta})$ and thus belongs to $CL_1$. If we can prove that segment $S_p$ is also broadcast at time slot $(j-1) \cdot 2^{a-\beta} + O^t \cdot 1 + p$ of $C_i$ for $j > 1$, which is in $TS(C_i, (j-1) \cdot 2^{a-\beta} + O^t, 2^{a-\beta})$ and belongs to $CL_j$, then the lemma follows. Since segment $S_p$ is broadcast every $P'$ ($= 2^{a-i+1}$) time slots on $C_i$ from time slot $O^t \cdot 1 + p$, this means that segment $S_p$ is also broadcast at time slot $y \cdot 2^{a-i+1} + O^t \cdot 1 + p$ for any integer $y \geq 0$. Let $y = (j-1) \cdot 2^{i-\beta-1}$. We can find that segment $S_p$ is also broadcast at time slot $(j-1) \cdot 2^{a-\beta} + O^t \cdot 1 + p$ on $C_i$. Finally, let us consider the catching channel $C_a$. Since only segment $S_i$ is broadcast at odd time slots over channel $C_a$, the transmitting pattern on $C_a$ of cluster $CL_j$ is the same as that of cluster $CL_w$. The lemma holds. \[\square\]
Without loss of generality, we shall discuss only the first cluster according to Lemma 10. Lemma 11 proves that a client can download all segments from only one cluster.

**Lemma 11** Clients starting to receive segments within a cluster only fetch the segments from the same cluster.

**Proof.** Assume that a client receives segment $S_{P^i}$ at the last time slot $O^i + 2^{\alpha - \beta} - 1$ of channel $C_i$, $1 \leq i \leq \alpha$, within the first cluster $CL_1$. According to the reception strategy, the next segment $S_{P^i+1}$ can be downloaded from channel $C_m$, $m < i$, before the time of finishing downloading segment $S_{P^i}$. If we can prove that segments $S_{P^i+1}, \ldots, S_{P^\alpha}$, which occur on channel $C_m$, are downloaded within $CL_1$, then the lemma follows. According to the server transmission rules and Definition 9, the transmitting pattern $S_1 \cdots S_{P^i}$ of channel $C_s$, $1 \leq s \leq \alpha - 1$, is broadcast $2^{\alpha - \beta} - 1$ times in $TS(C_s, O^i, 2^{\alpha - \beta})$ in order. Therefore, within $CL_1$ and $CL_2$, the last and first transmission of segment $S_{P^i+1}$ are at time slots $O^m + 2^{\alpha - \beta} + p^i - p^m$ and $O^m + 2^{\alpha - \beta} + p^i$, respectively. By the reception strategy, the client must download segment $S_{P^i+1}$ at time slot $O^m + 2^{\alpha - \beta} + p^i - p^m$. The lemma thus follows.

Lemmas 10 and 11 have shown the temporal relationship of two clusters of $MBC(\alpha, \beta)$; whereas Lemma 12 below exhibits the spatial relationship of two clusters of $MBC(\alpha, \beta)$ and $MBC(\alpha + k, \beta + k)$.

**Lemma 12** Let $CL_1$ and $CL_1'$ be clusters of $MBC(\alpha, \beta)$ and $MBC(\alpha + k, \beta + k)$ respectively, where $k$ is a positive integer satisfying $k \geq 1$. Let $C_i$ and $C_{i+k}$, $1 \leq i \leq \alpha$, the corresponding catching channels of $CL_1$ and $CL_1'$, respectively. Both clusters $CL_1$ and $CL_1'$ exhibit the following properties:

(1) the transmitting patterns of $C_i$ and $C_i'$ comprises the whole video, and are transmitted exactly once.

(2) the transmitting patterns of $C_i$ and $C_{i+k}$ are identical though their segment sizes are different.

