AUTOMATIC CAR STEERING CONTROL

BRIDGES OVER THE DRIVER REACTION TIME

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Abstract:
A car driver needs at least five hundred milliseconds before he can react to unexpected yaw motions. During this time the uncontrolled car may produce a dangerous yaw rate and sideslip angle. Automatic steering control for disturbance rejection is designed such that it bridges over the driver reaction time, but returns the full steering authority to the driver thereafter. The solution is robust with respect to uncertainties in the road-tire contact and in the mass and velocity of the vehicle. The model representation is scaled by methods of similarity mechanics.

1 Introduction

Critical car driving situations arise, when a disturbance torque acts on the vehicle. Examples are crosswind, flat tire, braking on a slippery road and gas release in a curve. It takes the driver at least five hundred milliseconds to react to the resulting yaw motion of his car. During this time the tire sideforce may already reach its physical limits. A delayed overreaction of the driver may also cause driver-induced oscillations. Such dangerous situations can be avoided by robust decoupling of car steering [1, 2]. On the other hand, the driver should not be cheated by the feedback control system. In particular he should feel a similar response to his steering commands as in the uncontrolled car. This applies both to the immediate reaction after a step input at the steering wheel and to the required steering-wheel angle for steady-state cornering. These requirements call for a modification of the decoupling control law.

In section 2 of this paper the car steering model is first scaled by application of similarity mechanics [3] in order to simplify the further analysis. In section 3 the properties of the robust decoupling control law are analyzed. Section 4 introduces the modified control law. In section 5 we discuss some of the benefits that come from the application of similarity mechanics and the meaning of the various similarity numbers. Some simulation results are shown in section 6 to compare the conventional car with both controlled cars.

2 Scaling of the car steering dynamics

Consider the vehicle of Fig. 1. Vehicle data are usually given

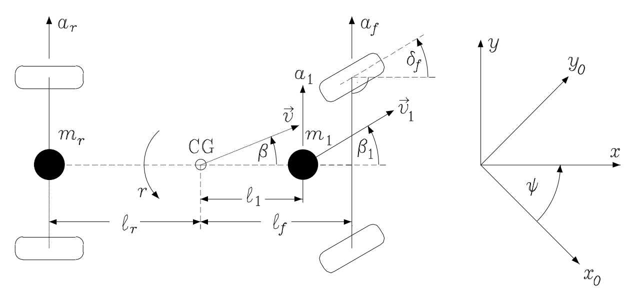

Fig 1: Vehicle with arbitrary mass distribution

in the form of ℓ_r, ℓ_f, vehicle mass m, and moment of inertia J w.r.t. a vertical axis through the center of gravity (CG). These quantities are related with the quantities m_1, m_r, ℓ_1 of Fig. 1 by

$$m = m_1 + m_r \tag{1}$$
$$J = m_1\ell_1^2 + m_r\ell_r^2 \tag{2}$$
$$m_1\ell_1 = m_r\ell_r \tag{3}$$

(2) may be expressed using (3) as

$$J = m_r\ell_r\ell_1 + m_1\ell_r\ell_1$$

and with (1)

$$J = m\ell_r\ell_1$$

The resulting length is

$$\ell_1 = \frac{J}{m\ell_r}$$

There is a one-to-one relationship between the parameter sets $\{m, J, \ell_r, \ell_f\}$ and $\{m, \ell_1, \ell_r, \ell_f\}$. We will use ℓ_1 in the derivation of the model. ψ is the yaw angle between the vehicle center line x and an inertially fixed direction x_0. Input to the system is the front-wheel steering angle δ_f, output is the yaw rate $r = \dot{\psi}$ measured by a gyro. For small sideslip angle β and small steering steering angle δ_f the standard single-track model of car steering [4] is

$$\begin{bmatrix} mv(\dot{\beta} + r) \\ J\dot{r} \end{bmatrix} = \begin{bmatrix} f_y \\ m_z \end{bmatrix} + \begin{bmatrix} 0 \\ m_d \end{bmatrix} \tag{4}$$

Only a disturbance torque m_d is assumed and no disturbance lateral force, because the latter is easily compensated by the driver in the course of his normal steering.
The steering force f_y and torque m_z are generated by the lateral tire forces $f_f(\alpha_f)$ and $f_r(\alpha_r)$ via

$$\begin{bmatrix} f_y \\ m_z \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \ell_f & -\ell_r \end{bmatrix} \begin{bmatrix} f_f(\alpha_f) \\ f_r(\alpha_r) \end{bmatrix} \tag{5}$$

The tire forces depend on the tire sideslip angles α_f and α_r as illustrated in Fig. 2 for the front wheel.

