

# Imaging Using Alternate Point Spread Functions: Lenslets with Pseudo-Random Phase Diversity

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**Abstract:** An optical imaging system's resolution can often be limited by the detector array instead of the optics. We present alternate non-impulse like optical point spread functions that overcome the distortions introduced by the detector array.

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The optical point spread function (PSF) is often viewed as the resolution-limiting element of an imager [1]. For this reason conventional optical design procedures typically strive to achieve an impulse-like PSF. When such a well-designed optical system is followed by a semiconductor detector array, the resulting pixel blur and/or aliasing can become the dominant distortions present in the overall imager. In this case it is necessary to revisit the optical design. The optical PSF may now be viewed as a method of encoding the image measurement so as to better tolerate the distortions introduced by the detector array. Within such a framework we find that an impulse-like PSF can be suboptimal.

Figure 1 depicts a schematic example of the problem discussed above. In figure 1a we show the image arising from a single point source in the object space of a conventional imager (i.e., one that uses an impulse-like PSF). Figure 1b shows the image of the same point object in a slightly different location. In this example we see that the large detector size (represented by the square grid) results in an identical measurement for these two object configurations and thus the resolution of this imager would be limited by the pixel spacing. Figures 1c and 1d show the analogous cases for an imager that employs an extended PSF. It is clear that given sufficient measurement SNR, simple neighborhood processing (e.g., correlation) can yield the centroid of this PSF with sub-pixel accuracy. Thus, the resolution achieved via use of an extended PSF can be superior to that of an impulse-like PSF.

We consider the optical PSF to be an encoding of the object prior to measurement. The overall imaging system must therefore be viewed as "computational," involving both the optical front end and electronic decoding [2]. The example shown in figure 1 suggests that the use of an alternate PSF can improve the condition of the decoding process, but that it may also be accompanied by an SNR cost. We will quantify this tradeoff for one simple class of alternate PSF. Our target imager design requires only modest resolution (0.1mrad) and field of view (0.1rad); however, it is characterized by an extreme form factor with an aperture of 36mm and a thickness of only 5mm. We assume the use of a detector array with 7.5 $\mu$ m pixels, full-well capacity of 45,000 electrons and 100% fill factor.

A single-lens solution to this imager design problem is clearly not feasible and a common alternate approach utilizes a lenslet array as shown in figure 2a. Such a lenslet array will produce an array of sub-images that can be post-processed to generate a high-resolution composite image [3]. The optical PSF of this system is simply an array of diffraction-limited spots and is one example of the class of alternate PSFs described above. It is critical to insure that the lenslet

## CMB1

spacing is not commensurate with the pixel spacing in order to insure a well-conditioned decoder. We set the aperture of each lenslet to 5.5mm in order to provide diffraction-limited resolution to each sub-image and to insure that the ratio of detector to lenslet spacing is not an integer. Defining  $\mathbf{x}$  to be the input object vector (i.e., a continuous object sampled at the desired resolution) and  $\mathbf{y}$  to be the measurement vector we use a 1D linear operator formalism to represent the imaging process (optics + detector array) as  $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$ , where  $\mathbf{n}$  is assumed to be additive white Gaussian noise (AWGN). Note that the forward imaging operator  $\mathbf{H}$ , will in general be blockwise shift-invariant so that the linear minimum-mean-squared-error (LMMSE) decoder  $\mathbf{W}$ , will not necessarily take the form of a convolution operator/filter. We define the output of the decoder to be  $\mathbf{z} = \mathbf{W}\mathbf{y}$  and we find the LMMSE decoder  $\mathbf{W}$ , by minimizing  $E\{|\mathbf{z} - \mathbf{x}|^2\}$ , where  $E\{\}$  denotes statistical expectation and is taken over the object class and noise.

We have evaluated the performance of such a lenslet-based computational imaging system in terms of the resolution achieved after decoding (i.e., the resolution of the composite image). We find that for a measurement SNR of 23dB (limited by the shot-noise associated with the full-well capacity of the detector,  $\text{SNR} = 10 \log_{10}(\sqrt{\#\text{photons}})$ ), this imager can achieve a resolution of 1.85mrad. Increasing the SNR to 34db (corresponding to 160 image frames) can improve the resolution only slightly to 1.81mrad. We conclude that the image encoding represented by a PSF consisting of an array of impulse-like functions, is not sufficiently well-conditioned to meet our specification of 0.1mrad.

