Multi-amplitude Differential Space-time Block Coding Scheme for Square/Non-Square Code Matrix in MIMO Systems

Xiangbin Yu, Xiaomin Chen, Yuyu Xin, Qiuming Zhu, Dazhuan Xu
Nanjing University of Aeronautics and Astronautics, Nanjing, China
Email: yxbxwy@gmail.com

Abstract—Differential space-time coding (DSTC) technique has become a good choice when channel estimations are difficult to obtain in multiple antennas system. On the basis of analyzing the existing DSTC schemes, by introducing multi-level quadrature amplitude modulation (MQAM) and matrix transform method, we develop a multi-amplitude differential space-time block coding (STBC) scheme for square or non-square code matrix in MIMO systems, and give the derivation of calculation formulae of the coding advantage in detail. The developed scheme can effectively avoid the performance loss of conventional DSTC schemes based on PSK modulation (i.e. single amplitude DSTC) in high spectrum efficiency. It can be applied to non-square code matrix case, and thus overcomes the shortcoming that existing DSTC schemes are only suitable for square code matrix. Compared with single amplitude DSTC schemes, our scheme has higher spectrum efficiency by carrying information not only on phases but also on amplitudes. Moreover, our scheme has linear decoding complexity, higher coding advantage, and higher code rate for more than two antennas. The simulations results show that the proposed scheme can provide lower BER than the existing single amplitude differential STBC schemes for both square and non-square code matrices.

Index Terms—Differential detection; space-time block code; coding advantage; multiple-input multiple-output (MIMO); multi-amplitude modulation

I. INTRODUCTION

Recently, transmit diversity technique has received considerable attention in multiple-input multiple-output (MIMO) communication systems due to its diversity gain [1-5]. MIMO systems can attain the transmit diversity by employing space-time block coding (STBC) schemes [2-5]. The STBC for two transmit antennas is firstly proposed by Alamouti [4], and then a general framework of space-time block codes by using the orthogonal design theory for different antennae is developed by Tarokh et al [3]. The STBC schemes can achieve full transmit diversity with the use of maximum-likelihood (ML) decoding. As a result, it has been adopted as the open-loop transmit diversity scheme in the current 3GPP standards. Considering that the encoding complexity of high-rate multi-antenna STBC schemes, two simple and effective orthogonal STBC schemes are developed in Refs.[6, 7]. Unfortunately, all these scheme designs are based on the assumption that channel state information (CSI) is perfectly known at the receiver, and unknown at the transmitter, whereas in practice, accurate CSI is difficult to achieve either due to rapid changes in the channel or due to the overhead needed to estimate a large number of parameters such as in a MIMO system [8]. Thus, the differential modulation scheme becomes an attractive alternative.

With differential detection, CSI is not required either at the transmitter or at the receiver. In [9] and [10], Hughes and Hochwald et al. independently developed differential unitary space-time coding (DUSTC) schemes based on group codes. These schemes can allow easy realization at the transmitter, but they require group structure and have exponential decoding complexity, which will make their use formidable in practice. In [11], Tarokh et al. proposed a simple differential space-time block coding (DSTBC) scheme based on conventional STBC [11], but the scheme is limited in 2 transmit antennas. For this, in a subsequent work, they also proposed a multi-antenna space-time coding scheme for more than 2 transmit antennas [12], but the scheme was still limited in existing STBC structure, and the code-rate is only 1/2, corresponding decoding method is also complex. Afterwards, Ganesan et al. presented a differential STBC scheme based on amicable orthogonal design [13]. Compared to the above scheme, the scheme has lower computational complexity, and need not the algebra group structure. By exploiting the rotational invariance of unitary matrices and the property of MPSK modulation, the constellation design for differential space-time modulation is provided in [14]. In [15], the error probability performance of differential orthogonal STBC is analyzed, and the simulation results confirm the accuracy of theoretical analysis. Based on the existing DUSTC, a differential space-time coding design for a MIMO system employing switch and stay combining at the receiver is presented in [16].

