



# Constructing $K$ -Connected $M$ -Dominating Sets in Wireless Sensor Networks

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# Outline

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- Introduction
- Centralized Algorithm – *CGA*
- Distributed Algorithm – *DDA*
- Simulation Results
- Conclusion



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## Introduction

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- A *Connected Dominating Set (CDS)*  $C$  of  $G$  is a dominating set of  $G$  which induces a connected subgraph of  $G$ . The nodes in  $C$  are called *dominators*, the others are called *dominatees*.
- A *CDS* is the earliest proposed candidate to serve as a virtual backbone in WSNs.



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# Introduction

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- Using this virtual backbone, a sender can send messages to its neighbouring dominator. Then along the *CDS*, the messages are sent to the dominator closest to the receiver. Finally, the messages are delivered to the receiver.
- *k*-Connected *m*-Dominating Set (*kmCDS*) is necessary for fault tolerance and routing flexibility.





# Introduction

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- $k$ -connected or  $k$ -vertex connected
  - A graph is  $k$ -connected if and only if it contains  $k$  independent paths between any two vertices.
- $m$ -dominating set
  - If each node not in  $C$  is dominated by at least  $m$  nodes in  $C$ , then  $C$  is a  $m$ -dominating set.
- $k$ -connected  $m$ -dominating set



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## Centralized Algorithm CGA

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- *CGA* is a greedy algorithm with time complexity of *CGA* is  $O(|V|^{3.5}|E|)$ .
- The main idea:
  - construct an  $m$ -dominating set  $C$
  - augment this set  $C$  for  $k$ -connectivity
  - remove redundant nodes (optional)



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# Centralized Algorithm CGA

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- Notations:
  - $N_i$ : The number of neighbors of a node  $i$ .
  - $e_i$ : The energy of a node  $i$ .
  - $N_i^c$ : The number of dominator neighbors of node  $i$ .
  - $ID_i$ : The ID of node  $i$ .
  - $C$ : A  $k$ -connected  $m$ -dominating set.
- Weight function  $w(N, e, ID)$



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# Centralized Algorithm CGA

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## Algorithm 1 CGA( $k, m, G(V, E)$ )

