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Supersymmetry through CP violation in $H^{\pm} \to W^{\pm} h^0$

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Abstract: The CP violating asymmetry between the partial decay rates of $H^+ \to W^+ h^0$ and $H^- \to W^- h^0$ is calculated in first order of the weak coupling constant $\alpha_\omega = g^2/4\pi$ in MSSM with complex parameters. The dependence on the phases of A_τ and M_1 is discussed. Different values of $\tan\beta$ are considered. The asymmetry is up to the order of 10^{-2}.

Keywords: Supersymmetric Standard Model, Higgs Physics, CP violation.

1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) implies the existence of a pair of charged Higgs bosons $H^\pm$. At tree level there are three possible decay modes of $H^\pm$ into ordinary particles: $H^+ \to t\bar{b}$, $H^+ \to \nu\tau^+$ and $H^+ \to W^+h^0$ where h^0 is the lightest neutral Higgs boson. The lighter fermions from the first two generations and the heavier neutral Higgs bosons are not considered. Loop corrections due to a Lagrangian with CP violating phases lead to decay rate asymmetries between the partial decay widths of H^+ and H^- and that would be a clear signal of CP violation. In refs.[1] and [2] such decay rate asymmetries are considered in the MSSM with complex phases for the quark decay mode $H^+ \to tb$ – the asymmetry δ_{tb}, and for the lepton decay mode $H^+ \to \nu\tau$ – the asymmetry $\delta_{\nu\tau}$. In order to finalize this investigation we consider here the decay rate asymmetry of $H^\pm \to W^\pm h^0$:

$$\delta_{Wh^0} = \frac{\Gamma(H^+ \to W^+h^0) - \Gamma(H^- \to W^-h^0)}{\Gamma(H^+ \to W^+h^0) + \Gamma(H^- \to W^-h^0)}. \tag{1.1}$$

We shall work in MSSM with complex parameters in first order of the weak coupling constant $\alpha_\omega = g^2/4\pi$. Within MSSM, after redefining the fields, the new sources of CP violation are the phase of the higgsino mass parameter μ, two of the phases of the gaugino masses M_i, $i = 1, 2, 3$ (we choose these to be the phases of M_1 and M_3), and the phases of the trilinear couplings of the fermions f, A_f. Especially the latter ones are practically unconstrained. Previously this asymmetry was considered in the two-Higgs doublet model in [3].

Discussing CP violation in the decay widths, we must keep in mind the branching ratios of the relevant decay modes. $H^+ \to \nu\tau^+$ is significant for low m_{H^+}, below the $\bar{t}b$ threshold. This determines the sensitivity of $\delta_{\nu\tau}$ to light $\tilde{\nu}\tilde{\tau}$ and $\tilde{\chi}^0\tilde{\chi}^\pm$ in the loops, i.e. to the phases ϕ_τ and ϕ_1 of A_τ and M_1. The decay $H^+ \to t\bar{b}$ dominates for high m_{H^+}, which determines the sensitivity of δ_{tb} to the phases of A_t and A_b. The complication with the decay $H^+ \to W^+h^0$ is that the final state h^0 is not observed yet and m_{h^0} is an unknown parameter. However, once m_{H^+} and $\tan\beta$ are fixed, the SUSY structure of the theory determines uniquely both m_{h^0} and the coupling $H^+W^-h^0$. There are two consequences that are important for us. First, increasing m_{H^+}, the mass m_{h^0} is saturated approaching its maximum value. At tree level this is particularly simple: $m_{h^0} \leq m_{h^0}^{max} = m_Z|\cos 2\beta|$ [4], while including QCD and SUSY radiative corrections $m_{h^0}^{max}$ can be increased considerably, $m_{h^0}^{max} \simeq 130$ GeV (for a recent review see [5] and the refs. therein). There is also an experimental lower bound for m_{h^0}, $m_{h^0} \geq 96$ GeV [6]. Thus, respecting both the experimental and theoretical bounds, we shall consider m_{h^0} in the range $96 \leq m_{h^0} \leq 130$ GeV. In this range of m_{h^0}, for $m_{H^+} > m_W + m_{h^0}$ we are already in the saturation regime where we may keep m_{H^+} and m_{h^0} as independent parameters. The second consequence concerns the $H^+W^-h^0$ coupling which determines the $\mathrm{Br}(H^+ \to W^+h^0)$. Increasing m_{H^+} it quickly falls down and, depending on $\tan\beta$, we can enter the so called decoupling limit, $\cos^2(\beta - \alpha) \to 0$, where the $\mathrm{Br}(H^+ \to W^+h^0)$ almost vanishes. This imposes severe restrictions on m_{H^+} and $\tan\beta$. In order to keep the value of $\mathrm{Br}(H^+ \to W^+h^0)$ at the level of few percents, we shall consider $200 \leq m_{H^+} \leq 600$ GeV and low $\tan\beta$, $3 \leq \tan\beta \leq 9$ ($\tan\beta \leq 3$ is already excluded from the Higgs searches at LEP[6]). In accordance with this, δ_{Wh^0} will receive contributions from $\tilde{\nu}\tilde{\tau}^{\pm}$, $\tilde{t}\tilde{b}$ and $\tilde{\chi}^0\tilde{\chi}^{\pm}$ in the loops. Due to the large mass of the top-quark, the radiative corrections with stops and sbottoms with low masses appear to be too large to be considered within the α_w approximation used here. This means that we assume that the squarks are heavy and will not contribute in the considered range of m_{H^+}. This will be discussed in the next Section 2. Thus, we shall consider the sensitivity of the asymmetry δ_{Wh} to the phases ϕ_τ and ϕ_1 of the chargino-neutralino and the slepton sectors. According to the experimental limits on the electric dipole moments of the electron and the neutron, we assume a zero phase for the Higgsino mass parameter μ, $\phi_\mu = 0$. However, as is shown in [7], a large phase ϕ_μ is not impossible, it would require fine-tuning between the phase ϕ_μ and the other SUSY parameters. We end up with a short discussion of the influence of ϕ_μ on δ_{Wh^0}.