We seek a method to improve the performance of this lenslet-based imager. Toward the goal of improving the condition number of the forward operator  $\mathbf{H}$ , we will (a) increase the extent of the individual lenslet PSFs (as suggested in figure 1) and (b) add optical PSF diversity to compensate potential MTF nulls. These two features may be achieved by use of pseudo-random phase perturbations as shown in figure 2b; an approach that is motivated by CDMA-based communication systems [4,5]. In our present application the use of pseudo-random phase perturbations will serve to encode the aliasing in such a way as to facilitate later decoding. Our phase perturbations are obtained by filtering a sequence of independent and identically distributed uniform random variables using a Gaussian low pass filter. The resulting phase patterns are parameterized by a roughness parameter  $\Delta$  and a correlation length  $\rho$ . Each lenslet in the array is perturbed by a different sample realization of this underlying random process. We refer to the resulting imager as a pseudo-random-phase enhanced lenslet (PRPEL) imager.

Figure 3 presents a graph of the resolution that is achieved by the PRPEL imager after LMMSE decoding. This figure presents the decoded angular resolution as a function of the phase roughness parameter  $\Delta$ , for several values of SNR. Two important features of this data should be noted. First we see the expected tradeoff between the extent of the PSF and the effect of noise. Because increasing  $\Delta$  increases the size of the optical PSF we find that small  $\Delta$  does not produce a sufficiently well-conditioned imaging operator; while, large  $\Delta$  results in significant SNR degradation. An optimal  $\Delta$  can be readily seen from the data in figure 3. The second important observation is that the phase functions that were applied to the lenslets have indeed improved the performance of the imager over a significant range of  $\Delta$ . We note that with sufficient measurement SNR (roughly 34dB corresponding to 160 image frames in this example), the PRPEL imager can actually meet the original imager specification of 0.1mrad resolution. The

CMB1

COSI presentation will extend these results to include (a) effect of prior knowledge concerning object class, (b) alternate forms of lenslet diversity, (c) iterative nonlinear decoding methods.

References:

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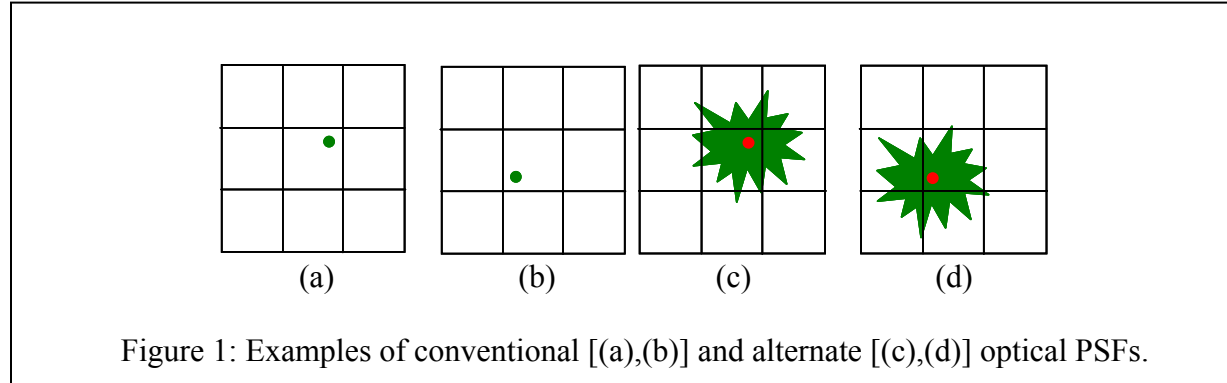


Figure 1: Examples of conventional [(a),(b)] and alternate [(c),(d)] optical PSFs.