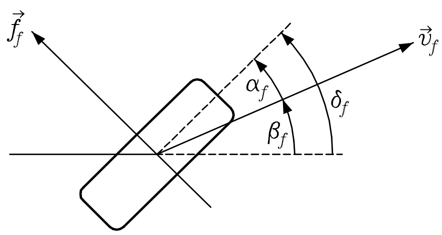

Fig 2: Variables of the tire model

The sideslip angle at the front mass β_1 will be introduced as a state variable. The sideslip angles at the CG (β) and at the axles $(\beta_r), (\beta_f)$ are related with β_1 by the kinematic relations for small angles

$$\begin{aligned} \beta &= \beta_1 - \frac{\ell_1}{v} r \\ \beta_f &= \beta_1 + \frac{\ell_f - \ell_1}{v} r \\ \beta_r &= \beta_1 - \frac{\ell_1 + \ell_r}{v} r \end{aligned} \tag{6}$$

The local velocity vector $\vec{v}_f$ forms the (chassis) sideslip angle β_f with the car body and the tire sideslip angle α_f with the tire direction, thus

$$\begin{aligned} \alpha_f &= \delta_f - \beta_f = \delta_f - \beta_1 - \frac{\ell_f - \ell_1}{v} r \\ \alpha_r &= \delta_r - \beta_r = \delta_r - \beta_1 + \frac{\ell_1 + \ell_r}{v} r \end{aligned} \tag{7}$$

In this paper we do not use rear wheel steering, i.e. $\delta_r \equiv 0$. The tire force characteristics are linearized as

$$\begin{aligned} f_f(\alpha_f) &= c_f \alpha_f \\ f_r(\alpha_r) &= c_r \alpha_r \end{aligned} \tag{8}$$

where the cornering stiffnesses c_f and c_r are uncertain parameters that vary with the road tire contact. We assume $c_f = \mu c_{f0},\ c_r = \mu c_{r0}$ where c_{f0} and c_{r0} are nominal values for the dry road and $\mu \in [\mu^- \,;\, 1],\ \mu^- > 0$ is an uncertain parameter. Other uncertain parameters are the vehicle mass $m \in [m^- \,;\, m^+]$ and velocity $v \in [v^- \,;\, v^+]$. The model (4), (5) with the above equations substituted becomes

$$\begin{bmatrix} mv(\dot{\beta}_1 - \frac{\ell_1}{v}\dot{r} + r) \\ m\ell_r\ell_1\dot{r} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \ell_f & -\ell_r \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} \mu c_{f0}(\delta_f - \beta_1 - \frac{\ell_f - \ell_1}{v} r) \\ \mu c_{r0}(-\beta_1 + \frac{\ell_1 + \ell_r}{v} r) \end{bmatrix} + \begin{bmatrix} 0 \\ m_d \end{bmatrix}$$

and, solving for $\dot{\beta}_1$ and $\dot{r}$,

$$\begin{bmatrix} \dot{\beta}_1 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{\ell_f + \ell_r}{m\ell_r v} & 0 \\ \frac{\ell_f}{m\ell_1\ell_r} & -\frac{1}{m\ell_1} \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} \mu c_{f0}(\delta_f - \beta_1 - \frac{\ell_f - \ell_1}{v} r) \\ \mu c_{r0}(-\beta_1 + \frac{\ell_1 + \ell_r}{v} r) \end{bmatrix} -$$

$$- \begin{bmatrix} 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} \frac{1}{m\ell_r v} \\ \frac{1}{m\ell_1\ell_r} \end{bmatrix} m_d \tag{9}$$

This form shows that $\dot{\beta}_1$ does not depend directly on the forces at the rear tires. An indirect coupling occurs through the r-terms in the equation for $\dot{\beta}_1$. These r-terms will be cancelled by the decoupling control law. The state equations of the system are

$$\begin{bmatrix} \dot{\beta}_1 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} \begin{bmatrix} \beta_1 \\ r \end{bmatrix} + + \begin{bmatrix} \frac{\mu c_{f0}\left(\ell_f + \ell_r\right)}{m\ell_r v} \\ \frac{\mu c_{f0}\ell_f}{m\ell_1\ell_r} \end{bmatrix} \delta_f + \begin{bmatrix} \frac{1}{m\ell_r v} \\ \frac{1}{m\ell_1\ell_r} \end{bmatrix} m_d \qquad (10)$$

$$\begin{aligned} c_1 &= -\frac{\mu c_{f0}\left(\ell_f+\ell_r\right)}{m\ell_r v} \\ c_2 &= -1-\frac{\mu c_{f0}\left(\ell_f+\ell_r\right)\left(\ell_f-\ell_1\right)}{m\ell_r v^2} \\ c_3 &= \frac{\mu\left(c_{r0}\ell_r - c_{f0}\ell_f\right)}{m\ell_1\ell_r} \\ c_4 &= -\frac{\mu\left(\ell_1\left(c_{f0}\ell_f - c_{r0}\ell_r\right) - c_{f0}\ell_f^2 - c_{r0}\ell_r^2\right)}{m\ell_1\ell_r v} \end{aligned}$$