(3) the starting offsets, in the number of time slots, of $C_i$ and $C_{i+k}$ are identical.
Proof. According to server transmission rules and Definition 9, the first property directly holds. The starting offset and $TP_i$ of each catching channel $C_i$ of cluster $CL_i$ of $MBC(\alpha, \beta)$ are $O_i$ and $S_i \odot S_2 \odot \cdots \odot S_{p_i}$, $\beta + 1 \leq i \leq \alpha$, respectively, whereas the starting offset and the transmitting pattern of each catching channel $C_{i+k}$ of cluster $CL_{i+k}$ of $MBC(\alpha + k, \beta + k)$ are $O_{i+k}$ and $S_i \odot S_2 \odot \cdots \odot S_{p_{i+k}}$, $\beta + 1 \leq i \leq \alpha$, respectively. Clearly, $O_i = O_{i+k}$ and $P_i = P_{i+k}$. The proof is completed.

For example, Table 6 below lists transmitting patterns of each catching channel of the first clusters in MBC(5,2) and MBC(6,3), respectively.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Transmitting pattern</th>
<th>Channel</th>
<th>Transmitting pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$S_1 \odot S_2 \odot S_3 \odot \cdots \odot S_{16}$</td>
<td>$C_1$</td>
<td>$S_1 \odot S_2 \odot S_3 \odot \cdots \odot S_{24}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$S_1 \odot S_2 \odot S_3 \odot \cdots \odot S_{8}$</td>
<td>$C_4$</td>
<td>$S_1 \odot S_2 \odot S_3 \odot \cdots \odot S_{8}$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$S_1 \odot S_2 \odot S_3 \odot S_4$</td>
<td>$C_5$</td>
<td>$S_1 \odot S_2 \odot S_3 \odot S_4$</td>
</tr>
<tr>
<td>$C_5$</td>
<td>$S_1$</td>
<td>$C_6$</td>
<td>$S_1$</td>
</tr>
</tbody>
</table>

Based on the temporal and spatial relationships of two clusters, as shown in Lemmas 10 and 12, Theorem 13 proves that MBC exhibits seamless channel transition property with zero heterogeneous scalability when PCT is performed.

**Theorem 13** The heterogeneous scalability of the proposed MBC scheme is zero when positive channel transition is conducted.

Proof. According to Lemmas 10, 11, and 12, for clusters $CL_4$ and $CL_{i}$ of $MBC(\alpha, \beta)$ and $MBC(\alpha + k, \beta + k)$, respectively, their corresponding catching channels will broadcast the same transmitting pattern with different segment sizes. Therefore, worst client bandwidth requirements of
CL and CL are identical. As a consequence, the heterogeneous scalability of the proposed MBC scheme is thus zero.

This raises an interesting problem: what is the heterogeneous scalability of the MBC scheme when NCT is performed? We thus slightly modify the definition of the heterogeneous scalability as the amount of bandwidth that a client has to add on average to benefit from a newly added/removed server channel. Since the worst client bandwidth requirements of CL and CL of MBC(α, β) and MBC(α − 1, β − 1) are identical as discussed in the proof of Theorem 13, the heterogeneous scalability of the MBC scheme is zero as well when transiting from MBC(α, β) to MBC(α − 1, β − 1).

As shown in [29], the heterogeneous scalability of BroadCatch scheme is b/4 by adding a catching channel. This implies that the average worst client bandwidths requirement associated with α channels is b/4 more than that with α − 1 channels. Thus, the heterogeneous scalability of the BroadCatch is −b/4 when a catching channel is removed.

We would like to make a point here. Due to the fact that transmitting patterns of clusters CL and CL of MBC(α, β) and MBC(α + k, β + k) are identical, minimum client bandwidth requirements of CL and CL remain the same. This implies that the low bandwidth capacity clients are still able to receive the video as the number of channels increases.

### 3.4 Channel Allocation Policy

In this subsection, we present a channel allocation policy which allows the service provider to be capable of determining the number of channels assigned to each video so as to make the most benefit from the broadcasting service according to the popularities of all videos. Suppose that A channels are allocated to broadcast m videos V₁, V₂, ..., Vₘ. Let αᵢ denote the number of channels allocated to video Vᵢ, and λᵢ,j, 1 ≤ i ≤ m and j ≥ 1, be the ratio of the number of clients with j · b communication capabilities requesting to video Vᵢ to the number of all clients. Since the service provider knows the information about the bandwidth capabilities for all clients in the current system, average startup latency experienced by all clients in MBC(αᵢ, βᵢ) for 1 ≤ βᵢ ≤ αᵢ can be
easily computed. Apparently, according to the distribution of clients’ capabilities, the service provider can broadcast video $V_i$ by the scheme MBC($\alpha_i$, $\beta_i$) that has the minimum average startup latency, denoted by $f_i(\alpha_i)$, so that the average startup latency experienced by all clients requesting to video $V_i$ is minimized.

