Optimal Monetary Policy Under Low Trend Inflation

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Abstract

In this paper we generalize the standard optimal monetary policy literature as in Galí (2003) to the case of positive trend inflation. We present a simple framework that provides straightforward analytical results directly comparable with the standard case. Optimal monetary policy is strongly influenced by trend inflation and becomes less effective in controlling inflation as trend inflation increases. Moreover: (i) under discretion, the rational expectations equilibrium is not always determinate and the efficient policy frontier worsens; (ii) under commitment, the degree of interest rate smoothing increases with trend inflation and the gains from commitment are highly sensitive to the level of underlying inflation. At the very least, our results serve as a caution against the use of existing New Keynesian models that cannot explain observed trend inflation. Moreover, an ECB-like stability oriented monetary policy (i.e., 2% target inflation rate in the medium term) determines a substantial percentage loss in welfare with respect to a zero inflation target policy.

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"The Council also clarifies that, in the pursuit of price stability, it aims to maintain inflation rates below, but close to, 2% over the medium term" (ECB, 2003, p.79)

1. Introduction

In May 2003 the ECB stated exactly what it meant by price stability: an inflation rate of 2% over the medium term. It is unfortunate that most of the theoretical models employed by the rapidly growing field of optimal monetary literature cannot say much about how the choice of a positive target level of inflation would affect the optimal short-run stabilization policy of the ECB. Indeed, with few notable exceptions (e.g., Khan et al., 2003 and Schmitt-Grohé and Uribe, 2004), the literature always used a version of the New Keynesian model log-linearized around a zero inflation steady state.

This paper aims to fill this gap and solve the inconsistency by addressing the question of how optimal monetary policy is affected by positive trend inflation.[1]

There are two important reasons why this literature has been concerned with zero inflation steady state. The first is certainly analytical convenience. The second is that zero inflation is argued to be optimal in a so-called cashless economy where relative price dispersion is the only distortion (see Goodfriend and King, 2001, and Woodford, 2003). The theoretical assumptions needed for the optimality of zero inflation in the long-run are however quite special, such that generally that would not be the case.

We think that there are much more compelling reasons to look at the case of *low and positive* trend inflation. First, the zero inflation case is plainly unrealistic, as the post-war economic history of industrialized countries shows. Schmitt-Grohé and Uribe (2004) use the average US GDP deflator growth in the period 1960-1998 to calibrate the steady state inflation rate to 4.2 percent. The average inflation rate for European countries in post-war years ranges from approximately 3% in Germany to almost 10% in Spain. Hence we should

[1] By trend inflation we mean the rate of inflation in the deterministic steady state.

model how an optimal stabilization policy would act in that environment. This is even more compelling if these models are to be used to empirically assess the behavior of central banks in post-war data. Second, the practice of many central banks suggests that a zero inflation steady state is not an actual target. In other words, zero inflation does not coincide with the central bankers' "price stability" concept, as the ECB case illustrates.

It is therefore important to check whether the results in the existing literature remain valid when the assumption of a zero inflation steady state is removed, that is, how much they are sensitive to *moderately* positive trend inflation levels.

This paper owes a lot to the seminal works by Clarida et al. (1999) and Galí (2003) and it can actually be interpreted as a generalization of those contributions to positive trend inflation. We are able to define a simple and neat framework that nests the standard one in Clarida et al. (1999) and Galí (2003) and allows us to deal with general steady state inflation. The main change to the standard framework is the addition of one equation which derives from a generalization of the New Keynesian Phillips Curve. This allows us to provide intuitive analytical results for the case of discretionary monetary policy and to develop a straightforward comparison with the standard case.

Our main finding is that optimal monetary policy is highly sensitive to low levels of trend inflation. In particular, monetary policy progressively loses efficacy in stabilizing inflation after a cost push shock, as trend inflation increases. Indeed, when firms reset their price, they are concerned with keeping up with the trend in inflation, that erodes their relative price. The optimal reset price becomes therefore less affected by the current level of economic activity. As a consequence, the New Keynesian Phillips Curve turns out to be flatter as trend inflation increases, so that, the current output gap decreases its influence on current inflation. Moreover, for the case of discretionary monetary policy, we find that: (i) the rational expectations equilibrium (REE, henceforth) is not always determinate under optimal monetary policy; (ii) the efficient policy frontier substantially worsens with trend inflation. Under commitment, we find that: (i) the impulse response functions and the gains from commitment are highly sensitive to the level of trend inflation; (ii) the degree of interest rate smoothing increases with trend inflation. Finally, our model is able to match the empirical positive correlation between average inflation and inflation variability.

As in the standard literature, we will assume that optimal monetary policy aims at stabilizing inflation and output gap around the long-run targets, through the control of the nominal interest rate. We then ask the model how the optimal monetary policy response to shocks changes, as trend inflation moves to values more in line with the empirical observations for developed economies. Our results serve as a caution against the application of existing New Keynesian models to empirical analysis and against the use of these models if they cannot explain observed trend inflation. Our different trend rates of inflation are, however, generally inconsistent with the long-run maximization of our assumed loss function. Hence, the true extent, and indeed very nature, of the problem analyzed in this paper will not be known until a model is derived from first principles that predicts a positive average rate of inflation (see also Schmitt-Grohé and Uribe, 2005). The features that would deliver an endogenously optimal positive long-run inflation are likely to further affect the short-run Phillips curve and thus change the results of this paper both quantitatively and possibly qualitatively. At the very least, therefore, our paper highlights the need for such a model.

This paper is linked to two contributions which have recently appeared in the literature. Khan et al. (2003) show that the optimal inflation rate in the long-run is actually negative, since it is basically a compromise between the Friedman rule and minimizing the relative price distortion. Schmitt-Grohé and Uribe (2004), instead, look at optimal monetary and fiscal policy taking the level of trend inflation as exogenous, as we do in this paper, and calibrating it to US post war data. Our paper is complementary to these two works, since it differs in two important respects. First, neither of the two papers investigates how optimal monetary policy is affected by changes in trend inflation. Second, providing a tractable framework, our paper largely hinges on analytics, while the two papers mentioned above rely mainly on numerical results.

2. Trend Inflation and the Basic New-Keynesian Model

In this section we describe a simple New Keynesian DSGE model, similar to Clarida et al. (1999), Galí (2003) and Woodford (2003), generalized to allow for positive trend inflation.

2.1. *The model*

Households

The economy is populated by infinitively lived households whose instantaneous utility function is increasing in the consumption of a final good (C_t) and real money balances (M_t/P_t) and decreasing in labor (N_t) according to

$$U\left(C,\frac{M}{P},N\right)=\frac{C_t^{1-\sigma_c}-1}{1-\sigma_c}+\chi_m\frac{\left(M_t/P_t\right)^{1-\sigma_m}-1}{1-\sigma_m}-\chi_n N_t \tag{1}$$

where the positive parameters σ_c and σ_m represent the inverse of the intertemporal elasticity of substitution in consumption and real money balances, respectively, while χ_m and χ_n are positive constants.[2] At a given period t, the representative household faces the following nominal flow budget constraint

$$P_tC_t+M_t+B_t\leq P_tw_tN_t+M_{t-1}+\left(1+i_{t-1}\right)B_{t-1}+D_t+T_t \tag{2}$$

where P_t is the price of the final good, M_t represents holding of nominal money balances, B_t represents holding of bonds offering a one-period nominal return i_t, w_t is the real wage and D_t are firms profits rebated to the households. In addition, each period the government

[2] Here we want to present the simplest possible model, and hence we will assume indivisibility of labor as in Hansen (1985). This assumption allows us to derive neat analytical results, providing the intuition for the main mechanisms at play. The results do not depend on this assumption.

makes lump-sum nominal transfers to households equal to T_t. The household's problem is to maximize the lifetime expected utility subject to budget constraint (2), that is

$$\max_{\left\{C_t, \frac{M_t}{P_t}, N_t, B_t\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma_c} - 1}{1 - \sigma_c} + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1 - \sigma_m} - \chi_n N_t \right) \tag{3}$$

$$s.t. \qquad C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq w_t N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{T_t}{P_t}$$

where $\beta \in (0,1)$ is the subjective rate of time preference and E_0 denotes the expectation operator conditional on time $t = 0$ information set. The resulting first order conditions yield

$$\textit{labor supply} \quad : \qquad \chi_n C_t^{\sigma_c} = w_t \tag{4}$$

$$\textit{money demand} \quad : \qquad \chi_m (M_t/P_t)^{-\sigma_m} C_t^{\sigma_c} = i_t / (1 + i_t) \tag{5}$$

$$\textit{consumption Euler eq.} \quad : \qquad C_t^{-\sigma_c} = \beta E_t \left[C_{t+1}^{-\sigma_c} (1 + i_t) P_t / P_{t+1} \right]. \tag{6}$$

Equations (4), (5) and (6) have the usual straightforward economic interpretation.