Unfortunately, the above schemes basically need to employ the multi-level phase shift-keying (MPSK) constellation, that is, the transmitted symbols are all from the unitary constellation and the corresponding encoded matrix has unit-level. As a result, the minimum distance between symbols become smaller when spectrum
efficiency gets higher, and the minimum product distance between the two encoded matrices also become smaller accordingly, which will bring about the obvious decrease of coding advantage (as defined in [9]) and the loss of system performance. These conclusions can also be achieved from Table I in [9] and Table I in [13]. Considering that multi-level quadrature amplitude modulation (MQAM) scheme has better performance than corresponding MPSK modulation under the same spectrum efficiency, multiple amplitudes DSTBC scheme using MQAM are presented in [17-20], respectively. They obtain better performance than the corresponding DSTBC scheme based on MPSK modulation. However, these schemes are limited to the square STBC scheme (i.e. corresponding code matrix is square) only, which is also main disadvantage in [17]. Moreover, the schemes from Refs.[19-20] are only designed for two transmit antennas and one receive antenna case, the scheme in [18] only suits square STBC with amicable orthogonal design. Motivated by the reason above, by introducing transform matrix and MQAM method as well as simple STBC [7,8] structure, we develop a simple and effective multi-amplitude DSTBC scheme for general STBCs on the basis of Ref.[17], and give specific decoding scheme for multiple receive antennas. The corresponding coding advantage is also analyzed and calculated in detail by means of derivation. The scheme not only suits general STBC schemes with square code matrix, but also can be applied in the case of non-square code matrix, such as the code matrices from 3,5,6,7 transmit antenna, etc. Similarly, it can make use of not only the phase but also amplitude to carry information to improve the spectrum efficiency. Moreover, by using MQAM method, the constellation matrices are no longer limited in unity constellation, they will have different amplitudes, and thus the minimum produce distance will also be increased. As a result, we can improve the performance of pervious code matrix and avoid the performance degradation in high spectrum efficiency. Compared with existing DSTBC based on MPSK modulation, the scheme has lower BER and implementation complexity, and higher spectrum efficiency and coding advantage.

Throughout the paper the superscripts \((\cdot)^T, (\cdot)^*, (\cdot)\) are used to stand for the transpose, complex conjugate, and Hermitian transpose respectively. We denote \(E\{\cdot\}\) and \(I_{N\times N}\) as expectation and \(N\times N\) identity matrix, respectively.

II. SYSTEM MODEL

In this section, we consider a wireless multi-antenna communication system with \(N\) transmit antennas and \(K\) receive antennas operating over a flat and quasi-static Rayleigh fading channel represented by a \(N\times K\) fading channel matrix \(H=[h_{n,k}]\). The complex element \(h_{n,k}\) denotes the channel gain from the \(n\)-th transmit antenna to the \(k\)-th receive antenna, which is assumed to be constant over a frame of \(P\) symbols and varied from one frame to another. The channel gains are modeled as independent complex Gaussian random variables with zero-mean and variance 0.5 per real dimension. Let \(V_i\) be the code matrix with \(P\times N\) at time \(i\), then at the receiver, the received signal matrix \(X_i\) can be expressed as

\[
X_i = \sqrt{\gamma} V_i H + Z_i
\]

where \(Z_i\) is \(P\times K\) complex Gaussian noise matrix, whose elements are independent, identically distributed \((i.i.d)\) complex Gaussian random variables with zero-mean and unit-variance. So at the receiver, the received signal for receive antenna \(k (k=1,2,\ldots,K)\) can be written as

\[
x_{i,k} = \sqrt{\gamma} \sum_{n=1}^{N} v_{i,n,k} h_{n,k} + z_{i,k}
\]

where the coded symbols from code matrix are normalized to obey \(E\{\sum_{n=1}^{N} |v_{i,n,k}|^2\} = 1\), and thus it can ensure that \(\gamma\) is the expected signal-to-noise ratio (SNR).

Let the code matrix index be \(i\) and time epoch index within the code matrix be \(t\).\{\(z_{i,k}\)\} are elements of noise matrix \(Z\). The structure diagram of a MIMO wireless communication system with \(N\) transmit antennas and \(K\) receive antennas is illustrated in Fig.1.

\[\text{Figure 1. Structure diagram of a MIMO wireless communication system}\]

III. MULTIPLE LEVELS CODE MATRIX

Let \(L, N\) and \(P\) be positive integers, a conventional STBC is defined by a \(P\times N\) dimensional transmission matrix \(D\), every entry of which is linear combination of the \(L\) input symbols \(d_1, d_2, \ldots, d_L\) and their conjugates \(d_1^*, d_2^*, \ldots, d_L^*\), and it satisfies the following complex orthogonal condition:

\[
D^H D = \sum_{l=1}^{L} |d_l|^2 I_{N\times N}
\]

where \(N\) and \(P\) are the numbers of transmit antennas and time slots used to transmit \(L\) input symbols, respectively. The input symbols \{\(d_l, l=1,\ldots,L\)\} represent the information-bearing binary bits to be transmitted. In a signal constellation such as PSK or QAM having \(2^b\) constellation points, \(b\) binary bits is used to represent a symbol \(d\). So a block of \(Lb\) binary bits is entered into the STBC encoder at a time. Considering that \(L\) symbols are transmitted over \(P\) time slots, the rate of the STBC is defined as \(R_{STBC} = L/P\) [3,5]. Here, we first consider that
the code matrix of STBC is square, i.e., \( N = P \). So \( D^TD = DD^H \) in terms of (3).

