5-axis Interpolation and Synchronous Servo Process with NURBS Method

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Abstract

Nowadays, NURBS (Non-Uniform Rational B-Spline) method is widely used in CAD/CAM (Computer-Aided Design/Computer-Aided Manufacturing) to represent sculptured curves or surfaces. In this paper, we develop a 5-axis NURBS real-time interpolator and realize it in our developing CNC (Computer Numerical Control) system. At first, we use two NURBS curves to represent tool-tip and tool-axis path respectively. According to feedrate and Taylor series extension, servo-controlling signals of 5 axes are obtained for each interpolating cycle. Then, generation procedure of NC (Numerical Control) code with the presented method is introduced and the method how to integrate the interpolator into our developing CNC system is given. And also, the servo-controlling structure of the CNC system is introduced. Through the illustration, it has been indicated that the proposed method can enhance the machining accuracy and the spline interpolator is feasible for 5-axis CNC system.

Keywords: 5-axis Interpolation, Synchronous Servo, NURBS

1. Introduction

Many commercial CAD systems currently provide their correspondents the capability of defining sculptured surfaces. Among the mathematical techniques used for designing specific surfaces, the NURBS is one that attracts a lot of attention[1~3]. NURBS method offers a common mathematical form for describing both standard analytical shapes and free-form surfaces[3].

In order to get a part with sculptured surfaces, a CAD system is firstly chosen to design the part with NURBS method, and then a CAM system is used to calculate tool paths for cutting the part[4]. The tool paths are typically approximated with piecewise line or circular segments. As a result, the corresponding NC codes are generated and the motion controller can be applied to guide the CNC machines to execute accurately machining[5].

However, the above approximation approach may result in several problems such as the large size of the CNC program, discontinuity of velocity and acceleration and so on[6].

In order to overcome the disadvantages of conventional CNC machining methods and also keep the advantages of CAD systems (use NURBS to represent curves), the need to develop general real-time NURBS command generators for CNC machines cannot be overlooked[7]. The real-time ability of a signal processing system had been presented in several research work to realize synchronous action[8~10]. The above method can also be applied in the field of CNC servo process. In addition, the high-speed and high-accuracy controlled machining of parts with NURBS surfaces is generally finished in 5-axis coordinated CNC machine tools[11,12]. Consequently, the CNC system should be provided with the ability to directly real-time generate 5-axis servo-controlling signals to drive the cutting tool along the NURBS curve relative the part[13].

The rest of the paper is organized in the following way. Section 2 describes how a NURBS curve is represented in a CAD system. Section 3 introduces the generating process of NC code. Section 4 shows the data structures of NC translators. Section 5 presents the realization method of 5-axis spline interpolator. Section 6 demonstrates a real example to validate the proposed method. Section 7 concludes the paper.
2. NURBS curve representation

Most CAD/CAM systems use parametric forms to represent curves or surfaces. The general form of a parametric curve in three-dimensional (3D) space can be expressed, in terms of a free parameter \( u \) (a real number between 0 and 1), as

\[
C(u) = x(u)i + y(u)j + z(u)k
\]  
(1)

where \( C(u) \) represents vector of a curve in 3D space and \( i, j, k \) is the unit vector of axis orientation of \( x, y, z \) respectively.

A \( p \)-degree NURBS curve is defined as follows

\[
C(u) = \sum_{i=0}^{n} N_{i,p}(u)W_i \mathbf{P}_i = \sum_{i=0}^{n} P_i R_{i,p}(u) \quad 0 \leq u \leq 1
\]  
(2)

where

\[
R_{i,p}(u) = \frac{N_{i,p}(u)W_i}{\sum_{j=0}^{n} N_{j,p}(u)W_j}
\]