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```
1: procedure FINDKMCDS( $k, m, G(V, E)$ )
2:   Sort nodes in non-increasing order in  $G$  based on
   their  $(N_i, e_i, ID_i)$ 
3:    $C \leftarrow \phi$ 
4:   for  $i = 1$  to  $|V|$  do
5:     if  $N_i^c < m$  then
6:        $C \leftarrow C \cup \{v_i\}$ 
7:     end if
8:   end for
9:   while  $C$  is not  $k$  connected do
10:    add a dominatee with highest  $(N_i, e_i, ID_i)$  into  $C$ 
11:  end while
12:  return  $C$ 
13: end procedure
```





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## Distributed Algorithm - DDA

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- *DDA* consists of three phases
  - Phase 1: Use one of the distributed *CDS* algorithms to construct a *CDS*  $C$ .
  - Phase 2: Augment  $C$  to a 1-connected  $m$ -dominating set by adding  $m-1$  *MISs*.
  - Phase 3: Connect set  $C$  for  $k$ -connectivity.



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## Distributed Algorithm – DDA phase 3

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- How to make  $C$  for  $k$ -connectivity in a distributed way?
- *Lemma 1:* If  $G$  is a  $k$ -connected graph, and  $G'$  is obtained from  $G$  by adding a new node  $v$  with at least  $k$  neighbors in  $G$ , then  $G'$  is also a  $k$ -connected graph.



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## Distributed Algorithm – DDA phase 3