Let $\lambda_i = \sum_j \lambda_{i,j}$ be the request rate of video $V_i$. Our goal is to minimize the average startup latency of all clients, i.e.,

$$\text{Minimize } \sum_{i=1}^{m} \lambda_i \cdot f_i(\alpha_i)$$

subject to

$$\sum_{i=1}^{m} \alpha_i = A, \text{ such that } \alpha_i \geq \alpha'$$

where $\alpha'$ is the minimum number of channels assigned to each video.

To solve this optimization problem, we apply the similar greedy approach as proposed in [19]. Initially, $\alpha'$ channels are allocated to broadcast each video. Since all videos are assumed to be highly demanded, a reasonable value is $\alpha' \geq 4$ so that the basic service for each video can be guaranteed. Assume that $V_i, 1 \leq i \leq m$, is broadcast by MBC($\alpha_i$, $\beta_i$) at current time. We now show the way to allocate those remaining unassigned channels. First, for each video $V_i, 1 \leq i \leq m$, we calculate its minimum average startup latency $f_i(\alpha_i + 1)$ together with the difference between $f_i(\alpha_i)$ and $f_i(\alpha_i + 1)$ to obtain the reduction of the average startup latency due to the increasing of one channel. We then find the maximum value among all $d_i = f_i(\alpha_i) - f_i(\alpha_i + 1)$, say $d_i$, and allocate one of the unassigned channels to video $V_i$. In this way, the overall reduction of the average startup latency is maximal. The process is repeated until all channels are assigned.

4 Performance Evaluation

In this section, we evaluate the performance of the proposed MBC($\alpha, \beta$) scheme and compare that with the performances of BroadCatch and Fast schemes, in the following four aspects: worst startup latency, average startup latency, buffering requirement, and heterogeneous scalability by varying the value of $\beta$. 

20
4.1 Worst Startup Latency

In this subsection, we focus on worst startup latencies for heterogeneous clients of MBC, BroadCatch, and Fast schemes. For example, let us first consider worst startup latency of a client with capability $2b$ in MBC(6,3). As shown in Figure 2 in Subsection 3.1, client bandwidth requirements associated with one cluster are 1, 2, 2, 2, 2, 2, 2, and 3. If the client arrives after the beginning of time slot 6, s/he has to wait for the length of two time slots. Thus, worst startup latency of the client will be $600 (=2*(7200/24))$ seconds for a 120-min video.

Assume that $\alpha$ server channels are allocated to broadcast a video. Recall that channel transition for MBC and BroadCatch are accomplished by increasing/decreasing the number of base and catching channels, respectively. We thus pick up BroadCatch with $\alpha + k$ server channels, i.e., MBC($\alpha + k,2$), and MBC($\alpha + k,2 + k$) as targets to compare, given that $k$ extra channels are added. Let $\alpha = 10$ and $k=1$. Figure 5 demonstrates worst startup latencies for heterogeneous clients with communication capabilities ranging from $2b$ to $10b$ for MBC(11,3), BroadCatch with 11 channels, i.e. MBC(11,2), and Fast schemes. Table 7 shows the corresponding times for a 120-min video. Here the notation $\rho$ is used to denote the client bandwidth in number of channels.