The paper is organized as follows. In the next section we present the analytic expression for the asymmetry. In Section 3 we discuss the numerical results in MSSM. We end up with a conclusion. There are two Appendicies - with the Lagrangian and with the analytic expressions for the imaginary parts of the Passarino Veltmann (PV) integrals.

2. The asymmetry

We write the matrix elements of $H^+ \to W^+h^0$ and $H^- \to W^-h^0$ in the form:

$$M_{H^\pm} = ig\varepsilon^\lambda_\alpha(p_W)p^\alpha_h Y^\pm, \tag{2.1}$$

where $\varepsilon^\lambda_\alpha(p_W)$ is the polarization vector of $W^\pm$, $Y^\pm$ are the loop corrected couplings:

$$Y^\pm = y + \delta Y^\pm_1 + \delta Y^\pm_2 + \delta Y^\pm_3 + ... \tag{2.2}$$

Here y is the tree level coupling:

$$y = \cos(\alpha - \beta), \tag{2.3}$$

and $\delta Y^\pm_k, k = 1, 2, 3, ...$ are the SUSY-induced loop corrections.
The decay rates of $H^\pm \longrightarrow W^\pm h^0$ are:

$$\Gamma(H^\pm \longrightarrow W^\pm h^0) = \frac{\alpha_\omega}{16}\frac{\lambda^{3/2}(m_H^2, m_h^2, m_W^2)}{m_H^3 m_W^2}|Y^\pm|^2 \tag{2.4}$$

Here:

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz, \tag{2.5}$$

$$|Y^\pm|^2 = y^2 + 2y\sum_k \Re e(\delta Y^\pm_k) + \mathcal{O}(\alpha^2_\omega). \tag{2.6}$$

At tree level the decay widths of H^+ and H^- are equal and there is no CP violation:

$$\Gamma_{tree}(H^\pm \longrightarrow W^\pm h^0) = \frac{\alpha_\omega}{16}\frac{\lambda^{3/2}(m_H^2, m_h^2, m_W^2)}{m_H^3 m_W^2}\cos^2(\alpha - \beta). \tag{2.7}$$

CP violation is induced by loop corrections. They have CP-invariant and CP-violating contributions:

$$\delta Y_k^{\pm} = \delta Y_k^{inv} \pm \delta Y_k^{CP} \tag{2.8}$$

and each one has real and imaginary parts:

$$\delta Y_k^{inv} = \Re e(\delta Y_k^{inv}) + i\Im m(\delta Y_k^{inv}), \quad \delta Y_k^{CP} = \Re e(\delta Y_k^{CP}) + i\Im m(\delta Y_k^{CP}) \tag{2.9}$$

The asymmetry δ_{Wh} is determined by $\Re e(\delta Y_k^{CP})$. Further we shall work in first order of the weak radiative coupling constant $\alpha_\omega = g^2/4\pi$. This approximation means that we neglect the CP-invariant loop corrections $\Re e(\delta Y_k^{inv})$ in the denominator in formula (1.1). Then from (1.1), (2.4) and (2.6) we obtain:

$$\delta_{Wh^0} \simeq \frac{2\sum\limits_k \Re e(\delta Y_k^{CP})}{y}. \tag{2.10}$$

Here the sum is over the loops with CP-violation. We work in MSSM with complex parameters. The Lagrangian is in Appendix A.

In general, there are two types of SUSY radiative corrections – self-energy loops and vertex corrections. We have self-energy loops on the W^+-line, on the h^0-line and on the H^+-line. The loops on the W^+-line are proportional to the 4-momentum of W^+, p_W^α, and because of the gauge invariance ($p_W^\alpha \epsilon_\alpha(p_W) = 0$), equal to zero. As δ_{Wh^0} is determined by the absoptive parts of the loops, the loops on the h^0-line will contribute only if the kinematic condition $m_{h^0}^2 \geq (\widetilde{m}_1 + \widetilde{m}_2)^2$ is satisfied, where $\widetilde{m}_1$ and $\widetilde{m}_2$ are the masses of the two particles in the loop. However, because of the upper theoretical bound on m_{h^0} ($m_{h^0} \leq 130GeV$) and the lower experimental bounds on the SUSY particles, the above condition cannot be fulfilled for any pair of SUSY particles ($\widetilde{m}_1 + \widetilde{m}_2$) and these loops will not contribute either. Thus, the only radiative corrections that will contribute are the self-energy loops on the H^+-line and all vertex corrections.

According to the particles in the loops, we have radiative corrections with sneutrinos and staus (Fig.1a with sleptons), with stops and sbottoms (Fig.1a with squarks) and with charginos and neutralinos (Fig.1b). (Note that the radiative corrections with ordinary quarks and with Higgs bosons are CP invariant.) The radiative corrections with scalar quarks are proportional to m_t and, in general, one would expect that they should give the main contribution to δ_{Wh^0}. This contribution is enhanced, in addition, by the colour factor 3 that multiplies each diagram with squarks. Having at our disposal the parameters of the scalar mass matrices, one can achieve masses of the squarks that are low enough to be kinematically allowed in the considered range of m_{H^+}, $m_{H^+}^2 \geq (m_{\tilde{b}_m} + m_{\tilde{t}_n})^2$, still respecting the experimental bounds. However, at such small masses of the squarks, the CP-invariant radiative corrections to the denominator in (1.1) also grow and one can no longer expect that our first order formula (2,10) would be a good approximation. The performed numerical analysis confirmed these arguments. That's why we shall not consider the contribution of loops with squarks. Physically, this means that we assume that they are heavy and the decay $H^+ \to \tilde{t}\tilde{b}$ is not allowed kinematically. In the commonly discussed models of SUSY breaking, the squarks are much heavier than sleptons, charginos and neutralinos.

Thus, finally we are left with the loop corrections on Fig.1a with sleptons and on Fig. 1b with chargino and neutralinos. The full analytic expressions for δY_k are rather lengthy but they considerably symplify for $\Re e(\delta Y_k^{CP})$.