For convenience of analysis, we define the amplitude of matrix based on Ref.[19]: For a \( N \times N \) matrix \( S \), if a nonnegative scalar \( \rho \) satisfy the \( SS^H = \rho^2 I_{N,N} \), then \( \rho \) is defined as the amplitude of matrix \( S \). Let the data symbols \( \{d_l, l=1,\ldots,L\} \) be taken from the QAM or PSK constellation with unit average energy. Then the code matrix is \( G = D/\sqrt{L} \) in terms of [19]. Thus according to (3), we have

\[
G^HG = \frac{1}{L} \sum_{l=1}^{L} |d_l|^2 I_{N,N} = \rho_G^2 I_{N,N}, \quad G=D/\sqrt{L}. \tag{4}
\]

where \( \rho_G = \sqrt{\sum_{l=1}^{L} |d_l|^2 / L} \), thus \( \rho_G \) is called the amplitude of \( G \) based on the above amplitude definition. Then the set of all possible code matrices will form a constellation \( \Psi \) [19]. If \( d_i \) \( (l=1,\ldots,L) \) is from MPSK symbol, \( \rho_G = 1 \) for all \( G \), and the corresponding \( \Psi \) is called a constellation with single amplitude, i.e., it is unitary constellation, the conventional differential space-time coding scheme is based on this. If \( d_i \) \( (l=1,\ldots,L) \) is from MQAM, \( \rho_G \) will take some discrete values, and \( \Psi \) is called a constellation with multiple amplitudes accordingly [19]. Thus \( G \) is called multi-level code matrix. The scheme in [19] is based on this, and it is different from other differential code schemes based on unitary code matrix. By introducing multi-amplitude constellation, the information is carried by means of not only the phase but also the amplitude of the code matrix. So the spectral efficiency is effectively improved.

IV. DIFFERENTIAL SPACE-TIME BLOCK CODING BASED ON MULTI-LEVEL CODE MATRIX

At the transmitter, we consider the case of \( N \times N \) square code matrices (i.e. \( P=N \)) firstly. According to Refs.[3,8,9], we have such \( N \times N \) matrices for \( N=2,4 \) and 8. The \( i \)th block to be transmitted is a differential code matrix \( V_t \) with \( N \times N \). At the start of the transmission, the transmitter sends a \( N \times N \) identity matrix as initial code matrix \( V_0 \) (i.e. \( V_0=I_{N,N} \)), which does not carry information. Then the information matrix to be transmitted (i.e. \( G \), which is defined by (4) and is from constellation \( \Psi \)) is differential encoded, the corresponding differential encoded matrix \( V_t \) at time \( i \) is obtained as follows:

\[
V_t = G \bar{V}_{t-1} = G V_{t-1} / \rho_{t-1} \tag{5}
\]

where \( \bar{V}_{t-1} \) is the normalized value of \( V_{t-1} \), \( \rho_{t-1} \) is the amplitude of previous differential code matrix \( V_{t-1} \), and \( \rho_0 = 1 \). From \( V_0=I_{N,N} \) and (5), we have:

\[
V_t^H V_t^H = G V_{t-1}^H V_{t-1}^H G^H / \rho_{t-1}^2 = G G^H \tag{6}
\]

Namely \( V_t \) has the same amplitude (i.e. \( \rho_t \)) with \( G_t \). So,

\[
\bar{V}_t V_t^H = V_t^H V_t^H / \rho_t^2 = I_{N,N} \tag{7}
\]

So at time \( i \), \( \bar{V}_t \) is a unitary matrix and \( V_t \) is multi-amplitude matrix. At the receiver, we assume that the channel gains remain constant at two consecutive time blocks, then the received matrix at time \( i \) is

\[
X_i = \sqrt{\lambda} V_t H + Z_i \tag{8}
\]

where \( T \) is our introduced transform matrix, which is identity matrix for square code matrix, and the related explanation will be seen in next section. So the received matrix at time \( i-1 \) is written as

\[
X_{i-1} = \sqrt{\lambda} V_{i-1} H + Z_{i-1} \tag{9}
\]

Substituting (5) and (9) into (8) yields:

\[
X_i = \bar{V}_t V_t^H H / \rho_{t-1} + Z_i = G_t X_{i-1} / \rho_{i-1} + \tilde{Z}_i \tag{10}
\]

where \( \tilde{Z}_i = Z_i - G_t Z_{i-1} / \rho_{i-1} \) is a \( N \times K \) matrix. Since \( Z_i \) and \( Z_{i-1} \) are both complex Gaussian random variables with zero mean, \( \tilde{Z}_i \) is also a Gaussian random variable with zero mean. Considering that \( G_t G_t^H = \rho_t I_{N,N} \), we can evaluate the variance of \( \tilde{Z}_i \) by

\[
E(\tilde{Z}_i \tilde{Z}_i^H) = (1 + \rho_t^2 / \rho_{i-1}^2) I_{N,N} \tag{11}
\]

Hence, \( \tilde{Z}_i \) is an equivalent noise matrix, and its elements are \( i.i.d. \) complex Gaussian random variables with mean zero and variance \( (1 + \rho_t^2 / \rho_{i-1}^2) \).