This equation contains seven independent parameters ($\ell_1, \ell_f, \ell_r, m, v, \mu c_{f0}, \mu c_{r0}$) involving three basic units (m, kg, s). Thus we know by Buckingham's theorem [3] that $7 - 3 = 4$ dimensionless similarity numbers suffice to characterize the system, provided that also the variables and the time are scaled appropriately. There are many ways to choose such similarity numbers. We have chosen a set of numbers with some physical meaning and the property that the uncertain operating point m, v, μ enters only into one number. Let

$$\begin{aligned} P_J &= \frac{\ell_1}{\ell_f} && \text{mass distribution number} \\ P_\ell &= \frac{\ell_r}{\ell_f} && \text{CG location number} \\ P_c &= \frac{c_{r0}}{c_{f0}} && \text{cornering stiffness ratio} \\ P_v &= v\sqrt{\frac{m}{\mu c_{f0}\ell_f}} && \text{operating point number} \end{aligned} \qquad (11)$$

Also introduce the dimensionless state and input variables respectively

$$\begin{aligned} \tilde{r} &= \frac{\ell_f}{v} r \\ \tilde{\beta}_1 &= \beta_1 \\ \tilde{\delta}_f &= \delta_f \\ \tilde{m}_d &= \frac{1}{\mu c_{f0}\ell_f} m_d \end{aligned} \qquad (12)$$

and scale the time by

$$\tilde{t} = t\sqrt{\frac{\mu c_{f0}}{m\ell_f}} \qquad (13)$$

In the Laplace transformed equation (10) with $s\beta_1(s)$ and $sr(s)$ on the left hand side, the scaled Laplace variable becomes

$$\tilde{s} = s\sqrt{\frac{m\ell_f}{\mu c_{f0}}} \qquad (14)$$

such that $\tilde{s}\tilde{t} = st$. With the above substitutions (11),(12),(14), the Laplace transformed state equations (10) become

$$\begin{bmatrix} \tilde{s}\tilde{\beta}_1(\tilde{s}) \\ \tilde{s}\tilde{r}(\tilde{s}) \end{bmatrix} = \begin{bmatrix} c_5 & c_6 \\ c_7 & c_8 \end{bmatrix} \begin{bmatrix} \tilde{\beta}_1(\tilde{s}) \\ \tilde{r}(\tilde{s}) \end{bmatrix} + + \begin{bmatrix} \frac{1}{P_v}\left(1+\frac{1}{P_l}\right) \\ \frac{1}{P_J P_l P_v} \end{bmatrix} \tilde{\delta}_f(\tilde{s}) + \begin{bmatrix} \frac{1}{P_l P_v} \\ \frac{1}{P_J P_l P_v} \end{bmatrix} \tilde{m}_d(\tilde{s}) \quad (15)$$

$$\begin{aligned} c_5 &= -\frac{1}{P_v}\left(1+\frac{1}{P_l}\right) \\ c_6 &= \frac{P_J - 1 + P_l\left(P_J - 1 - P_v^2\right)}{P_l P_v} \\ c_7 &= \frac{1}{P_v P_J}\left(P_c - \frac{1}{P_l}\right) \\ c_8 &= \frac{P_J - 1 - P_c P_l\left(P_l + P_J\right)}{P_J P_l P_v} \end{aligned}$$

3 Robust decoupling control law

Robust decoupling [1] is achieved by the feedback control law

$$\delta_f(s) = \frac{1}{s}\left(F(v) i_L \delta_L(s) - r(s) + \frac{\ell_f - \ell_1}{v} s\, r(s)\right) \quad (16)$$

δ_L is the steering wheel input, i_L denotes the transmission ratio of the steering gear. $F(v)$ is the steady-state yaw rate response to a unit step input for a nominal model ($\mu = \mu_0$

and $m = m_0$). It can be computed by (10) with $\dot{\beta}_1 = \dot{r} = m_d = 0$ and $\delta_f = 1$.

$$F(v) = \frac{c_{f0}c_{r0}(\ell_f + \ell_r)v}{c_{f0}c_{r0}(\ell_f + \ell_r)^2 + (c_{r0}\ell_r - c_{f0}\ell_f)\dfrac{m_0}{\mu_0}v^2} \quad (17)$$