For the loops with staus and sneutrinos we have:

$$\begin{aligned}
\Re e(\delta Y_1^{CP})(\widetilde{\tau}\widetilde{\nu}\widetilde{\nu}) &= \frac{\alpha_\omega m_z \sin(\alpha+\beta)}{8\pi m_W \cos\theta_W} \sum_{m=1,2} \Im m((g_4^{\widetilde{\tau}})_m \mathcal{R}_{Lm}^{\widetilde{\tau}*}) \Im m(C_0^{(1)} + C_1^{(1)} + C_2^{(1)}) \\
C_X^{(1)} &= C_X(m_H^2, m_W^2, m_h^2, m_{\widetilde{\nu}}^2, m_{\widetilde{\tau}_m}^2, m_{\widetilde{\nu}}^2)
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
\Re e(\delta Y_2^{CP})(\widetilde{\nu}\widetilde{\tau}\widetilde{\tau}) &= -\frac{\alpha_\omega}{4\pi m_W} \sum_{m,n=1,2} \Im m((g_4^{\widetilde{\tau}})_n \mathcal{R}_{Lm}^{\widetilde{\tau}*} c_{mn}^{\widetilde{\tau}}) \Im m(C_0^{(2)} + C_1^{(2)} + C_2^{(2)}) \\
C_X^{(2)} &= C_X(m_{H^+}^2, m_W^2, m_h^2, m_{\widetilde{\tau}_n}^2, m_{\widetilde{\nu}}^2, m_{\widetilde{\tau}_m}^2)
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
\Re e(\delta Y_3^{CP})(\widetilde{\tau}\widetilde{\nu}) &= \frac{\alpha_\omega}{8\pi m_{H^+}^2 m_W^2} \sin(\beta-\alpha) \sum_{m=1,2} (m_{\widetilde{\tau}_m}^2 - m_{\widetilde{\nu}}^2) \Im m((g_4^{\widetilde{\tau}})_m \mathcal{R}_{Lm}^{\widetilde{\tau}*}) \Im m\, B_0^{(3)} \\
B_0^{(3)} &= B_0(m_{H^+}^2, m_{\widetilde{\nu}}^2, m_{\widetilde{\tau}_m}^2)
\end{aligned} \tag{2.13}$$

The loops with charginos and neutralinos give:

$$\begin{aligned}
\Re e(\delta Y_4^{CP})(\widetilde{\chi}^+\widetilde{\chi}^0\widetilde{\chi}^0) = -\frac{\alpha_\omega}{2\pi} \sum_{\substack{i=1,2\\ k,l=1,2,3,4}} &\Big\{ \Im m(f_{ik}^L A_{kl} O_{li}^R + f_{ik}^R A_{kl}^* O_{li}^L) \Big[m_{\widetilde{\chi}_k^0}^2 \Im m(2C_0^{(4)} + C_1^{(4)} + C_2^{(4)}) \\
&\qquad + m_{H^+}^2 \Im m(C_1^{(4)}) + m_h^2 \Im m(C_2^{(4)}) \Big] \\
&+ m_{\widetilde{\chi}_k^0} m_{\widetilde{\chi}_i^+} \Im m(f_{ik}^L A_{kl}^* O_{li}^L + f_{ik}^R A_{kl} O_{li}^R) \Im m(C_0^{(4)} + C_1^{(4)} + C_2^{(4)}) \\
&+ m_{\widetilde{\chi}_k^0} m_{\widetilde{\chi}_l^0} \Im m(f_{ik}^L A_{kl}^* O_{li}^R + f_{ik}^R A_{kl} O_{li}^L) \Im m(C_0^{(4)} + C_1^{(4)} + C_2^{(4)}) \\
&+ m_{\widetilde{\chi}_i^+} m_{\widetilde{\chi}_l^0} \Im m(f_{ik}^L A_{kl} O_{li}^L + f_{ik}^R A_{kl}^* O_{li}^R) \Im m(C_1^{(4)} + C_2^{(4)}) \Big\} \\
C_X^{(4)} = C_X(m_{H^+}^2, m_W^2, m_h^2, m_{\widetilde{\chi}_k^0}^2, m_{\widetilde{\chi}_i^+}^2, m_{\widetilde{\chi}_l^0}^2)&.
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
\Re e(\delta Y_5^{CP})(\widetilde{\chi}^0\widetilde{\chi}^+\widetilde{\chi}^+) = \frac{\alpha_\omega}{2\pi} \sum_{\substack{i,j=1,2\\ k=1,2,3,4}} &\Big\{ \Im m(f_{ik}^L O_{kj}^L \widetilde{A}_{ij} + f_{ik}^R O_{kj}^R \widetilde{A}_{ji}^*) [m_{\widetilde{\chi}_i^+}^2 \Im m(2C_0^{(5)} + C_1^{(5)} + C_2^{(5)}) \\
&\qquad + m_{H^+}^2 \Im m(C_1^{(5)}) + m_h^2 \Im m(C_2^{(5)})] \\
&+ m_{\widetilde{\chi}_i^+} m_{\widetilde{\chi}_j^+} \Im m(f_{ik}^L O_{kj}^L \widetilde{A}_{ji}^* + f_{ik}^R O_{kj}^R \widetilde{A}_{ij}) \Im m(C_0^{(5)} + C_1^{(5)} + C_2^{(5)}) \\
&+ m_{\widetilde{\chi}_k^0} m_{\widetilde{\chi}_i^+} \Im m(f_{ik}^L O_{kj}^R \widetilde{A}_{ji}^* + f_{ik}^R O_{kj}^L \widetilde{A}_{ij}) \Im m(C_0^{(5)} + C_1^{(5)} + C_2^{(5)}) \\
&+ m_{\widetilde{\chi}_k^0} m_{\widetilde{\chi}_j^+} \Im m(f_{ik}^L O_{kj}^R \widetilde{A}_{ij} + f_{ik}^R O_{kj}^L \widetilde{A}_{ji}^*) \Im m(C_1^{(5)} + C_2^{(5)}) \Big\} \\
C_X^{(5)} = C_X(m_{H^+}^2, m_W^2, m_h^2, m_{\widetilde{\chi}_i^+}^2, m_{\widetilde{\chi}_k^0}^2, m_{\widetilde{\chi}_j^+}^2)&.
\end{aligned} \tag{2.15}$$

