Decentralized Control of Nonlinear Large scale systems Using Dynamic Output Feedback

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Abstract: To control the large scale systems is important. In this paper, a multi variable non-linear system (two inverted pendulum coupled by a spring) is output feedback linearized and the system is generalized in two subsystems and decentralized dynamic output feedback basis on lyapunov equation is applied. Using this model, the large scale system can be formulated, designed and generalized to be controlled.

Key-words: Large scale System, Decentralized Control, Dynamic Output Feedback

1. Introduction

Power systems and multimachine systems are modeled as large nonlinear highly structured systems [1]. Despite the importance and potentials of large-scale systems, it seems that impact of the research in this area is not as great as it could be. There are indeed many successful applications of large-scale systems control, for example to electrical power systems [2]. However, these applications are mainly developed by domain experts. All applications in this area are "large-scale", i.e. the number of state variables is very big and special knowledge is normally required for the formulation of the problem. In the general control communities, due to lack of simple meaningful examples, the interests in this area are not matched with its importance and potentials [3]. Conventional linear control for large-scale systems is limited since it can only deal with small disturbances about an operating point. Since differential geometric tools were introduced to nonlinear control system design, various stabilizing control results based on nonlinear power system models have been obtained for single machine systems and for multimachine systems. Two important issues for power systems control are robustness and a decentralized structure. The robustness issue arises to deal with sources of uncertainties which mainly come from the varying network topology and the dynamic variation of the load. Since physical limitation on the system structure makes information transfer among subsystems unfeasible, decentralized controllers for multimachine systems must be used [1]. Therefore, decentralized control is considered as an effective method to deal with large-scale interconnected systems. In addition, it is often used to utilize the system structural characteristics, such as symmetric structure [4], cascaded structure [5] or similar structure [6] to study special large-scale systems as a first step toward general large-scale systems [7]. A double system pendulums coupled by a spring (fig 1) was used in [8] to demonstrate some important theoretical results achieved in decentralized control. This system is used in this paper to be studied using dynamic output feedback. A character for modeling this system is that by simply adding more pendulums and springs to the existing system, this can be extended to a system of n-inverted-pendulums coupled by (n-1)-springs. Such a system can be observed as an attractive example for the current research on "characterization of problems in decentralized control" [3].

2. Problem Formulation

2.1 Notation and Preliminaries [7]

For a given matrix A, let $A^T$ denote its transport; let $\sigma(A)$ denotes the maximum singular values of A; when A is real symmetric, let $\lambda(A)$ and $\bar{\lambda}(A)$ denote its minimum and maximum eigenvalues, respectively; let $A > 0$ denotes that A is positive define. Let $L$ denotes Lipschitz constant of the function $F(x)$ in its domain of definition; also, let
\[ \| \| \] denote the Euclidean norm or its induced norm.

Consider the following two systems:

\[
\begin{align*}
\dot{x} &= f_1(x) + g_1(x)u \\
y_1 &= h_1(x)
\end{align*}
\]  
\(\Sigma_1\)  
\(\dot{x} = f_2(x) + g_2(x)u \)  
\(y_2 = h_2(x) \)  
\(\Sigma_2\)

Where \(x, \dot{x} \in \mathbb{R}^n, y_1, y_2 \in \mathbb{R}^l\) and \(u \in \mathbb{R}^m\) are the state vectors, outputs, and inputs of systems.

Definition 2.1. \(\Sigma_1\) is said to be similar to \(\Sigma_2\) in the domain \(E\) if there exists a diffeomorphism \(T: x \rightarrow \dot{x}\) defined in \(E\) such that in the coordinate \(\dot{x}\) defined by \(T\), the system \(\Sigma_1\) possesses the same form as \(\Sigma_2\). In this case, \(T(x)\) is called a similarity transformation from \(\Sigma_1\) to \(\Sigma_2\).

Remark 2.1. Similarity between systems is an equivalence relationship. That is, similarity possesses the properties of reflectivity, symmetry and transitivity; it is an extension of equivalence between linear systems.