Based on the above-mentioned analysis, we can obtain the decision matrix by employing ML detector as follows:

\[
\hat{G}_t = \arg \min_{G_t \in \Psi} \| X_t - G_t X_{t-1} / \rho_{i-1} \|^2 \tag{12}
\]

In this paper, we assume that the data symbols \( \{d^*_l\} \) are from MQAM constellation \( \Phi \). Considering that information matrix \( G_t \) is defined by (4), its elements are linear combination of \( L \) input symbols \( \{d^*_l, l=1,2,\ldots,L\} \) and their conjugates \( \{d^*_l\} \) in terms of space-time code matrix \( D_t \). Thus the (12) can be equivalent as:

\[
\hat{d}^*_i = \arg \min_{d^*_i \in \Phi} \| X_{t-1} - \rho_{i-1} G_t X_{t-1} / \rho_{i-1} \|^2 \tag{13}
\]

\[
= \arg \min_{d_1,\ldots,d_L \in \Phi} \| (X_{t-1} - \rho_{i-1} G_t X_{t-1})^H (X_{t-1} - \rho_{i-1} G_t X_{t-1}) \|
\]

V. DSTBC SCHEME FOR NON-SQUARE MATRIX

A. Encoding scheme

The scheme presented in above section is valid for \( N=2,4 \) and 8 transmit antennas, and corresponding code matrix is square. Now we consider the scheme in the case of \( N=3,5,6 \) and 7 transmit antennas, where the code matrix will be non-square matrix. This is also an open problem that needs to be solved in future work in [19]. For simplicity of analysis, we only focus on the 3 transmit antennas case; similar analysis can be extended to other multiple antennas cases. For 3 transmit antennas, we can transmit the first three columns of the differential code matrix of 4 transmit antennas to perform the data
transmission, but the corresponding code matrix is not square. For this reason, we introduce a transform matrix to realize data transmission of 3 antennas as follows:

\[ V_{3,j} = V_{4,j}T \]  

where \( V_{3,j} \) and \( V_{4,j} \) denote the transmitted differential code matrices for 3 and 4 transmit antennas, respectively. \( V_{4,j} \) is a 4x4 square matrix, and \( V_{3,j} \) is a 4x3 non-square matrix. \( T \) is our introduced transform matrix, and used to solve the data transmission in the case of non-square code matrix. Here, \( T = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1] \) is a 4x4 matrix for 3 antennas case.

According to the above transform and using Eq.(8), we can implement the data transmission of 3 transmit antennas case by transmitting \( V_{3,j}T \). Thus with Eqs.(1) and (8), the corresponding received signal matrix at time \( i \) is written by

\[ X_i = \sqrt{\rho}V_{3,i}H + Z_i \]  

where \( H \) is an equivalent channel gain matrix. After equivalent transform, \( T \) can be absorbed in the channel gain matrix. Considering that \( T^H = I_{3x3} \), then \( H^TH = H'^TH \). Thus, the same diversity performances are obtained. Moreover, after the above transform, we can use the previous 4-antenna analysis method to detect the received data for 3-antenna systems at the receiver, and the system performance is not affected. From Ref.[3] and [19], we have the channel gain matrix. Considering that

\[ |H| = \begin{bmatrix} d_1 & d_2 & d_3 & 0 \\ -d_2 & -d_1 & 0 & -d_3 \\ d_3 & d_1 & d_2 & 0 \end{bmatrix} \]  

we can express the equivalent channel gain matrix as

\[ G_{eq} = \frac{1}{\sqrt{3}} \begin{bmatrix} d_1 & d_2 & d_3 & 0 \\ -d_1 & -d_2 & 0 & -d_3 \\ d_2 & d_1 & d_3 & 0 \end{bmatrix} \]  

where \( d_i \) (\( i = 1,2,3 \)) denotes \( d_0 \) in fact, we drop the time index \( i \) for convenience. Considering that the above code matrix has lots of complex addition and multiplication, here we will use the improved STBC for 3 transmit antennas [8], this STBC has full diversity, the orthogonality and the same rate of 3/4 as \( H_0 \) STBC in [3,19]. The improvement is the simpler encoding and decoding of STBC. This STBC code matrix is:

\[ H'_{3} = \begin{bmatrix} d_1 & d_2 & d_3 & 0 \\ -d_2 & -d_1 & 0 & -d_3 \\ d_3 & d_1 & d_2 & 0 \end{bmatrix} \]  

So the corresponding information matrix for 4-antenna is

\[ G_{4} = H'_{3} / \sqrt{3} \]  

Since 3 symbols are transmitted over 4 time slots, the corresponding scheme will have a code rate of 3/4. Moreover, our proposed scheme is based on the above information matrix, so our scheme has the same rate of 3/4. Thus in the case of 3 or 4 transmit antennas, the code rate of our scheme is higher than that of the scheme in [14], which can not be used for 3/4-rate STBCs, and only has a code rate of 1/2.

