OPTIMAL TRAJECTORY GENERATION OF COMPASS-GAIT BIPED BASED ON PASSIVE DYNAMIC WALKING

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ABSTRACT

This work aims at finding optimal trajectory of compass-gait biped walking robot in terms of energy consumption and passive dynamic walking. The planar biped model, an inverted double pendulum, has a free hinge at all joints, i.e., the hip and stance ankle. The impulsive external force is exerted on the swing ankle for short initial period. With the equation of motion, the power of external force is minimized to generate optimal trajectory. The finite element method, where the solution is parameterized by a piecewise second order polynomial, is applied for numerical implementation. The validity of the proposed trajectory is investigated through numerical simulations.

KEY WORDS

Biped robot, passive dynamic walking, compass-gait biped, finite element method

1. Introduction

In recent years, many autonomous walking robots have been realized and the most famous one is the ASIMO, Honda. It can not only walk alongside a human while holding hands, but also run at 3km/hr. However, the ASIMO does not move quite like people do and is inefficient in terms of energy consumption. In other word, ASIMO can operate only for about 30 minutes on a single battery.

McGeer [1] pioneered a class of walking robots, known as passive dynamic walkers. Some of his machines, powered only by gravity, can walk stably and somewhat anthropomorphically down shallow slopes. McGeer shows that walking robots based on passive-dynamic principles can have the human-like efficiency and thus more practical energy use requirements [2]. Many researchers have continued the study of the dynamics of McGeer-like physical and mathematical biped models that have little or no actuation or control. Collins and Ruina et al. [4] built the three-dimensional passive-dynamic walker which has knees and arms. The simpler models have been built: the compass-like model [5], and kneed model [6].

On the other hand, many studies have reported on the gait optimization problem of biped robots with respect to the system’s energy consumption. Optimization of dynamical biped walking has been studied by Hardt [12]. However, the optimized trajectory of biped robot which is based on PDW on a flat plane such as Collins and Ruina built [3] has yet been established. Most of the previous works for optimal gait generation have been done for legged locomotion of dynamical model or passive-dynamic walker in an inclined plane. Ono and Liu [13] built a 3-degree-of-freedom (DOF) walking model that composed of a knee-less stance leg and a 2-DOF swing leg. Divided one swing phase into two sections: controlled swing motion of 2-DOF swing leg with straight stance leg and free motion of straight swing leg with straight stance one. And they obtained an energy efficient gait with minimum input torque of the controlled section by using optimal trajectory planning method [14].

In this paper, we find minimized energy use for passive biped walking on a flat terrain. A robot with a very similar structure has been built by Camp [4]. It has two modes, passive and powered. Each mode has its own set of governing dynamical equations. Instead of having two different dynamical equations in a stride, we suggest unified dynamic equations. The analysis is built around simulation of a single walking step. One step, or cycle of motion, starts at an arbitrary point; say just after a foot collision. A cycle then include the motion between foot collisions. Subject to dynamical equations of the model, the actuated force is minimized to generate optimal trajectory. The validity of the gait proposed is demonstrated in numerical simulations.
2. Compass-Gait Biped Robot

In this paper, a simple planar biped robot shown in Fig. 1, is considered. It has identical rigid legs connected by a passive hinge at the hip and an actuator at the end of trailing leg which generates linear force to move the legs. The trailing leg mostly swings like a free pendulum, but with the boost that the hinge of the pendulum is moving forward, while stance leg is an inverted pendulum on a level, and rigid floor. When the swing foot strikes the floor the plastic (no-slip, and no-bounce) collision is assumed. At about mid-stance, the swing foot is briefly allowed to pass through the floor level, which is inevitable for a walker with straight legs.

The gait consists of a single-support or swing stage and double-support phase which is an instantaneous transition stage [11]. During the single-support phase the robot behaves exactly like an inverted double pendulum with its support point being analogous to the point of suspension of the pendulum. During the double-support phase the support is transferred from one leg to the other. Our model has two modes, active and passive. Unlike the model of Camp [9], we only use one governing dynamical equations during single support phase that have nonlinear constraints that separates two modes.

The governing equations of motion are derived using the iterative Newton-Euler approach [10] as

\[ M(\theta) \ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) = \tau \]

where \( \theta = [\theta_1 \ \theta_2]^T \), \( \tau = [\tau_1 \ \tau_2]^T \). The details of the matrices are

\[ M(\theta) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \]

\[ V(\theta, \dot{\theta}) = \begin{bmatrix} m_l \dot{\theta}_1^2 \dot{s}_2 \\ -m_h \dot{\theta}_2 \dot{s}_2 \end{bmatrix} \]

(3)

\[ G(\theta) = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \]

(4)

where \( M(\theta) \) is a \( 2 \times 2 \) mass matrix of the robot, \( V(\theta, \dot{\theta}) \) is an \( 2 \times 1 \) vector of centrifugal and Coriolis terms, and \( G(\theta) \) is an \( 2 \times 1 \) vector of gravity terms. \( F_x \) is the force on x-axis and \( F_y \) is the force on y-axis. Those are the linear forces to actuate the walking robot. And during the passive-mode, \( F_x \) and \( F_y \) are set to zero, which means no external force on the robot.