Definition 2.2. the system \(\Sigma_1\) is said to be output feedback linearizable in the domain \(E\) if there exists a diffeomorphism \(T: x \rightarrow \dot{x}\) defined in \(E\) such that in the coordinate \(\dot{x}\) defined by \(T\), the closed-loop system resulting from the input \(u = \alpha(y) + \beta(y)v\) to \(\Sigma_1\) is described by

\[ \dot{z} = Az + Bv \]  
\(y = Cz \)

With the realization \((A, B, C)\) both controllable and observable.

Remark 2.2. It should be noticed that output feedback linearizability defined above does not imply static output feedback stabilizability. However, the fact that a system is output feedback linearizable. It should be mentioned that static output feedback stabilizability is a very strong condition and it remains an open problem even for linear systems.

Lemma 2.1. Suppose that \(\Sigma_1\) is similar to \(\Sigma_2\) in the domain \(E\). Then, \(\Sigma_1\) is output feedback linearizable if and only if \(\Sigma_2\) is output feedback linearizable.

Proof. Necessity, by output feedback linearization of \(\Sigma_1\), it follows that there exists a similarity transformation \(T: x \rightarrow \dot{x}\), \(\alpha(y) \in \mathbb{R}^m\) and a nonsingular matrix \(\beta(y) \in \mathbb{R}^{m \times m}\) such that in the coordinate \(z\) defined by \(T\), the closed-loop system

\[ \begin{align*}
\dot{x} &= f_1(x) + g_1(x)[\alpha(y_1) + \beta(y_1)v] \\
y_1 &= h_1(x)
\end{align*} \]

Has the following form

\[ \begin{align*}
\dot{\Sigma} &= f_1(x) + \Delta \psi_1(x) \| u_i + \Delta \psi_i(x) \| v \\
y_1 &= h_1(x)
\end{align*} \]

It is obvious that the systems (2)-(3) is similar to (4)-(5) with similarity transformation \(T_1\). Now, suppose that \(\Sigma_1\) is similar to \(\Sigma_2\) with similarity transformation \(T_2\). Then, it is observed that the system

\[ \begin{align*}
\dot{\Sigma} &= f_1(x) + g_1(x)[\alpha(y_1) + \beta(y_1)v] \\
y_1 &= h_1(x)
\end{align*} \]

is similar to (2)-(3) with similarity transformation \(T_2^{-1}\). By the properties of similarity, it follows that (6)-(7) is similar to (4)-(5) with similarity transformation \(T_2^{-1}\) to \(T_1\). Therefore, for \(\Sigma_2\), there exists a composition of transformation given by \(T_2^{-1} T_1\) and output feedback \(u = \alpha(y_1) + \beta(y_1)v\) such that the resulting closed-loop system is linearizable.

Sufficiently, it may be obtained directly from the necessity proof and the symmetry property of the similarity transformation. Hence, the result follows.

2.2 System Description [7]

Consider a nonlinear large-scale interconnected system described by

\[ \begin{align*}
\dot{x}_i &= f_i(x_i) + g_i(x_i)u_i + \Delta \psi_i(x_i) \\
N_{j=1}^N H_i(x_i) + \Delta H_i(x) \\
y_i &= h_i(x_i)
\end{align*} \]

(8)  
(i = 1, 2, ..., N)

Here \(x_i \in \Omega_i \in \mathbb{R}^n(\Omega_i)\) is a neighborhood of \(x_i = 0\), \(u_i, y_i \in \mathbb{R}^m\) are the state vector, input and output vector of the \(i\)th subsystems, respectively; \(f_i(x_i), g_i(x_i)\) are both smooth vectors, \(h_i\) is a smooth function in \(\Omega_i\), \(\Delta \psi_i(x_i)\) is the matched uncertainty of the \(i\)th isolated subsystem; \(\Delta H_i(x)\) is the known interconnection, the uncertain interconnection \(\Delta H_i(x)\) includes all unmatched uncertainties, and they are all continuous in their arguments. Without loss of generality, it is supposed that \(f_i(0) = 0\) and \(h_i(0) = 0\). Also, we write

\[ x = col(x_1, x_2, ..., x_N) \in \Omega_1 \times \Omega_2 \times ... \Omega_N = \Omega \]

Definition 3.1. consider the system (8)-(9). The systems

\[ \begin{align*}
\dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\
y_i &= h_i(x_i)
\end{align*} \]

(i = 1, 2, ..., N)

are called nominal subsystems of the system (8)-(9); the systems

\[ \begin{align*}
\dot{x}_i &= f_i(x_i) + g_i(x_i)[u_i + \Delta \psi_i(x_i)] \\
y_i &= h_i(x_i)
\end{align*} \]

(i = 1, 2, ..., N)
are called isolated subsystems of the system (8)-(9).