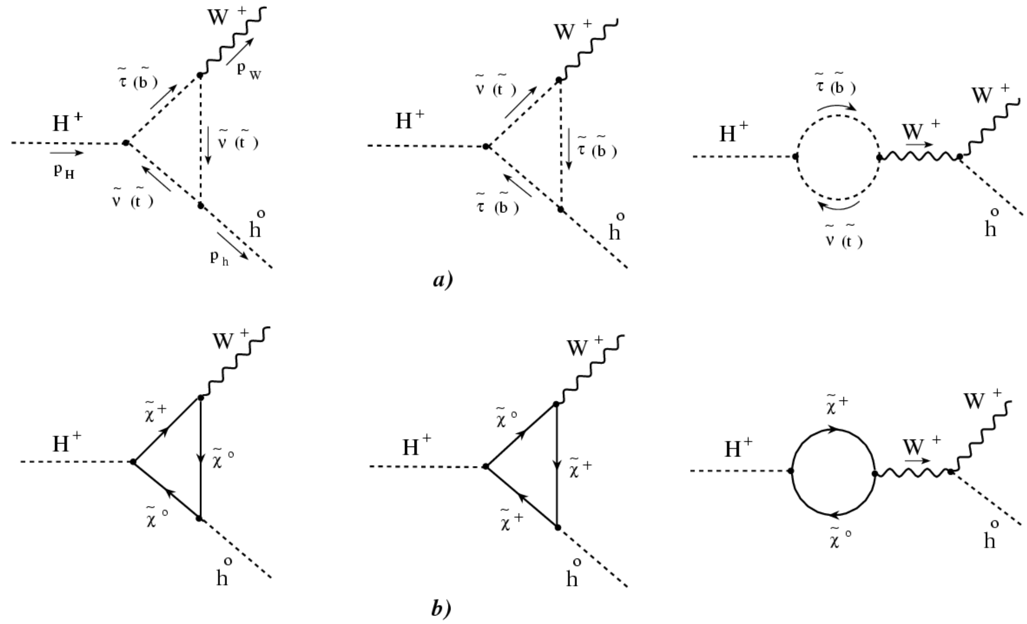

Figure 1: The 1-loop diagrams in MSSM with complex parameters that contribute to δ_{Wh^0}

$$\begin{aligned}\Re e(\delta Y_6^{CP})(\widetilde{\chi}^+\widetilde{\chi}^0) &= -\frac{\alpha_\omega}{4\pi}\frac{\sin(\beta-\alpha)}{m_{H^+}^2 m_W}\sum_{\substack{i=1,2\\k=1,2,3,4}}\{m_{\widetilde{\chi}_k^0}(m_{H^+}^2+m_{\widetilde{\chi}_i^+}^2-m_{\widetilde{\chi}_k^0}^2)\Im m(f_{ik}^L O_{ki}^R+f_{ik}^R O_{ki}^L)\\
&\quad - m_{\widetilde{\chi}_i^+}(m_{H^+}^2-m_{\widetilde{\chi}_i^+}^2+m_{\widetilde{\chi}_k^0}^2)\Im m(f_{ik}^L O_{ki}^L+f_{ik}^R O_{ki}^R)\}\Im m(B_0^{(6)})\\
B_0^{(6)} &= B_0(m_{H^+}^2, m_{\widetilde{\chi}_k^0}^2, m_{\widetilde{\chi}_i^+}^2)\end{aligned}\tag{2.16}$$

Here the imaginary parts of the PV integrals $C_X(m_{H^+}^2, m_W^2, m_h^2, m_0^2, m_1^2, m_2^2)$ and $B_0(m_{H^+}^2, m_0^2, m_1^2)$ enter. The appropriate analytic expressions are given in Appendix B.

3. Numerical Results

Here we present our numerical analysis for the dependence of δ_{Wh^0} on the MSSM parameters. Taking into account the lower experimental and the upper theoretical bounds on the mass m_{h^0}, we consider the range $96 \le m_{h^0} \le 130$ GeV. Our analysis showed a very weak

$\tan\beta$	$m_{\tilde{\nu}}$	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^+}$	$m_{\tilde{\chi}_2^+}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$
3	105	119	130	116	291	96	139	162	291
6	102	118	133	123	288	100	139	167	287
9	102	115	136	126	286	101	139	169	286

Table 1: The masses of the superparticles for the parameters (3.1) and $\phi_\tau = \phi_1 = \pi/2$.

dependence on m_{h^0} and the results here are presented for

$$m_{h^0} = 125 \text{ GeV}.$$

As explained in Section 2 the diagrams with squarks cannot be considered within the α_w-approximation used here and our analysis will be based on the diagrams on Fig. 1a with sleptons, and on Fig.1b. Thus our numerical results will depend on the parameters of the slepton, chargino and neutralino sectors. In order not to vary too many parameters we fix part of the SUSY parameter space:

$$M_2 = 250 \text{ GeV}, \; M_E = M_L - 5 \text{ GeV}, M_L = 120 \text{ GeV}, \\ |A_\tau| = 500 \text{ GeV}, \; |\mu| = 150 \text{ GeV}. \tag{3.1}$$

Assuming the GUT relation only for the absolute values of M_1 and M_2 ($|M_1| = \frac{5}{3}\tan\theta_W\,|M_2|$), we keep ϕ_1, the phase of M_1, as a physical phase. The phase of M_2 can be rotated away. The other phase relevant for our considerations is the phase ϕ_τ of A_τ. The varied parameters thus, are the charged Higgs mass, $\tan\beta$ and the CP-violating phases. We consider $\tan\beta$ in the interval

$$3 \leq \tan\beta \leq 9.$$

As it is well known, in order δ_{Wh^0} to be nonzero we need both new decay channels opened and CP violating phases. In accordance with this we have three cases: i) When

