Robust Adaptive Control of Manipulators in the Task Space by Dynamical Partitioning Approach

M. R. Soltanpour
Department of Electrical Engineering, Shahid Sattari University, Tehran, Iran, phone: +989124863071; e-mail: m_r_soltanpour@yahoo.com

S. E. Shafiei
Department of Electrical and Robotic Engineering, Shahrood University of Technology, Shahrood, Iran, phone: +989133187993; e-mail: sehshf@yahoo.com

Introduction

It is well known that the kinematics and dynamics of robots are highly nonlinear with existing coupling between joints. To cope with the nonlinearity and uncertainty of the robot dynamics, it has been shown in [1, 2] that a simple joint space controller such as the PD or PID feedback is effective for set-point control. However, in some applications, it is necessary to specify the motion in much more details than simply stating the desired final position.

In trajectory tracking control, a model-based robot controller that is tuned or calibrated to work perfectly using exact models of the system may give rise to very good control performance [3–5]. However, the assumption of having exact models of the robot system also means that the robot is not able to adapt to any changes and uncertainties in its models and environment. For example, when a robot picks up several tools of different dimensions, unknown orientations or gripping points, the overall dynamics of robot changes and is therefore difficult to derive exactly.

The way by which human manipulates his arms easily and skillfully shows that we do not need the exact knowledge of the lengths and dynamics of our arms, the desired joint angles to reach for an object and the exact geometric relationship between our eyes and arms.

In the most of robot applications, a desired position for the end-effector is usually specified in task space or Cartesian space. In order to move the robot end-effector to the desired position, the exact knowledge of the kinematics is required to solve the inverse kinematics problem to generate the desired position in joint space [6–8]. When the control problem is formulated directly in task space, the need to solve the inverse kinematics problem is eliminated [6-8].

To overcome the problem of parameter uncertainty several set-point controllers [6, 7] were proposed in the task space recently. Using the proposed controllers, other open problems such as force control with uncertainties [9] and control of robot fingers with uncertain contact points [10] can be resolved in a unified formulation. However, the results in [6, 7] are focusing on parameter uncertainty in set-point control of robot.

Recently, an adaptive jacobian controller was proposed for trajectory tracking control of robot manipulators in the task space [11, 12]. The controller does not require the exact knowledge of jacobian matrix and dynamic parameters. However in dynamics of robot manipulators, there are unstructured uncertainties such as friction, disturbance and un-model dynamics that may cause an unstable closed loop control system.

In this paper, we propose a task space robust adaptive tracking control scheme based on dynamical partitioning approach that can deal with the uncertainties in both kinematics and dynamics of rigid-link robots. The proposed control scheme does not need accurate information about robot kinematics and dynamics. Sufficient conditions to guarantee system stability are provided and simulation results are presented to show the effectiveness of the control scheme proposed.

Robot dynamics and problem formulation

The joint space dynamics of an \( n \)-link rigid-body robot manipulator can be described by the following second order nonlinear vector differential equation, so-called Euler-Lagrange equation [13]

\[
M(q)\ddot{q} + V_n(q, \dot{q})\dot{q} + G(q) + F_\tau(q) + F_\epsilon(q) + T_q = \tau(t),
\]

where \( q(t) \in \mathbb{R}^n \) denotes the joint angles of the manipulator; \( \dot{q}(t) \) and \( \ddot{q}(t) \) – the vectors of joint velocity and joint acceleration, respectively; \( M(q) \in \mathbb{R}^{n \times n} \) – the inertia matrix which is symmetric and positive definite; \( V_n(q, \dot{q}) \in \mathbb{R}^n \) – a vector function containing coriolis and centrifugal forces;
\[ G(q) \in R^n \] – a vector function consisting of gravitational forces; \( F_q \in R^{n \times n} \) – a diagonal matrix of viscous and dynamic friction coefficients; \( F_t(q) \in R^n \) – the vector of unstructured friction effects such as static friction terms; \( T_q \in R^n \) – the vector of any generalized input due to disturbances or un-modelled dynamics; \( \tau(t) \in R^n \) – the vector function consisting of applied generalized torques.

According to [11, 12], the robot dynamics described above has the following properties:

**Property 1.** The inertia matrix \( M(q) \) is symmetric and positive definite for all \( q \in R^n \) and \( M(q) \) is uniformly bounded above and below. That is

\[
\mu_1 I \leq M(q) \leq \mu_2 I \text{ or } \mu_1 \leq |M(q)| \leq \mu_2 ,
\]

where \( |\cdot| \) stand for the Euclidean norm; \( \mu_1 \) and \( \mu_2 \) – positive constant.