For a general investigation, again the scaled (dimensionless) representation is employed. Therefore we introduce the dimensionless variable

$$\tilde{\delta}_L = i_L \delta_L \quad (18)$$

The introduction of a nominal virtual mass m_0/μ_0 makes it necessary to define a new similarity number.

$$P_m = \frac{m_0}{\mu_0}\frac{\mu}{m} \qquad \text{virtual mass number} \quad (19)$$

The dimensionless form of (17) is

$$\tilde{F} = \frac{\ell_f}{v}F(v)$$

$$= \frac{P_c(1+P_l)}{P_c(1+P_l)^2 + (P_c P_l - 1)P_m P_v^2} \quad (20)$$

With the above substitutions (11),(12),(14),(18),(20) the control equation (16) becomes

$$\tilde{\delta}_f(\tilde{s}) = \frac{1}{\tilde{s}}\left[((1-P_J)\,\tilde{s} - P_v)\,\tilde{r}(\tilde{s}) + P_v\,\tilde{F}\,\tilde{\delta}_L(\tilde{s})\right] \quad (21)$$

Consider now the lateral acceleration a_1 at the front mass m_1, see Fig. 1. It is related with the model quantities by

$$a_1 = a_{CG} + \ell_1\dot{r} = \frac{f_f(\alpha_f) + f_r(\alpha_r)}{m} + \ell_1\dot{r} \quad (22)$$

We substitute (7) and (8) to obtain the output equation for a_1 (Laplace-transformed)

$$a_1(s) = -\frac{\mu(c_{f0}+c_{r0})}{m}\beta_1(s) + \frac{\mu c_{f0}}{m}\delta_f(s) +$$

$$+\left(\ell_1 s + \mu\frac{c_{r0}(\ell_r+\ell_1) - c_{f0}(\ell_f - \ell_1)}{mv}\right) r(s) \quad (23)$$

The dimensionless lateral acceleration at the front mass is defined as

$$\tilde{a}_1 = \frac{\ell_f}{v^2}a_1 \quad (24)$$

Equation (23) now can be rewritten in its dimensionless representation

$$P_v^2\tilde{a}_1(\tilde{s}) = -(1+P_c)\tilde{\beta}_1(\tilde{s}) + \tilde{\delta}_f(\tilde{s}) +$$

$$+ (P_J - 1 + P_c(P_J + P_l) + P_J P_v\tilde{s})\tilde{r}(\tilde{s}) \quad (25)$$

The transfer function from the yaw disturbance torque to the yaw rate ($\tilde{\delta}_L(\tilde{s}) \equiv 0$) is obtained by substituting (21) into (15) and solving for $\tilde{r}(\tilde{s})$.

$$\tilde{r}_{dec}(\tilde{s}) = \frac{R_{dec}(\tilde{s})}{D_{dec}(\tilde{s})}\,\tilde{m}_d(\tilde{s}) \quad (26)$$

$$R_{dec}(\tilde{s}) = \tilde{s}R(\tilde{s}) = \tilde{s}\,(P_v\tilde{s} + 1 + P_c)$$

$$D_{dec}(\tilde{s}) = (P_l P_v\tilde{s} + 1 + P_l)\cdot$$
$$\cdot\left(P_J P_v\tilde{s}^2 + (P_J + P_l)\,P_c\tilde{s} + P_c P_v\right)$$

The properties of the decoupling control law are

- The coupling from the yaw mode into the lateral acceleration a_1 has been removed. Thereby the steering transfer function from $\tilde{\delta}_L(\tilde{s})$ to $\tilde{a}_1(\tilde{s})$ is of first order ($\tilde{m}_d(\tilde{s}) \equiv 0$).

$$\tilde{a}_1(\tilde{s}) = \frac{(1+P_l)\tilde{F}}{1 + P_l + P_l P_v\tilde{s}}\tilde{\delta}_L(\tilde{s}) \quad (27)$$

  The influence of the uncertain parameters m,v and μ has been drastically reduced by decoupling. Theoretically the steering dynamics can be made arbitrarily fast by additional feedback of a_1, measured by an accelerometer, to δ_L. The task of the driver is only to keep the point mass m_1 on top of his planned path by commanding a_1. He does not have to care about the stable yaw motion [1].

- For a step input at the yaw disturbance m_d (e.g. from crosswind, μ-split braking) the conventional car has a nonzero steady state value of the yaw rate, whereas for the decoupled car the yaw rate quickly returns to zero [2]. The steady-state effect can be seen from (26), where $\lim_{s\to 0}\dfrac{R_{dec}(s)}{D_{dec}(s)} = 0$. Considerable safety advantages have been shown in [2].

On the other hand, the decoupling control law (16) also has some disadvantages that motivate the modifications in the next section.