3. Optimal Trajectory Generation

To maximize the energy efficiency, a total of energy use should be minimized [14]. In our model, it can be expressed mathematically as the form

\[ J = \int_0^T (F_x^2 + F_y^2) \, dt \]

(5)

We use Finite Element Method (FEM) [8], in order to find the optimal node positions of robot gait in terms of time. Period-one gait cycle is discretized by N time slices; 2nd order polynomial is used to cover one time slice. Each slice is represented in the form

\[ f_n = \frac{1}{2} a_n t^2 + b_n t + c_n \]

(6)

Using the node point continuity of the line and the node point derivative continuity of the line, the following form can be derived as

\[ a_n = \frac{b_{n+1} - b_n}{\Delta t} \]

(8)

\[ \Delta t \left( b_{n+1} + b_n \right) = \theta_{n+1} - \theta_n \]

(9)
Minimization of a scalar function of several variables starting at an initial estimate, which is generally referred to as constrained nonlinear optimization. The boundary conditions are the following

\[
\begin{align*}
\theta_1(0) &= \theta_{1s}, \quad \theta_1(T) = \theta_{1e} \\
\theta_2(0) &= \theta_{2s}, \quad \theta_2(T) = \theta_{2e} \\
\dot{\theta}_1(0) &= \dot{\theta}_{1s}, \quad \dot{\theta}_1(T) = \dot{\theta}_{1e} \\
\dot{\theta}_2(0) &= \dot{\theta}_{2s}, \quad \dot{\theta}_2(T) = \dot{\theta}_{2e}
\end{align*}
\]

where T is the time for a stride. Nonlinear equality constraints are used for separating between active mode and passive mode. In order to prevent the leg off the ground, nonlinear inequality constraints are used as well.

4. Experimental Results

The experiment of the walking on the flat and level ground proposed in the previous section is conducted. The structural, physical and actuation/control parameters are manually tuned. The physical parameters of the robot in all of the following numerical simulations are chosen as \( m = 1.0 \, \text{kg}, l_1 = 1.0 \, \text{m}, \) and \( l_2, h_2 \) have same values with link1. These variables in time over the trajectory are discretized by 20 slices in our trials. The initial condition is assumed same as that for fully passive walking on a slope. Fig. 2 shows the results of passive dynamic walking on a level by energy constraint control. We see that the robot walked passively and had a natural looking gait. Note that the foot-scuffing during the single support phase is ignored. Fig. 3 shows that the impulsive external force is exerted on the swing ankle for active mode. We set this period as first 0.15 sec.

To verify the feasibility of trajectories, the simulation of the dynamic system is performed. The external forces \( F_x \) and \( F_y \) generated by optimization method is applied to the system input. Provided with the initial conditions, it should produce completely the same result as the
The MATLAB built-in ODE-Solver (ODE45) can solve first-order differential equation of the type

\[ M(t, a) \dot{a} = f(a) \]  

In our model, in order to parameterize to ODE-Solver, the equations can be expressed in the matrix form as

\[ M(t, a) \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \dot{a}_3 \\ \dot{a}_4 \end{bmatrix} = \begin{bmatrix} A_{11} \\ A_{12} \\ A_{13} \\ A_{14} \end{bmatrix} \]  

The details of the matrix are

\[ A_1 = a_2 \]
\[ A_2 = \tau_1 - m h_1 g c_1 \]
\[ -l_1 (s_{12} F_x + c_{12} F_y + m (g s_{12} s_2 + g c_{12} c_2 - h_2 (a_3 + a_4)^2 s_2)) \]
\[ A_3 = a_4 \]
\[ A_4 = \tau_1 - m h_2 (s_1 a_2 s_2 + g c_{12}) - l_1 (s_3 F_x + c_3 F_y) \]

Where \( M(t, a) \) is same as (2), \( t \) is the time, and \( a_1 = \theta_1, \ a_2 = \dot{\theta}_1, \ a_3 = \theta_2, \ a_4 = \dot{\theta}_2 \). Fig. 3 shows the

simulated results of trajectory versus time. The initial conditions are set as \( a = [2.0944 \ 0 \ 2.0944 \ 0]^T \). From Fig. 3, we can see that walking pattern is nearly equivalent to that we have solved in the implementation of optimization.

5. Conclusion and Future Work

We have studied the optimality of the passive motion of a simple biped machine, the compass gait walker. Our investigation into the generation of minimum energy symmetric, periodic gaits of passive-dynamic walker gathers together several different research areas in the modelling and control of complex, nonlinear systems. We have presented the strategy for minimizing energy use in passive-walking on a flat plane in the sense of control.

The first obvious task related to this system would be to evaluate the stability of gait cycles – as we have only explored the efficiency of walking robot. The design process for our model has been done iteratively. At each iteration small extensions are made to increase the
complexity of the robot. The next step in this research would be to add knees and curved feet to the current model.

Acknowledgement

This research was supported as a Brain Neuroinformatics Research Program sponsored by the Ministry of Commerce, Industry and Energy; and the Ubiquitous Autonomic Computing and Network Project sponsored by the Ministry of Information and Communication 21st Century Frontier R&D Program in Korea.

References