Final good producers

In each period t, a final good Y_t is produced by perfectly competitive firms, combining a continuum of intermediate inputs $Y_t(i)$, according to the following standard CES production function: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$, with $\theta > 1$. Taking prices as given the final good producer chooses the quantities of intermediate goods $Y_t(i)$ that maximize its profits, resulting in the usual demand schedule: $Y_t(i) = [P_t(i)/P_t]^{-\theta} Y_t$. The zero profit condition in the final good sector brings about the following expression for the aggregate price index $P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$.

Intermediate goods producers

Intermediate inputs $Y_t(i)$ are produced by a continuum of firms indexed by $i \in [0,1]$, with the following constant returns to scale to labor production technology: $Y_t(i) = N_t(i)$. Intermediate goods sector is characterized by the fact that prices are sticky. In particular, intermediate goods producers act as monopolistic competitors and set prices according to a standard discrete version of the mechanism put forward by Calvo (1983). In each period, there exists a fixed probability $(1-\alpha)$ according to which a firm can re-optimize its nominal price. With probability α the firm cannot set a new price and must keep the price unchanged. The problem of a price-resetting firm can thus be formulated as

$$\max_{p_t^*(i)} \quad E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[\frac{p_t^*(i)}{P_{t+j}} Y_{t+j}(i) - \Gamma_{t+j}(i) \right], \quad \text{s.t.} \quad Y_{t+j}(i) = \left[\frac{p_t^*(i)}{P_{t+j}} \right]^{-\theta} Y_{t+j} \qquad (7)$$

where $p_t^*(i)$ denotes the new optimal price, $\Gamma_{t+j}(i)$ the real total cost function and $\Delta_{t,t+j}$ the stochastic discount factor. The solution to this problem yields the familiar formula for the optimal reset price in a Calvo's setup

$$p_t^*(i) = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[P_{t+j}^{\theta} Y_{t+j} \Gamma'_{t+j}(i) \right]}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[P_{t+j}^{\theta-1} Y_{t+j} \right]} \qquad (8)$$

where $\Gamma'_t(i)$ denotes the real marginal costs function, which, given the production function, is simply equal to the real wage.[3] To see how trend inflation affects intermediate firms' optimizing behavior, and thereby inflation, it is revealing to rewrite (8) by making explicit the *cumulative gross inflation rates* (CGIR, hereafter)[4]

$$\frac{p_t^*(i)}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} Y_{t+j} \left[(\Pi_{t+1} \times \Pi_{t+2} \times \cdots \times \Pi_{t+j})^{\theta} \Gamma'_{t+j}(i) \right]}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} Y_{t+j} \left[(\Pi_{t+1} \times \Pi_{t+2} \times \cdots \times \Pi_{t+j})^{\theta-1} \right]}. \qquad (9)$$

For the sake of argument, look at the steady state, where: $\Pi_{t+j} = \gamma$, for $j = 1, 2, \cdots, \infty$. Then, in the standard case of zero trend inflation, i.e., $\gamma = 1$, the CGIRs attached to future expected terms in (9) are, at all times, equal to one. Future expected terms are discounted by $\alpha\beta$. With positive trend inflation, i.e. $\gamma > 1$, two effects come into play. First, the structure

[3] In a deterministic steady state, equation (8) converges if and only if $\alpha\beta\gamma^{\theta} < 1$ (see Ascari, 2004). We will then assume that this condition holds in what follows, implying that annual trend inflation has to be lower than 11.4%, for the benchmark calibration values in Table 1.

[4] We define the CGIR between time $t+1$ and $t+j$ as $\Pi_{t+1,t+j} = \Pi_{t+1} \times \Pi_{t+2} \times \cdots \times \Pi_{t+j}$, where $\Pi_{t+j} = P_{t+j}/P_{t+j-1}$. Also, notice that CGIRs are raised to the power of θ in the numerator and to the power of $\theta - 1$ in the denominator.

of CGIRs at different time horizons shifts upwards changing the effective discount factor: $\alpha\beta\gamma^{\theta}$ in the numerator and $\alpha\beta\gamma^{\theta-1}$ in the denominator. Accordingly, when free to adjust, intermediate firms will set higher prices in the attempt to offset the fact that trend inflation mechanically erodes relative prices and profits, as long as their price is fixed. Second, future terms in (9) are progressively multiplied by higher CGIRs. This means that with trend inflation the optimal relative price will reflect future economic conditions to a larger extent, and short-run cyclical variations to a lesser extent. Price setting firms become "more forward-looking" and so does inflation. At the aggregate level, because current demand is relatively less important in price setting, current output has a weaker effect on inflation. As we will see, these effects of trend inflation will drive the main results of the paper.

Government

The government injects money into the economy through nominal transfers, such that $T_t = M_t^s - M_{t-1}^s$, where M^s is aggregate nominal money supply. Most importantly, we assume that in the steady state money supply evolves according to the following fixed rule $M_t^s = \gamma M_{t-1}^s$, where γ is the (gross) rate of nominal money supply growth.

Market clearing conditions

The market clearing conditions in the goods markets, in the money market and in the labour market are simply given by: $Y_t = C_t$; $Y_t^s(i) = Y_t^D(i) = [P_t(i)/P_t]^{-\theta} Y_t$, which has to hold $\forall i$; $M_t = M_t^s$ and $N_t = \int_0^1 N_t(i)di$.

2.2. A generalized New Keynesian Phillips Curve

Log-linearizing (5), (6) and using the market clearing condition $\hat{Y}_t = \hat{C}_t$, we obtain

$$\beta\hat{\imath}_t = -\sigma_m(\gamma-\beta)\hat{m}_t + \sigma_c(\gamma-\beta)\hat{Y}_t \tag{10}$$

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma_c^{-1}(\hat{\imath}_t - E_t\hat{\pi}_{t+1}) \tag{11}$$

where hatted variables denote percentage deviations from the deterministic steady state.

Regarding equation (8), the log-linearization under trend inflation is definitely more cumbersome. It is possible to describe a generalized New Keynesian Phillips Curve (NKPC, henceforth) with trend inflation as a system of two first-order expectational difference equations

$$\begin{cases} \hat{\pi}_t = \left[\frac{(1-\gamma)(1-\sigma_c)}{1-\alpha\beta\gamma^{\theta}} + \sigma_c\right] \bar{\lambda}(\gamma) \hat{Y}_t + \beta\gamma E_t \hat{\pi}_{t+1} + \frac{\bar{\lambda}(\gamma)(\gamma-1)}{1-\alpha\beta\gamma^{\theta}} \hat{\phi}_t \\ \hat{\phi}_t = \left(1-\alpha\beta\gamma^{\theta-1}\right)(1-\sigma_c)\hat{Y}_t + \alpha\beta\gamma^{\theta-1}\left[(\theta-1) E_t\hat{\pi}_{t+1} + E_t\hat{\phi}_{t+1}\right] \end{cases} \tag{12}$$

where $\bar{\lambda}(\gamma) \equiv \frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta}\right)}{\alpha\gamma^{\theta-1}}$ and $\hat{\phi}_t$ is just an auxiliary variable with no obvious interpretation. Our generalized version encompasses the standard NKPC as in Galí (2003). Indeed, when $\gamma = 1$ then (12) reads as $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)\sigma_c}{\alpha}\hat{Y}_t$ and the auxiliary variable $\hat{\phi}_t$ becomes irrelevant for inflation dynamics.

Three main points are worth stressing. First, and most importantly, trend inflation dramatically alters inflation dynamics with respect to the usual Calvo's model with $\gamma = 1$. The first equation in (12) can equivalently be written as

$$\hat{\pi}_t = \kappa\hat{Y}_t + \beta\gamma E_t\hat{\pi}_{t+1} + (\gamma-1)\beta\left(1-\alpha\gamma^{\theta-1}\right)\left[(\theta-1)E_t\hat{\pi}_{t+1} + E_t\hat{\phi}_{t+1}\right] \tag{13}$$

where $\kappa \equiv (\gamma-1)(\sigma_c-1)\beta\left(1-\alpha\gamma^{\theta-1}\right) + \sigma_c\bar{\lambda}(\gamma)$. As usual, the elasticity of inflation to current output gap, i.e. κ, is a function of α, β, θ and σ_c, but now also depends on γ. For standard calibration values in the relevant parameter space (i.e., $\alpha\beta\gamma^{\theta} < 1$), the higher the trend inflation the smaller the coefficient on the current output gap and the higher the coefficients on future expected inflation and on $E_t\hat{\phi}_{t+1}$. In other words, higher trend inflation flattens the short-run NKPC. For given future expectations, the less current inflation reacts to a given change in current output (or the more current output has to vary to cause a given change in current inflation). The intuition rests again on the more forward-looking behavior of intermediate firms when setting prices, as explained in section 2.1. As trend inflation increases, future economic conditions have relatively higher weights.