- By the integration in the control law there is no immediate reaction of a_1 after a step command at δ_L. The driver feels that the car is not reacting as promptly as the conventional car. Therefore a direct throughput from δ_L to δ_f will be provided, i.e. $\delta_f = i_L\delta_L + \delta_c$, where δ_c is produced by the controller (16). This change does not affect the feedback path from r to δ_f and therefore the disturbance attenuation properties of the control law (16) are preserved.

- The yaw damping is decreased at high velocity. In [2] a good yaw damping was achieved by rear-wheel steering, which is not available under the assumptions of this paper. The driver is not so much concerned about the first overshoot in r (which is similar to the conventional car) but about the following undershoot, which occurs about 0.8 seconds after a steering wheel step input. One remedy is a velocity scheduled feedback of the lateral rear axle acceleration a_r [5]. Another one is to provide the decoupling action only for the first 0.5 seconds after a step input and thereafter to return smoothly to the properties of the conventional car, i.e. δ_c should follow the integral action only initially and then return to zero. Regarding a disturbance step m_d this concept helps the driver during the first half second by providing an additional steering angle but thereafter it returns the full authority gradually back to the driver. Reasonable yaw damping can be achieved by tuning the parameters of the filter which is employed to implement the fading decoupling action.
- The feedback from r provides an additional term to δ_f. In steady-state cornering there is no need for this term, the controlled car should behave like the conventional car in this situation, i.e. the driver should apply the same δ_L. The above modification takes care of this requirement.

4 The modified control law

The modified control law is

$$\delta_f(s) = i_L \delta_L(s) + \delta_c(s) \tag{28}$$

$$\delta_c(s) = \frac{s}{s^2 + 2D\omega_0 s + \omega_0^2} x_1(s)$$

$$x_1(s) = F(v) i_L \delta_L(s) - r(s) + \frac{\ell_f - \ell_1}{v} s\, r(s)$$

Now there is a direct throughput of the input by the steering wheel δ_L to the front wheel steering angle. The pure integrator has been replaced by a dynamical filter. The initial behaviour ($s \to \infty$) of this filter to any input is the same as the response of an integrator. But the steady-state output of the filter ($s \to 0$) is zero. In the meantime the filter is unloaded so that after some time the responsibility is softly returned to the driver. It also leads to the same steady-state cornering as of the conventional car.
Hence by the modified control law both the immediate and the long term reaction to any driver input is the same for the controlled car as for the uncontrolled car. But the transient behaviour is different as well as the disturbance rejection property of both systems. Since we add additional dynamics to the system by applying the modified control law, new parameters are introduced to the system. They are independent of each other and of the already introduced parameters. This increases the number of independent parameters to ten ($\ell_1, \ell_f, \ell_r, m, v, \mu c_{f0}, \mu c_{r0}, m_0/\mu_0, D, \omega_0$). According to Buckingham's theorem the number of similarity numbers which suffice to characterize the system now is $10 - 3 = 7$. In addition to (11) and (19) we define two more similarity numbers

$$P_D = D \qquad \text{filter damping number}$$

$$P_\omega = \omega_0 \sqrt{\frac{m_0 \ell_f}{\mu_0 c_{f0}}} \qquad \text{filter bandwidth number} \tag{29}$$

With the substitutions (11),(12),(14),(18),(29) the control equation (28) becomes

$$\tilde{\delta}_f(\tilde{s}) = \tilde{\delta}_L(\tilde{s}) + \frac{\tilde{s}}{\tilde{s}^2 + \frac{2P_D P_\omega}{\sqrt{P_m}} \tilde{s} + \frac{P_\omega^2}{P_m}} \tilde{x}_1(\tilde{s}) \tag{30}$$

$$\tilde{x}_1(\tilde{s}) = ((1 - P_J)\, \tilde{s} - P_v)\, \tilde{r}(\tilde{s}) + P_v\, \tilde{F}\, \tilde{\delta}_L(\tilde{s})$$

5 Meaning of the similarity numbers

Using a dimensionless representation of a system's equations and introducing dimensionless similarity numbers has several benefits. Most interesting with respect to robust control theory is the fact, that the number of uncertain parameters of a system possibly can be reduced. Thus the analysis and design of controlled systems are simplified. Without employing similarity mechanics, for the analysis of the model (10) a two-dimensional operating domain has to be checked. The three uncertain parameters (μ, m, v) have to be varied in order to cover the whole operating domain. Since μ and m only occur in the combination of m/μ, the variation of uncertain parameters of this plant is a two-dimensional problem.
After making use of Buckingham's theorem, we succeeded to put all uncertain parameters of the conventional car into one similarity number P_v. Hence all possible operating points of the system in its dimensionless form can be represented by one single number. The other similarity numbers (P_J, P_l, P_c) are invariant for a specific car due to construction, tire properties etc.
If for one value of P_v an analysis is performed (e.g. a simulation or an eigenvalue computation), the results stand for an infinite number of uncertain parameter values, i.e. one can obtain the same P_v for either large values of v or m or a small value of μ. The dimensionless time-domain results of a simulation can be transformed to original time-domain

