Monte Carlo simulation based sensitivity analysis of multi-echelon supply chains

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Abstract: Since supply chain performance impacts the financial performance of services and companies, it is important to optimize and analyze the performances of supply chains. Simulation provides a way to get closer to real life complex situations and uses less simplifications and assumptions than needed with analytical solutions. This paper proposes the application of Monte Carlo simulation based sensitivity analysis of supply chains to handle modeling uncertainties and stochastic nature of the supply chains and to extract and visualize relationship among the decision variables and the Key Performance Indicators. In this article the authors utilize an in-house developed interactive simulator, SIMWARE, capable to simulate complex multi-echelon supply chains based on simple configurable connection of building blocks. The proposed sensitivity analysis technique is based on the improved method used to extract gradients from Monte Carlo simulation. The extracted gradients (sensitivities) are visualized by a technique developed by the authors. The results illustrate that the developed sensitivity analysis tool is flexible enough to handle complex situations and straightforward and simple enough to be used for decision support.

Keywords: Monte Carlo simulation, sensitivity analysis, multi-echelon supply chain, SIMWARE, visualization

1 Introduction

Since supply chain performance impacts the financial results of services and companies, it is important to analyze and optimize supply chain processes. To get a better understanding of the relationships between decision variables and Key Performance Indicators (KPI) it is necessary to perform sensitivity analysis.

Simulation models can handle complex dynamic systems, and can simulate scenarios in a short timeframe providing results for different investment and process development ideas. They use less simplifications and assumptions than what analytical solutions need.

In this article the authors introduce an interactive simulator, SIMWARE, capable to simulate complex multi-echelon supply chains based on simple configurable connections of building blocks [1]. The simulator uses normally distributed random variables to make proper simulation of inventory movements week by week. These random variables represent the demands in a week. Hence, the evaluation of many simulations is needed to check the effect of any change in decision variables.

The local sensitivity analysis method can be only used to determine the local sensitivity near a fixed nominal point. However, they do not account for interactions between variables and the local sensitivity coefficients. A global sensitivity analysis method can be applied to determine the effect of an input while all the inputs are varied [2-4]. These methods do not depend on the chosen nominal point.

This paper proposes the application of Monte Carlo (MC) simulation to analyze supply chains based on sensitivity analysis to discover the connections between the input and output variables. Fig. 1 shows the simplified data flow diagram of the propose method. The proposed sensitivity analysis technique is based on the improved method used to extract gradients from MC simulations [5-7]. It applies the linear least squares fit method to identify the gradients numerically. A hyperplane is fitted to the output values generated by MC simulation. The
parameters of the hyperplane give the partial derivatives of the investigated output functions. The extracted gradients (sensitivities) are visualized by a technique developed by the authors. The results illustrate that the developed sensitivity analysis tool is flexible enough to handle complex situations and straightforward and simple enough to be used for decision support.

\[ Q = d \cdot T \], i.e. it is the ordered quantity in a purchase order. We denote the Safety Stock by \( S \) which is needed if the demand is higher than expected (line \( d \) in Fig.1). In our special case, \( S \) is to cover the stochastic demand changes, and for a given Service Level this is the maximum demand can be satisfied over \( L \). We also have Maximum Stock Level which is the stock level necessary to cover the Expected Demand (dashed line) in period \( T \).

The Reorder Point \( R \) is the stock level where the next purchase order has to be issued. It is used for materials where the inventory control is based on actual stock levels. In an ideal case \( R \) equals to total of safety stock and average demand over lead time: \( (R = d_L + S) \).

Average Stock \( K \), assuming constant demand pattern over the cycle time, can be calculated as follows:

\[ K = \frac{Q}{2} + S \quad (1) \]

It is calculated as a weighted average of stock levels over the cycle time.

Service Level (\( SL \)) is the ratio of the satisfied and the total demand (in general it is the mean of a probability distribution), or in other words it is the difference between the 100% and the ratio of unsatisfied demand:

\[ SL = 100 - 100 \left( \frac{d_L - R}{Q} \right) \quad (2) \]
We assume that all demand is satisfied from stock until available stock exists. When we reach stock level R the demand over the lead time \( (d_L) \) will be satisfied up to \( R \). Consequently, if \( d_L > R \), we are getting out of stock and there will be unsatisfied demand. Therefore the service level will be lower than 100%. \( d_i \) is unknown and it is a random variable. The probability of a certain demand level is \( P(d_L) \). Based on this, the service level is calculated as follows:

\[
SL = 100 - 100 \frac{\int_{d_L}^{d_{\text{max}}} P(d_L)(d_L - R) \, dL}{Q}
\]

where \( d_L \) is continuous random variable, and \( d_{\text{max}} \) is the maximum demand over lead time.