Current output becomes relatively less important as a determinant of reset prices, thus the contemporaneous relation between $\hat{\pi}_t$ and $\hat{Y}_t$ progressively weakens, and the inflation rate becomes less sensitive to variations in the output gap.

Second, the system of equations composed by (11) and (12) is a neat and compact generalization of the standard New Keynesian model popularized by the work of Clarida et al. (1999), Galí (2003) and Woodford (2003). Through the definition of the auxiliary variable $\hat{\phi}_t$, we express the NKPC under trend inflation by just adding one equation to the model. This allows us to derive some analytical results and provide intuitive comparisons with the standard model. On the other hand, one extra dynamic equation enlarges the model dynamics, forcing us to resort to numerical results more often than in the standard case.

Third, we will assume log-utility in consumption, i.e., $\sigma_c \to 1$, so that the system (12) is greatly simplified. The equation for $\hat{\phi}_t$ depends in this case only on future expected variables. Hence, if the monetary authority is unable to influence expectations of future variables (i.e., under discretionary monetary policy) then we can simply ignore $\hat{\phi}_t$.

2.3. Transmission of monetary policy shocks

Before analyzing the effects of trend inflation on optimal monetary policy, it is instructive to look at the monetary policy transmission mechanism. For the sake of comparison, we choose the benchmark calibration values in Galí (2003), as in Table 1.[5] Figure 1 displays impulse responses functions (IRFs, hereafter) of output gap, inflation, real interest rate and real money balances to a unit shock to the growth rate of money.[6] In each panel, we report IRFs for six levels of trend inflation: $\gamma = 1.0$, $\gamma = 1.005$, $\gamma = 1.01$, $\gamma = 1.015$, $\gamma = 1.02$ and $\gamma = 1.025$, corresponding respectively to 0%, 2%, 4%, 6%, 8% and 10% trend inflation rates.

[5] Changes in the values of θ and σ_c have the usual effects as in a standard model (see equation (13)). The results in the paper regarding the effects of trend inflation, however, qualitatively do not change. Therefore,we present numerical results only for the benchmark calibration.

[6] As in Galí (2003), we calibrate to 0.5 the persistence parameter of the AR(1) process governing the money supply growth rate.

Consider first the case of a zero inflation steady state. In response to the shock, staggered price adjustment of individual prices leads to an upward sluggish adjustment of aggregate price level, thus causing an increase both in inflation and in real money balances. Output and the nominal interest rate need to adjust to satisfy the money demand equation (10). For the sake of argument, assume for the moment an interest rate inelastic money demand (i.e., a quantity theory money demand). Then, the persistent increase in money growth and the consequent future higher expected inflation produce a long period of negative (ex-ante) real interest rates. Given the Euler equation (11), households anticipate consumption, increasing demand and therefore output. Indeed, solving the Euler equation (11) forward, i.e., $\hat{Y}_t = -\sigma_c^{-1} E_t \sum_{i=0}^{\infty} \left(\hat{\imath}_{t+i} - \hat{\pi}_{t+1+i}\right)$, shows that the monetary authority will have an effect on output only to the extent that it will influence the current or future expected short term real interest rate. For our benchmark calibration, the rise in output on impact is so high that the nominal interest rate actually jumps up rather than down following a positive money shock, according to (10).[7] On impact, output jumps up by 1.48% and inflation by 2%, and then they both progressively return to steady state values, as prices sluggishly adjust.

Keeping in mind the discussion in section 2.3, it is then easy to explain the main result shown in Figure 1: positive levels of trend inflation have notable effects on the transmission mechanism. More precisely, as trend inflation takes up higher values, the money growth shock produces increasingly higher rises in real money balances, because of the more muted response of newly reset prices to current economic conditions. Accordingly, the size of the output gap response increases with the level of trend inflation, due to the money demand and the consumption Euler equation, while the inflation response decreases.

There is also a second effect of higher trend inflation: the persistence of output and inflation IRFs substantially increases with trend inflation. As shown in Table 2, if steady state inflation is zero, the half-life of output and inflation is 3.5 quarters for both, while at

[7] See Galí (2003) on the lack of liquidity effect in these kind of models.

10% steady state inflation is more than doubled, rising to 8.7 and 8.3 quarters, respectively. The intuition for this result rests again on the effect that trend inflation has on the output-inflation trade-off.[8]

Following Galí (2003) we add a cost-push shock u_t to the first equation in (12), whose law of motion is $u_t = \rho u_{t-1} + \varepsilon_t$, and ε_t is a i.i.d. random variable with zero mean and unit variance. Figure 2 shows the IRFs after a unit cost push shock, with $\rho = 0.5$, keeping the growth rate of money constant. The same kind of effects as in Figure 1 are clearly visible. In particular, the higher reaction shown by inflation and the lower one shown by output as trend inflation increases demonstrate that the link in the model between these two variables becomes weaker with trend inflation. Again persistence also increases with trend inflation since inflation is less sensitive to output movements. It is important to stress that trend inflation therefore substantially alters the dynamics of the model, by modifying its key equation, namely the NKPC. Not surprisingly the response of the optimal policy will be affected.

3. Optimal Monetary Policy Under Discretion

As in the optimal policy literature, (e.g., Clarida et al., 1999, and Galí, 2003) the monetary authority controls the nominal interest rate in order to minimize the discounted sum of expected instantaneous loss functions defined over inflation and output gap according to

$$\mathcal{W} = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left(\hat{\pi}_{t+j}^2 + \chi \hat{Y}_{t+j}^2 \right) \tag{14}$$

where χ is the relative weight placed on output gap stabilization with respect to inflation. The policy problem is that of choosing the optimal time path for the nominal interest rate, $\hat{\imath}_t$, and engineering the time paths of the target variables $\hat{\pi}_t$ and $\hat{Y}_t$ that minimize (14) subject to the IS curve (11) and the NKPC (12).

[8] See also Amano, Ambler, and Rebei (2005).

Under discretion, the monetary authority cannot make credible announcements about future policy actions and it reoptimizes (14) each period, taking as given future expectations

$$\min_{\hat{\pi}_t, \hat{Y}_t} \quad \frac{1}{2}\left(\hat{\pi}_t^2 + \chi \hat{Y}_t^2 + F_t\right), \text{ s.t. } \hat{\pi}_t = \bar{\lambda}\left(\gamma\right)\hat{Y}_t + f_t + u_t \tag{15}$$

where F_t and f_t contain all future expected variables. The solution to this simple problem is

$$\hat{Y}_t = -\frac{\bar{\lambda}\left(\gamma\right)}{\chi}\hat{\pi}_t. \tag{16}$$

Condition (16) simply states that the general prescription for a discretionary policy is to "lean against the wind". The solution is very similar to the standard one obtained with the zero inflation steady state, as in Clarida et al. (1999) and Galí (2003). There is, however, a crucial difference: the degree of "aggressiveness" with which the output gap ought to respond to inflation along the optimal path now depends on trend inflation. The higher the trend inflation the less aggressively the central bank will fight inflation and the more it will stabilize output. The intuition rests on the slope of the short-run NKPC, i.e., $\bar{\lambda}\left(\gamma\right)$: a higher level of trend inflation lowers the gain in reduced inflation per unit of output loss. The worsening in the inflation/output trade off induces a less aggressive policy response to inflation. In sum, the higher the trend inflation, the more the shock will be passed on to inflation and less to output. Hence the higher the relative variability between inflation and output (σ_π/σ_Y).

Notice this result has an appealing empirical implication. It delivers a positive correlation between the average inflation level and the variance of inflation, which is a very robust empirical feature both over time and across countries (see, e.g., Friedman, 1977, Ball and Cecchetti, 1990, and Caporale and McKiernan, 1997). As we will see, the same implication holds for the optimal monetary policy under commitment. It is worth noting that the positive correlation between γ and σ_π stems from the optimal response of monetary policy.

3.1. An indeterminacy problem

Substituting the optimal condition (16) into (12), we obtain a system of two first order difference equations for $\hat{\pi}_t$ and $\hat{\phi}_t$, from which we can compute the optimal solution $\hat{\pi}_t$ as function of the only state variable, i.e., u_t. Then, we can recover $\hat{Y}_t$ exploiting condition (16) and finally the time path for $\hat{\imath}_t$ using the IS curve. However, provided that $\gamma \in \left[1, (1/\alpha\beta)^{1/\theta}\right)$, the necessary and sufficient condition is not always satisfied for the REE to be unique.