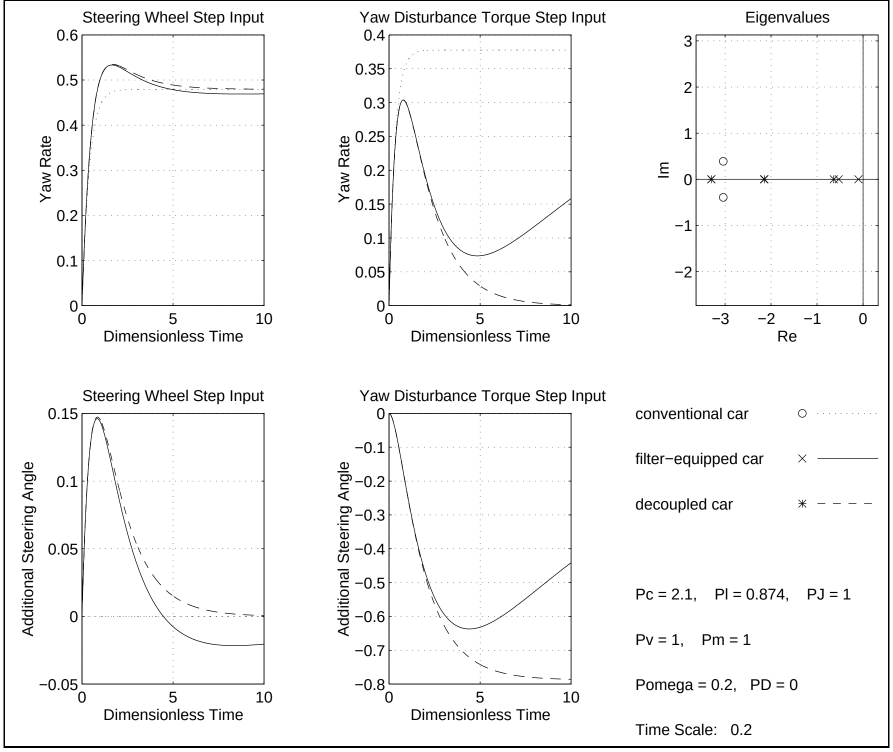

Fig 3: Response to steering wheel step and yaw disturbance torque step for $P_v = 1$

by rescaling the time and variables.
With the robust decoupling control law a new similarity number was needed, because the use of a nominal model introduced a new scale to the system. Every new independent parameter which is added to the system brings with it a new similarity number, unless it brings a new physical basic unit with it also (Buckingham). By the modified control law two more filter parameters and similarity numbers are needed to sufficiently characterize the system.
In the following simulation study we used the data of a BMW 735i passenger car to determine the values of the certain similarity numbers ($P_l = 0.874,\ P_c = 2.1,\ P_J = 1.0$) as well as the operating domain of the car ($1.0 \leq P_v \leq 8.0$). A lower limit for P_v is necessary since the controller is switched on only after a certain velocity is reached. The car is not controllable for $v = 0$, i.e. $P_v = 0$. The virtual mass number P_m is a new uncertain similarity number although it can be influenced by the choice of the nominal values (μ_0, m_0). Concerning the modified control law, the values of P_ω and P_D can be set in the controller design for a specific car. We choose a filter bandwith $\omega_0 = 1/sec$ to provide the above described properties (the decoupling action is supposed to fade out after ca. 0.5 seconds). With the data of the car we have $P_\omega = 0.2$. The damping of the filter is tuned by the value of D. It turned out, that a scaling with velocity of this parameter is useful to achieve a reasonable damping at all velocities.

$$
\begin{aligned}
D &= f(v) = f^*\left(v\sqrt{m_0/(\mu_0 c_{f0}\ell_f)}\right) \\
P_D &= f^*\left(P_v\sqrt{P_m}\right) \qquad (31)
\end{aligned}
$$

6 Simulation results

In the following simulation plots of the scaled yaw rate $\tilde{r}$ and additional steering angle $\tilde{\delta}_c$ the three controller versions are compared.

- The conventional car. Equation (15) is employed to compute the yaw rate response to yaw disturbance torque step inputs or steering wheel step inputs respectively. The "control law" is just $\tilde{\delta}_f(\tilde{s}) = \tilde{\delta}_L(\tilde{s})$. The additional steering angle is zero. Dotted line style is used for the dimensionless time response plots and circles denote the eigenvalue position in the dimensionless $\tilde{s}$-plane.