When the number of transmit antennas \( N=5 \), we transmit the first five columns of differential code matrix of 8 transmit antennas to perform the corresponding data transmission, which can be realized by introducing 8x5 transform matrix \( T = [1 0 0 0 0; 1 0 0 0 0; 0 1 0 0 0; 0 0 1 0 0; 0 0 0 1 0] \). Similar method can be applied in the case of 6 or 7 transmit antennas. Besides, our multiple amplitudes differential STBC scheme may be easily extended in the case of \( N>8 \).

Based on the above analysis, we can give a general form of received signal matrix for multi-amplitude DSTBC scheme. Namely, \( X_i = \sqrt{\rho}V_{3,i}T H + Z_i \), where \( T \) is an identity matrix for square code matrix, and it becomes the above specific transform matrix for non-square code matrix in the case of \( N=3,5,6,7 \).

B. Decoding scheme

In this subsection, we give the decoding scheme of the above differential space-time block coding scheme for non-square code matrix. For simplicity, we will focus on 3 transmit antennas and 2 receive antennas case here, and other cases can be analyzed by using similar methods. Let \( X_i = [x_i^{1,1}, x_i^{1,2}, x_i^{2,1}, x_i^{2,2}, x_i^{3,1}, x_i^{3,2}, x_i^{4,1}, x_i^{4,2}] \), and \( X_{n+1} = [x_{n+1}^{1,1}, x_{n+1}^{1,2}, x_{n+1}^{2,1}, x_{n+1}^{2,2}, x_{n+1}^{3,1}, x_{n+1}^{3,2}, x_{n+1}^{4,1}, x_{n+1}^{4,2}] \), then according to Eq.(15), the ML detection corresponding to (13) can be changed as:

\[ \hat{d}_i = \arg \min_{\{d_i\}_0} \left[\|x_i^{1,1} - (d_i x_i^{1,1} + d_i x_i^{1,2} + d_i x_i^{1,3})\rho_1 / \sqrt{3}\|^2 + \|x_i^{1,2} - (d_i x_i^{1,2} + d_i x_i^{1,3} - d_i x_i^{4,1})\rho_1 / \sqrt{3}\|^2 + \|x_i^{2,1} - (d_i x_i^{4,1} + d_i x_i^{4,2} - d_i x_i^{3,1})\rho_1 / \sqrt{3}\|^2 + \|x_i^{2,2} - (d_i x_i^{4,2} + d_i x_i^{4,1} - d_i x_i^{3,2})\rho_1 / \sqrt{3}\|^2 \right] \]  

We expand the above minimal metric in the right of Eq.(19) and delete the terms that are independent of the transmitted symbols \( d_i \), \( i = 1,2,3 \), then the Eq.(19) is equivalent to

\[ \hat{d}_i = \arg \min_{\{d_i\}_0} \left[ \sum_{n=0}^{N-1} \left( x_{n+1}^{1,1} - x_{n+1}^{1,2} + d_i \rho_1 / \sqrt{3} \right)^2 + \left( x_{n+1}^{1,2} - x_{n+1}^{1,3} - d_i \rho_1 / \sqrt{3} \right)^2 + \left( x_{n+1}^{2,1} - x_{n+1}^{2,2} - d_i \rho_1 / \sqrt{3} \right)^2 + \left( x_{n+1}^{2,2} - x_{n+1}^{2,3} + d_i \rho_1 / \sqrt{3} \right)^2 \right] \]  

© 2012 ACADEMY PUBLISHER
\[
- \sum_{i, j} x_{i, j} y_{i, j} + x_{i, j} y_{i, j} - x_{i, j} y_{i, j} + x_{i, j} y_{i, j} (\rho_{i, j} / \sqrt{3}) d_i^2
\]
\[
- \sum_{i, j} x_{i, j} y_{i, j} + x_{i, j} y_{i, j} - x_{i, j} y_{i, j} + x_{i, j} y_{i, j} (\rho_{i, j} / \sqrt{3}) d_i^2
\]

(20)

Based on Eq.(20), we can divide the above minimal decision metric into three parts, the first part is only a function of \(d_1\), the second part is only a function of \(d_2\), and the third part is only a function of \(d_3\). Thus Eq.(20) is equivalent to these three separate parts. By equivalent transformation, the three parts can be further equivalent to minimizing the following three independent decision metrics. Namely, Eq.(19) has following equivalent decision form:

\[
\hat{d}_1 = \arg \min_{d_1 \in \Phi} \rho_{d_1}^2 d_1 - \frac{\rho_{d_1}^2}{3} \sum_{k=1}^{\frac{\rho_{d_1}}{2}} x_{k, 1} y_{k, 1} + x_{k, 1} y_{k, 1} - x_{k, 1} y_{k, 1} + x_{k, 1} y_{k, 1} - \rho_{d_1}^2 / 3 (k) [d_1]^2
\]

(21)