Proposition 1 *Let $\sigma_c = \beta = 1$. Then, the dynamic system defined by the optimal monetary policy under discretion admits a unique REE if and only if: $\chi\theta(\gamma - 1) < \left[\bar{\lambda}(\gamma)\right]^2$.*

Corollary 1. *Comparative statics. Ceteris paribus, indeterminacy arises: (i) the higher the level of trend inflation, γ; (ii) the higher the elasticity of substitution among goods, θ; (iii) the higher the weight on output in the monetary authority loss function, χ; (iv) the higher the probability of non-adjusting prices, α.*

Corollary 2. *If $\gamma = 1$, then the REE is always unique in the admissible parameters space.*

In particular, note that higher trend inflation brings about the possibility of indeterminacy of the REE. While under the standard assumption of zero trend inflation indeterminacy never arises, we show that, instead, the optimal policy under discretion could lead to indeterminacy in the general case, even for very low levels of trend inflation. However, the more conservative the central banker, i.e., the lower the χ, the less likely that indeterminacy will arise.

Corollary 3. *The higher the value of trend inflation, the more "conservative" a central bank needs to be to guarantee the uniqueness of the REE under optimal discretionary policy. Moreover, whatever the value of trend inflation (among the admissible ones), there is always a sufficiently low value of χ to ensure that the REE is unique.*[9]

[9] $\bar{\lambda}(\gamma)$ tends to zero as γ tends to its upper bound (defined by the condition $\alpha\beta\gamma^{\theta} < 1$), so that it has a finite value within the range of admissible values of γ.

Figure 3 shows the feasible combinations of (γ, χ) that ensure uniqueness of the REE for our benchmark calibration in Table 1. The values of γ range from 1 to 1.02 and the values of χ range from 0 to 1, that is from a "pure inflation targeting" central bank to a central bank that gives equal weight to inflation and output stabilization. The plot evidently shows the analytical results of Proposition 1 and its corollaries.

It is important to stress the following points. First, the ECB stability-oriented monetary policy target inflation rate of 2%, corresponding to $\gamma = 1.005$, would require a value of χ lower than 0.078 to make the optimal discretionary monetary policy implementable. In other words, if it has to be true that "the medium-term orientation also allows monetary policy to take into account concerns about output fluctuations, without prejudice to attaining the primary objective" (p. 80, ECB, 2003), then the ECB should put a weight on inflation fluctuations which is at least ten times higher than the one on output fluctuations.[10]

Second, from a theoretical point of view, Galí (2003) calibrates the microfounded value of χ equal to 0.0078. In this case only levels of trend inflation lower than 4.7 % annually (i.e., $\gamma = 1.0116$) can support optimal discretionary monetary policy.

Third, Schmitt-Grohé and Uribe (2004), instead, calibrate trend inflation to be 4.2%, equal to the U.S. average GDP deflator growth rate in the period 1960-1989. This means that the Fed's weight on inflation should have been roughly seventy-seven times higher than the one on output fluctuations to keep its discretionary monetary policy implementable.

Finally, it is clear that very low levels of χ are necessary for determinacy as soon as trend inflation reaches values which are in line with the historical experience of many developed countries after the second world war. This casts some shadows on the monetary policy of many developed countries in the 70's and 80's, when the level of trend inflation would have required basically a pure inflation target central banker. If we are willing to reasonably

[10] Unless one is willing to make the Euro Area monetary policy rest on the risky assumption of commitment, see next section.

assume that many central banks had no commitment power and were not purely concerned with inflation in that historical period (see, e.g., Clarida et al., 1999), then we must conclude that their monetary policy was simply not implementable. And indeed in many countries inflation got out of hand, following the oil shocks in the 70's.

3.2. The efficient frontier

The case of a purely transitory cost push shock, i.e., $\rho = 0$, yields the following analytical closed form solution

$$\hat{Y}_t = -\hat{\imath}_t = -\frac{\bar{\lambda}(\gamma)}{\bar{\lambda}(\gamma)^2 + \chi} u_t \qquad \text{and} \qquad \hat{\pi}_t = \frac{\chi}{\bar{\lambda}(\gamma)^2 + \chi} u_t. \tag{17}$$

Note that (17) exactly parallels the solutions in Clarida et al. (1999) (i.e., equations (3.4) and (3.5) at p. 1672) and in Galí (2003) (i.e., equations (39) and (40)), showing once more that our framework is able to generalize those results in a very simple and intuitive way. It is evident that trend inflation modifies the response of output gap and inflation under the optimal policy. The incentive to split the effects of the shock between inflation and output now depends on the loss function parameter χ but also on trend inflation through $\bar{\lambda}(\gamma)$.

Proposition 2 *As trend inflation increases, the reaction of inflation to the cost push shock is higher and the one of output is instead ambiguous, depending on the relative importance of the gain, measured by χ, and the cost, measured by $\bar{\lambda}(\gamma)$.*

The effectiveness of a fall in output gap to curb inflation decreases quite strongly with trend inflation. Note that: $\frac{\partial \widehat{\imath}_t}{\partial \gamma} < 0$ if $\chi > 2\bar{\lambda}(\gamma)$. Assume that for zero trend inflation $\chi < 2\bar{\lambda}(1)$, as for the benchmark calibration. Then as trend inflation increases the response of monetary policy is firstly more aggressive (i.e., $\frac{\partial \widehat{\imath}_t}{\partial \gamma} > 0$), but then starts decreasing, because as γ increases, $\bar{\lambda}(\gamma)$ diminishes and thus the above condition switches sign, i.e., $\chi > 2\bar{\lambda}(\gamma) \Longrightarrow \frac{\partial \widehat{\imath}_t}{\partial \gamma} < 0$. Numerically this would happen for something less than 1% annual inflation in the benchmark calibration. There is a simple reason to do that: monetary policy becomes less effective as trend inflation increases, so the optimal response is to be increasingly cautious and passive. Indeed, the current output cost necessary to correct inflation is increasing with trend inflation. Low values of inflation variability can be obtained only at the expense of great output variability.

As in Clarida et al. (1999), it is very revealing to calculate the efficient policy frontier to fully understand the effects of low levels of trend inflation on the monetary policy trade-off under discretion. The efficient policy frontier links the output/inflation variability for different values of χ in (14).

Proposition 3 *Let $\rho = 0$ and $\sigma_\varepsilon = 1$. Then, the efficient policy frontier, expressed in standard deviations, is given by: $\sigma_Y = 1/\bar{\lambda}(\gamma) - \sigma_\pi/\bar{\lambda}(\gamma)$. Thus, as trend inflation takes up higher values, the efficient policy frontier gets worse: it progressively moves north-east and becomes steeper ($\bar{\lambda}(\gamma)$ is a decreasing function of γ).*

In other words, the attainable points with zero trend inflation in the space (σ_π, σ_Y) are not anymore so as γ rises: either a higher value of σ_Y is necessary for the same σ_π or viceversa. Moreover, the change in the slope of the efficient frontier once more confirms the output/inflation trade-off tends to vanish because the monetary policy is losing its efficacy.

Figure 4 shows the efficient policy frontiers in the case of $\rho = 0.5$. Each line corresponds to different values of trend inflation. Consider the black line with circles that displays the output/inflation variability frontier under zero trend inflation. For a given value of χ, we have a particular combination of σ_Y and σ_π, hence a circle on that line. By varying χ from 0 to 1 (with a step of 0.01), the circle moves from left to right along the line. As χ rises Figure 4 clearly indicates that output (inflation) variability monotonically decreases (increases). More importantly, the figure illustrates that as trend inflation increases efficient policy frontiers tilt upwards and become progressively steeper, thus leading to worse outcomes for both inflation and output variability. For instance, attainable points with 2% trend inflation (the black line with stars) are not more attainable with 4% trend inflation (the black line with diamonds).

In addition, most of the points that compose the efficient policy frontiers are concentrated on the right-lower corner, suggesting that only very low values of χ can deliver low values of inflation variability, Furthermore, the number of points that composes the frontier decreases with γ, since the model enters the indeterminacy region in Figure 3. In sum, as trend inflation increases, the frontier tilts upwards and becomes steeper and shorter.

4. Optimal Monetary Policy Under Commitment

In the presence of a credible commitment mechanism, the monetary authority does not take private future expectations as exogenously given but recognizes instead that its policy actions effectively influence such beliefs. In this case, we cannot provide analytical results and are forced to rely on numerical simulations. A first important result is the following.