**Property 2.** The matrix \( \dot{M}(q) \) is skew-symmetric. That is

\[
y^T \dot{M}(q)y = 2y^T V_m(q, q)\dot{y}, \forall y, q, \dot{q} \in R^n .
\]

**Property 3.** The left side of (1) can be linearly parameterized. This property may be expressed as

\[
M(q) \dot{q} + V_m(q, q) \dot{q} + G(q) = W(q, \dot{q}, \ddot{q})p ,
\]

where \( p \in R^n \) – a parameter vector; \( W(q, \dot{q}, \ddot{q}) \) – a known matrix of robot function depending on the joint variables, joint velocities and joint accelerations.

In most applications of robot manipulators, a desired path for the end-effector is specified in task space such as visual space or Cartesian space. Let \( X \in R^n \) be a task space vector defined by [11]

\[
X = h(q),
\]

where \( h(\cdot) \in R^n \rightarrow R^n \) – generally a nonlinear transformation describing the relation between the joint space and the task space. According to [11, 12], the task space velocity \( \dot{X} \) is related to joint space velocity \( \dot{q} \) as

\[
\dot{X} = J(q) \dot{q} ,
\]

where \( J(q) \in R^{n \times n} \) is the Jacobian matrix from joint space to task space. From (6) we have

\[
\dot{q} = J^{-1}(q) \dot{X} .
\]

**Dynamical partitioning approach**

In the presence of uncertainty such as unknown parameters, frictions, load variation, disturbances and un-model dynamics, dynamics of robotic systems are usually not totally known. All the terms in (1) can be reduced without loss of any generality into two parts:

\[
\begin{bmatrix}
M(q) = M_k(q) + M_u(q) \\
V_m(q, q) = V_{m_k}(q, q) + V_{m_u}(q, q) \\
G(q) = G_k(q) + G_u(q)
\end{bmatrix}
\]

where \( M_k(q) \), \( V_{m_k}(q, q) \) and \( G_k(q) \) – the known parts; \( M_u(q) \), \( V_{m_u}(q, q) \) and \( G_u(q) \) denote the unknown parts of \( M(q) \), \( V_m(q, q) \) and \( G(q) \) respectively. For design of robust adaptive controller, the following assumptions should be established.

**Assumption 1.** The terms on frictions are bounded as

\[
\|F_q \| \leq \xi_{\tau},
\]

where \( \xi_{\tau} \), \( \xi_{\tau} \) and \( \xi_{\tau} \) – known and positive constants. It is worth mentioning that, although there are several models for representing the influence of friction, frictions are very difficult to determine and all existing models are at best approximate [14–15]. It is also important that, although frictions are passive, but dynamic friction must be compensated for in set-point regulation. Thus, due to the lack of structural information, frictions as well as disturbances must be bounded.

**Robust adaptive control**

From Let us define a vector \( \dot{X}_p \in R^n \) as

\[
\dot{X}_p = \alpha (X_d - X) + \dot{X}_d
\]

where \( \alpha \) – positive constant; \( X \) – measured from a position sensor; \( X_d \in R^n \) – a desired trajectory specified in task space; \( \dot{X}_d \) – the desired velocity specified in task space. Many commercial sensors are available for measurement of \( X \), such as vision systems, electromagnetic measurement systems, position sensitive detectors or laser tracking systems. To design of robust adaptive control, we define a task space sliding vector as

\[
Z_s = \dot{X}_p - \dot{X} = \alpha (X_d - X) + \dot{X}_d - \dot{X}.
\]

We define the task space position error \( X_d - X = e(t) \) and velocity error \( \dot{X}_d - \dot{X} = \dot{e}(t) \) therefore we have

\[
Z_s = \alpha e(t) + \dot{e}(t) .
\]

From (7), we have

\[
\dot{q}_p = J^{-1}(q) \dot{X}_p .
\]

The derivative of \( Z_s \) respect to time can be written as
\[
\ddot{q}_p = J^{-1}(q)\dot{X}_p + J^{-1}(q)\dot{X}_p.
\]

We define a joint space sliding vector as
\[
Z_q = \dot{q}_p - \dot{q}.
\]

Jacobian matrix is multiplied by both side of (16) and from (6) and (12), we have
\[
J(q)Z_q = J(q)\dot{q}_p - J(q)\dot{q} = \dot{X}_p - \dot{X} = Z_q.
\]

The torque \(r(t)\) is related to force \(f(t)\) as [13]
\[
\tau(t) = J^T(q)f(t).
\]

According to (8), (13), (14), (15) and (18) robust adaptive control is proposed to the following form
\[
M_k(q)\ddot{Z}_q + V_m(k,q,\dot{q})\dot{Z}_q + KZ_q + J^T(q)e(t) = W(q,\dot{q},\dot{q})P + \Delta A - u_r.
\]

where \(K\) is positive constant; \(J^T(q)\in R^{m\times n}\) is transpose Jacobian matrix; \(u_r\in R^n\) is adaptive control vector and \(u_r\) is robust control vector. (19) is substituted into (1), thus we have
\[
M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d = M_k(q)\dot{q}_p + V_m,k(q,\dot{q})\dot{q}_p + G_k(q) + KZ_q + J^T(q)e(t) + u_a + u_r.
\]