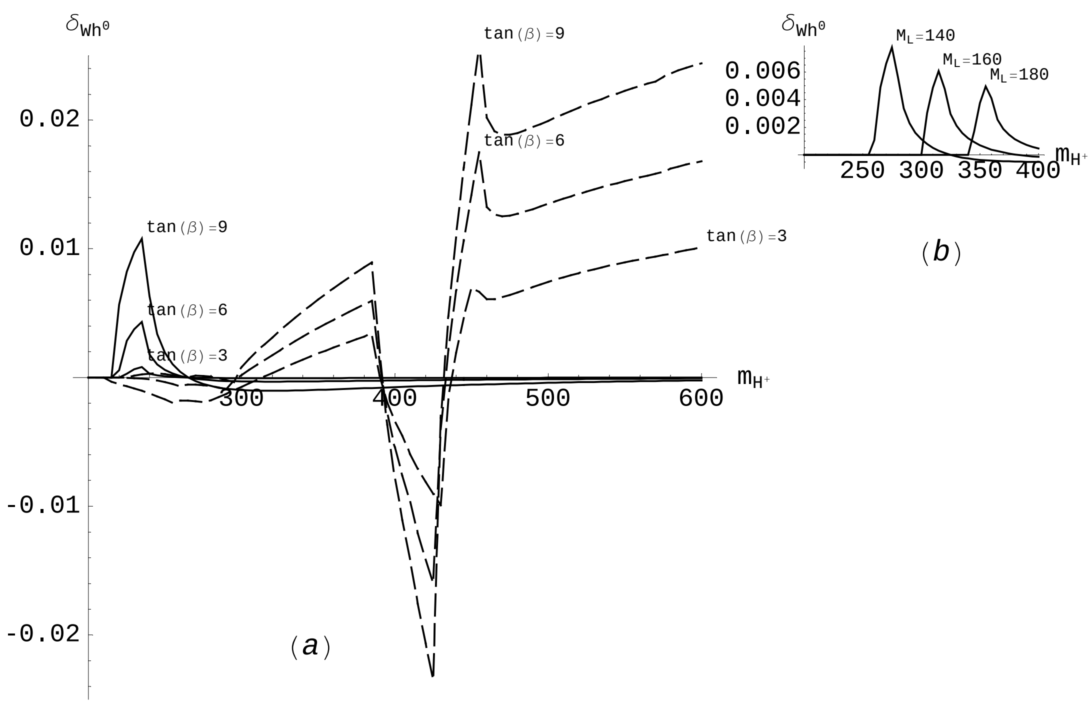

Figure 2: δ_{Wh^0} as a function of m_{H^+} **a)** for different values of $\tan\beta$, $M_L = 120$ GeV; solid lines are for $\phi_\tau = -\pi/2,\ \phi_1 = 0$; dashed lines are for $\phi_\tau = 0,\ \phi_1 = -\pi/2$. **b)** for different values of M_L, at $\tan\beta = 9,\ \phi_\tau = -\pi/2,\ \phi_1 = 0$.

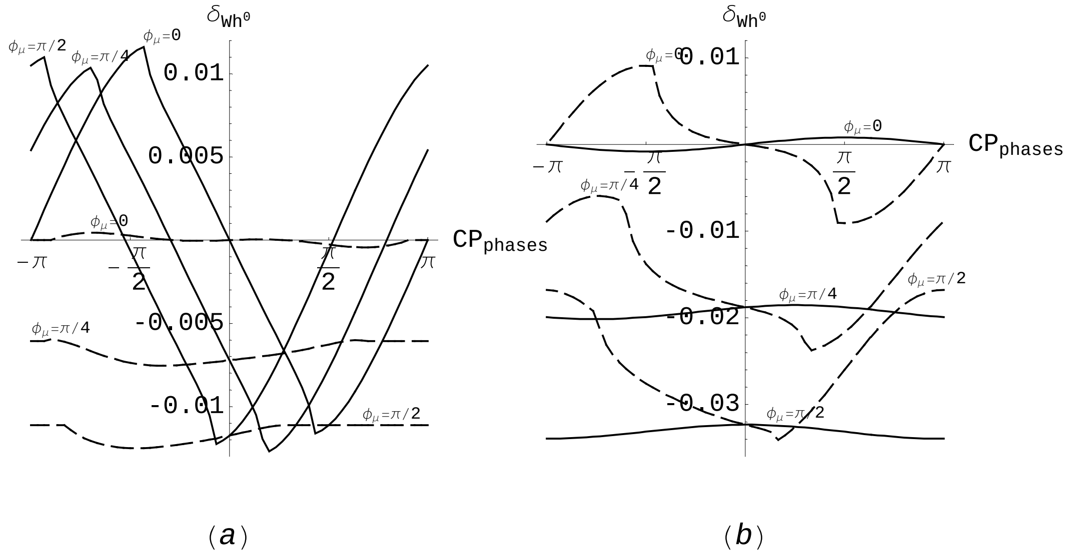

Figure 3: δ_{Wh^0} at $\tan\beta = 9$ versus the CP violating phases ϕ_τ and ϕ_1 for different values of ϕ_μ [$\phi_\mu = 0, \pi/4, \pi/2$]. The solid lines are for $\phi_\tau = [-\pi, \pi]$ while $\phi_1 = 0$; the dashed lines are for $\phi_1 = [-\pi, \pi]$ while $\phi_\tau = 0$, **a)** for $m_{H^+} = 237$ GeV ($= m_{\tilde{\nu}} + m_{\tilde{\tau}_2^+}$) and **b)** for $m_{H^+} = 387$ GeV ($= m_{\tilde{\chi}_2^+} + m_{\tilde{\chi}_1^0}$).

only the decay channels $H^+ \to \tilde{\nu}\tilde{\tau}_n^+$ are open (Fig. 1a). Then the phase ϕ_τ is responsible for CP violation. ii) When the decay channels $H^+ \to \tilde{\chi}_i^+\tilde{\chi}_k^0$ are open only (Fig. 1b). In this case CP violation is due to the phase ϕ_1, and iii) When both $H^+ \to \tilde{\nu}\tilde{\tau}_n^+$ and $H^+ \to \tilde{\chi}_i^+\tilde{\chi}_k^0$ decay channels are kinematically allowed (all diagrams of Fig.1). In this case the two phases ϕ_τ and ϕ_1 contribute.