![Figure 5. Worst startup latencies for heterogeneous clients of MBC(11,3), BroadCatch with 11 channels, i.e. MBC(11,2), and Fast schemes.](image-url)
Table 7. Comparison of worst startup latencies for heterogeneous clients of MBC(11,3), MBC(11,2), and Fast schemes

<table>
<thead>
<tr>
<th></th>
<th>$\rho = 2$</th>
<th>$\rho = 3$</th>
<th>$\rho = 4$</th>
<th>$\rho = 5$</th>
<th>$\rho = 6$</th>
<th>$\rho = 7$</th>
<th>$\rho = 8$</th>
<th>$\rho = 9$</th>
<th>$\rho = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BroadCatch</td>
<td>900 s</td>
<td>450 s</td>
<td>225 s</td>
<td>113 s</td>
<td>56 s</td>
<td>28 s</td>
<td>14 s</td>
<td>7 s</td>
<td>7 s</td>
</tr>
<tr>
<td>Fast scheme</td>
<td>1203 s</td>
<td>517 s</td>
<td>243 s</td>
<td>120 s</td>
<td>60 s</td>
<td>32 s</td>
<td>18 s</td>
<td>11 s</td>
<td>7 s</td>
</tr>
<tr>
<td>MBC(11,3)</td>
<td>600 s</td>
<td>300 s</td>
<td>150 s</td>
<td>75 s</td>
<td>38 s</td>
<td>19 s</td>
<td>10 s</td>
<td>10 s</td>
<td>10 s</td>
</tr>
</tbody>
</table>

As compared with BroadCatch and Fast schemes in Figure 5 and Table 7, MBC(11,3) greatly reduces worst startup latencies by around 30%–50% for clients with communication capabilities ranging from $2b$ to $8b$ while slightly sacrifices those for high-end clients. By increasing the number of base channels when performing channel transition, worst startup latencies for low-end clients of MBC are shorter than those of BroadCatch and Fast schemes.

Assume that MBC allows users with variant bandwidth capabilities from $b$ to $\text{Max} \cdot b$. To give a clear picture, Table 8 shows startup latencies for $\rho = 2$ and $\rho = \text{Max}$ of $\text{MBC}(10 + k, 2 + k)$, $\text{MBC}(10 + k, 2)$, and Fast scheme with $10 + k$ channels by varying the value of $k$.

Table 8. Comparison of worst startup latencies for $\rho = 2$ and $\rho = \text{Max}$

<table>
<thead>
<tr>
<th>$k$</th>
<th>schemes</th>
<th>$\rho = 2$</th>
<th>$\rho = \text{Max}$</th>
<th>schemes</th>
<th>$\rho = 2$</th>
<th>$\rho = \text{Max}$</th>
<th>schemes</th>
<th>$\rho = 2$</th>
<th>$\rho = \text{Max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fast</td>
<td>1203 s</td>
<td>4 s</td>
<td>MBC(11,2)</td>
<td>900 s</td>
<td>7 s</td>
<td>MBC(11,3)</td>
<td>600 s</td>
<td>10 s</td>
</tr>
<tr>
<td>2</td>
<td>Fast</td>
<td>1201 s</td>
<td>2 s</td>
<td>MBC(12,2)</td>
<td>900 s</td>
<td>4 s</td>
<td>MBC(12,4)</td>
<td>450 s</td>
<td>7 s</td>
</tr>
<tr>
<td>3</td>
<td>Fast</td>
<td>1201 s</td>
<td>1 s</td>
<td>MBC(13,2)</td>
<td>900 s</td>
<td>2 s</td>
<td>MBC(13,5)</td>
<td>360 s</td>
<td>6 s</td>
</tr>
<tr>
<td>4</td>
<td>Fast</td>
<td>1200 s</td>
<td>0.4 s</td>
<td>MBC(14,2)</td>
<td>900 s</td>
<td>1 s</td>
<td>MBC(14,6)</td>
<td>300 s</td>
<td>5 s</td>
</tr>
</tbody>
</table>

Here it is easy to find that startup latencies are shorter for low-end clients ($\rho = 2$) by increasing the number of base channels instead of that of catching channels. On the other hand, startup latencies for high-end clients ($\rho = \text{Max}$) are relatively acceptable.

4.2 Average Startup Latency
In this subsection, we focus on average startup latencies for heterogeneous clients of MBC, BroadCatch, and Fast schemes. For example, let us first consider average startup latency of a client with capability $2^b$ in MBC(6,3). As shown in Figure 2 in Subsection 3.1, client bandwidth requirements associated with one cluster are 1, 2, 2, 2, 2, 2, 2, and 3. If the client arrives during one time slot before the beginning of the first time slot of one cluster, s/he has to wait for the length of a half of one time slot on average, i.e., 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 1.5 and 0.5 lengths of time slots 1 to 8, respectively. Thus, average startup latency of the client will be 188 $(=5/8)\times (7200/24)$ seconds for a 120-min video. To be easily verified by hand, we list corresponding times of MBC(6,3) and BroadCatch for a 120-min video in Table 9. Figure 6 demonstrates average startup latencies for heterogeneous clients with communication capabilities ranging from $2^b$ to $10^b$ for MBC(11,3), BroadCatch, and Fast schemes for more information.