Among which \( \mathbf{P}_i \) is the \( i \)th 3D control point; \( W_i \) is the corresponding weight factor of \( \mathbf{P}_i \); \((n+1)\) is the number of control points; \( N_{i,p}(u) \), B-spline basis function with degree of \( p \), can be calculated by the following formula

\[
N_{i,p}(u) = \begin{cases} 
1 & (u_i \leq u < u_{i+1}) \\
0 & \text{otherwise} 
\end{cases}
\]

\[
N_{i,p}(u) = \frac{u - u_{i+p}}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+1} - u}{u_{i+1} - u_{i+p-1}} N_{i+p-1}(u)
\]

\[
N_{i,p}(u) = 0 = 0 \quad \text{(prescribed)}
\]

where \([u_0, u_1, \ldots, u_{n+p+1}]\) is knot vector[4].

3. Tool paths and NC code generation

General CAD/CAM software such as Unigraphics cannot generate NC code including the introduced 5-axis coordinated double NURBS spline interpolation format, thus the CAD/CAM module needs to be developed by ourselves. The flow chart of generating NC code by the CAD/CAM module is shown as Figure 1.
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The CAD part in the module supplies interfaces connecting other CAD/CAM software to make full use of the geometry modeling functions of excellent CAD/CAM softwares. After pre-process procedure such as tool path generation, interference checking, the CAM part in the module calculates the cutter center point coordinates and the corresponding tool-axis unit vectors, which are then fitted into cutter center point spline and orientation spline in the post-process procedure and the NC code including the double NURBS spline curve interpolation format is generated and output.