Definition 3.2. The system (8)-(9) is said to be a similar interconnected large-scale system or to posses a similar structure if all its nominal subsystems are similar to one another.

Remark 3.1. It should be noticed from remark 2.1 that there exists one system such that each nominal subsystem of the system (8)-(9) is similar to the system, if (8)-(9) is a similar interconnected large-scale system.

Assumption A1. The system (8)-(9) possesses a similar structure and there exists one output feedback linearizable nominal subsystem. It should be noticed from Definition 2.2 that Assumption A1 does not imply output feedback linearizability of the nominal subsystem (10)-(11) of the system (8)-(9). In fact, we do not require that the nominal subsystem (10)-(11) of the system (8)-(9) is output feedback stabilizable in this paper; on the other hand, the nominal subsystem is required to be linear and output feedback stabilizable in ref 8.

By Lemma 2.1, it is observed that, under Assumption A1, all nominal subsystems of the system (8)-(9) are similar. Therefore, there exists diffeomorphisms $T_1; x_i \rightarrow z_i$ and output feedback $u_i = \alpha(y_i) + \beta(y_i)\nu$ \hspace{1cm} (14)

In $\Omega_i$, for $i = 1, 2, \ldots, N$ such that, in the new coordinate $z = \text{col}(z_1, z_2, \ldots, z_N)$, the system (8)-(9) is described by

$\dot{z}_i = Az_i + B[v_i + \Delta \Phi_i(z_i)] + \sum_{j=1}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z), \hspace{1cm} (15)$

$y_i = Cz_i, \hspace{1cm} i = 1, 2, \ldots, N \hspace{1cm} (16)$

Where the realization $(A, B, C)$ is controllable and observable and where

$\Delta \Phi_i(z_i) = [\beta_i^{-1}(y_i)\Delta \Psi_i(x_i)]_{x_i=T_i^{-1}(z_i)} \hspace{1cm} (17)$

$\Delta M_i(z) = \left[ \frac{\partial T_i(x_i)}{\partial x_i} \right]_{x_i=T_i^{-1}(z_i)} \Delta H_i(T^{-1}(z)) \hspace{1cm} (18)$

$M_{ij}(z_i, z_j) = \left[ \frac{\partial T_i(x_i)}{\partial x_i} \right]_{x_i=T_i^{-1}(z_i)} \cdot H_{ij} \left(T_j^{-1}(z_j) \right), \hspace{1cm} i \neq j \hspace{1cm} (19)$

with $i, j = 1, 2, \ldots, N$ and $z = T(x) = \text{col}(T_1(x_1), T_2(x_2), \ldots, T_N(x_N))$

Assumption A2. $M_{ij}(z_i, z_j), i \neq j$ is Lipschitz in $T_i(\Omega_i) \times T_j(\Omega_j)$ with Lipschitz constants $L_{M_{ij}}$ and $L_{M_{ij}}^{ij}$. That is, for any $z_i, \bar{z}_i \in T_i(\Omega_i)$ and $z_j, \bar{z}_j \in T_j(\Omega_j), \hspace{1cm} \| M_{ij}(z_i, z_j) - M_{ij}(\bar{z}_i, \bar{z}_j) \| \leq L_{M_{ij}} \| z_i - \bar{z}_i \| + L_{M_{ij}}^{ij} \| z_j - \bar{z}_j \|, \hspace{1cm} i \neq j \hspace{1cm} (20)$

with $\Pi_{ij} \in \mathbb{R}^{n \times n}$ for all $i = 1, 2, \ldots, N$.

