Electric Field-induced dynamical evolution of spiral wave in the regular networks of Hodgkin–Huxley neurons

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Abstract

An additional gradient force is often used to simulate the polarization effect induced by the external field in the reaction–diffusion systems. The polarization effect of weak electric field on the regular networks of Hodgkin–Huxley neurons is measured by imposing an additive term $V_E$ on physiological membrane potential at the cellular level, and the dynamical evolution of spiral wave subjected to the external electric field is investigated. A statistical variable is defined to study the dynamical evolution of spiral wave due to polarization effect. In the numerical simulation, 40000 neurons placed in the $200 \times 200$ square array with nearest neighbor connection type. It is found that spiral wave encounters death and the networks become homogeneous when the intensity of electric field exceeds the critical value, otherwise, spiral wave keeps alive completely. On the other hand, breakup of spiral wave occurs as the intensity of electric field exceeds the critical value in the presence of weak channel noise, otherwise, spiral wave keeps robustness to the external field completely. The critical value can be detected from the abrupt changes in the curve for factors of synchronization vs. control parameter, a smaller factor of synchronization is detected when the spiral wave keeps alive.

1. Introduction

Spiral waves are peculiar spatiotemporal patterns far from equilibrium state and are observed in the reaction–diffusion systems, networks of neurons and oscillators. The propagation of chemical wave in the Belousov–Zhabotinsky reaction is often investigated to detect the dynamical evolution of spiral wave in an experimental way. On the other hand, numerical studies are also effective to study the dynamics of these nonlinear waves, for example, Amdjadi presented an effective numerical scheme to study the dynamics and stability of spiral waves. Ramos investigated the propagation of spiral waves in two-dimensional reactive–diffusive media subject to a velocity field with straining and pattern formation in two-dimensional excitable media subject to Robin boundary conditions in numerical way. It does show some significance to study the spiral wave formation, selection, breakup and control in the reaction–diffusion systems or coupled arrays, because it is helpful to remove spiral wave in the cardiac tissue and thus prevent the occurrence of ventricular fibrillation. Many interesting works have been carried out and most of them are confirmed to be effective to remove spiral wave and prevent the breakup of spiral wave. Sinha et al. have proposed to suppress the spiral wave and prevent the breakup of spiral wave.
simulated the formation of spiral wave in the excitable media with small-world connection type, and other interesting works [5-10,32] were also carried out to explain the mechanism and potential role of spiral wave in the networks of neurons since the important experimental results [33,34]. It is confirmed that spiral waves emerge in disinvested mammalian cortex and the appearance of spiral wave could play positive role in signal communicating among neurons. For example, the author of this paper discussed the transition of spiral wave to other coherent states by changing some bifurcation parameters, noise-induced transition of spiral wave etc. [5,6,32].

Neuronal diseases occur when the normal electric signals communication among neurons and domains are destructed abnormally. The original Hodgkin–Huxley (HH) equations are reliable theoretical model to measure the electric activities of neurons [35]. The response of neurons depend on the collective behaviors of all neurons because each domain can consist a large number of neurons, and these neurons can couple with each other in different connection types. Stable rotating spiral waves in rat neocortical slices visualized by voltage-sensitive dye imaging are observed in experiments [33,34], and it is thought that spiral waves might serve as emergent population pacemakers to generate periodical activities in a nonoscillatory network without individual cellular pacemakers. However, the theoretical study about the potential mechanism for formation and breakup of spiral wave in neocortical slices keeps open and it is challengeable to simulate the development and transition of spiral wave in the networks of neurons in theoretical way [9,10,19]. Jr and Brunnet [8] ever investigated the formation and breakup of spiral wave in neocortical slices keeps open and it is challengeable to simulate the development and transition of spiral wave in the networks.

It is confirmed that spiral waves emerge in disinvested mammalian cortex and the appearance of spiral wave could play positive role in signal communicating among neurons. For example, the author of this paper discussed the transition of spiral wave to other coherent states by changing some bifurcation parameters, noise-induced transition of spiral wave etc. [5,6,32].