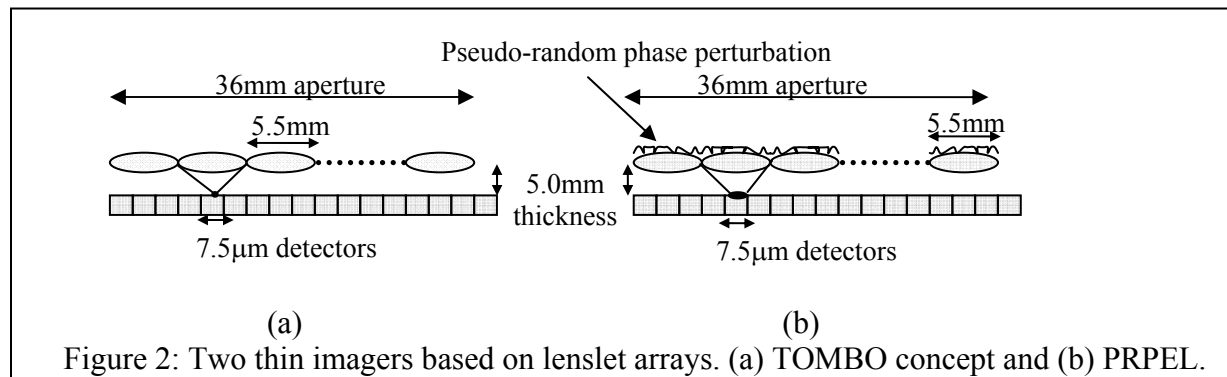


Figure 2: Two thin imagers based on lenslet arrays. (a) TOMBO concept and (b) PRPEL.

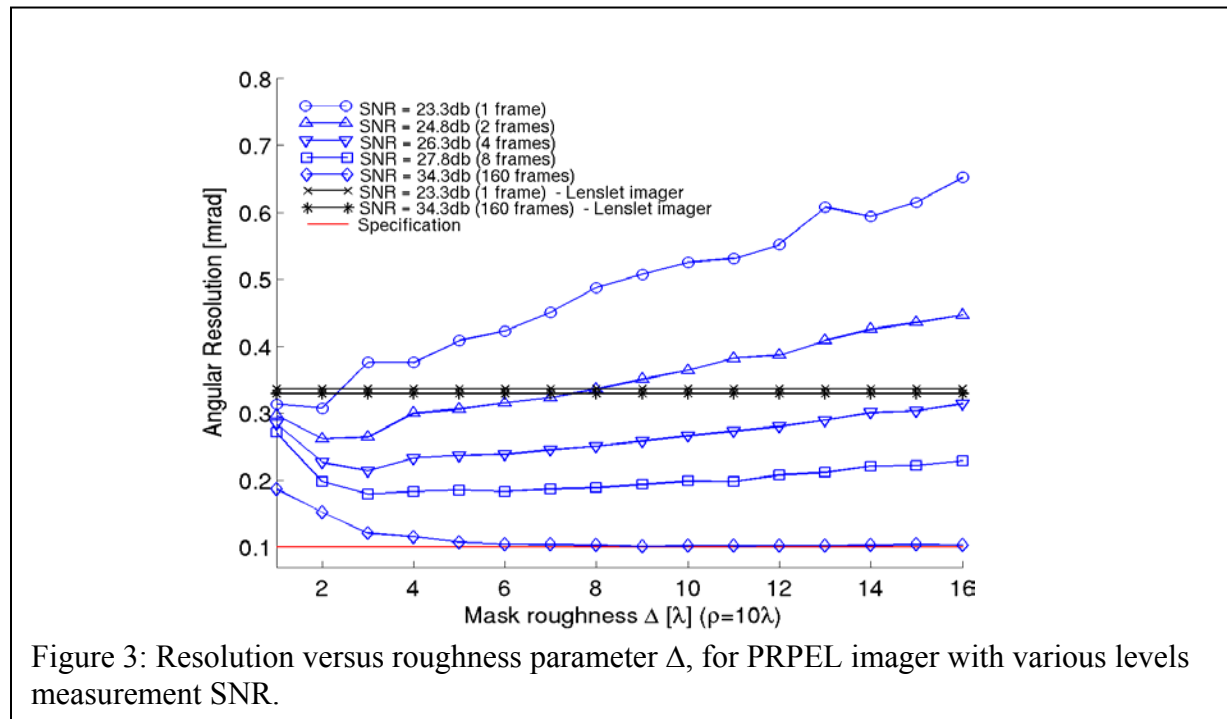


Figure 3: Resolution versus roughness parameter  $\Delta$ , for PRPEL imager with various levels measurement SNR.