Assumption A3. There exist known continuous functions $\rho_i(\cdot)$ and $\gamma_i(\cdot)$, defined in their domains of definition, such that, for $i=1, 2, \ldots, N$,

$\| \Delta \Phi_i(z_i) \| \leq \rho_i(\| y_i \|) \| y_i \| \hspace{1cm} (21)$

$\| \Delta M_i(z) \| \leq \gamma_i(\| y_i \|) \| z \| \hspace{1cm} (22)$

4. Dynamic Output Feedback Controller Design

Consider the system (15)-(16). From the controllability and observability of the realization $(A, B, C)$, it follows that there exist $K, L$ such that, for any $Q > 0$ and $S > 0$, the following Lyapunov equations:

$(A - BK)^T P + P(A - BK) = -Q \hspace{1cm} (21)$

$(A - LC)^T R + R(A - LC) = -S \hspace{1cm} (22)$

have unique solutions $P > 0$ and $R > 0$, respectively.

Assumption A4. There exists matrix $F$ such that $B^T P = FC$, with $P$ defined by (21).

Consider the system (8)-(9). Construct the controller described by

$\hat{x}_i = f_i(\hat{x}_i) + g_i(\hat{x}_i)u_i + \left[ \frac{\partial T_i(x_i)}{\partial x_i} \right]^{-1} L[y_i - h_i(x_i) + \sum_{j=1}^{N} M_{ij}(z_i, z_j) + \Delta M_i(z)], \hspace{1cm} (23)$

$y_i = h_i(x_i) \hspace{1cm} (24)$

Where $K$ satisfies (21) and $\eta_i(\cdot)$ is defined by, for $i=1,2,\ldots,N$,

$\eta_i(y_i) = \begin{cases} \frac{F_{y_i} y_i}{\| F_{y_i} y_i \|} \rho_i(\| y_i \|) \| y_i \|, & F_{y_i} \neq 0 \\ 0, & F_{y_i} = 0 \end{cases} \hspace{1cm} (25)$

With $\rho_i(\| y_i \|)$, for $i=1,2,\ldots,N$ and $F$ defined respectively by assumptions A3 and A4. Now, we have the following result.
Theorem 4.1. Under assumptions A1-A4, the system (8)-(9) is stabilized by the controller (23)-(25) if there exists a neighborhood about the origin $\Omega \subseteq \Omega$ such that $W + W^T > 0$ in $\Omega \setminus \{0\}$. where $W = [w_{ij}]_{2N \times 2N}$ is defined by

$$w_{ij} = \frac{\lambda(j) - 2\lambda(i)\lambda(j)}{1 \leq i \leq N, i \neq j},$$

$$w_{ij} = \frac{\lambda(j) - 2\lambda(i)\lambda(j)}{1 \leq i \leq N}, i = j,$$

$$w_{ij} = \frac{1}{2}\left(\frac{\lambda(j) - 2\lambda(i)\lambda(j)}{1 \leq i \leq N, i \neq j},\right),$$

$$w_{ij} = \frac{1}{2}\left(\frac{\lambda(j) - 2\lambda(i)\lambda(j)}{1 \leq i \leq N}, i = j,\right),$$

$$w_{ij} = \frac{1}{2}\left(\frac{\lambda(j) - 2\lambda(i)\lambda(j)}{1 \leq i \leq N, i \neq j},\right),$$

$$w_{ij} = \frac{1}{2}\left(\frac{\lambda(j) - 2\lambda(i)\lambda(j)}{1 \leq i \leq N}, i = j,\right).$$

Where P, Q, R, S are defined by (21)-(22) [7].

3. Problem Solution [3]

A system of two inverted pendulum coupled by a spring is shown in figure 1. The variables of the system are:

- $\theta_i$: angular displacement of pendulum I ($i=1, 2$)
- $\tau_i$: torque input generated by the actuator for pendulum I ($i=1, 2$)
- $F$: spring force
- $\phi$: angular of the spring to the earth and the constants are:
  - $m_i$: mass of pendulum
  - $L$: distance of two pendulums
  - $\kappa$: spring constant

The mass of each pendulum is uniformly distributed. The length of spring is chosen so that $F=0$ when $\theta_1 = 0$, which implies that $(\theta_1, \theta_2, \theta_3)^T = 0$ is an equilibrium of the system if $\tau_i = 0$. For simplicity, we assume that the mass of spring is zero.