By substituting (8) into (20), it is simplified as
\[
M_k(q)\dot{q}_p + V_m,k(q,\dot{q})\dot{q}_p + G_k(q) + KZ_q + J^T(q)e(t) + u_a + u_r.
\]

From (14), we have
\[
\dot{q} = \dot{q}_p - Z_q, \quad \ddot{q} = \ddot{q}_p - \dot{Z}_p.
\]

Now, we define the following equation
\[
\Delta A = F_d\dot{q} + F_s(\dot{q}) + T_d,
\]

where \(\Delta A\) is containing unstructured uncertainties. (16), (22) and (23) are substituted into (18) and it is simplified as:
\[
M_k(q)\ddot{Z}_q + V_m,k(q,\dot{q})\dot{Z}_q + KZ_q + J^T(q)e(t) = M_u(q)\dot{q} + V_m,u(q,\dot{q})\dot{q} + G_u(q) + \Delta A - u_a - u_r.
\]

By using of property 3 in section 2, we have
\[
M_u(q)\dot{q} + V_m,u(q,\dot{q})\dot{q} + G_u(q) = W(q,\dot{q},\dot{q})P.
\]

We propose adaptive control \(u_a\) to the following form:
\[
u_a = W(q,\dot{q},\dot{q})\dot{P},
\]

where \(\dot{P}\in R^l\) is estimation of \(P\). \(P - \dot{P} = \tilde{P}\) is defined as estimation error. Then, (23) and (26) are substituted into (24) as:
\[
M_k(q)\ddot{Z}_q + V_m,k(q,\dot{q})\dot{Z}_q + KZ_q + J^T(q)e(t) = W(q,\dot{q},\dot{q})P + \Delta A - u_r.
\]

Stability proof

To prove the stability of closed loop system, the Lyapunov function candidate is presented as
\[
V = \frac{1}{2}e^T e + \frac{1}{2}Z_q^T M(q)Z_q + \frac{1}{2}\tilde{P}^T \tilde{P}.
\]

The derivative of (30) respect to time can be written as
\[
\dot{V} = e^T \dot{e} + \frac{1}{2}Z_q^T M(q)Z_q + Z_q^T M(q)\dot{Z}_q - \tilde{P}^T \dot{\tilde{P}}.
\]

If (27) is substituted in to (29) then by using of property 2 in section 2, we have
\[
\dot{V} = e^T \dot{e} - K\|Z_q\|^2 - (J(q)Z_q)^T e(t) + Z_q^T W\tilde{P} + Z_q^T (\Delta A - u_r) - \tilde{P}^T \dot{\tilde{P}}.
\]

According to (23) and assumptions 1 and 2, we have
\[
\|\Delta A\| \leq \xi_n + \xi_0 \|\| + \xi_0.
\]

We define the following equation
\[
\rho = \xi_n + \xi_0 \|\| + \xi_0.
\]

According to (32) and (33), we have
\[
\|\Delta A\| \leq \rho.
\]

According to (31), (32), (33) and (34), we have
\[
\dot{V} \leq -\alpha \|e(t)\|^2 - K\|Z_q\|^2 + Z_q^T W\tilde{P} + \|Z_q\|\rho - Z_q^T u_r - \tilde{P}^T \dot{\tilde{P}}.
\]

We propose robust control \(u_r\) and adaptive law \(\dot{\tilde{P}}\) to the following forms
\[
\dot{u}_r = \begin{cases} \begin{array}{l} Z_q \rho \|Z_q\|^2 \neq 0 \\ 0 \end{array} \\ \end{cases}, \quad \dot{\tilde{P}} = W^T(q,\dot{q},\dot{q})Z_q.
\]

(36) is substituted into (35) and it is simplified as
\[
\dot{V} \leq -\alpha \|e(t)\|^2 - K\|Z_q\|^2.
\]
According to (37), trajectory tracking error in task space converge to zero therefore, the closed loop system is global asymptotic stable by proposed control. Hence, (19), (26) and (36) are robust adaptive controller.

\[
\begin{align*}
\tau(t) &= M(q)\ddot{q} + V_{m,k}(q,\dot{q})\dot{q} + G_k(q) + KZ_q + \\
&\quad + J^T(q)(\dot{q} + u_r) + u_e, \\
\dot{u}_r &= W(q,\dot{q},\ddot{q})\hat{p}, \\
\dot{u}_d &= W(q,\dot{q},\ddot{q})\hat{p}.
\end{align*}
\tag{38}
\]

The control law (38) is formed by measuring joint positions \( q \), the joint velocities \( \dot{q} \) and the end-effector positions \( X \) and the end-effector velocities \( \dot{X} \). A joint position is commonly measured by an optical encoder and many commercial sensors are available for measurement of \( X \), such as vision systems, electromagnetic measurement systems, position sensitive detectors or laser tracking systems. However, \( \dot{X} \) is rarely measured in robotic applications while vision technique was used to measure the end-effector position \( X \) precisely and then \( \dot{X} \) can be computed by (6).