### 2.1 The stochastic model

Analyzing actual supply chain systems it can be discovered that the probability patterns of material flow and demand are different from the theoretical functions. Consequently we will find a difference between the theoretical (calculated) and the actual inventory movements, therefore it makes sense using an approach based on “actual” distribution function.

The applied simulator, SIMWARE uses this approach to calculate inventory movements. Modeling inventory movements using stochastic differential equations turns out to be more successful than based on the theoretical assumption that movements are following normal distribution. We propose the following model:

\[
x_{t+i} = x_t - W_i + u(x, R, L),
\]

where \( x_t \) is stock level on the \( i \)th week, \( W_i \) is a stochastic process to represent consumption. This stochastic process can be based on the empirical cumulative distribution function. \( u \) is the quantity of material received on week \( i \), based on purchase orders. Purchase orders are calculated based on the actual inventory level \( x \), reorder point, and the replenishment lead-time.

### 2.2 Calculation of sensitivities

In this subsection the method used for the estimation of the sensitivity is described. The utilized method and its description are based on [7].

The expected system performance can be given calculated as the average of the outputs in every simulation:

\[
\bar{L}(v) = \frac{1}{N} \sum_{i=1}^{N} L(x, v)
\]

where \( v \) contains the parameters of distribution functions of variables in \( x, L \) is the output function at a given input and \( N \) is the number of MC simulations. A response surface can be constructed from the realizations (or a subset) from a Monte Carlo analysis with a best fit, using standard criteria, at a near nominal point.

The fitting of a tangent plane is a known robust procedure called linear least squares method. The n-dimensional tangent hyperplane can be defined by the following expression:

\[
L_i(x) = a_0 + \sum_{i=1}^{n} a_i x_i
\]

where \( a_0 \) and \( a_i \) are the coefficients of the fitted hyperplane, \( x_i \) is the \( i \)th input and \( L_i \) is the fitted hyperplane. The \( a_i \) coefficients are the partial derivatives of the analyzed output functions respect to \( x_i \).

This method applies criteria to select the applicable input and output values to fit the hyperplane from a large number of MC simulations. The selection is based on the maximum likelihood of each input variable. If all the values in an input combination are in the following region, that input is used in fitting:

\[
x_{i,ml} \pm \sigma_i
\]

where \( x_{i,ml} \) is the maximum likelihood of \( x_i \) (in normal distribution it equals to the mean value), \( \sigma_i \) is the deviation of \( i \)th input.

### 2.3 Visualization of sensitivities

The result of sensitivity analysis is a sensitivity matrix which contains the partial derivatives of output values (e.g. key performance indicators) based on the input variables (e.g. decision variables). If we have many variables and outputs it is hard to effectively rank the inputs because the
large number of possible combinations between the inputs and outputs, therefore we developed a simple visualization technique to support the ranking process.

In the first step we defined the normalized sensitivity matrix:

\[
\hat{S}(i, j) = \frac{S(i, j)}{\sum_{j=1}^{n}[S(i, j)]}
\]

where \( i \) and \( j \) are the row and the column numbers in the sensitivity matrix, \( n \) is the number of inputs, \( \hat{J} \) is the normalized sensitivity matrix. The sum of the absolute values of a column in the new matrix is 1.

Based on the normalized sensitivity matrix a color code from blue to red is assigned to every matrix element. Blue denotes a negative connection between input and output, red shows positive cause-effect relationship.

The following is a simple example to show the proposed visualization technique. The analyzed model has three inputs and three outputs:

\[
\begin{align*}
  f_1(x) &= 15 - 2x_1 - 5x_2 - x_1x_2 \\  f_2(x) &= 40 - (x_1 + 2)^2 - (x_2 + 3)^2 \\  f_3(x) &= 1000 + 250x_1 + 100x_2 - 200x_3 + 12.5x_1x_2
\end{align*}
\]

All the input variables have normal distribution. The parameters of the distribution functions are summarized in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean value</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>50</td>
<td>15</td>
</tr>
</tbody>
</table>

The analytically calculated sensitivity matrix at the expected values of inputs is shown in Table 2.

<table>
<thead>
<tr>
<th>Function</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f1(x) )</td>
<td>-22</td>
<td>-15</td>
<td>0</td>
</tr>
<tr>
<td>( f2(x) )</td>
<td>-24</td>
<td>-46</td>
<td>0</td>
</tr>
<tr>
<td>( f3(x) )</td>
<td>500</td>
<td>225</td>
<td>-200</td>
</tr>
</tbody>
</table>

The visualized numerically calculated sensitivities are shown in Fig.4. As it can be interpreted easily, \( x_1 \) has the biggest impact on \( f1(x) \) and it has negative effect, \( x_2 \) has the second biggest impact, while \( x_3 \) has no effect. This example was used to validate our methodology and show the graphical representation of the analysis.