Result 1. *Let* $\chi \in [0,1]$ *and* $\gamma \in [1, 1.03]$. *Then, the REE is always determinate under commitment.*[11]

Unlike the discretionary case, if a credible commitment mechanism exists, the optimal path for the nominal interest rate ensures uniqueness of the REE, regardless of the underlying parameterization of the model economy.

Moreover, it is well-known that optimal monetary policy shows a certain degree of inertia under commitment (see Woodford, 1999). Interestingly, our numerical results clearly illustrate that as trend inflation increases, optimal monetary policy inertia, proxied by the magnitude of the stable eigenvalues, increases. We can therefore state the following result.

Result 2. *In response to a cost-push shock, the monetary authority, optimally acting under commitment, chooses a trajectory for the short term nominal interest rate whose degree*

[11] For robustness, we consider the sensitivity of the results to different parametrization of the intertemporal elasticity of substitution (i.e., $\sigma_c = 5$) and the degree of nominal stickiness (i.e., $\alpha = 0.5$).

of persistence is positively correlated to the level of trend inflation.

Hence, with positive trend inflation, the nominal interest rate, as well as the inflation and output gap, will take several quarters before they settle back to steady state, even if the shock is purely transitory. The intuition for this result rests again on the effects of trend inflation on the NKPC. First, trend inflation makes the NKPC more forward-looking, and thus, since commitment influences future expectations, the typical feature of optimal monetary policy under commitment (i.e., persistence) is reinforced. Second, we have already seen in section 2.3. that the transmission mechanism becomes more inertial with trend inflation, since current inflation is increasingly less responsive to changes in the output gap.

Impulse response functions to a cost push shock

We now turn to the analysis of the IRFs of output gap, inflation rate, nominal and real interest rates to a unit cost-push shock under commitment.[12] In each simulation, we stick to the benchmark parameterization in Table 1 and put $\chi = 0.0078$, following Galí (2003).

Figure 5 displays IRFs to a purely transitory cost-push shock. Consider first the standard case of zero inflation steady state. In the event of a cost-push shock, the monetary authority responds by engineering an aggressive deflation and a persistent adjustment pattern in output gap. Within one period, inflation goes from 1.57 to -0.95 while output gap, on impact, jumps down to -4.32 and remains under steady state for several quarters. Such a pattern for $\hat{Y}$ and $\hat{\pi}$ is achieved by raising the nominal interest rate and keeping it above steady state for a few quarters. As known, compared to the case of discretion, the IRFs of the interest rate and of the output gap are smaller, but more prolonged. Indeed, the forward looking price setters anticipate the protracted period of tight monetary policy and therefore do not increase prices too much in the first place.

As trend inflation comes about, the overall picture substantially changes both qualitatively and quantitatively. Recall that trend inflation produces two effects. First, it makes

[12] In computing the impulse response function we employed the algorithm proposed by Soderlind (1999).

price setters and inflation dynamics more forward-looking (see Section 2.2.). As such, we would expect the ability to commit, and thus to influence future expectations, to become even more important with trend inflation, and the features of optimal monetary policy under commitment, i.e., lower impact effect and more persistent policy, to be strengthened. Second, trend inflation makes monetary policy less effective. From this, instead, we would expect the monetary policy to react progressively less as trend inflation increases. Overall, intuition would therefore predict that both effects tend to decrease the impact multipliers on $\hat{\imath}$ and $\hat{Y}$, while the first one tends to increase the degree of inertia.

Figure 5 clearly illustrates that this is indeed the case. Increasing levels of trend inflation smooth IRFs and dampen the impact effects on $\hat{\imath}$ and $\hat{Y}$. For levels of trend inflation between 0% and 6% the patterns of the endogenous variables are quite intuitive and plausible. As trend inflation increases, the interest rate responds less aggressively, implementing a milder output contraction and hence the response of inflation is higher. Moreover, the paths of the endogenous variables show higher persistence. In correspondence to 8% or 10% trend inflation, however, the dynamic properties of the system are quite striking. For these levels of trend inflation the gain in reducing inflation per unit of output loss is very low. Therefore, the policy maker finds it optimal to keep the output gap almost constant, setting the interest rate slightly above expected future inflation. This policy obviously produces a more volatile time pattern for inflation.

Figure 6 displays IRFs in the case of persistent cost-push shock, i.e., $\rho = 0.5$. The persistence of the shock makes the same features described above more evident. Again, for very high trend inflation rates the monetary authority simply quits controlling inflation.

Gains from commitment relative to discretion

In this section we assess the welfare implications of positive levels of trend inflation under discretion and under commitment. We do so by calculating the unconditional loss function, i.e., $E(W) = Var(\hat{\pi}_t) + \chi Var(\hat{Y}_t)$, and the percentage gain from commitment relative to discretion, i.e., $100 \times (1 - L_c/L_d)$, where L_c and L_d denote the loss under commitment and under discretion respectively. Table 3 reports unconditional variances of $\hat{\pi}$ and $\hat{Y}$, the value of the expected loss and the percentage gain from commitment, for different values of trend inflation and of the autoregressive coefficient of the cost push shock.

Several interesting features are worth stressing. First, discretionary policy always delivers an expected welfare loss greater than the one obtained under commitment. The economic intuition for this finding is well known and impinges on the capacity to influence future expectations under commitment. In fact, as current inflation crucially depends on future output gaps, the monetary policy can better stabilize inflation by credibly engineering a long-lived recession, even when the shock has died out. Such a promise to spread the shock over multiple periods is thus welfare enhancing.

Second, and most importantly for the matter of this paper, the welfare loss is always increasing in the level of trend inflation both under discretion and commitment and regardless of the shock persistence. With respect to a policy that targets zero inflation, an ECB-like stability oriented monetary policy (i.e., 2% target inflation rate in the medium term) determines a substantial percentage loss in welfare. The size of this loss is about 36% under discretion and 32% under commitment, if $\rho = 0$; and 81% under discretion and 56% under commitment, if $\rho = 0.5$. Hence, even very low levels of trend inflation quite substantially deteriorate the performance of the optimal monetary policy.

Third, the last column of Table 3 reports the percentage gain from commitment. From previous discussions one would expect the gains from commitment to increase with trend inflation, because of the ability to influence future expectations. This is true only for moderate levels of trend inflation. For instance, in the case of $\rho = 0$ the percentage gain from commitment is increasing for levels of trend inflation up to 2.4% annual. After such a level the gain is still positive but starts to decline. In the case of $\rho = 0.5$ this threshold shifts to 4.84% annual. The economic intuition for this finding, which is apparently at odds with our insights, rests on the fact that there is another effect induced by positive trend inflation, as discussed above. Trend inflation, indeed, reduces the effectiveness of monetary policy. Above a certain level of trend inflation, this latter effect dominates and monetary policy starts disregarding inflation, because it is too costly. This is true both under discretion and under commitment: the two policies get close one another, hence the gain is reduced. This result is well captured by the behavior of the unconditional variances of $\hat{\pi}$ and $\hat{Y}$.

Fourth, Table 3 also reveals that trend inflation tends to reinforce the fact that the percentage gain from commitment is more relevant when the shock exhibits some degree of persistence. If $\rho = 0$ the gain from commitment increases by about 9.5% moving from 0% to 2% trend inflation, and by 5% moving from 0% to 4%. If $\rho = 0.5$, the percentage gain is much higher: 26% and 41% respectively. Once again, the economic intuition for this finding impinges on the effect of trend inflation in making the dynamics of inflation more forward looking. Discounting future monetary policy actions firms do not increase prices too much in the first place, thus reducing the overall variability of the endogenous variables and enhancing welfare.

Pure inflation targeting

Finally, it is instructive to analyze the effects of appointing a central bank who pursues a strict inflation targeting policy. Recall that in the case $\chi \to 0$ the REE under discretion is always determinate. Actually there is basically no difference between the optimal policy under discretion and commitment. Both policies achieve zero inflation variability thus fulfilling the objective. As a result, output variability is much higher and increasing with trend inflation: monetary policy becomes less effective with trend inflation and thus needs to engineer severe reductions in output gap to completely stabilize inflation, at any output cost. What is surprising, however, just by looking at Table 4, are the numbers: output variability increases esponentially with trend inflation. Under strict inflation targeting, changing the target from 0% to 2% more than doubles output variability, while passing from 0% to 4%

actually amplifies output variability by a factor of almost 6.