**Simulation results**

In order to verify the performance of proposed control schemes, as an illustration, we will apply the above presented controller to a two-link elbow robot manipulator. The dynamic of the two-link elbow robot manipulator can be described in the following differential equation

\[
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
h_1(q, \dot{q}) \\
h_2(q, \dot{q})
\end{bmatrix} = \begin{bmatrix}
u_1(t) \\
u_2(t)
\end{bmatrix},
\tag{39}
\]

where \( h_1(q, \dot{q}) = \) \( -m_2l_2q_2^2 + 2m_2l_2\cos(q_2)q_2 - 2m_2l_2\sin(q_2)q_2 \), \( h_2(q, \dot{q}) = \) \( m_2l_2g\cos(q_1 + q_2) + (m_1 + m_2)g\cos(q_1), \)

\[

F_{d1}q_1 + F_{d2}q_2 + T_d, \\
F_{d2}q_2 + F_{d2}q_2 + T_d
\]

where \( l_1 \) and \( l_2 \) — lengths of the first and second links respectively; \( m_1 \) and \( m_2 \) — masses of the first and second links respectively; \( g \) — the gravitational force; \( F_d \) — dynamic friction; \( F_s \) — static friction; \( T_d \) — disturbance and un-modeled dynamics. \( u_1 \) and \( u_2 \) are input torques of the first and second links respectively. Robot parameters which have been used in this simulation are given in Table 1.

<table>
<thead>
<tr>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( g )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>9.8</td>
<td>5</td>
</tr>
<tr>
<td>( F_{d1} )</td>
<td>( F_{d2} )</td>
<td>( F_{s1} )</td>
<td>( F_{s2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameters of Controller are shown in Table 2.

<table>
<thead>
<tr>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( g )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.5</td>
<td>9</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Regression matrix is shown in Table 3. Physical parametric vector of adaptive control is presented as:

\[
\hat{p} = \begin{bmatrix}
w_{1,1} & l_1 & m_1 & m_1l_1 & (m_1 + m_2)l_1 & m_1u_x & m_1u_y & m_1u_z & u_x & u_y & u_z
\end{bmatrix}.
\tag{45}
\]

The Jacobian matrix is in the form of

\[
J(q) = \begin{bmatrix}
-l_1\sin(q_1) - l_1\sin(q_1 + q_2) - l_1\sin(q_1 + q_2)
\end{bmatrix}.
\tag{46}
\]

The kinematic equation is given by

\[
X = \begin{bmatrix}
l_1\cos(q_1) + l_2\cos(q_1 + q_2)
\end{bmatrix}.
\tag{47}
\]

Desired paths in the task space and initial conditions are shown in Fig. 1 and Fig. 2. The task space control is simulated to track the desired path. According to Fig. 3 and Fig. 4, the performance of control system is satisfactory and trajectory tracking errors in task space.

As one can see from these two figures, less than one second, tracking errors converges to zero from 45 mm and 26 mm at joint 1, 2, respectively. Fig. 5 and Fig. 6 illustrate the input control commands of the first and the second joints, respectively which both of them are at acceptable range regarding manipulator parameters and practical standpoint. Physical parameters of adaptive control are estimated as shown in Fig. 7 and Fig. 8.
Fig. 1. Desired path $X_d$ in task space

Fig. 2. Desired Path $X_d$ in Task Space

Fig. 3. Trajectory tracking error $X_d$ in task space

Fig. 4. Trajectory tracking error $X_d$ in task space

Fig. 5. Input control of the first joint

Fig. 6. Input control of the second joint

Fig. 7. Parameter estimations of adaptive control

Fig. 8. Parameter estimations of adaptive control
Conclusions

In this paper, based on the physical properties of the robot manipulator, the adaptive control is designed for compensation of parameter uncertainties. But the robot dynamics have structured and unstructured uncertainties therefore adaptive control cannot succeed in presence of unstructured uncertainties. Therefore, by addition of a robust control based on dynamical partitioning approach to adaptive control, we can design a robust adaptive control for compensation of structure and unstructured uncertainties in the task space. In this method, trajectory tracking errors in task space are directly observed by sensor and we do not necessary to use of inverse kinematic. It was proven that the closed loop system has global asymptotic stability. Simulation results are shown good performance of proposed control.

References