- The decoupled car with direct throughput. To obtain comparable results with the two other versions of the

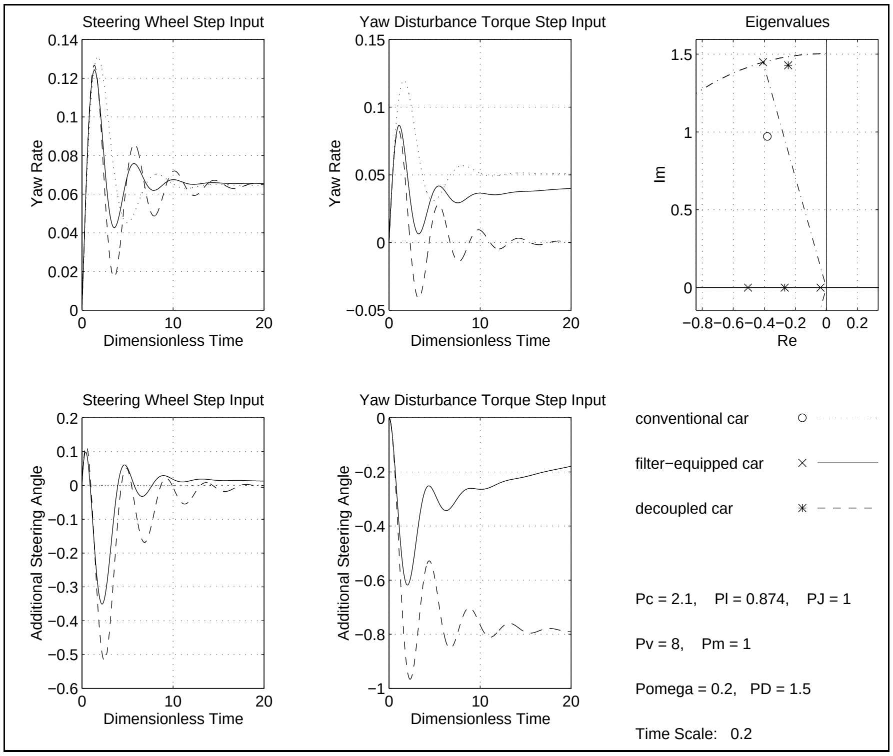

Fig 4: Response to steering wheel step and yaw disturbance torque step for $P_v = 8$

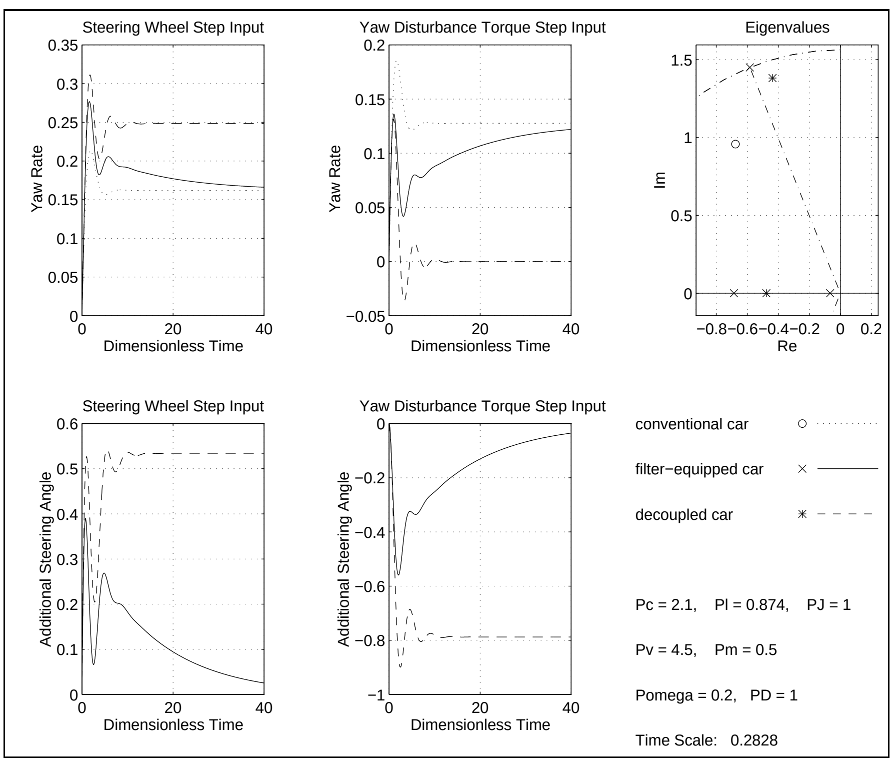

Fig 5: Response to steering wheel step and yaw disturbance torque step for $P_m = 0.5$

car, the direct throughput is combined with the robust decoupling control law (21). Thus the steering angle equation reads

$$\tilde{\delta}_f(\tilde{s}) \;=\; \tilde{\delta}_L(\tilde{s}) + \frac{1}{\tilde{s}}\left[((1-P_J)\,\tilde{s} - P_v)\,\tilde{r}(\tilde{s}) + \right.$$

$$\left. + P_v\;\tilde{F}\;\tilde{\delta}_L(\tilde{s})\right]$$

Dashed line style for the resonses and star markers for the eigenvalues are employed.