Finally, we considered robustness analysis with respect to two extensions of this basic model: (*i*) elastic labor supply; (*ii*) strategic complementarity, stemming from sector-specific labor markets. All the results presented above are robust to these modifications, and they are indeed greatly strengthened from a quantitative point of view, especially in the case of strategic complementarity.[13]

5. Conclusion

In the post-war data, average inflation for developed countries is quite far from zero. Central banks inflation targets are also different from zero. Despite that, the literature concentrates on the case of a zero inflation steady state. It is therefore very important to check the robustness of the main model to a general trend inflation level different from zero.

In order to do that, this paper generalizes the seminal works by Clarida et al. (1999) and Galí (2003). We were able to provide a simple and neat framework that nests the standard one in Clarida et al. (1999) and Galí (2003) and allows it to deal with a generic steady state inflation rate. Our main finding is that optimal monetary policy is highly sensitive to the trend inflation level. In particular, monetary policy loses efficacy with trend inflation in stabilizing the economy after a cost push shock. This happens because the New Keynesian Phillips Curve becomes flatter as trend inflation increases, so that, the current output gap level decreases its influence on current inflation. Moreover, for the case of discretionary monetary policy, we find that: (i) the rational expectations equilibrium is not always determinate under optimal monetary policy; (ii) the efficient policy frontier substantially worsens with trend inflation. Under commitment, we find that: (i) the impulse responses and the gains from commitment are highly sensitive to the level of trend inflation; (ii) the degree of interest rate smoothing increases with trend inflation.

[13] Results are available in the working paper version of this paper on the authors' webpages.

This paper shows that the results of the standard model are very sensitive to the special assumption of zero trend inflation. At the very least, our results serve as a caution against the application of existing New Keynesian models to empirical analysis and against the use of these models if they cannot explain observed trend inflation.

Our different trend rates of inflation are, however, generally inconsistent with the long-run maximization of our assumed loss function. Hence, the paper can be read as stressing the urgency of a model that endogenously delivers an optimal positive long-run inflation level. This unknown model, moreover, is likely to further affect the short-run NKPC and thus change the results of this paper both quantitatively and possibly qualitatively.

Finally, our results are obviously sensitive to two features of the standard model: no indexation and fixed contract length. A relaxation of these two standard assumptions would weaken our results. With respect to the former, Ascari (2004) shows that any kind of indexation reduces the effect that trend inflation has on the NKPC and that these effects actually vanishes with full indexation. It must be said however that we are concerned with *low levels* of trend inflation, like those that post world war II data exhibit in developed economies. In such an environment the sticky price assumption is usually believed to be valid. Moreover: (i) in reality we do not observe indexed prices; (ii) we know, at least since Gray (1976), that full indexation is not optimal; (iii) the theoretical microfoundations of such a price indexation scheme is rather questionable and indeed the Christiano et al. (2005) justification for using it is mainly empirical. Besides, given that our concern is with moderate level of trend inflation, we also keep the expected duration of prices exogenously fixed.

To conclude, in our view the results in this paper strongly call for a rethinking of the standard New Keynesian model: either by changing the assumed price setting mechanism to one that is more realistic and less sensitive to trend inflation, or by thinking of a model that delivers optimal positive inflation.

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6. Tables

Parameter	β	σ_c	σ_m	α	θ
Value	0.99	1	1	0.75	11

Table 1: Benchmark Calibration (quarterly)

Annual Trend Inflation	0%	2%	4%	6%	8%	10%
Output Half Life	3.5	3.9	4.4	5.1	6.3	8.7
Inflation Half Life	3.5	3.8	4.3	5	6.1	8.3

Table 2: Half-Life of Output and Inflation (quarters)

Parameter values		Discretion			Commitment			%Gain
		Var($\hat{\pi}_t$)	Var($\hat{Y}_t$)	Loss	Var($\hat{\pi}_t$)	Var($\hat{Y}_t$)	Loss	
$\gamma = 1$	$\rho = 0.0$	0.265	32.025	0.514	0.222	22.096	0.394	23.397
$\gamma = 1.005$	$\rho = 0.0$	0.489	26.949	0.699	0.361	20.411	0.520	25.609
$\gamma = 1.01$	$\rho = 0.0$	0.741	15.373	0.861	0.530	15.302	0.649	24.601
$\gamma = 1.015$	$\rho = 0.0$	0.916	5.248	0.957	0.692	7.766	0.753	21.333
$\gamma = 1.02$	$\rho = 0.0$	0.987	0.842	0.993	0.795	1.605	0.807	18.743
$\gamma = 1.025$	$\rho = 0.0$	1	0.019	1	0.839	0.044	0.840	16.016
$\gamma = 1$	$\rho = 0.5$	0.635	76.843	1.234	0.283	67.597	0.811	34.313
$\gamma = 1.005$	$\rho = 0.5$	1.563	86.1	2.235	0.605	84.986	1.268	43.243
$\gamma = 1.01$	$\rho = 0.5$	3.18	65.992	3.695	1.216	89.19	1.912	48.256
$\gamma = 1.015$	$\rho = 0.5$	4.831	27.669	5.047	2.151	60.169	2.620	48.08
$\gamma = 1.02$	$\rho = 0.5$	5.639	4.809	5.677	2.956	14.213	3.067	45.973
$\gamma = 1.025$	$\rho = 0.5$	5.707	0.108	5.708	3.369	0.425	3.372	40.922

Table 3. Welfare Analysis

Annual Trend Inflation	Var($\hat{Y}_t$)
0%	135.73
2%	298.42
4%	792.02

Table 4. Pure Inflation Targeting

7. Figures

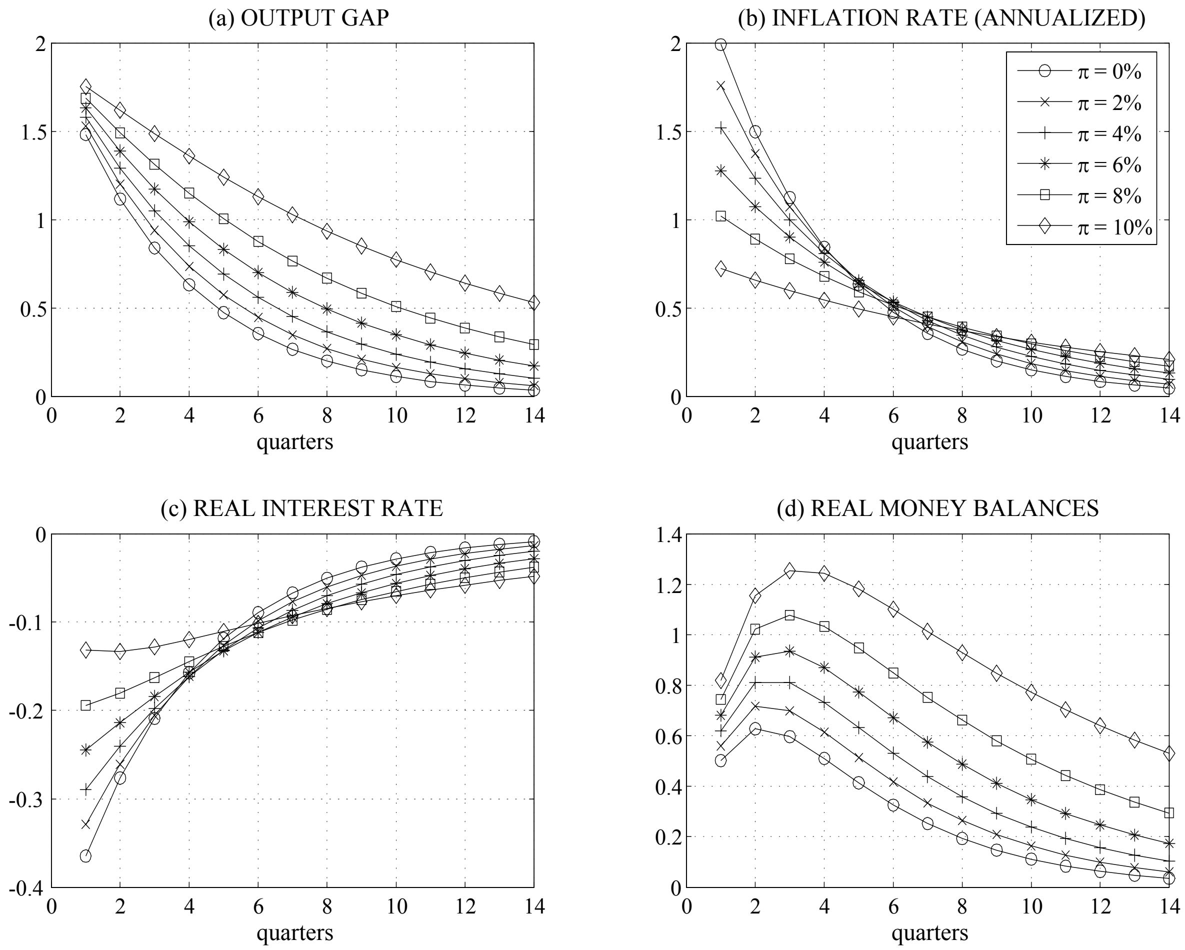

Figure 1. Impulse responses functions to a monetary shock.