<table>
<thead>
<tr>
<th></th>
<th>$\rho = 1$</th>
<th>$\rho = 2$</th>
<th>$\rho = 3$</th>
<th>$\rho = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBC(6,3)</td>
<td>1200 s</td>
<td>188 s</td>
<td>150 s</td>
<td>150 s</td>
</tr>
<tr>
<td>BroadCatch</td>
<td>1800 s</td>
<td>211 s</td>
<td>127 s</td>
<td>113 s</td>
</tr>
</tbody>
</table>

Figure 6. Average startup latencies of MBC(11,3), BroadCatch and Fast schemes.
As compared with BroadCatch and Fast schemes in Figure 6, MBC(11,3) reduces average startup latencies by around 34%~45% for clients with communication capabilities ranging from 2b to 5b while provides relatively acceptable startup latencies for some high-end clients.

4.3 Buffering Requirement

Without loss of generality, consider the situation that a client arrives during the first $1/\beta$ portion of video, in which she/he catches up to the first base channel. The buffering requirement of the client will be the total length of the segments she/he has missed. Recall that a client must wait for the suitable schedule that fits her/his bandwidth capability before she/he starts the downloading process. The maximum buffering space will be $(2^{\alpha-\beta}-1) \cdot \frac{L}{\beta \cdot 2^{\alpha-\beta}}$.

Figure 7 shows the buffering requirements of MBC($\alpha,3$), BroadCatch, and Fast schemes with $\alpha$ channels for $\alpha$ ranging from $3b$ to $12b$. Recall that the maximum buffering space of Fast scheme with $\alpha$ channels is $(2^{\alpha-1}-1) \cdot \frac{L}{2^{\alpha-1}}$. Note that the buffering requirement for MBC($\alpha,\beta$) never exceeds $1/\beta$ portion of the video. Hence the buffering requirement of MBC($\alpha,\beta$) will get down as $\beta$ increases. As compared with BroadCatch and Fast schemes in Figure 7, MBC reduces buffering requirements by more than 30%.

![Figure 7. Buffering requirement vs. server bandwidth for MBC($\alpha,3$), MBC($\alpha,2$), and Fast schemes.](image)
4.4 Heterogeneous Scalability

Figure 8 shows average client bandwidth requirements for the Fast, BroadCatch, MBC(11,3), and MBC(11,6) schemes when performing channel transition by adding $k$ server channels, where negative values of $k$ indicate the NCT operations. As one can see, average client bandwidth requirements of the MBC scheme remain stable while those of the BroadCatch and Fast schemes are fluctuated.

![Figure 8: Average client bandwidth requirements vs. server channels added for MBC(11,3), MBC(11,6), BroadCatch, and Fast schemes.](image)

5 Concluding Remarks

In this paper, we have presented the MBC scheme and proposed a seamless channel transition enhancement on top of it. The proposed scheme is not only adaptive to heterogeneous receivers but also with zero heterogeneous scalability when channel transition is conducted. With the scheme, service provider is capable of adjusting bandwidth allocated to a video according to its popularity so as to make the most benefit out of the available bandwidth. Furthermore, clients can benefit from the conducting of PCT without paying extra cost. As compared with BroadCatch and Fast schemes,
MBC greatly reduces worst startup latencies by around 30%~50% for low-end clients while slightly sacrifices those for high-end clients; MBC reduces buffering requirements by more than 30%.

A Proof of Lemma 4

We consider the following two cases.