The dynamic equations for the system of fig.1 are given as

$$[m_1(l_1)^2/3]\dot{\theta}_1 = \tau_1 + m_1 g (l_1/2) \sin \theta_1 + l_1 F \cos (\theta_1 - \phi)$$

$$[m_2(l_2)^2/3]\dot{\theta}_2 = \tau_2 + m_2 g (l_2/2) \sin \theta_2 + l_2 F \cos (\theta_2 - \phi)$$

For simplicity, we assume that

where $g = 9.8 m/s^2$ is the constant of gravity and

$$F = \kappa (l_s - [l_1^2 + (l_2 - l_1)^2]^{1/2})$$

$$l_s = [(l + l_2 \sin \theta_2 - l_1 \sin \theta_1)^2 + (l_2 \cos \theta_2 - l_1 \cos \theta_1) / 2]$$

$$\Phi = \tan^{-1} \left( \frac{l_1 \cos \theta_1 - l_2 \cos \theta_2}{L + l_1 \cos \theta_1 - l_2 \cos \theta_2} \right)$$

(27)

The following variables are used: $l_1 = 1 m$, $l_2 = 0.8 m \cdot m_1 = 1 kg \cdot m_2 = 0.8 kg \cdot L = 1.2 m$, $\kappa = 0.04 N/m$ [8].

Now, the system of two inverted pendulum coupled by spring with governing equations (1)-(2) is set decentralized nonlinear control using dynamic output feedback. First, the state equations can be written considering the definition of state variables:

$$x_1 = \theta_1, x_2 = \dot{\theta}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2$$

then

$$[\dot{x}_1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} [x_1] + \begin{bmatrix} 14.7 \sin x_1 \\ 0 \\ 0 \\ 18.37 \sin x_3 \end{bmatrix}$$

$$[\dot{\tau}_1] = \begin{bmatrix} 0 \\ 3 \end{bmatrix} [\tau_1] + \begin{bmatrix} 3F \cos (x_1 - \phi) \\ 0 \end{bmatrix}$$

$$[\dot{\tau}_2] = \begin{bmatrix} 0 \\ 5.86 \end{bmatrix} [\tau_2] + \begin{bmatrix} -4.6875F \cos (x_2 - \phi) \end{bmatrix}$$
The second assumption is confirmed by becoming zero of interconnection terms. Control inputs (applied torque) is achieved by equation (14). Consider as $\rho_i(|y_i|)$ is zero, so $\xi_i(y_i) = 0.$

\[
\tau_1 = \left( -Kz_1 + 14.7 \sin(x_1) \right) / 3 \tag{39}
\]

\[
\tau_2 = \left( -Kz_2 + 18.37 \sin(x_3) \right) / 5.86 \tag{40}
\]

In which K is defined by equation (7). Also, consider that K must be chosen in order to have asymptotic stability for matrix $A = BK.$ So, $K = [3 \ 6]$ is defined. In Lyapunov equation, by considering the fourth assumption and choosing $S = [2 0]$, and $Q = [2 0]$, the response of equations are as follows:

\[
P = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 0.0859 & 0.0312 \\ 0.0312 & 0.0859 \end{bmatrix} \tag{41}
\]

To satisfy assumption A4, $F=1$ is chosen. To satisfy assumption A3, $\gamma_1(y) = 0.075$, $\gamma_2(y) = 0.075$ and $\rho_i(.)$ is set zero, too.

So, theorem 4.1 can be applied by mentioned data for the system of two inverted pendulum coupled by spring. Thus, matrix W must be constructed and related condition must be tested in the theorem.

\[
W \cong \begin{bmatrix} 1.4885 & 0.2557 & -1.6440 & -0.2340 & \\
0.2557 & 1.7443 & -0.2340 & -1.6440 & \\
-1.6440 & -0.2340 & 2.0000 & 0.0000 & \\
-0.2340 & -1.6440 & 0.0000 & 2.0000 \end{bmatrix} \tag{42}
\]

That the condition $W + W^T > 0$ is satisfied.

4. Conclusion

We have presented a dynamic output feedback control scheme to stabilize a class of nonlinear interconnected systems. A system of two inverted pendulum coupled by spring is controlled using output feedback decentralized control. The results obtained by applying control to the system using output feedback decentralized control are shown in figure 2-7. As shown in figures, the variation of angle $\theta$ and angular velocity is stabilized. We mention that this method may be extended to the case where the dimensions of each subsystem are different.
by introducing a new similar structure. Last, but not the least, it is demonstrated that the system structure plays an important role in reducing the computation effect for the Lyapunov equation.

References


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