- The filter-equipped car. It is described by equation (15) extended by the steering angle equation of the modified control law (30). Plots use solid line style for the responses and x-marks for the eigenvalues.

The time scale $t/\tilde{t} = P_\omega/(\omega_0\sqrt{P_m})$ (see (13)) is printed below the eigenvalue plot. Original time scale is obtained by multiplying the dimensionless time of the plots with the specified time scale and original eigenvalue scale is obtained by dividing the dimensionless $\tilde{s}$ by the same time scale such that $\tilde{s}\tilde{t} = st$.
In the first simulation of Fig. 3 a steering wheel step input is applied to the three cars at a small value of $P_v = 1$ (low car mass at low velocity on dry road). For the choosen value of P_D there is not much difference between the three versions. Both the decoupled and the filter-equipped car perform a little overshoot in the yaw rate. With the simulation in Fig. 3 for a disturbance torque step input the concept of fading out the decoupling action is well recognizeable. Within the first 0.5 seconds both the decoupled and the filter-equipped car provide the same disturbance rejection activity. However after 0.5 seconds for the filter-equipped car the steering authority is softly returned to the driver and the additional steering angle $\tilde{\delta}_c(\tilde{s}) = \tilde{\delta}_f(\tilde{s}) - \tilde{\delta}_L(\tilde{s})$ goes to zero. Only now the driver is supposed to react to the yaw disturbance. The right upper plot of Fig. 3 shows the eigenvalues for $P_v = 1$. They are well damped for all three controllers.
Concerning Figs. 4 and 5, for every complex eigenvalue pair of the filter-equipped car three more curves (dash-dotted) are added to the eigenvalue plot to simplify comparison. One is a natural frequency circle around the origin through the eigenvalues. The other curves are damping lines which connect the origin with the eigenvalues.
Fig. 4 shows the responses for $P_v = 8$ which corresponds e.g. to higher speed on wet road. The yaw damping for all controllers is worse than at lower values of P_v. The worst damping occurs for the decoupled car. The oscillations of the filter-equipped car after a steering wheel step input decrease as quickly as those of the conventional car although the damping of the corresponding eigenvalues is less but their bandwidth is higher. The yaw disturbance responses show again how the initial decoupling action is fading out. Fig. 5 shows the result of deviations ($P_m \neq 1$) of the real car from the nominal model w.r.t. the virtual mass at a medium velocity ($P_v = 4.5$). Here P_m equals 0.5. The steady-state responses of the car with filter-feedback are the same as for the conventional car, whereas the decoupled car has a nonzero steady-state value of the additional steering angle.

7 Conclusions

The filter-equipped car can be considered as a compromise between the conventional car which has bad yaw disturbance attenuation properties and the decoupled car which exhibits poor yaw damping at higher velocities. Within the minimum driver reaction time of 0.5 seconds, the filter equipped car behaves like the decoupled car and then it returns smoothly to the steady-state behaviour of the conventional car. By using the modified control law of the filter-equipped car instead of the pure decoupling control law the yaw damping properties are considerably improved.

References

[1] J. Ackermann, "Robust decoupling of car steering dynamics with arbitrary mass distribution," in *Proc. American Control Conference*, vol. 2, (Baltimore, USA), pp. 1964–1968, 1994.

[2] J. Ackermann and W. Sienel, "Robust yaw damping of cars with front and rear wheel steering," *IEEE Trans. on Control Systems Technology*, vol. 1, no. 1, pp. 15–20, 1993.

[3] G. Rumpel and H. Sondershausen, "Ähnlichkeitsmechanik," in *Dubbel, Taschenbuch für den Maschinenbau, 15. Auflage* (W. Beitz and K.-H. Küttner, eds.), pp. 175–178, Berlin: Springer Verlag, 1986.

[4] M. Mitschke, *Dynamik der Kraftfahrzeuge*, vol. C. Berlin: Springer, 1990.

[5] W. Sienel, "Design of robust gain scheduling controllers," in *Proc. Robust and Adaptive Control Tutorial Workshop*, (Dublin), 1994.