2. Mathematical model and discussion

The regular networks of Hodgkin–Huxley neurons in the presence of channel noise [39] is described by

\[
C_m \frac{dV_{ij}}{dt} = g_K n_{ij}^3(V_L - V_{ij}) + g_Na m_{ij}^2 h_{ij}(V_{Na} - V_{ij}) + g_L(V_L - V_{ij}) + I_{ij} + D(V_{i-1,j} + V_{i+1,j} + V_{i,j-1} + V_{i,j+1} - 4V_{ij}),
\]

\[
\frac{dm_{ij}}{dt} = a_m(V_{ij})(1 - m_{ij}) - \beta_m(V_{ij})m_{ij} + \xi_m(t),
\]

\[
\frac{dh_{ij}}{dt} = a_h(V_{ij})(1 - h_{ij}) - \beta_h(V_{ij})h_{ij} + \xi_h(t),
\]

\[
\frac{dn_{ij}}{dt} = a_n(V_{ij})(1 - n_{ij}) - \beta_n(V_{ij})n_{ij} + \xi_n(t),
\]

\[
am = \frac{0.1(V_{ij} + 40)}{1 - \exp(-(V_{ij} + 40)/10)};
\]

\[
\beta_m = 4\exp(-(V_{ij} + 65)/18);
\]
\[
\begin{align*}
    a_h &= 0.07 \exp(-(V_{ij} + 65)/20); \\
    \beta_h &= \frac{1}{1 + \exp(-(V_{ij} + 35)/10)}; \\
    a_n &= \frac{0.01(V_{ij} + 55)}{1 - \exp(-(V_{ij} + 55)/10)}; \\
    \beta_n &= 0.125 \exp(-(V_{ij} + 65)/80);
\end{align*}
\]

where the variable \( V_{ij} \) describes the membrane potential of the neuron in the site \((i,j)\), and all the neurons are arranged in a two-dimensional square array in space. \( m, n \) and \( h \) are parameters for gate channel, the capacitance of membrane is \( C_m = 1 \mu F/cm^2 \) and the temperature of the membrane is fixed at \( T = 6.3^\circ C \). \( D \) is the intensity of coupling, the maximal conductance of potassium is \( g_K = 36 \, ms/cm^2 \), the maximal conductance of sodium is \( g_{Na} = 120 \, ms/cm^2 \), the conductance of leakage current is \( g_L = 0.3 \, ms/cm^2 \) and the external injection current \( I_{ij} = 0 \). The reversal potential \( V_K = -77 \, mV \), \( V_{Na} = 50 \, mV \) and \( V_L = -54.4 \, mV \). \( \zeta_m(t), \zeta_n(t), \zeta_h(t) \), are independent Gaussian white noise and the statistical properties [39] of the channel noise are defined by

\[
\begin{align*}
    \langle \zeta_m(t) \rangle &= 0, \langle \zeta_m(t) \zeta_m(t') \rangle = \frac{2\sigma_m \beta_m}{N_{Na}(\alpha_m + \beta_m)} \delta(t - t'), \\
    \langle \zeta_n(t) \rangle &= 0, \langle \zeta_n(t) \zeta_n(t') \rangle = \frac{2\sigma_n \beta_n}{N_K(\alpha_n + \beta_n)} \delta(t - t'), \\
    \langle \zeta_h(t) \rangle &= 0, \langle \zeta_h(t) \zeta_h(t') \rangle = \frac{2\sigma_h \beta_h}{N_{Na}(\alpha_h + \beta_h)} \delta(t - t'),
\end{align*}
\]

where \( D_m, \sigma_m, \beta_m \) and \( D_n, \sigma_n, \beta_n \) describes the intensity of noise, respectively. The function \( \delta(t - t') = 1 \) at \( t = t' \) and \( \delta(t - t') = 0 \) at \( t \neq t' \). \( N_{Na}, N_K \) is the total numbers of sodium and potassium channels presented in a given patch of the membrane, respectively. In the case of homogeneous ion channel density, \( \rho_{Na} = 60 \, \mu \text{m}^{-2} \) and \( \rho_K = 18 \, \mu \text{m}^{-2} \), the total channels number is decided by \( N_{Na} = \rho_{Na}s \) and \( N_K = \rho_Ks \), and \( s \) describes the membrane patch. Based on the mean field theory, a statistical variable is defined to study the collective behaviors and statistical property