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- Main idea:
  - The leader builds a  $k$ -connected component.
  - In order to join the  $k$ -connected component, one black node adds at most  $k^2$  connectors according to the previous lemma.



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## Distributed Algorithm – DDA phase 3

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- Important messages used in *DDA*:
  - *K-ConnectedComponent (KC) message*
  - *RequireConnector (RC) message*
  - *ACKConnected (AC) message*
  - *ConfirmSuccess (CS) message*
  - *ConfirmUnuse (CU) message*

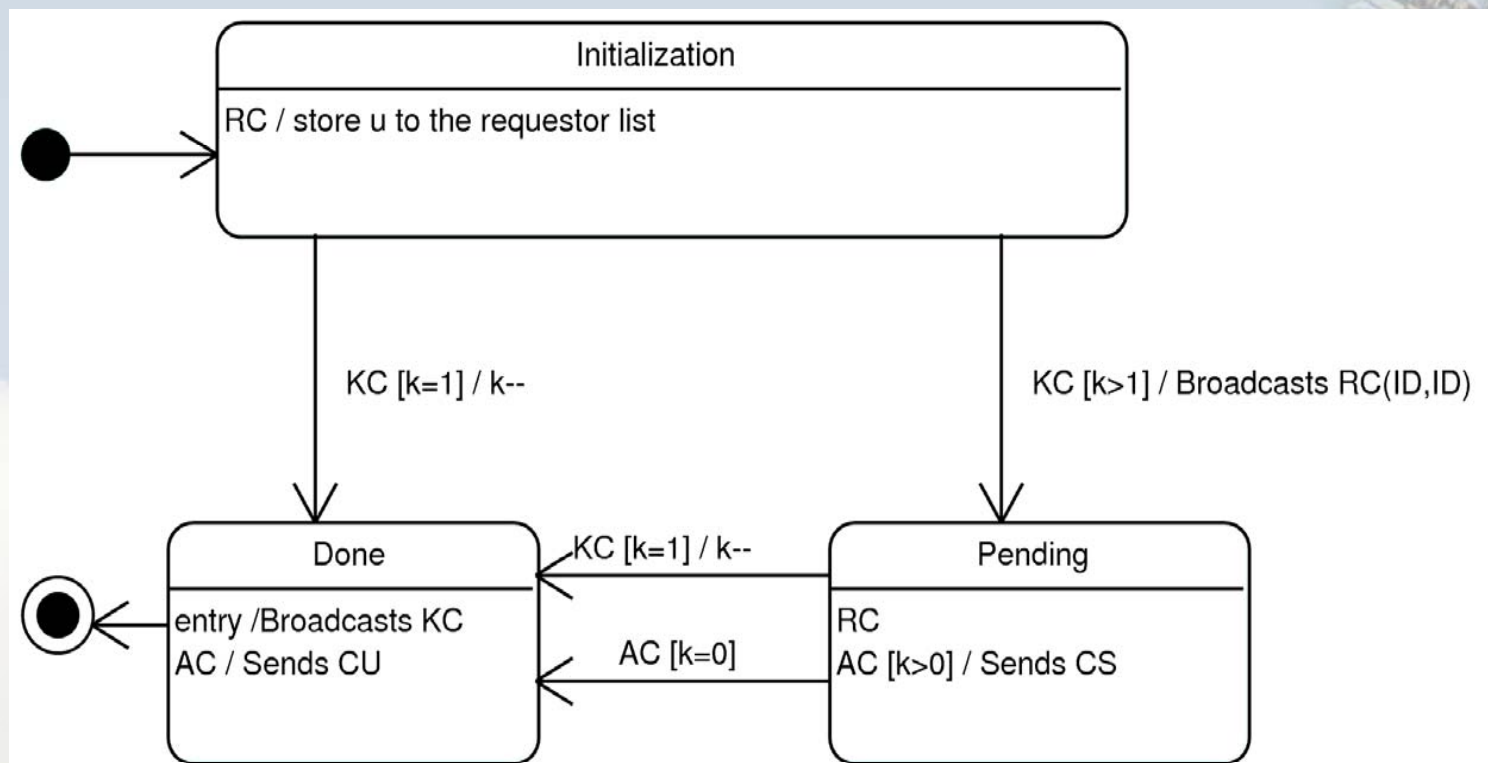


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# Distributed Algorithm – DDA phase 3