By employing the above decoding algorithm, we can obtain the symbols decision variables for the encoding scheme with information matrix being \(H_x\). Namely, for input symbols \(d_1, d_2, d_3\), their decisions are:

\[
\hat{d}_1 = \arg \min_{d_1 \in \Phi} \rho_{d_1}^2 d_1 - \frac{\rho_{d_1}^2}{3} \sum_{k=1}^{\frac{\rho_{d_1}}{2}} x_{k, 1} y_{k, 1} + x_{k, 1} y_{k, 1} - x_{k, 1} y_{k, 1} + x_{k, 1} y_{k, 1} - \rho_{d_1}^2 / 3 (k) [d_1]^2
\]

\[
\hat{d}_2 = \arg \min_{d_2 \in \Phi} \rho_{d_2}^2 d_2 - \frac{\rho_{d_2}^2}{3} \sum_{k=1}^{\frac{\rho_{d_2}}{2}} x_{k, 2} y_{k, 2} + x_{k, 2} y_{k, 2} - x_{k, 2} y_{k, 2} + x_{k, 2} y_{k, 2} - \rho_{d_2}^2 / 3 (k) [d_2]^2
\]

\[
\hat{d}_3 = \arg \min_{d_3 \in \Phi} \rho_{d_3}^2 d_3 - \frac{\rho_{d_3}^2}{3} \sum_{k=1}^{\frac{\rho_{d_3}}{2}} x_{k, 3} y_{k, 3} + x_{k, 3} y_{k, 3} - x_{k, 3} y_{k, 3} + x_{k, 3} y_{k, 3} - \rho_{d_3}^2 / 3 (k) [d_3]^2
\]

(22)

From the Eqs.(21) and (22), we can see that the calculation complexity of (22) is slightly higher than that of (21), so we will adopt the encoding scheme with \(H'_x\) code for 3 and 4 antennas in simulation. Besides, the above decoding algorithm can be easily extended to more than 2 receive antennas case (i.e \(K > 2\)). When \(K > 2\), Eq.(21) will be changed as follows:

\[
\hat{d}_1 = \arg \min_{d_1 \in \Phi} \rho_{d_1}^2 d_1 - \frac{\rho_{d_1}^2}{3} \sum_{k=1}^{\frac{\rho_{d_1}}{2}} x_{k, 1} y_{k, 1} + x_{k, 1} y_{k, 1} - x_{k, 1} y_{k, 1} + x_{k, 1} y_{k, 1} - \rho_{d_1}^2 / 3 (k) [d_1]^2
\]

(23)

VI. CODING ADVANTAGE

For a constellation constructed from multiple data symbols, a good metric to judge the performance is the square of the minimum distance between two points in the constellation. If the distance is bigger, then the performance is better. Similarly, for the constellation constructed by code matrices, the coding advantage is a good metric to judge the performance of corresponding constellation in terms of the BER analysis in [11] and [12]. By maximizing the coding advantage; the optimal group codes are obtained in [11] and [12]. The coding advantage is defined as in [11]:

\[
\Lambda_p = \min_{m \in M_1} N \times \Lambda_p (G_m, G_s)
\]

(24)

where \(\Lambda_p (G_m, G_s) = \det((G_m - G_s)^T (G_m - G_s))^{1/N}\) is minimum product distance between two code matrices \(G_m\) and \(G_s\), \(\det()\) represents determinant operator, \(N\) is the number of transmit antennas.

Let \(G_m\) and \(G_s\) be the information matrices (as defined in (4)) constructed from the data symbols set \(\{d_{1m}, \ldots, d_{Lm}\}\) and \(\{d_{1l}, \ldots, d_{Ll}\}\), respectively, where symbols \(\{d_{1m}\}\) and \(\{d_{1l}\}\) are both from constellation \(\Phi\). Considering that the entries of \(G_m\) (i.e. \(g_{muv}\), \(u, v\) are row index and column index of matrix \(G_m\) respectively) are linear combinations of input symbols \(\{d_{1m}\}\) and their conjugates, without loss of generality, we can assume that
\[ g_{mn} = \sum_{i=1}^{L} \mu_{m} d_{mi} + \sum_{i=1}^{L} \beta_{m} d_{mi}^* \]  

where \( \mu_m \) and \( \beta_m \) are the constant coefficients used for linear combinations. Similar method can be applied to \( g_s \) due to the same definition, and thus its entries has the same form as (25), i.e.,

\[ g_{rs} = \sum_{i=1}^{L} \mu_{n} d_{ri} + \sum_{i=1}^{L} \beta_{n} d_{ri}^* \]

Let matrix \( A = G_{mn} G_s \), and \( d_{mn} = d_{mn} - d_{ns} \) then the entry of \( A, a_{mn} \) is

\[ a_{mn} = g_{mn} - g_{ns} = \sum_{i=1}^{L} \mu_{mn} d_{mi} + \sum_{i=1}^{L} \beta_{mn} d_{mi}^* \]  

Thus the matrix \( A \) has the same expression form as information matrix \( G_s \) (i.e., they have same definition), the only difference is that their input symbols are different, the former symbols are \( \{d_{ns}\} \), and the latter symbols are \( \{d_{nl}\} \). So according to (4), we will have following equation:

\[ A^H A = \left( \sum_{i=1}^{L} d_{mi} d_{mi}^*/L \right) I_{KL} = \left( \sum_{i=1}^{L} d_{ml} - d_{dl}^2 /L \right) I_{KL} \]