Examples of the asymmetry as function of m_H^+ for cases i) and ii) are shown on Fig.2a for different values of $\tan\beta$. The relevant SUSY mass spectrum is presented in Table 1. Numerically the two cases are obtained taking $\phi_\tau \neq 0$, ϕ_1=0 for case i), and $\phi_\tau = 0$, $\phi_1 \neq 0$ for case ii)[1]. It is clearly seen that in both cases the asymmetry strongly increases with $\tan\beta$. The positions of the spikes correspond to the threasholds of the decay modes to the intermediate particles in the loops. It is seen that for $m_{H^+} \leq 300$ GeV the asymmetry is dominated by the light staus and sneutrinos in the SUSY mass spectrum - the solid lines on Fig. 2a, while for $m_{H^+} \geq 300$ GeV the asymmetry is determined by the charginos and neutralinos - the dashed lines on Fig. 2a. In both cases, i) and ii), the asymmetry reaches up to 10^{-2}. The dependence on M_L for case i) is seen on Fig.2b. Case iii), when all relevant SUSY particles can be light, is described by the algebraic sum of the two graphs at a given $\tan\beta$ and we don't present it separately. In all these cases the asymmetry does not exceed few percents.

Up to now, all presented results are for the phase of μ zero, $\phi_\mu = 0$, i.e. for positive μ's. Negative μ roughly speaking just flips the sign of δ_{Wh^0}. The effect of a non zero phase

[1] The two cases i) and ii) can, surely, be obtained alternatively varying the mass parameters M_2, μ and M_L.

ϕ_μ is seen on Fig.3, where we show the dependence of the asymmetry on ϕ_τ (solid lines), and on ϕ_1 (dashed lines) for $\phi_\mu = 0, \pi/4$ and $\pi/2$. The values of m_{H^+} are chosen to be near the threasholds of the decay channels: $m_{H^+} = m_{\tilde{\nu}} + m_{\tilde{\tau}_2^+} = 237$ GeV on Fig.3a, and $m_{H^+} = m_{\tilde{\chi}_2^+} + m_{\tilde{\chi}_1^0} = 378$ GeV on Fig.3b. In all cases a CP violating phase of μ does not change the form of the curves but rather shifts the positions of the maximum and, in general, increases the absolute value of the asymmetry. Note that even for $\phi_\mu = 0$ the maximal effect is not achieved for $\phi_\tau(\phi_1) = -\pi/2$.

4. Conclusions

We have considered the CP violating asymmetry δ_{Wh^0} of the decay rate difference between $H^+ \to W^+ h^0$ and $H^- \to W^- h^0$ induced by one loop radiative corrections in MSSM with complex parameters. This decay is important for relatively low m_{H^+} and $\tan\beta$. This in turn determines the importance of δ_{Wh^0} only if there are relatively low SUSY masses. We have considered the contribution from $m_{\tilde{\nu}}$ and $m_{\tilde{\tau}}$, and/or $m_{\tilde{\chi}^+}$ and $m_{\tilde{\chi}^0}$ in the loops, and thus the sensitivity to the phases of A_τ, M_1 and μ. We work in first order of the weak coupling constant α_w and this approximation is not enough to consider the contribution of light stops and sbottoms. Typical values for the asymmetry are about $10^{-2} \div 10^{-3}$, the main contributions being from $\tilde{\nu}$ and $\tilde{\tau}$ for $m_{H^+} < 300$ GeV, and from $\tilde{\chi}^+$ and $\tilde{\chi}^0$ for $m_{H^+} \geq 300$ GeV. The dependence on different values of $\tan\beta$ is examined.

The approximate number of $H^\pm$'s needed to measure δ_{Wh^0} is $N_{H^\pm} \geq 1/[\delta^2_{Wh^0}\, Br(H^+ \to W^\pm h^0)]$, which, for $\delta_{Wh^0} \simeq 10^{-2}$ and a branching ration $\simeq 10\%$, implies $N_{H^\pm} \geq 10^5$.

Charged $H^\pm$ will be produced at the Tevatron in FermiLab if $m_{H^+} \leq 300$ GeV, and at LHC in CERN if $m_{H^+} \leq 1000$ GeV. As the cross sections for $H^\pm$ production at pp and $p\bar{p}$ collisions decreases strongly for low $\tan\beta$, the required number $N_{H^\pm}$ is too large for the planned luminosities at the hadron colliders [9]. For example, at LHC with integrated luminosity per year $L = 100\ fb^{-1}$, at $m_{H^+} = 500$ GeV and $\tan\beta = 10$, with an efficiency for the signal $\epsilon = 2,6\%$, using the results of [9], for the ratio of the signal (S) and the background (B) events, we obtain $S/B \simeq 65/3770$.

More promising are the linear e^+e^- colliders. In this case the charged Higgs will be copiously produced, the main production mechanism being $e^+e^- \to H^+H^-$. Thus, the only parameter for the production cross section, at tree level, is the Higgs boson mass m_{H^+}. For a collider at $\sqrt{s} = 800$ GeV with luminosity $L = 500\ fb^{-1}$, the cross section is $\sim 29\ fb$ for $m_{H^+} = 200$ GeV, and $\sim 12\ fb$ for $m_{H^+} = 300$ GeV, which corresponds to 1.5×10^4 and 6×10^3 H^+H^--pairs, respectively [10]. For the CLIC collider, at $\sqrt{s} = 3$ TeV, the cross section is $3\ fb$ for $m_{H^+} = 400$ GeV which, for $L = 800\ fb^{-1}$, corresponds to 2.4×10^3 charged Higgs pairs. This implies that at the NLC higher luminosities will be needed for such an asymmetry to be measured.

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A. Lagrangian with complex couplings

The mass matrices and their diagonalization matrices are defined in the Appendicies A of [1] – for charginos and neutralinos and of [2] – for staus and sneutrinos. Here we give only the pieces of the interaction Lagrangian we use.