**Case 1:** $1 \leq i \leq \alpha - 1$, and $i \neq e$.

Suppose that channel $C_i$ starts to broadcast segment $S_i$ at $T_1$. According to Definition 3, $x$ is equal to $I(i, \delta) = (\delta - O') \mod P' + 1$. Recall that $C_i$ broadcasts segments $S_1, S_2, \ldots, S_p$, periodically. To finish the last $TP_i$ before channel transition, there are $P' - I(i, \delta) + 1$ remaining segments needed to broadcast in order to satisfy the reception strategy. For $1 \leq i \leq \beta, i \neq e$, we can derive that

$$P' - I(i, \delta) + 1 = P' - ((\delta - O') \mod P' + 1)$$

$$= P' - ((O' - O') \mod P')$$

$$= 2^{a - \beta} \cdot \beta - ((2^{a - \beta} \cdot (e - i)) \mod (2^{a - \beta} \cdot \beta))$$

$$= O((i - e) \mod \beta + 1).$$

Similarly, for $\beta + 1 \leq i \leq \alpha - 1$, we can derive that

$$P' - I(i, \delta) + 1 = P' - ((\delta - O') \mod P' + 1)$$

$$= P' - (-O') \mod P'$$

$$= P' - (P' - O')$$

$$= O'.$$

**Case 2:** $i = \alpha$.

According to Definition 3 again, segment $S_i$ is broadcast at odd time slots in $C_\alpha$. Since $\delta$ is even, the last $S_i$ broadcast at $C_\alpha$ before channel transition is at time slot $\delta - 1$. Thus, $T_\alpha' = T_\delta$. The proof is completed.

B Proof of Theorem 7

Assume that $T_\delta$ is the time that channel $C_1$ has finished transmitting $TP_1$. Let $ST(p_j)$ be the starting time for transmitting new segments on channel $p_j$ and $CT(e_j)$ be the time for ceasing to
transmit old segments on channel $e_j$, for $1 \leq j \leq \alpha + k$, where $e_1 e_2 \cdots e_{\alpha + k}$ and $p_1 p_2 \cdots p_{\alpha + k}$ are E-sequence and P-sequence, respectively. Note that $P(e_j) = p_j$ when PCT is performed. If the inequality $ST(p_j) \geq CT(e_j)$ holds at any time, then we can conclude that there exists no overlap between the placements of old and new segments. This guarantees that our proposed PCT protocol is seamless.

We now discuss how $\text{MBC}(\alpha, \beta)$ is seamless transited to $\text{MBC}(\alpha + k, \beta + k)$. Since the inequality $ST(p_j) \geq CT(e_j)$ directly holds for all $1 \leq j \leq k + 1$ which are channels $C_1, A_1, A_2, \cdots, A_k$, we only focus on the mapping of old channels $e_j$ and new channels $p_j$ for $k + 2 \leq j \leq \alpha + k$ (see Figure 11 for the mapping of new and old channels). First, as one can see, all old base channels are reallocated as new base channels when PCT is conducted. Secondly, as to old catching channels, the relationship between $k$ and $\alpha - \beta$ must be taken into consideration. In the case that $k \geq \alpha - \beta$, all old catching channels are reallocated as new base channels; whereas in the case that $k < \alpha - \beta$, $k$ and $\alpha - \beta - k$ old catching channels are reallocated as new base and catching channels, respectively. We discuss these three cases below.

**Case 1**: Channels $e_j$ and $p_j$ with $\alpha + k - \beta + 2 \leq j \leq \alpha + k$.

In this case, all old base channels $C_2, C_3, \cdots, C_{\beta}$ are reallocated as new base channels $C_{2 + k}, C_{3 + k}, \cdots, C_{\beta + k}$, respectively. To show that $ST(p_j) - CT(e_j) \geq 0$ is equivalent to showing that $ST(C_{i + k}) - CT(C_i) \geq 0$, for $2 \leq i \leq \beta$. By Lemma 4 and the definitions of $O'$ and $l$, we have

$$CT(C_i) = T_\delta + O'^{(i-1) \mod \beta + 1} \cdot l = T_\delta + O' \cdot l = T_\delta + (i - 1) \cdot 2^{\alpha - \beta} \cdot \frac{L}{\beta \cdot 2^{\alpha - \beta}} = T_\delta + \frac{i - 1}{\beta} \cdot L. \quad (1)$$

According to the PCT protocol, we know that

$$ST(C_{i + k}) = T_\delta + O'^{i \cdot \mod \beta + 1} \cdot l = T_\delta + (k + i - 1) \cdot 2^{\alpha - \beta} \cdot \frac{L}{(\beta + k) \cdot 2^{\alpha - \beta}} = T_\delta + \frac{k + i - 1}{\beta + k} \cdot L. \quad (2)$$

Clearly, Eq. (2) $-$ Eq. (1) $\geq 0$.