Based on this, we can obtain the minimum product distance \( \Lambda_m(G_{mn}, G_s) \):

\[ \Lambda_m(G_{mn}, G_s) = \det(A^H A)^{1/N} \left( \sum_{i=1}^{L} d_{ml} - d_{dl}^2 /L \right) \]  

For \( G_m \neq G_s \), (29) is minimized when \( \{d_{ml}\} \) and \( \{d_{dl}\} \) differ in just one symbol, while the other corresponding symbols are same; namely the minimal value corresponds to the minimal distance between constellation points from \( \Phi \). Hence, we can evaluate the coding advantage in terms of \( d_{min} \) as follows:

\[ \Lambda_p = \min_{d_{ml}, d_{dl} \epsilon \Phi} (N/L) |d_{ml} - d_{dl}|^2 = (N/L)d_{min} \]  

where \( d_{min} \) is the minimal distance between constellation points.

<table>
<thead>
<tr>
<th>Constellation</th>
<th>Coding advantage ((N=2, L=2))</th>
<th>Coding advantage ((N=3, L=3))</th>
<th>Coding advantage ((N=4, L=4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>16PSK</td>
<td>0.1522</td>
<td>0.1522</td>
<td>0.203</td>
</tr>
<tr>
<td>16QAM</td>
<td>0.2222</td>
<td>0.2222</td>
<td>0.2963</td>
</tr>
<tr>
<td>64PSK</td>
<td>0.0096</td>
<td>0.0096</td>
<td>0.0128</td>
</tr>
<tr>
<td>64QAM</td>
<td>0.0408</td>
<td>0.0408</td>
<td>0.0544</td>
</tr>
</tbody>
</table>

Considering that the MQAM method is employed in our scheme, and the minimal distance between constellation points from MQAM is larger than that from MPSK under same spectrum efficiency [23], we can conclude that proposed DSTBC scheme based on MQAM outperforms the corresponding MPSK based DSTBC scheme in terms of Eq.(30). From Eq.(30), we can calculate the coding advantage values of the proposed multi-amplitude DSTBC and MPSK based single amplitude DSTBC respectively, and give their coding advantage comparisons under the same spectrum efficiency in Table I. From the Table I, we can see that the coding advantage of the proposed multi-amplitude DSTBC is indeed higher than that of single amplitude DSTBC, and this superiority becomes more significant with the increase of spectrum efficiency. Hence, the performance of our scheme is obviously superior to the corresponding single amplitude DSTBC scheme, which will also be testified by the following simulations.

**VII. SIMULATION RESULTS**

In this section, we will evaluate the performance of the developed scheme for multiple antennas systems by computer simulation. The channel is assumed to be quasi-static flat Rayleigh fading. Every data frame includes 480 information bits. The Monte-Carlo method is employed for simulation and Gray code is used to map the data bits to symbol constellations. The number of transmit antennas is set as 2, 3, 4 or 5, and the number of receive antennas is set as 1 or 2. 16PSK and 16QAM are used for the purpose of comparison under the same spectrum efficiency. Similarly, 64PSK and 64QAM are also employed for comparison. In the following figures, \( \times T \) denotes a MIMO system with \( x \) transmit antennas and \( y \) receive antennas.
64PSK and 64QAM are also used for single amplitude DSTBC and multiple amplitudes DSTBC respectively for comparison. Thus they obtain the same spectrum efficiency of 6 bit/s/Hz.

It is shown in Fig.2 that our proposed scheme has significant advantage over the conventional single amplitude DSTBC scheme, the “DSTBC3T1R64Q” scheme gives about 6dB gains over “DSTBC3T1R64P” scheme at a BER of 0.03, and the “DSTBC3T1R16Q” scheme achieves about 3 dB gains over “DSTBC3T1R16P” scheme at BER=10⁻³. When compared to the “DSTBC3T1R16Q” scheme with coherent decoding, our “DSTBC3T1R16Q” scheme can obtain the same order of antenna diversity, but has performance loss of 3-4dB as expected. However, to implement the coherent STBC scheme, the perfect CSI is required at the receiver, whereas our differential scheme do not need any CSI. Besides, the “DSTBC3T1R16Q” scheme performs better than the “DSTBC3T1R64Q” scheme. It is because when compared with 16QAM scheme, the 64QAM signal constellation points are more densely packed, thus they are more prone to errors in fading channel.