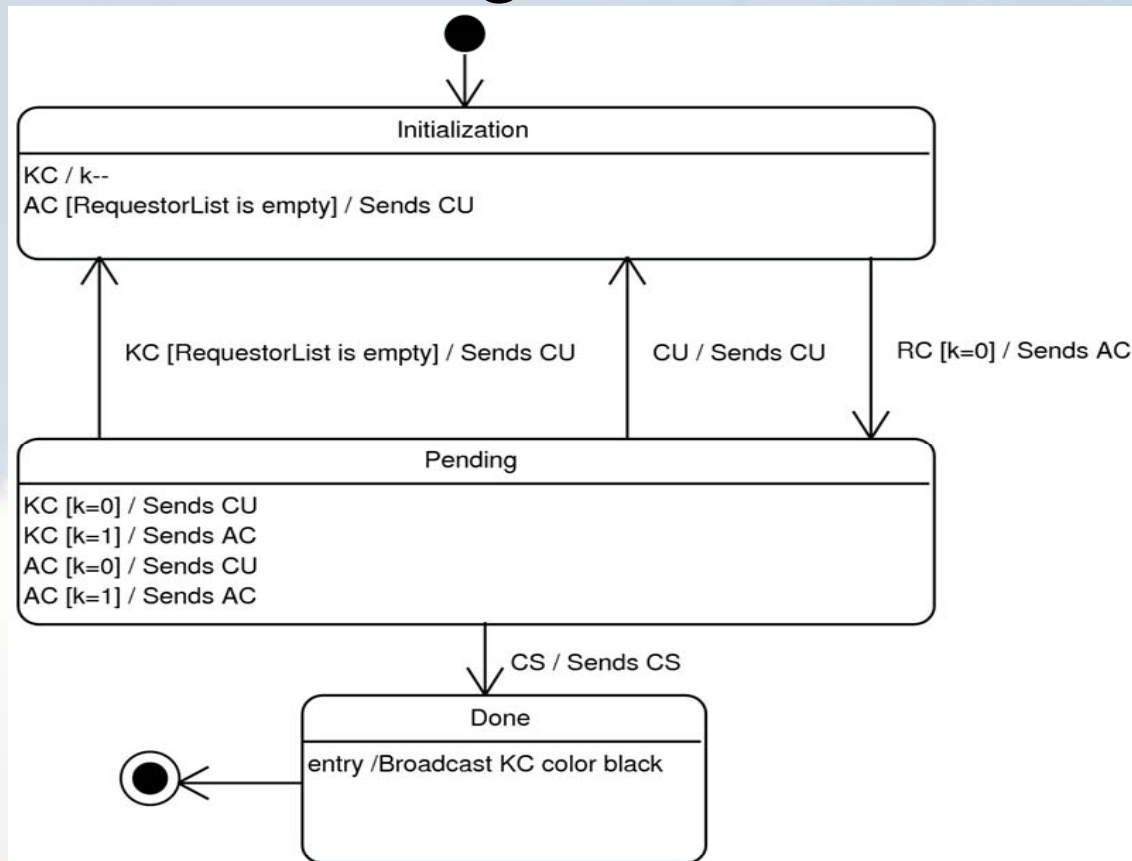
- State transition diagram for black nodes





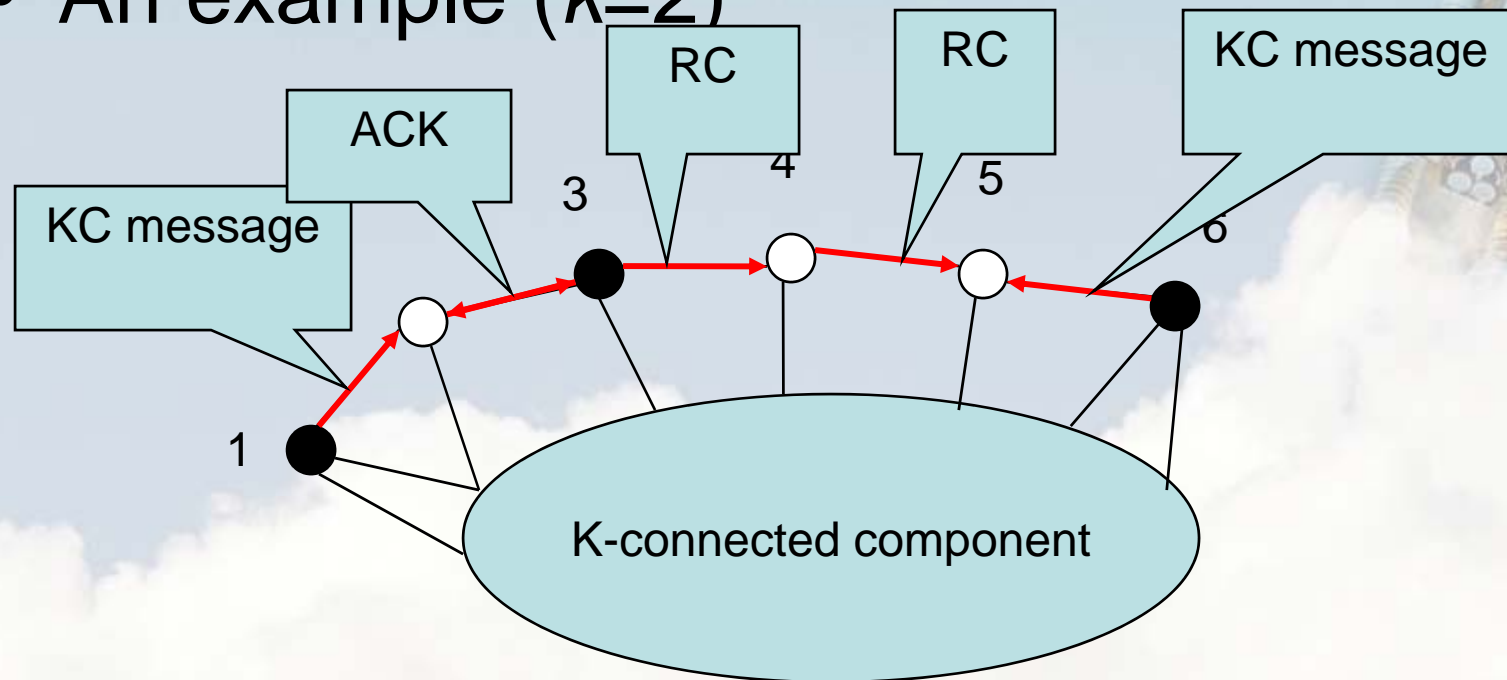
# Distributed Algorithm – DDA phase 3

- State transition diagram for white nodes



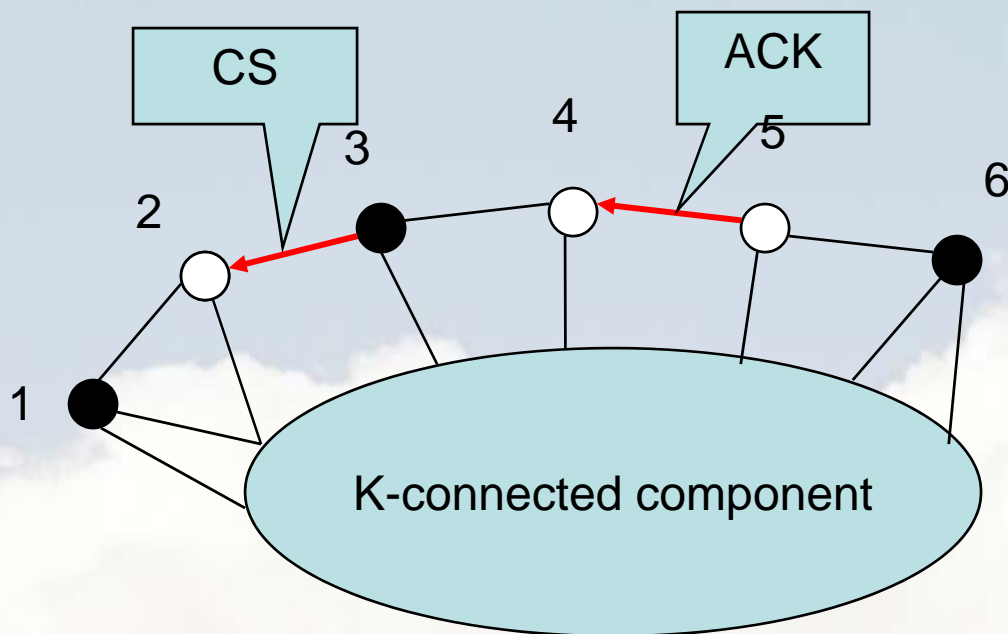
# Distributed Algorithm – DDA

- An example ( $k=2$ )



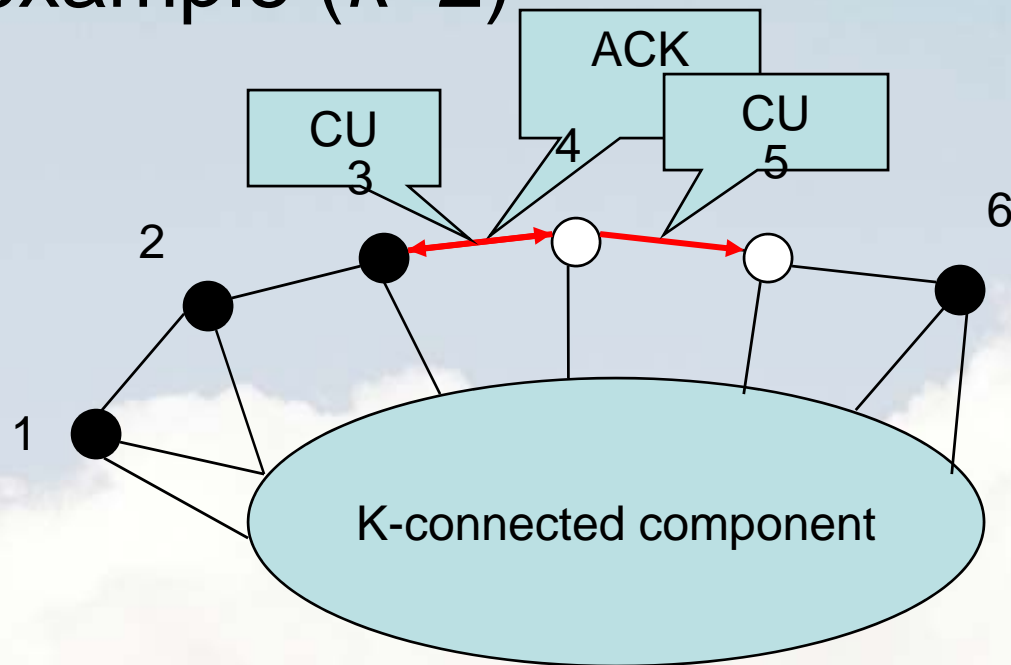
# Distributed Algorithm – DDA

- An example ( $k=2$ )



# Distributed Algorithm – DDA

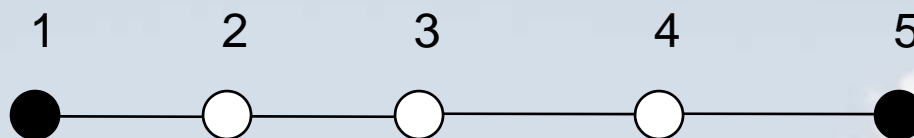
- An example ( $k=2$ )





## Distributed Algorithm – DDA

- *Lemma 2:* Every subset of an *MIS* is at most three hops away from its complement.