1. Lagrangian with neutral Higgses

$$\mathcal{L}_{h^0\,\tilde{\tau}^*_m\tilde{\tau}_n} = g \sum_{m,n=1,2} c^{\tilde{\tau}}_{mn}\tilde{\tau}^*_m\tilde{\tau}_n\, h^0\,, \qquad c^{\tilde{\tau}}_{mn} = c^{\tilde{\tau}*}_{nm} \tag{A.1}$$

$$\mathcal{L}_{h^0\,\widetilde{\nu}\widetilde{\nu}} = g\,\frac{m_z}{2\cos\theta_W}\sin(\alpha+\beta)\,\widetilde{\nu}^*_L\widetilde{\nu}_L\,h^0 \tag{A.2}$$

$$\mathcal{L}_{h^0\tilde{\chi}^0_l\tilde{\chi}^0_k} = g\,\bar{\tilde{\chi}}^0_l\,(A^*_{lk}\,P_L + A_{lk}\,P_R)\,\tilde{\chi}^0_k\,h^0 \tag{A.3}$$

$$\mathcal{L}_{h^0\tilde{\chi}^+_i\tilde{\chi}^+_j} = g\,\bar{\tilde{\chi}}^+_i\left(\widetilde{A}^*_{ij}\,P_L + \widetilde{A}_{ji}\,P_R\right)\tilde{\chi}^+_j\,h^0 \tag{A.4}$$

$$\mathcal{L}_{h^0WW} = g m_W\sin(\beta-\alpha)W^+_\mu W^{-\mu}h^0 \tag{A.5}$$

where

$$c^{\tilde{\tau}}_{mn} = c^{\tilde{\tau}}_{LL}\mathcal{R}^{\tilde{\tau}*}_{Lm}\mathcal{R}^{\tilde{\tau}}_{Ln} + c^{\tilde{\tau}}_{RR}\mathcal{R}^{\tilde{\tau}*}_{Rm}\mathcal{R}^{\tilde{\tau}}_{Rn} + c^{\tilde{\tau}}_{RL}\mathcal{R}^{\tilde{\tau}*}_{Rm}\mathcal{R}^{\tilde{\tau}}_{Ln} + c^{\tilde{\tau}}_{LR}\mathcal{R}^{\tilde{\tau}*}_{Lm}\mathcal{R}^{\tilde{\tau}}_{Rn}$$

$$\begin{aligned}
c^{\tilde{\tau}}_{LL} &= \frac{m_z}{\cos\theta_W}(-\frac{1}{2}+\sin^2\theta_W)\sin(\alpha+\beta) + \frac{m^2_\tau}{m_W}\frac{\sin\alpha}{\cos\beta},\\
c^{\tilde{\tau}}_{RR} &= -\frac{m_z}{\cos\theta_W}\sin^2\theta_W\sin(\alpha+\beta) + \frac{m^2_\tau}{m_W}\frac{\sin\alpha}{\cos\beta},\\
c^{\tilde{\tau}}_{LR} &= \frac{m_\tau}{2m_W\cos\beta}(\mu\cos\alpha + A^*_\tau\sin\alpha)\\
c^{\tilde{\tau}}_{RL} &= \frac{m_\tau}{2m_W\cos\beta}(\mu^*\cos\alpha + A_\tau\sin\alpha) = c^{\tilde{\tau}*}_{LR}.
\end{aligned}$$

$$\begin{aligned}
A_{lk} &= \frac{1}{2}(\sin\alpha\,Q''_{lk} + \cos\alpha\,S''_{lk}),\\
Q''_{lk} &= \frac{1}{2}[N_{l3}(N_{k2} - N_{k1}\tan\theta_W) + (l \longleftrightarrow k)],\\
S''_{lk} &= \frac{1}{2}[N_{l4}(N_{k2} - N_{k1}\tan\theta_W) + (l \longleftrightarrow k)],
\end{aligned}$$

$$\begin{aligned}
\widetilde{A}_{ij} &= \sin\alpha\,Q_{ij} - \cos\alpha\,S_{ij},\\
Q_{ij} &= \frac{1}{\sqrt{2}}U_{i2}V_{j1}, \qquad S_{ij} = \frac{1}{\sqrt{2}}U_{i1}V_{j2}.
\end{aligned}$$

2. Lagrangian with charged Higgses[2]

$$\mathcal{L}_{h^0H^+W^-} = -\frac{ig}{2}\cos(\alpha-\beta)\left\{(H^-\overleftrightarrow{\partial}_\alpha h^0)\,W^{+,\alpha} - (H^+\overleftrightarrow{\partial}_\alpha h^0)\,W^{-,\alpha}\right\} \tag{A.6}$$

[2]The correspondence with the notation in refs.[1] and [2] is $gf^{L,R} = F^{L,R}$, $g/(\sqrt{2}m_W)g^{\tilde{\tau}}_4 = G^{\tilde{\tau}}_4$, $g/(\sqrt{2}m_W)g^{\tilde{t}}_4 = G^{\tilde{t}}_4$

$$\mathcal{L}_{H^+\tilde{\chi}_j^+\tilde{\chi}_k^0} = g\left\{H^-\bar{\tilde{\chi}}_k^0\,(\,f_{kj}^{R*}P_L + f_{kj}^{L*}\,P_R\,)\,\tilde{\chi}_j^+ + H^+\bar{\tilde{\chi}}_j^+\,(\,f_{jk}^{R}P_R + f_{jk}^{L}\,P_L\,)\,\tilde{\chi}_k^0\right\} \quad \text{(A.7)}$$

$$\mathcal{L}_{H^+\tilde{\tau}_n\tilde{\nu}} = \frac{g}{\sqrt{2}\,m_W}\left[(g_4^{\tilde{\tau}})_n H^+\tilde{\nu}_\tau^*\tilde{\tau}_n + (g_4^{\tilde{\tau}})_n^* H^-\tilde{\tau}_n^*\tilde{\nu}_\tau\right] \quad \text{(A.8)}$$

where

$$\begin{aligned}
f_{kj}^L &= -\sin\beta\left[N_{k3}^*U_{j1}^* - \frac{1}{\sqrt{2}}(N_{k2}^* + N_{k1}^*\tan\theta_W)U_{j2}^*\right]\\
f_{kj}^R &= -\cos\beta\left[N_{k4}V_{j1} + \frac{1}{\sqrt{2}}(N_{k2} + N_{k1}\tan\theta_W)V_{j2}\right]\\
(g_4^{\tilde{\tau}})_n &= a_{LL}^{\tilde{\tau}}\mathcal{R}_{Ln}^{\tilde{\tau}} + a_{LR}^{\tilde{\tau}}\mathcal{R}_{Rn}^{\tilde{\tau}}\,,\\
a_{LL}^{\tilde{\tau}} &= m_\tau^2\tan\beta - m_W^2\sin 2\beta\\
a_{LR}^{\tilde{\tau}} &= m_\tau(A_\tau^*\tan\beta + \mu)
\end{aligned}$$