\[
F = \frac{1}{N^2} \sum_{j=1}^N \sum_{i=1}^N V_{ij} = \langle V_{ij} \rangle,
\]

\[
R = \frac{\langle F^2 \rangle - \langle F \rangle^2}{\frac{1}{N^2} \sum_{j=1}^N \sum_{i=1}^N (V_{ij}^2 - \langle V_{ij} \rangle^2)^2},
\]

where \( R \) is factor of synchronization, the number of neurons is \( N^2 \) and the variable \( V_{ij} \) is the membrane potential of neuron in site \((i,j)\). As reported in the previous works, optimized channel noise and multiplicative noise are helpful to support the survival of spiral wave in the networks of neurons, which can play positive role in signal communicating in breakthrough these quiescent areas; and the curve for factor of synchronization vs bifurcation parameters measures the phase transition of spiral wave by detecting the abrupt changing points in this curve. By using the modified Hodgkin–Huxley neuron model exposed to external weak electric field [41,42], the networks of the modified Hodgkin–Huxley neuron model can be rewritten by

\[
\begin{align*}
    C_m \frac{dV_{ij}}{dt} &= D(V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1} - 4V_{ij}) + \bar{g}_K h_m^2 (V_K - V_{ij} - V_L) + \bar{g}_{Na} m^2 h_n (V_{Na} - V_{ij} - V_L) - I_e + \bar{g}_L (V_L - V_{ij} - V_E), \\
    \frac{dy_{ij}}{dt} &= a_y(V_{ij})(1 - y_{ij}) - b_y(V_{ij})y_{ij} + \zeta_y(t). (y = m, n, h),
\end{align*}
\]

where the parameter \( V_L \) is the induced transmembrane potential from the external electric field which could be calculated in Refs. [41–45] and \( I_e = C_m \frac{dV_{ij}}{dt} \) is used to measure the parameter current flowing through the capacitor due to the electric field stimulus.

### 3. Numerical results and discussion

In this section, the external electric field-induced changes of stable rotating spiral wave in the networks of improved Hodgkin–Huxley neurons model is investigated. An appropriate initial states are used to develop a stable rotating spiral wave in the networks of neurons. In the numerical studies, the time step \( h = 0.001 \), coupling intensity \( D = 1.0 \), membrane temperature \( T = 6.3^\circ C \), external forcing current \( I_e = 0 \), 40000 neurons are arranged in a two-dimensional array with 200 \times 200 sites, and no-flux boundary condition is used. In Fig. 1, the developed stable rotating spiral wave in the networks is shown in gray and this stable rotating spiral wave will be regarded as initial state to be controlled by the external electric field.

The stable rotating spiral wave presented in Fig. 1 can be developed with certain transient period in the networks of Hodgkin–Huxley neuron model. Spiral wave becomes sparse at fixed bigger coupling intensity \( D \), the selection of initial values for the four variables can refer to our previous works[32].
The curve in Fig. 2 illustrates the correlation of synchronization factor and the intensity of external electric field, and abrupt changing point close to the peak is observed. Spiral wave is removed and the networks become homogenous completely when the intensity of electric field exceeds certain threshold, otherwise, the spiral wave keeps alive. As a result, the corresponding snapshots are plotted in Fig. 3 to illustrate the death and robustness of spiral wave at different conditions.

The results in Fig. 3 confirm that spiral wave encounters death when the intensity of electric field exceeds certain threshold about $V_E = -2 \times 10^{-7}$, and the time series of average membrane potentials of neurons in the networks decrease to certain stable values, otherwise, the spiral wave keeps alive and the time series of the average membrane potentials of neurons in the networks oscillate periodically vs. time. In fact, channel noise plays important role in changing the electric activities of neurons, therefore, it is important to study this problem in the presence of channel noise. As it is known, breakup of spiral wave occurs when the channel noise exceeds certain thresholds. Our aim in this paper is to study the electric field-induced transition of spiral wave, therefore, weak channel noise is considered, for example, the membrane patch $s = 4$ is used to develop a stable rotating spiral wave, then the effect of external electric field is considered.

The curve in Fig. 4 shows that correlation between the factors of synchronization and the intensity of electric field in the presence of channel noise, and peak is also observed. Spiral wave keeps alive when the intensity of field is less than the critical value about $V_E = -7 \times 10^{-8}$, otherwise, breakup of spiral wave occurs. Furthermore, snapshots of spiral wave are plotted to illustrate the state when the spiral wave is controlled by the external electric field with the intensity close to the critical value.