- *Lemma 3:* Let  $G = (V, E)$  be any *UDG* and  $m$  be any constant such that  $\delta_G \geq m - 1$  where  $\delta_G$  is the minimum node degree of graph  $G$ . Let  $D_m^*$  be any optimal  $m$ -domination of  $G$  and  $S$  be any *MIS* of  $G$ . Then  $|S| \leq 5m|D_m^*|$ .

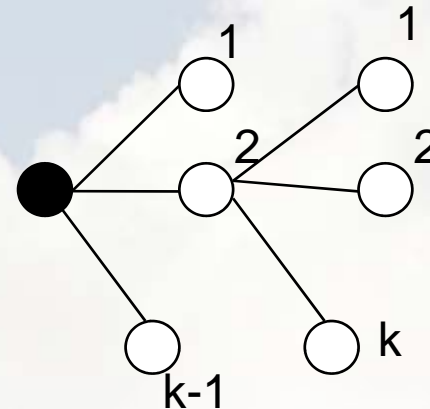


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## Distributed Algorithm – DDA

- *Theorem:* If  $C$  is a  $kmCDS$  obtained by DDA, then  $|C| \leq 5m(k^2 + 1)(m+42)opt$ , where  $opt$  is the size of any optimal  $kmCDS$  of the network.



- Proof:
  - Phase 1:  $43|S|$
  - Phase 2:  $(m - 1)|S|$
  - Phase 3:  $k^2$  for each black node in phase 1 and 2