3. Lagrangian with $W^\pm$

$$\mathcal{L}_{W^+\tilde{\chi}_j^+\tilde{\chi}_k^0} = g\left\{\bar{\tilde{\chi}}_k^0\,\gamma^\alpha\,(O_{kj}^L P_L + O_{kj}^R P_R)\,\tilde{\chi}_j^+\,W_\alpha^- + \bar{\tilde{\chi}}_j^+\,\gamma^\alpha\,(O_{jk}^{L*}P_L + O_{jk}^{R*}P_R)\,\tilde{\chi}_k^0\,W_\alpha^+\right\} \quad \text{(A.9)}$$

$$\mathcal{L}_{W\tilde{\nu}\tilde{\tau}_n} = \frac{-ig}{\sqrt{2}}\left\{\mathcal{R}_{Lm}^{\tilde{\tau}*}\,W_\alpha^-\left(\tilde{\tau}_m^*\,\overleftrightarrow{\partial}^\alpha\,\tilde{\nu}\right) + \mathcal{R}_{Lm}^{\tilde{\tau}}\,W_\alpha^+\left(\tilde{\nu}^*\,\overleftrightarrow{\partial}^\alpha\,\tilde{\tau}_m\right)\right\} \quad \text{(A.10)}$$

Here

$$O_{kj}^L = -\frac{1}{\sqrt{2}}N_{k4}\,V_{j2}^* + N_{k2}\,V_{j1}^*, \qquad O_{kj}^R = \frac{1}{\sqrt{2}}N_{k3}^*\,U_{j2} + N_{k2}^*\,U_{j1}$$

B. Absorptive parts of the Integrals

If we use the notation:

$$\mathcal{D}^0 = q^2 - m_0^2, \qquad \mathcal{D}^j = (q+p_j)^2 - m_j^2,$$

then the Passarino-Veltman two- and three-point functions [11], when the loop integrals are given in 4-dimentions, are:

$$B_0(p_1^2, m_0^2, m_1^2) = \frac{1}{i\pi^2}\int d^4q\frac{1}{\mathcal{D}^0\mathcal{D}^1}, \quad \text{(B.1)}$$

$$C_0(p_1^2,(p_1-p_2)^2,p_2^2,m_0^2,m_1^2,m_2^2) = \frac{1}{i\pi^2}\int d^4q\frac{1}{\mathcal{D}^0\mathcal{D}^1\mathcal{D}^2}, \quad \text{(B.2)}$$

$$C_\mu(p_1^2,(p_1-p_2)^2,p_2^2,m_0^2,m_1^2,m_2^2) = \frac{1}{i\pi^2}\int d^4q\frac{q_\mu}{\mathcal{D}^0\mathcal{D}^1\mathcal{D}^2} = p_{1\mu}C_1 + p_{2\mu}C_2 \quad \text{(B.3)}$$

For our diagrams in the considered H^+ decay we have:

$$p_1 = p_{H^+} \Rightarrow p_1^2 = m_{H^+}^2, \qquad p_2 = p_h \Rightarrow p_2^2 = m_h^2, \qquad (p_1-p_2)^2 = m_W^2.$$

Then for the the absorptive parts of the integrals, when the particles with masses m_0 and m_1 are put on mass shell we obtain:

$$\Im m\, B_0(m_{H^+}^2, m_0^2, m_1^2) = \frac{2\pi|\overrightarrow{k}|}{m_{H^+}} \tag{B.4}$$

$$\Im m\, C_0(m_{H^+}^2, m_W^2, m_h^2, m_0^2, m_1^2, m_2^2) = \frac{-\pi}{2\, m_{H^+}|\overrightarrow{p_h}|}\ \ln|\frac{a+b}{a-b}| \tag{B.5}$$

$$\Im m\, C_1(m_{H^+}^2, m_W^2, m_h^2, m_0^2, m_1^2, m_2^2) = \frac{m_h^2\, A - (p_{H^+}p_h)\, B}{\Delta} \tag{B.6}$$

$$\Im m\, C_2(m_{H^+}^2, m_W^2, m_h^2, m_0^2, m_1^2, m_2^2) = \frac{-(p_{H^+}p_h)\, A + m_{H^+}^2\, B}{\Delta}. \tag{B.7}$$

Here

$$\begin{aligned}
\Delta &= m_{H^+}^2 m_h^2 - (p_{H^+}p_h)^2, \qquad (p_{H^+}p_h) = \frac{m_{H^+}^2 + m_h^2 - m_W^2}{2} \\
|\overrightarrow{k}| &= \frac{\lambda^{1/2}(m_{H^+}^2, m_0^2, m_1^2)}{2m_{H^+}}, \qquad |\overrightarrow{p_h}| = \frac{\lambda^{1/2}(m_{H^+}^2, m_h^2, m_W^2)}{2m_{H^+}}, \\
\lambda(x,y,z) &= x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \\
A &= \frac{-\pi\, k^0}{2\,|\overrightarrow{p_h}|}\ \ln|\frac{a+b}{a-b}|, \qquad B = \frac{-\pi}{2\, m_{H^+}\,|\overrightarrow{p_h}|}\ \left\{\left[k^0\, p_h^0 - \frac{a}{2}\right] \ln|\frac{a+b}{a-b}| - 2\,|\overrightarrow{k}|\,|\overrightarrow{p_h}|\right\} \\
a &= m_h^2 + m_0^2 - m_2^2 + 2\, p_h^0 k^0, \qquad b = -2|\overrightarrow{k}|\,|\overrightarrow{p_h}|\,, \\
k^0 &= \frac{m_1^2 - m_0^2 - m_{H^+}^2}{2m_{H^+}}, \qquad p_h^0 = \frac{m_{H^+}^2 - m_W^2 + m_h^2}{2m_{H^+}}.
\end{aligned}$$

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