In Fig.3, we give the performance comparison of the proposed multi-amplitude DSTBC scheme with different transmit antennas and 1 receive antenna. As shown in Fig.3, when 2 receive antennas are employed, the proposed multiple amplitudes DSTBC scheme still performs better than the corresponding single amplitude DSTBC scheme. At a BER of 10⁻⁴, the “DSTBC3T2R16Q” scheme gives about 3dB gains over “DSTBC3T2R16P” scheme, and the “DSTBC3T2R64Q” scheme achieves about 7dB gains over “DSTBC3T2R64P” scheme at a BER of 0.03. Moreover, in the case of 2 receive antennas, the multiple amplitudes DSTBC schemes with more transmit antennas still outperforms those with fewer antennas. For our 5-antenna scheme, it can achieve about 2dB gains at a BER of 10⁻⁵ when compared with 4-antenna scheme, but our 3-antenna scheme performs worse than 4-antenna scheme due to lower space diversity gain. Thus these results are consistent with our expected conclusions, and further testify that the developed schemes for non-square code matrix are effective and reasonable. In addition, compared with the DSTBC schemes in Fig.2 and Fig.3, the corresponding DSTBC schemes in Fig.4 all achieve performance improvement because of using multiple receive antennas, which also further testifies the effectiveness of our given decoding algorithm.

To further comparison, we give the performance comparison of the different DSTBC schemes with two receive antennas in Fig.4. As shown in Fig.4, when 2 receive antennas are employed, the proposed multiple amplitudes DSTBC scheme still performs better than the corresponding single amplitude DSTBC scheme. At a BER of 10⁻⁴, the “DSTBC3T2R16Q” scheme gives about 3dB gains over “DSTBC3T2R16P” scheme, and the “DSTBC3T2R64Q” scheme achieves about 7dB gains over “DSTBC3T2R64P” scheme at a BER of 0.03. Moreover, in the case of 2 receive antennas, the multiple amplitudes DSTBC schemes with more transmit antennas still outperforms those with fewer antennas. For our 5-antenna scheme, it can achieve about 2dB gains at a BER of 10⁻⁵ when compared with 4-antenna scheme, but our 3-antenna scheme performs worse than 4-antenna scheme due to lower space diversity gain. Thus these results are consistent with our expected conclusions, and further testify that the developed schemes for non-square code matrix are effective and reasonable. In addition, compared with the DSTBC schemes in Fig.2 and Fig.3, the corresponding DSTBC schemes in Fig.4 all achieve performance improvement because of using multiple receive antennas, which also further testifies the effectiveness of our given decoding algorithm.

VIII. CONCLUSIONS
By introducing quadrature amplitude modulation and transform matrix method, a multi-amplitude DSTBC scheme for square/non-square code matrix is developed in this paper, and the corresponding coding advantage is derived in detail. The developed scheme can effectively avoid the performance degradation of conventional differential space-time coding scheme based on MPSK modulation in high spectrum efficiency, and overcome the shortcoming that some existing DSTBC schemes only suit square code matrix. Moreover, it can carry information via not only the phase but also amplitude. Thus the system performance and spectrum efficiency are both improved effectively. Theoretical analysis shows that the developed scheme has linear decoding complexity and high coding advantage. Besides, it can obtain higher code rate in the case of more than two antennas. The simulation results show that our multi-amplitude DSTBC scheme has lower BER than the corresponding single amplitude DSTBC under the same spectrum efficiency, and the presented encoding and decoding scheme of the multiple amplitudes DSTBC are also valid.

ACKNOWLEDGMENT

This work was supported in part by NUAA Research Funding (NS2010113, NP2011036), Doctoral Fund of Ministry of Education of China (20093218120021).

REFERENCES


Xiangbin Yu received his Ph.D. in Communication and Information Systems in 2004 from National Mobile Communications Research Laboratory at Southeast University, China. He has been an Associate Professor with the Nanjing University of Aeronautics and Astronautics since May 2006. Dr. Yu has served as a technical program committee of Globecom 2006, International conference on communications systems 2008 (ICCS’08), ICCS’10, and reviewer of some conferences and journals. He has been a member of IEEE ComSoC Radio Communications Committee (RCC) since May 2007. His research interests include space-time coding, adaptive modulation and space-time signal processing.

Xiaoming Chen received the M.S. and Ph.D. degrees in Communication and Information Systems from Nanjing University of Aeronautics and Astronautics in 2001 and 2010, respectively. She is currently working as an associate professor at Nanjing University of Aeronautics and Astronautics. Her research interests include MIMO systems, power adaptation, iterative detection and Turbo-blast.

Yuyu Xin is currently working towards the B.S. degree at Nanjing University of Aeronautics and Astronautics, Nanjing, China.

Qiuming Zhu received the M.S. degrees in Communication and Information Systems from Nanjing University of Aeronautics and Astronautics in 2005. He is currently working as a lecturer at Nanjing University of Aeronautics and Astronautics. Also, he is pursuing the Ph.D. degree in Communication and Information Systems at Nanjing University of Aeronautics and Astronautics.
Dazhuan Xu received the M.S degrees and Ph.D. in Communication and Information Systems from Nanjing University of Aeronautics and Astronautics in 1986 and 2001, respectively. He is now a full professor in College of Information Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing, China. Prof. Xu is a Senior Member of China Institute of Electronics (CIE). His research interests include digital communications, software radio, coding theory, medical signal processing.