## Distributed Algorithm – DDA

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- The message complexity of *DDA* is  $O(|V|\Delta^2)$  and time complexity is  $O(m\Delta + Diam)$ , where  $\Delta$  is the maximum node degree and *Diam* is the diameter of the network.



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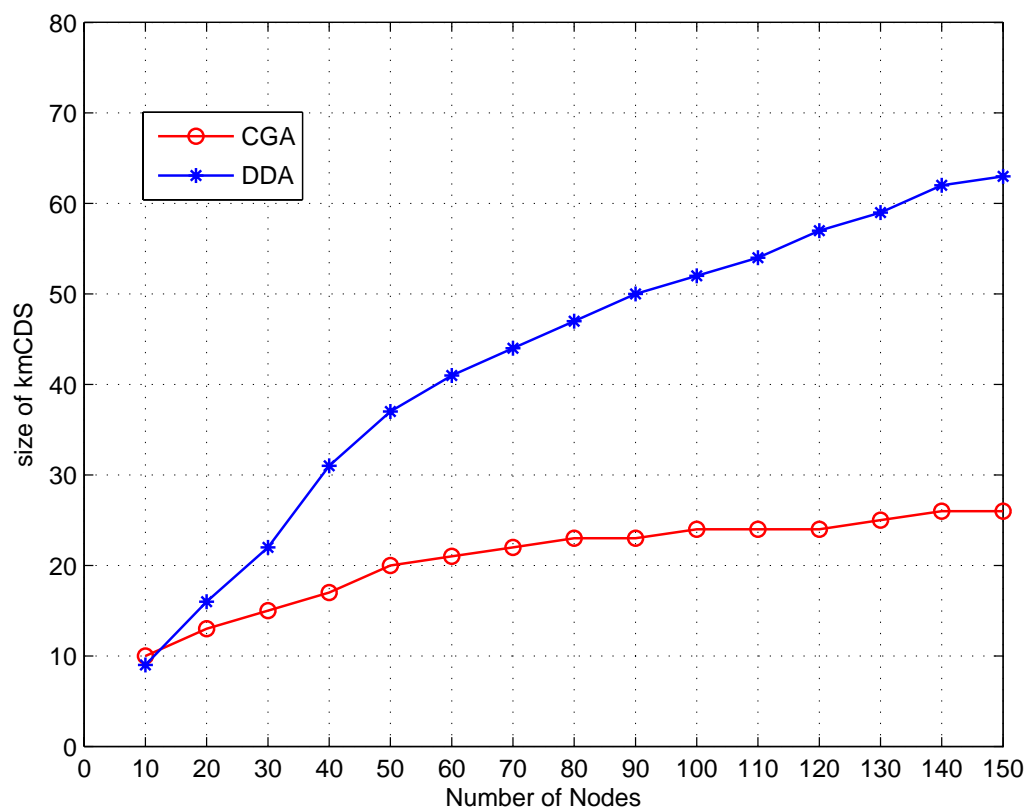
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# Simulation and Results

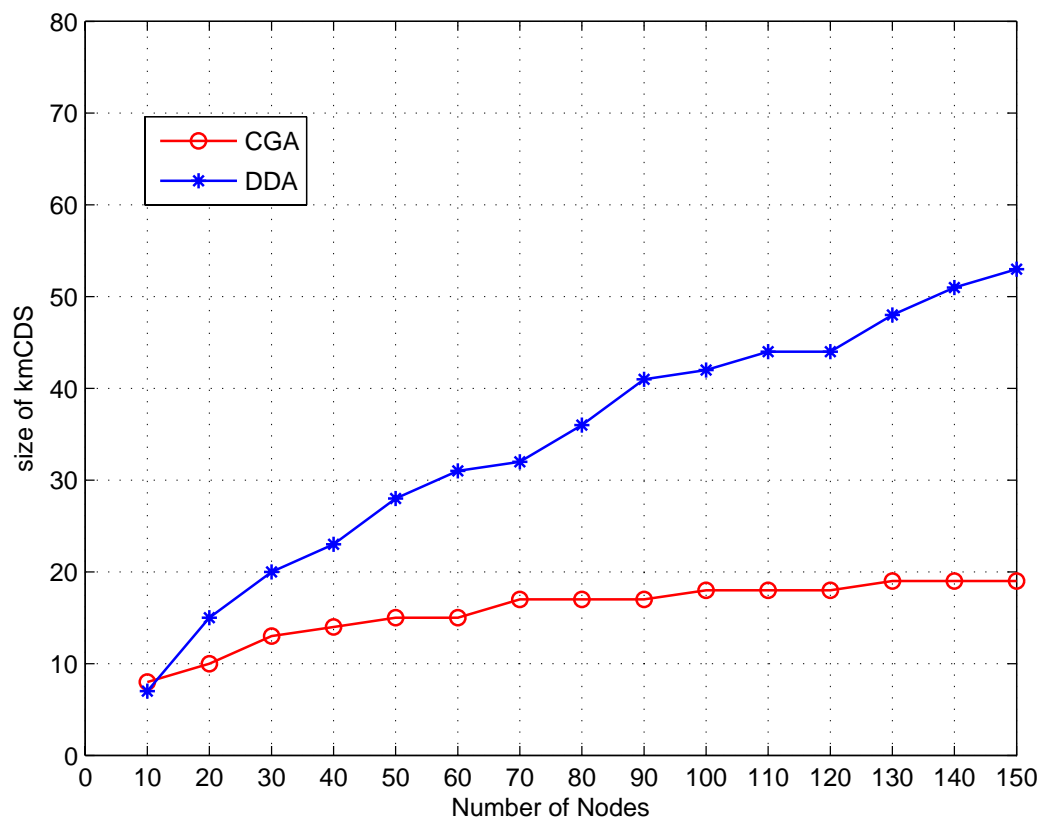
- $1000m \times 1000m$  area; transmission range = 250m





# Simulation and Results

- $1000m \times 1000m$ , transmission range = 350m

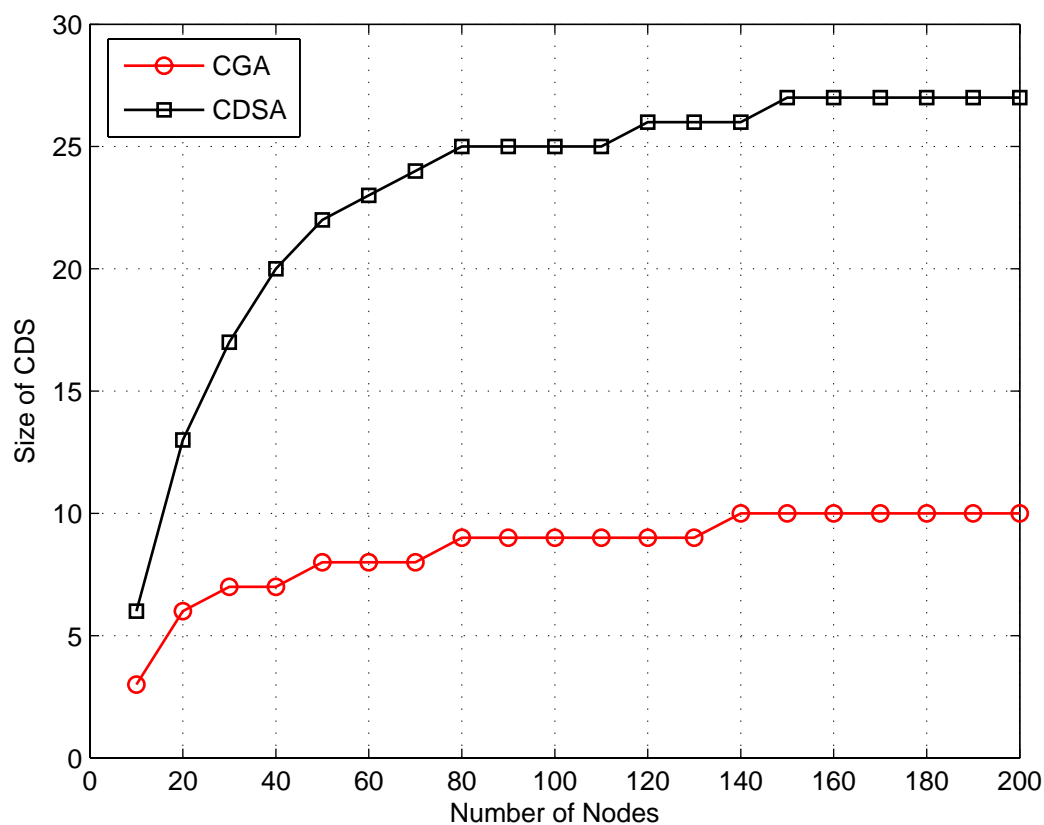


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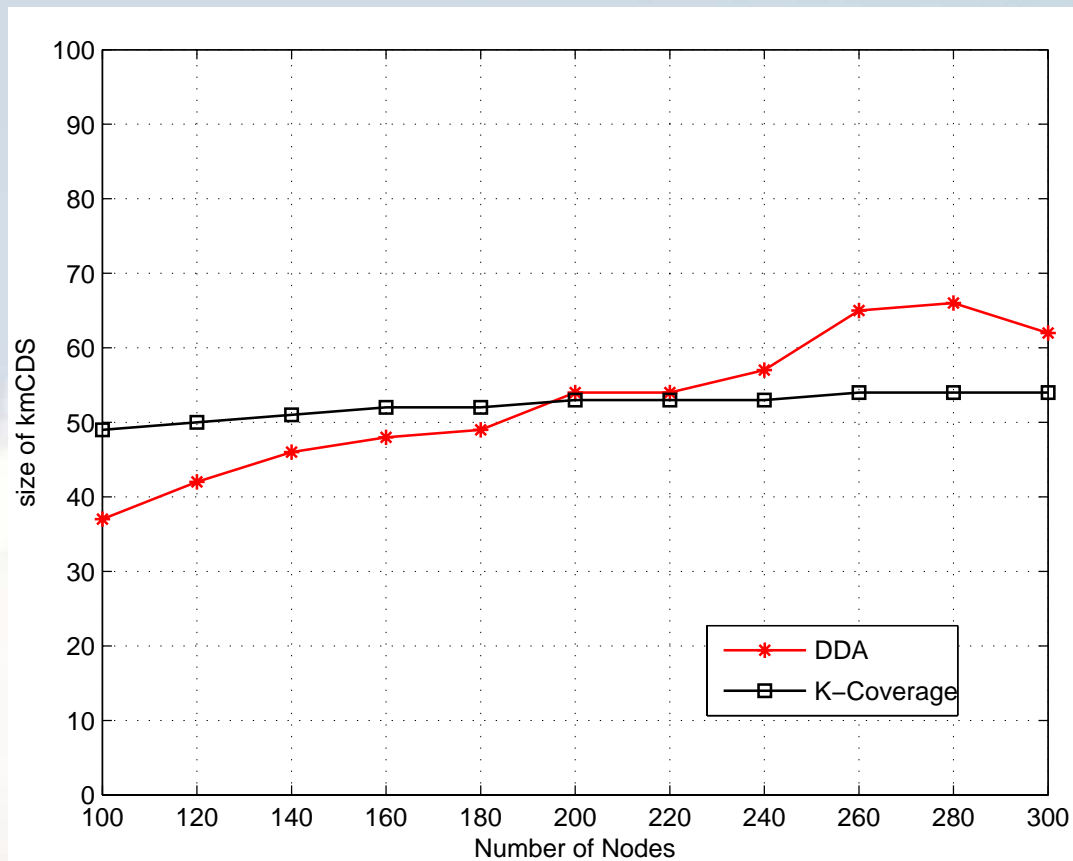