**Case 2**: $k \geq \alpha - \beta$.

In this case, all old catching channels $C_{\beta + 1}, C_{\beta + 2}, \cdots, C_{\alpha}$ are reallocated as new base channels $C_{\beta + 1}, C_{\beta + 2}, \cdots, C_{\alpha - \beta + 2}$, respectively. Again, to show that $ST(p_j) - CT(e_j) \geq 0$ is equivalent to...
showing that $ST(C_{k-1}) - CT(C_{\beta+1}) \geq 0$, for $1 \leq i \leq \alpha - \beta$. According to the PCT protocol and Lemma 4, we can obtain Eqs. (3) and (4), respectively, as follows.

$$ST(C_{k-1}) = T_\delta + O^{k-2}_a \cdot i = T_\delta + \frac{k-i+1}{\beta+k} \cdot L.$$  \hspace{1cm} (3)

$$CT(C_{\beta+1}) = T_\delta + O^{\beta} \cdot i = T_\delta + \frac{L}{2^i \cdot \beta}.$$ \hspace{1cm} (4)

To prove that Eq. (3) — Eq. (4) $\geq 0$ is equivalent to proving $(k-i+1)/(\beta+k)-1/(2^i \cdot \beta) \geq 0$ or, more simply, $2^i \cdot \beta \cdot (k-i+1)-(\beta+k) \geq 0$ for $1 \leq i \leq \alpha - \beta$. Since $k \geq \alpha - \beta$ and $2^{i-1} - i \geq 0$ for positive integer $i$, we can have the following derivation.

$$2^i \cdot \beta \cdot (k-i+1)-(\beta+k) = (k-i)(2^i \cdot \beta - 1) + \beta(2^{i-1}-1) + 2^{i-1} \cdot \beta - i \geq 0.$$

Hence the inequality $ST(C_{k-1}) - CT(C_{\beta+1}) \geq 0$, for $1 \leq i \leq \alpha - \beta$ holds.

**Case 3**: $k < \alpha - \beta$.

In this case, the $k$ old catching channels $C_{\beta+k}, C_{\beta+k-1}, \ldots, C_{\beta+1}$ are reallocated as new base channels $C_2, C_3, \ldots, C_{k+1}$, respectively, and the other $\alpha - \beta - k$ old catching channels $C_\alpha, C_{\alpha-1}, \ldots, C_{\beta+k+1}$ are reallocated as new catching channels $C'_\alpha, C'_{\alpha-1}, \ldots, C'_{\beta+k+1}$, respectively. Thus, we consider the following two subcases.

**Subcase 3.1**: Channels $e_j$ and $p_j$ with $\alpha - \beta + 2 \leq j \leq \alpha + k - \beta + 1$.

In this subcase, to show that $ST(p_j) - CT(e_j) \geq 0$ is equivalent to showing that $ST(C_{k-1}) - CT(C_{\beta+1}) \geq 0$, for $1 \leq i \leq k$. With a similar reasoning as the proof of Case 2, we can find that $ST(p_j) - CT(e_j) \geq 0$ holds.

**Subcase 3.2**: Channels $e_j$ and $p_j$ with $k + 2 \leq j \leq \alpha - \beta + 1$.

In this subcase, to show that $ST(p_j) - CT(e_j) \geq 0$ is equivalent to showing that $ST(C_{\beta+k+1}) - CT(C_{\beta+k+1}) \geq 0$, for $1 \leq i \leq \alpha - \beta - k$. According to the PCT protocol and Lemma 4, we can obtain Eqs. (5) and (6) as follows.
\[ ST(C_{\beta+k}) = T_\delta + O_{\alpha+\beta+k} \cdot l = T_\delta + \frac{L}{2^l \cdot (\beta + k)}. \]  
\[ CT(C_{\beta+k}) = T_\delta + O_{\alpha+\beta+k} \cdot l = T_\delta + \frac{L}{2^l \cdot \beta}. \]  

Since \( 2^k \cdot \beta \geq 2^l \cdot (\beta + k) \), this subcase holds directly. This completes the proof.