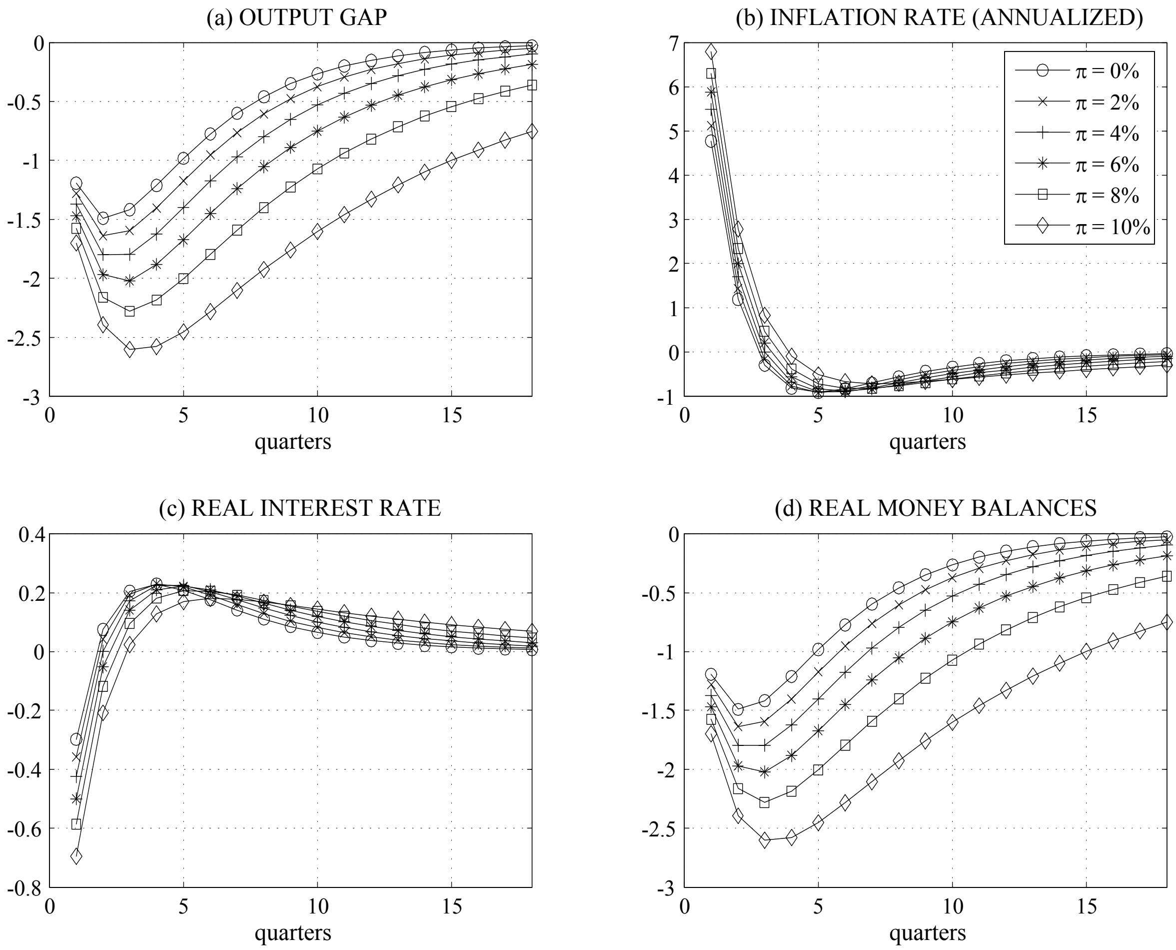

Figure 2. Impulse responses following a cost push shock ($\rho = 0.5$) under constant rate of growth of money supply

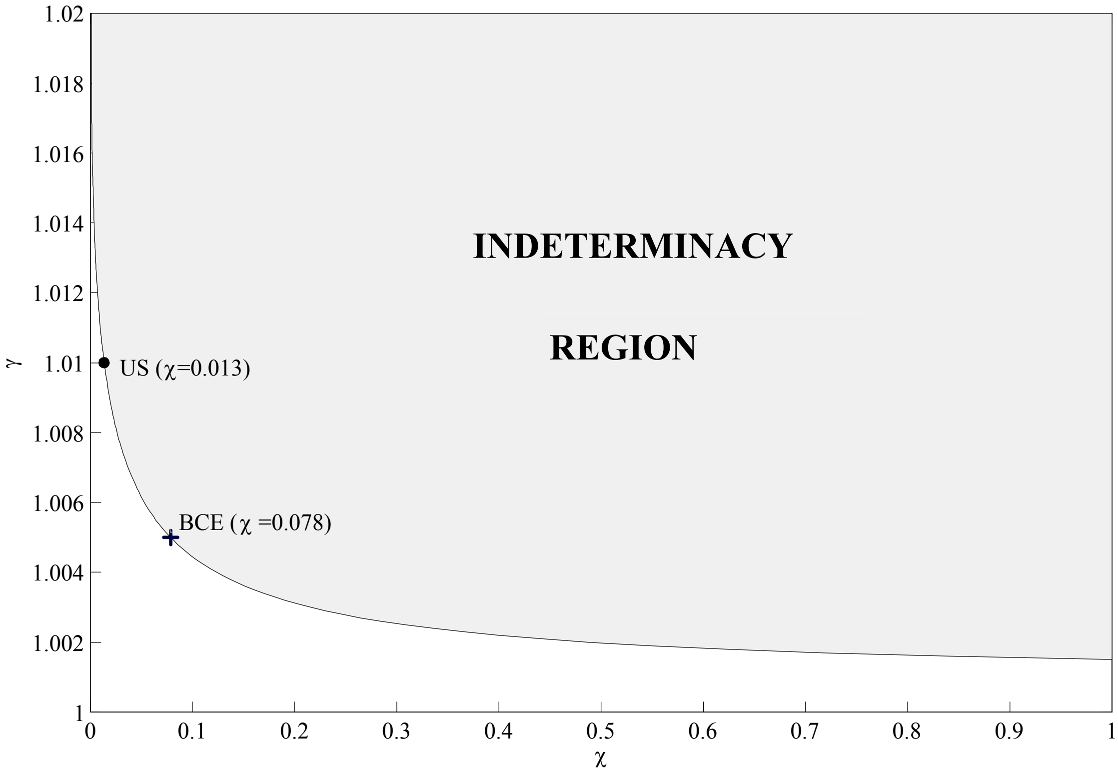

Figure 3. The indeterminacy region as γ and χ changes.

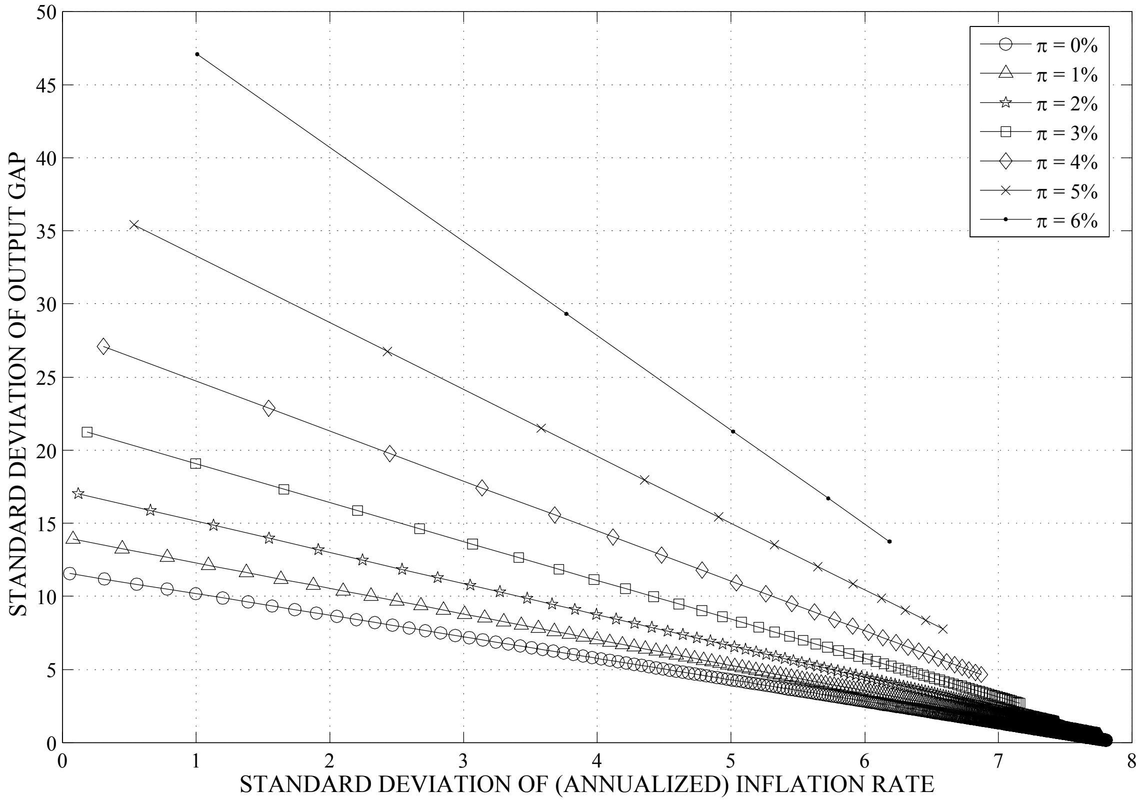

Figure 4. Efficient frontier as trend inflation varies ($\rho = 0.5$)