4. Realization of the translator

The modular structure of the developing CNC system is shown as Figure 2, among which, TaskGenerator module is realized in Microsoft’s COM technology and is responsible for the translation of NC programs, and the AxisGroup module is realized in VenturCom’s RTDLL (real-time dynamical link library) technology and is responsible for the real-time interpolation and other real-time tasks.
It is to be integrated a 5-axis NURBS translator to the CNC system. Modules need to be changed are only TaskGenerator module and AxisGroup module. Data structures (described in C++) used in TaskGenerator module is given as follows:

```cpp
typedef enum {LINE, DNURBS} SingleStep_TYPE; //defining motion segment type
typestruct dnurbs_struct
{
derive start_rate, traverse_rate, end_rate;  //feedrate of the motion segment
degree p;  //degree of the NURBS curve
double r[20], kv[26];  //array for storing weight and knot vector
double clx[20], cly[20], clz[20];  //array for storing coordinates of control points of cutter
    //center
double tclx[20], tcly[20], tclz[20];  //array for storing coordinates of control points of tool
    //axis orientation
    ...... //definitions of the other members
}
dnurbs;
typestruct singleStep_struct
{
    SingleStep_TYPE singleStep_type;
dnurbs _dnurbs_struct;  //structure for storing double NURBS motion information
    ......  //definitions of the other structures
} singleStep;
typestd::deque<singleStep> singleStep_deque;

The dnurbs struct is added in the module for the translation of 5-axis NURBS code. The information extracted from each motion segment after translation is put into singleStep_deque and is transferred into the AxisGroup module for interpolation.

5. 5-axis interpolation process

The task of real-time NURBS spline interpolation is to calculate next interpolation period’s CL data that including tool-tip location coordination and tool-axis orientation and then transfer the CL data into machine tool’s motion commands such as X,Y,Z,A,C through post-procession.

5.1. Real-time calculation of CL data

As shown in Figure 3, \( C_0(u) \) and \( C_1(u) \) is the cutter center point spline and the orientation spline, respectively. The two splines are constructed in the same knot vector. Assumed that parameter \( u \) is function of time \( t \), that is, \( u = u(t) \). By using Taylor’s expansion of the parameter \( u \) with respect to time \( t \) to obtain the first order approximation interpolation algorithm, the first order approximation up to the first derivatives is
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\[ u_{k+1} = u_k + T_s \frac{du}{dt} \bigg|_{t=k} \]  

(3)

where \( T_s \) is interpolation period, \( u_k \) and \( u_{k+1} \) are corresponding parameters of current and next time \( t_k \) and \( t_{k+1} \).

The feedrate of tool-tip point along \( C_0(u) \) is defined by

\[ V(u) = \frac{dC_0(u)}{du} \left( \frac{du}{dt} \right) \]

(4)

and the curve speed at the parameter \( u_k \) is

\[ V(u_k) = \left| \frac{dC_0(u)}{du} \right|_{u=u_k} \]

(5)

According to Eq.s (4) and (5), the first derivative of \( u \) with \( t \) is obtained as

\[ \frac{du}{dt} = \frac{V(u_k)}{\left| \frac{dC_0(u)}{du} \right|_{u=u_k}} \]

(6)

Therefore, the first-order interpolation algorithm is obtained by substituting Eq.(6) into Eq.(3), and the latter can be expressed as follows

\[ u_{k+1} = u_k + T_s V(u_k) \left| \frac{dC_0(u)}{du} \right|_{u=u_k} \]

(7)

The first derivative of \( C_0(u) \) with \( u \) is obtained as

\[ \frac{dC_0(u)}{du} = \sum_{i=0}^{n} N_{i,p}''(u)W_iP_i \]

(8)

where the general algorithm for 1st order derivative of \( N_{i,p}(u) \) is

\[ N_{i,p}''(u) = p \left[ \frac{N_{i,p-1}(u)}{u_{i,p}-u} - \frac{N_{i+1,p-1}(u)}{u_{i+1,p}-u} \right] \]

(9)

With Eq.(7), \( C_0(u_{k+1}) \) and \( C_1(u_{k+1}) \) can be respectively obtained by substituting the calculated \( u_{k+1} \) into \( C_0(u) \) and \( C_1(u) \). Suppose the tool-axis unit vector is \( T_{k+1} \), so

\[ T_{k+1} = \frac{C_1(u_{k+1}) - C_0(u_{k+1})}{\left| C_1(u_{k+1}) - C_0(u_{k+1}) \right|} \]

(10)
5.2. Post-processing of CL data

Through the above calculations, we assume that we have the following CL data:

\[ C_0(u_{k+1}) = (o_x, o_y, o_z) \quad T_{k+1} = (t_x, t_y, t_z) \]

For 5-axis machine of table tilting/rotating type as shown in Figure 4, the following post-processing method is presented to transfer the CL data into the machine’s motion commands in a real-time cycle.

**Figure 4.** A type of 5-axis machine with tilting/rotating table

(a) if \( t_x = t_y = 0 \) and \( t_z = 1 \), then

\[ A' = 0, \quad C' = 0 \]

(b) else if \( t_x = 0 \) and \( t_y \neq 0 \), then

\[ A' = \arcsin(-t_y), \quad C' = 0 \]

(c) else if \( t_y = 0 \) and \( t_x \neq 0 \), then

\[ A' = \arcsin(t_x), \quad C' = \pi/2 \]

(d) else

\[ A' = \arcsin(t_z) \]

\[ C' = \begin{cases} \arccos(-t_z / \sin A') & t_z > 0 \\ 2\pi - \arccos(-t_z / \sin A') & t_z < 0 \end{cases} \]

For the calculations of the three translation coordinates, the Cramer rule is used here.

\[
\begin{align*}
X' &= \frac{d_1 b_1 c_1}{a_1 b_1 c_1} \\
Y' &= \frac{d_2 b_2 c_2}{a_2 b_2 c_2} \\
Z &= \frac{d_3 b_3 c_3}{a_3 b_3 c_3}
\end{align*}
\]

where

\[
\begin{align*}
a_i &= \cos C' \\
b_i &= -\cos A' \sin C' \\
c_i &= t_x \\
d_i &= o_x - (l_1 + l_2)t_x \\
a_2 &= \sin C' \\
b_2 &= \cos A' \cos C' \\
c_2 &= t_y \\
d_2 &= o_y - (l_1 + l_2)t_y \\
a_3 &= 0 \\
b_3 &= \sin A' \\
c_3 &= t_z \\
d_3 &= o_z + l_2 - (l_1 + l_2)t_z
\end{align*}
\]
In Eq.(11), $l_1$ and $l_2$ represent vertical distances of tool-tip point and intersection point of A and C axis to rotation table.

### 5.3. Synchronous servo process

The proposed format’s double NURBS NC code is generated in CAD/CAM and disposed by the translator. Then the obtained data is sent to the interpolator in which it is disposed according to the given interpolation period and thereafter, the calculated target position is treated with control law. For the CNC system adopts SoftSERCANS technology (IEC61491-SERCOS technology’s software realization, as shown in Figure 2), the final data is put into an internal telegram buffer (a block of shared memory), and the SoftSERCANS card exchanges data with it. Then the data telegram containing moving instructions is sent to drivers by SoftSERCANS through fiber optic ring and thus motors are driven. The controlling structure is shown as Figure 5.

![Figure 5. Synchronous Servo structure of the CNC system](image)

The experimental 5-axis NURBS NC code is shown as follows:

```
G06.5 P3 F300 R1.0000
X47.7773 Y71.0202 Z58.8006 U47.8066 V71.9791 W59.0828 K0.0000
X44.3229 Y75.6066 Z54.4778 U44.3651 V76.5494 W54.8138 K0.0000
X37.2164 Y84.6443 Z44.5755 U37.2554 V85.5421 W45.0342 K0.0000
X28.7567 Y101.5650 Z29.7151 U28.7081 V102.354 W30.3361 K0.0000
X-1.6963 Y163.680  Z 5.8369  U-1.8572 V163.991 W 6.7762  K0.5046
K1.0000
K1.0000
K1.0000
K1.0000
```

It’s obvious the proposed method can greatly reduce the size of NC files; especially when in high precision machining, the size change will be more obvious. Meanwhile, the CNC system’s pre-reading one part program segment equals pre-reading tens or even hundreds of linear interpolation segments; thus, the “prospective property” of CNC system can be solved easily.
In Figure 6, the curve inflection point is near the parameter 0.42, where its curvature is the smallest. Compared with traditional linear approximation machining mode, the proposed method can greatly ameliorate the machining performance, improve machining accuracy and the machining accuracy is far less influenced by the curve curvature change.

![Figure 6. Error comparative of two methods](image)

The proposed method’s real-time property is tested in a computer (Pentium IV 2.4GHz, RAM 512M) and the result is shown in Figure 7, the calculation is based on the second order approximation algorithm and the beginning of a time segment is the calculation of the basis function and the end is the finish of each axis’ position calculation. It can be found the average run time in a double NURBS interpolation period is near 28μs and the time expense occupies a small proportion in a CNC system of which the interpolation period is 1 ms or even 500μs.

![Figure 7. Run time of the proposed method](image)

The proposed 5-axis NURBS interpolation method has been realized in the developing 5-axis CNC system. The type of the 5-axis machine is table tilting/rotating. Figure 8 shows that the CNC system is controlling cutting tool along a NURBS curve path to machine an impeller.

![Figure 8. Realization of the proposed method](image)
7. Conclusions

On the basis of analyzing the defects of the existing 5-axis linear interpolation method used in the machining of sculptured surface, 3-axis coordinated NURBS interpolation format is extended and a spline expressing the tool-axis orientation is added in the paper, and double NURBS curve interpolation method is put forward, the NC code generating method is introduced and the method to realize the interpolation function in an open CNC system developed by ourselves is given. The tool-axis orientation of each interpolation point can preferably accord with the orientation computed in CAM, meanwhile the machining accuracy, efficiency, and the size of NC code are greatly ameliorated compared with linear interpolation. Therefore, the proposed double NURBS interpolation method can not only be used in the real CNC system, but also have prospective future.

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9. References