The results in Fig. 5 confirm that spiral wave used to keep alive when the intensity of electric field is less than the critical value about $V_E = -7 \times 10^{-8}$, and the time series of average membrane potentials of neurons in the networks oscillate regu-
Fig. 3. The developed state of the stable rotating spiral wave at $t = 600$ time units under different intensities of external electric field, for $V_E = -8 \times 10^{-6}$ (a,e1), $V_E = -9 \times 10^{-6}$ (b,e2), $V_E = -1 \times 10^{-7}$ (c,e3) and $V_E = -2 \times 10^{-7}$ (d,e4); the time series of average membrane potentials of neurons in the networks are plotted vs. time under different intensities of external electric field close to the critical value, which can be measured from Fig. 2. The snapshots are plotted in gray from black (about $-80$ mV) to white (about 40 mV) and the coupling coefficient $D = 1$.

Fig. 4. Factors of synchronization under different intensities of external electric field ($V_E$) within a transient period about 800 time units are plotted, and the channel noise intensity is measured by membrane patch $s = 4$. The critical point is about $V_E = -7 \times 10^{-6}$, the red line with arrow indicates that higher factor of synchronization is associated to the breakup of spiral wave in this case. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
larly, otherwise, spiral wave encounters breakup and the time series of average membrane potentials of neurons oscillate irregularly when the channel noise is being considered. In summary, the death, breakup and survival of spiral wave depends on the electric field intensity greatly. In the case of no channel noise being considered, the spiral wave is removed when the intensity of electric field exceeds the critical value about \( V_E = \frac{1}{C_0} \cdot \frac{1}{C_2} \cdot \frac{10}{C_0} \), whiles it encounters breakup when the intensity of electric field exceeds the critical value about \( V_E = \frac{1}{C_0} \cdot \frac{9}{C_2} \cdot \frac{10}{C_0} \); otherwise, the spiral wave used to keep alive completely. On the other hand, the factor of synchronization can denote the development of spiral wave, smaller factor of synchronization often indicates that spiral wave keeps alive, and the critical value can be measured from the curve of factors of synchronization.

4. Conclusions

The external electric field is introduced into the regular networks of HH neurons, the polarization effect of field is described by an additive membrane potential perturbation at the cellular level. A statistical factor of synchronization is used

Fig. 5. The developed state of the stable rotating spiral wave at \( t = 800 \) time units under different intensities of external electric field, for \( V_E = -5 \times 10^{-8} (a,e1), V_E = -6 \times 10^{-8} (b,e2), V_E = -7 \times 10^{-8} (c,e3) \) and \( V_E = -8 \times 10^{-8} (d,e4) \); the time series of average membrane potentials of neurons in the networks are plotted vs. time under different intensities of external electric field close to the critical value, which can be measured from Fig. 4, the membrane patch \( s = 4 \) is used to denote the intensity of channel noise. The snapshots are plotted in gray from black (about \(-80 \text{ mV}\)) to white (about \(40 \text{ mV}\)) and the coupling coefficient \( D = 1 \).
to measure the abrupt phase transition of spiral wave in the networks, the curve for the factors of synchronization vs. the intensity of electric field are calculated and used to detect the critical threshold of electric field (breakup or death of spiral wave), which is associated with an abrupt changing point. It is found that spiral wave encounters death and the networks become homogeneous when the intensity of electric field exceeds the critical threshold, otherwise, the spiral wave keeps alive completely. In the presence of weak channel noise, spiral wave encounters breakup when the intensity of electric field exceed the certain critical intensity of electric field, otherwise, the spiral wave keeps alive well. The curve for factors of synchronization vs. control parameter (e.g. intensity of electric field etc) can be calculated and a stable rotating spiral wave often presents a smaller factor of synchronization. When the biological body is exposed to the electromagnetic field, the normal electric activities of neurons are often perturbed, some neuronal disease could be induced by the electromagnetic radiation. This work could give some potential explanation about the abnormality of neuronal activity because the spiral wave in the domain of neuron system is destroyed and thus normal signal communication is disturbed.

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References