Figure 11. The mapping of new and old channels when performing PCT.

## C Proof of Theorem 8

Assume that \( T_\delta \) is the time that channel \( C_i \) has finished transmitting \( TP_i \). Let \( ST(n_j) \) be the starting time for transmitting new segments on channel \( n_j \) and \( CT(f_j) \) be the time for ceasing to transmit old segments on channel \( f_j \), for \( 1 \leq j \leq a-k \), where \( f_1f_2\cdots f_{a-k} \) and \( n_1n_2\cdots n_{a-k} \) are \( F \)-sequence and \( N \)-sequence, respectively. Note that \( N(f_j) = n_j \) when NCT is performed. If the inequality \( ST(n_j) \geq CT(f_j) \) holds at any time, then we can conclude that there exists no overlap between the placements of old and new segments. This guarantees that our proposed NCT protocol is seamless. Since the inequality \( ST(n_j) \geq CT(f_j) \) directly holds for \( C_i \) and released channels, we only consider the inequality on old channels \( f_j \) and new channels \( n_j \) for \( 2 \leq j \leq a-k \). There are two cases to consider.

**Case 1:** Channels \( f_j \) and \( n_j \) with \( 2 \leq j \leq a - \beta + 1 \).
In this case, to show that \( ST(n_i) - CT(f_j) \geq 0 \) is equivalent to showing that \( ST(C_{i-k}) - CT(C_i) \geq 0 \), for \( \beta + 1 \leq i \leq \alpha \). By Lemma 4, we have

\[
CT(C_i) = T_\delta + O^i \cdot l = T_\delta + 2^{a-i} \cdot \frac{L}{\beta \cdot 2^{a-\beta}} = T_\delta + \frac{L}{\beta \cdot 2^{i-\beta}}.
\] (7)

According to the NCT protocol, we can obtain

\[
ST(C_{i-k}) = T_\delta + O^{i-k}_{\alpha-k} \cdot l' = T_\delta + 2^{a-i} \cdot \frac{L}{2^{a-\beta} \cdot (\beta - k)} = T_\delta + \frac{L}{2^{i-\beta} \cdot (\beta - k)}.
\] (8)

Subtracting Eq. (7) from Eq. (8), we get

\[
ST(C_{i-k}) - CT(C_i) = \frac{L}{2^{i-\beta} \cdot (\beta - k)} - \frac{L}{\beta \cdot 2^{i-\beta}} \geq 0.
\]

**Case 2:** Channels \( f_j \) and \( n_j \) with \( \alpha - \beta + 2 \leq j \leq \alpha - k \).

In this case, to show that \( ST(n_j) - CT(f_j) \geq 0 \) is equivalent to showing that \( ST(C_{i}) - CT(C_{i}) \geq 0 \), for \( 2 \leq i \leq \beta - k \). By Lemma 4, we have

\[
CT(C_i) = T_\delta + O^i \cdot l = T_\delta + (i-1) \cdot 2^{a-\beta} \cdot \frac{L}{\beta \cdot 2^{a-\beta}} = T_\delta + \frac{(i-1) \cdot L}{\beta}.
\] (9)

According to the NCT protocol, we know that

\[
ST(C_i) = T_\delta + O^{i-k}_{\alpha-k} \cdot l' = T_\delta + (i-1) \cdot 2^{a-\beta} \cdot \frac{L}{(\beta - k)2^{a-\beta}} = T_\delta + \frac{(i-1) \cdot L}{(\beta - k)}.
\] (10)

Subtracting Eq. (9) from Eq. (10), we get

\[
ST(C_i) - CT(C_i) = \frac{(i-1) \cdot L}{\beta - k} - \frac{(i-1) \cdot L}{\beta} \geq 0.
\]

Consequently, we conclude that our proposed NCT protocol exhibits the property of seamless channel transition.

\[ \Box \]

**References**