# Simulation and Results

- Compare *CGA* with *CDSA* when  $k = 2, m = 1$



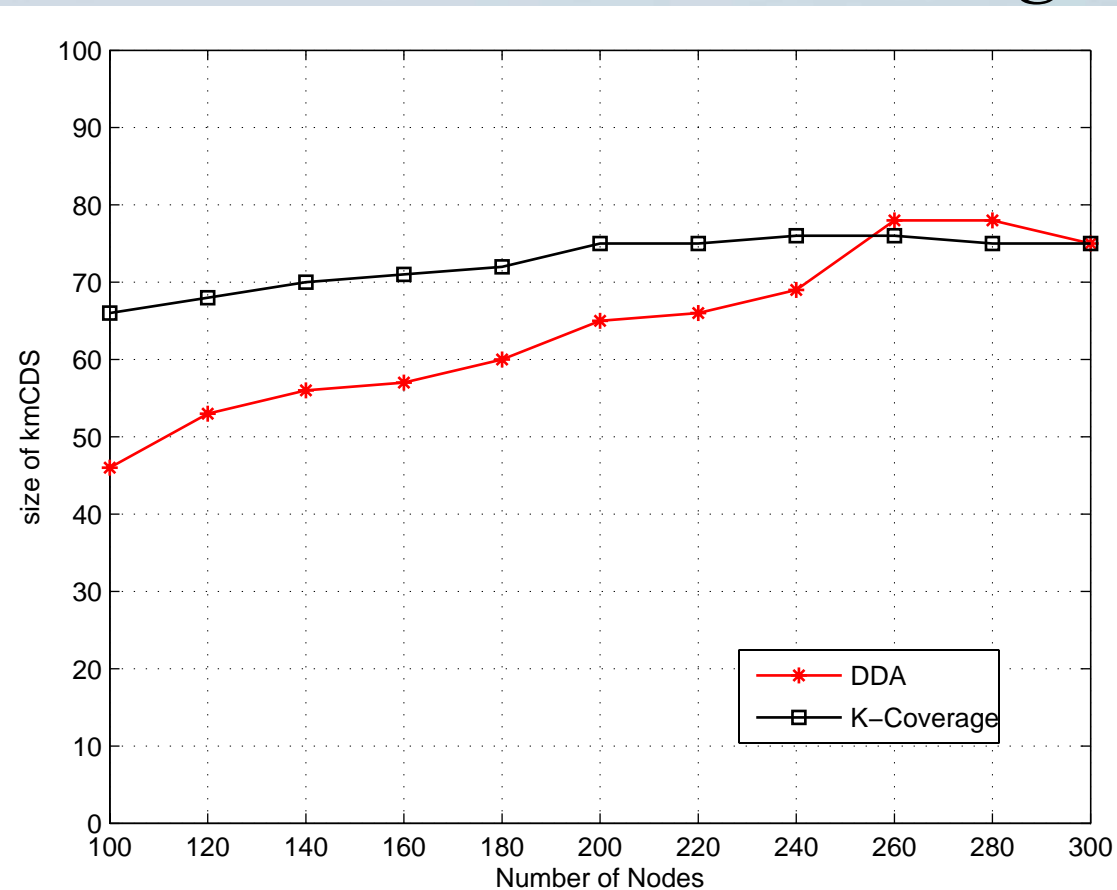
# Simulation and Results

- Comparison of *DDA* with *k-Coverage* ( $k=2$ ).



# Simulation and Results

- Comparison of *DDA* with *k-Coverage* ( $k=3$ ).





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## Conclusion

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- We investigate the problem of constructing a  $km$ CDS for general  $k$  and  $m$ .
- We propose one centralized algorithm  $CGA$  and one distributed algorithm  $DDA$ .
- Our algorithms can obtain good results.



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Questions?

Thanks!



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