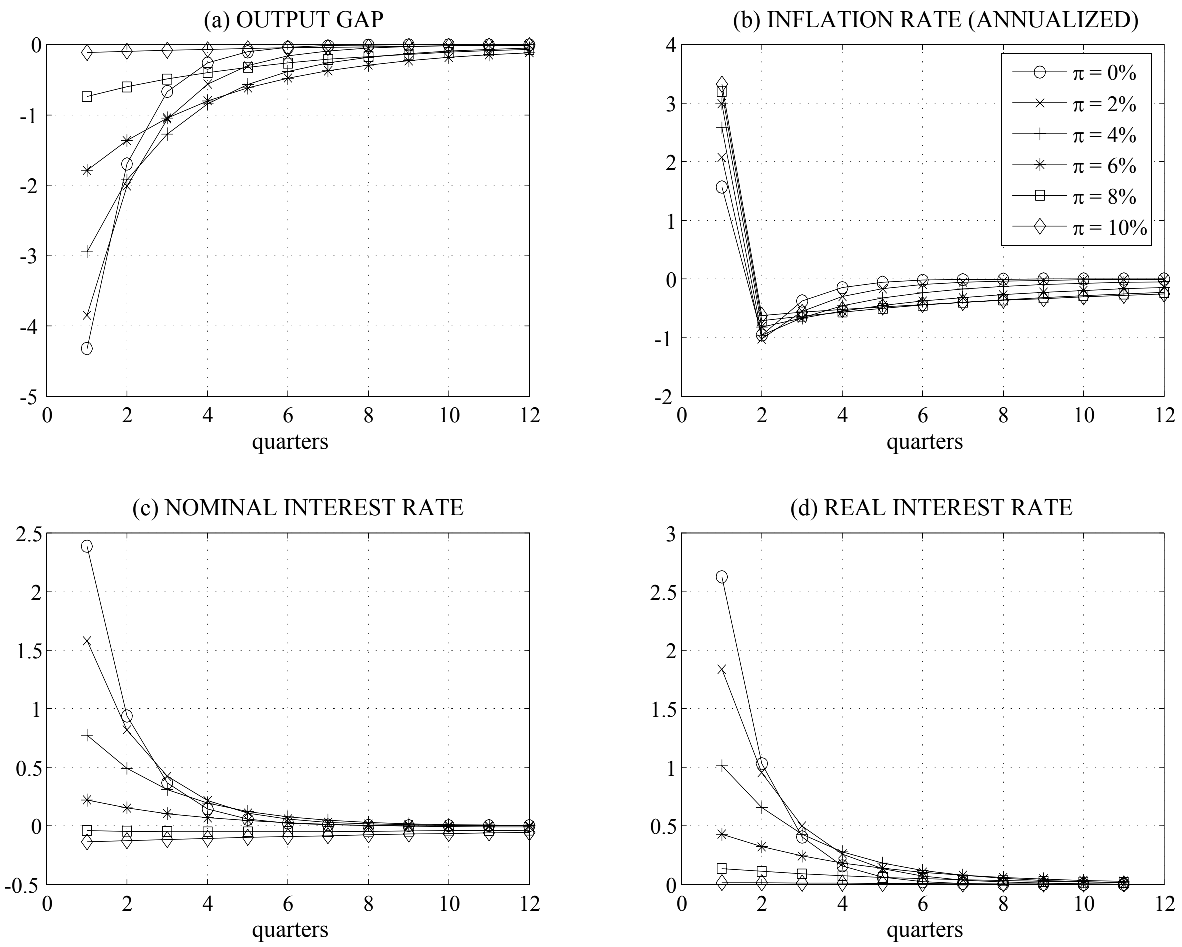

Figure 5. Optimal impulse responses under commitment ($\rho = 0$)

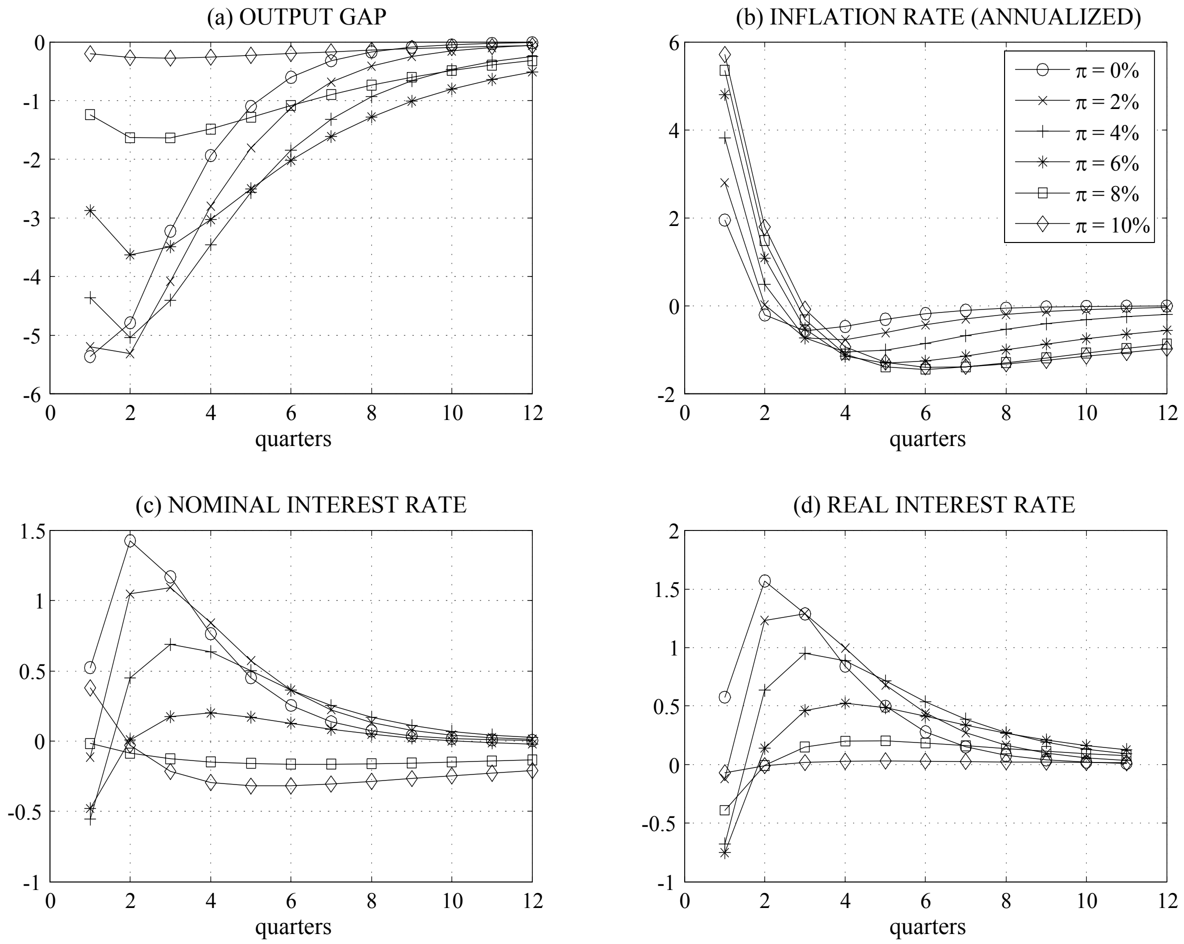

Figure 6. Optimal impulse responses under commitment ($\rho = 0.5$)