### 3 Problem Solution

The developed multi-echelon system model introduced in Section 2.1 is implemented in MATLAB. It can be used to analyze the impact of changes in the stochastic input variables on the desired outputs. We developed a MATLAB function based on the sensitivity analysis introduced in Section 2.2 to calculate the sensitivity matrix of a given variable selection.

![Figure 4. The visualization of sensitivities](image)

The proposed visualization technique introduced in Section 2.3 is implemented in MATLAB too, to visualize the resulted sensitivity matrices. In this study a multi-echelon system consisting two warehouses is investigated. Warehouse 01 is the distribution center, Warehouse 02 is a local warehouse. The local warehouse can order only from the central warehouse. Both warehouses store only one material in this case.

MC simulations are performed to simulate the stochastic behavior of the analyzed warehouses. After the simulations the average properties of the warehouses are calculated. The fluctuations in the average inventory levels after ten MC simulations are shown in Fig. 5. The investigated period is 50 weeks. The service levels of both warehouses are determined, they are 0.97 and 0.89. We selected reorder points to make sure that none of the warehouses running out of stock during the investigated period.

We calculated the sensitivity of the outputs to the change of the input variables. In the next example the effect of following inputs are investigated:

1. the mean value (\( x_1 \)) and the deviation (\( x_2 \)) in the normal distribution function of the demand in the central warehouse;
2. the mean value (\( x_1 \)) and the deviation (\( x_4 \)) in the normal distribution function of the demand in the local warehouse.
To analyze the sensitivity of these inputs, normal distribution functions need to be defined for all the four variables. The parameters of the distribution functions are summarized in Table 3. The given deviations of the inputs can be interpreted as the uncertainty of the variables.

Table 3. The investigated demand function parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean value</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>$x_2$</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>$x_3$</td>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>$x_4$</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

The analyzed output functions are the following:

1. the average holding cost during the investigated period ($f_1(x)$);
2. the average inventory level in Warehouse 01 ($f_2(x)$);
3. the average service level in Warehouse 01 ($f_3(x)$);
4. the average inventory level in Warehouse 02 ($f_4(x)$);
5. the average service level in Warehouse 02 ($f_5(x)$).

In the output functions, $x$ is a vector like [$x_1 \ x_2 \ x_3 \ x_4$]. 100000 input combinations are generated based on the given distribution functions. The output functions are evaluated next to these combinations. However, only 59 input combinations are used to calculate the gradient based on the introduced method, because only these combinations are in the given region.

The resulted Sensitivity matrix is summarized in Table 4. In this case the connection between four input variables and five output functions are determined. However, to rank the input variables based on which one has the most impact in the change of the output function is time consuming. Hence, the developed visualization technique is applied to show the force of the connections (see Fig. 6).

As it is shown in Fig.6 the uncertainties in the deviation of the demands have the biggest impact on all of the chosen KPIs. The mean values of the demands have very small impacts.
In Fig. 6 next to the input variables the histograms of that inputs can be seen. As it defined earlier, values of all these inputs follow normal distribution. However, the proposed method can be applied any kind of distributions. Below the output functions, the histograms of the outputs can be seen. These histograms show the expected values and the distributions of the analyzed output functions near by the given input values.

This type of visualization technique simplifies considerably the decision of Key Performance Indicators. The figure shows unambiguously which input has the utmost effect to the outputs, e.g. to the cost function, which is the most important objective function for most of the companies. Therefore, this technique can function as a decision support tool for managers.

4 Conclusion

Since supply chain performance impacts the financial performance of services and companies, it is important to optimize and analyze the performances of supply chains. This paper proposes the application of Monte Carlo simulation based sensitivity analysis of supply chains to handle modeling uncertainties and stochastic nature of the supply chains and to extract and visualize relationship among the decision variables and the Key Performance Indicators.

In this article the authors introduce an interactive simulator, SIMWARE, capable to simulate complex multi-echelon supply chains based on simple configurable connection of building blocks. To support the evaluation of the extracted gradients (sensitivities) a novel visualization technique is developed. The proposed method is applied in case of multi-echelon system built from two warehouses. We validated our solution by simulating four stochastic input variables. The results illustrate that the developed sensitivity analysis tool is flexible enough to handle complex situations and straightforward and simple enough to be used for decision support.

In further research the improvement of the applied sensitivity analysis method is one of our aims with the modification of the selection of input combinations for fitting hyperplane. The other important step is the application of constraints for KPIs in the calculation of sensitivity analysis.

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References: