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Emission of Radio-Frequency Waves from Plasmas*

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Observations of the radio-frequency emission from extraterrestrial plasmas and plasmas produced in the laboratory are described, and various attempts at interpretation of the results are reviewed. Estimates are made of the probable loss of radiant energy from plasmas in proposed thermonuclear reactors.

I. INTRODUCTION

OBSERVATIONS of the radio-frequency noise from plasmas produced in the laboratory and from plasmas of extraterrestrial origin have much in common. There appear to be two distinct components of such radiation that differ from each other in their spectral composition and in their relative magnitudes. A component of low intensity is observed which is considered to result from random emissions, absorptions, and scattering of electromagnetic radiation, and thus attains a certain equilibrium with the radiating matter. An intense component is also measured, which sometimes appears in the form of bursts of radiation, and is associated with disturbances in the plasma whose origin is, to a great extent, a matter of conjecture. However, large radiation intensities or prominent bursts of noise are not a prerequisite of nonequilibrium emission processes, and the distinction between equilibrium (or thermal) radiation and nonequilibrium (or nonthermal) radiation is often not easy to make.

Our ability to interpret the observations rests on knowledge of the basic radiating processes and on our understanding of the transport of radiant energy from within the plasma to the external observer. At radio frequencies (in which we include the microwave spectrum), the emitting processes are basically simple and few in number, as compared to the emission at optical wavelengths. The two most common mechanisms are bremsstrahlung and cyclotron emission by the free electrons of the plasma. However, the transport of radiant energy through the plasma is in many respects more complicated than at optical wavelengths. Strong interactions between the radiators and the radiation make the plasma highly dispersive, and in the presence of an external magnetic field, anisotropic in its dielectric properties. Furthermore, Coulomb interactions between the charged particles of the plasma can give rise to cooperative motions of the charges that not only modify the elementary emission processes, but can give rise to new forms of radiation.

II. THERMAL RADIATION

A. Transport of Radiant Energy and Kirchhoff's Laws

As a first step towards a calculation of the transport of radiation, we take a plasma whose dimensions are large compared with the wavelength of the radiation. This ensures that the concept of a ray trajectory is a meaningful representation of the direction of the energy flow. It also implies that whatever inhomogeneities occur within the plasma (e.g., as a result of a varying charge concentration), their spatial variation must be small compared with the wavelength.

Let **r** of Fig. 1 be the direction of the energy flow somewhere within the ionized medium and

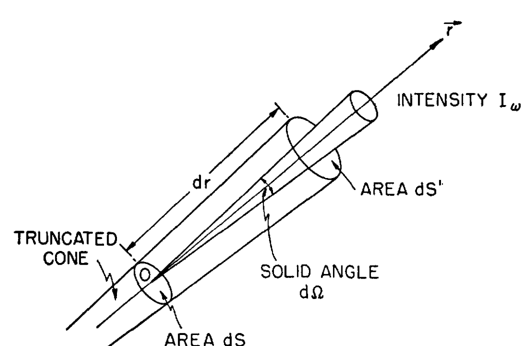

FIG. 1. Intensity of radiation along a ray **r** in the plasma.

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let dS be a small area drawn perpendicular to $\mathbf{r}$. dP_ω is the power, in the radian frequency interval between ω and $\omega+d\omega$, flowing through dS, in the direction $\mathbf{r}$, within the truncated cone of solid angle $d\Omega$. The intensity of radiation I_ω at a point 0 within dS is then defined as:

$$I_\omega = dP_\omega/dSd\omega d\Omega. \tag{1}$$

In the reception and measurement of the radio-frequency noise, generally only one mode of polarization of the radiation is examined at a time. For this reason the intensity I_ω, the power P_ω, and all other pertinent quantities will refer to a single polarization. For the purpose of differentiating between the two modes, sometimes called the ordinary and extraordinary waves, we shall use letters o and x. When the medium is isotropic, and the noise radiation is randomly polarized, the intensity of emission is the same for each characteristic wave: $I_\omega^{(o)} = I_\omega^{(x)}$, and the total intensity is $2I_\omega$.

The rate of increase of energy in the direction of the ray $\mathbf{r}$, within the small volume bounded by the areas dS and dS', and the truncated cone, is equal to the emission from this volume element (in the $\mathbf{r}$ direction), less the absorption suffered by the radiation in traversing the distance dr. If j_ω is the power radiated per unit solid angle (in one polarization) in the direction $\mathbf{r}$, and $\alpha_{\omega T}$ is the total absorption coefficient per unit length of path, we find for each characteristic wave that,[1,2]

$$\mu^2(d/dr)(I_\omega/\mu^2) = j_\omega - \alpha_{\omega T} I_\omega, \tag{2}$$

where μ is the refractive index for the mode in question. A solution of Eq. (2) for the intensity outside the plasma (where μ is one) is

$$I_\omega = \int_0^{\tau_0} (j_\omega/\mu^2\alpha_{\omega T}) \exp(-\tau) d\tau. \tag{3}$$

Here τ is the optical depth of the medium defined as $\tau = \int_0^r \alpha_{\omega T} dr$.

While radio-frequency noise j_ω originates almost entirely from the random accelerations of the free plasma electrons in the field of atoms, ions, and externally applied fields, the loss of electromagnetic energy $\alpha_{\omega T}$ from the ray results from one of two processes:

(a) Absorption of photons and a subsequent conversion of their energy into kinetic energy of the electrons, atoms, and ions. The magnitude of this absorption coefficient α_ω in a fully ionized plasma, for instance, is of the order of $10^{-9}N^2T^{-\frac{3}{2}}\omega^{-2}$ m^{-1}, where N is the electron concentration (m^{-3}) and T is the electron temperature in degrees Kelvin (see Sec. II. B).

(b) The removal of photons from the ray as a result of scattering by electrons. The classical scattering coefficient α_s is $(8\pi/3)(e^2/4\pi\epsilon_0 mc^2)^2 N$, where e and m are the electronic charge and mass, c is the velocity of light, and ϵ_0 the electric permittivity. The numerical value of α_s is 6.65×10^{-29} N m^{-1}.

Elementary scattering becomes important only at low electron densities and high temperatures.[3] When $T=10^5$ °K and the frequency of observation $\omega=10^{10}$ rad sec^{-1}, $\alpha_s/\alpha_\omega \approx 10^9/N$, and in this case scattering can be neglected for densities greater than approximately 10^3 electrons cm^{-3}. We shall neglect it henceforth.

By neglecting scattering in the expression for the total absorption, $\alpha_{\omega T} = \alpha_s + \alpha_\omega$, we not only simplify the solution of Eq. (3), but we can introduce the concept of temperature as a basic parameter of the problem of noise emission. We suppose that in every small volume element of the plasma the emission j_ω and the absorption α_ω are the same as if the whole system were in thermodynamic equilibrium. In other words, the radiators (the free electrons) have a Maxwellian distribution of energies at a temperature T, and the radiation is in thermal equilibrium with the electrons. In this case, one of Kirchhoff's radiation laws states that

$$j_\omega/\alpha_\omega = \mu^2 B(\omega,T), \tag{4}$$

where $B(\omega,T)$ is the blackbody intensity for one polarization. It is given by the Planck formula, which at radio frequencies becomes,

$$B(\omega,T) = kT\omega^2/8\pi^3c^2 \quad \text{wm}^{-2}\ \text{s}^{-1}. \tag{5}$$

Here k is Boltzmann's constant. We substitute Eqs. (4) and (5) in Eq. (3) and obtain the in-

[1] R. v. d. R. Woolley, Australian J. Sci., Suppl. 10, No. 2 (1947).
[2] R. v. d. R. Woolley and D. W. N. Stibbs, *The Outer Layers of a Star* (Clarendon Press, Oxford, England, 1953).
[3] V. A. Ambartsumyan, *Theoretical Astrophysics* (Pergamon Press, New York, 1958).

tensity outside the plasma:

$$I_\omega = \int_0^{\tau_0} B(\omega,T) \exp(-\tau)d\tau. \qquad (6)$$

When the plasma has a uniform temperature and a constant absorption coefficient, Eq. (6) reduces to,

$$I_\omega = B(\omega,T)[1-\exp(-\alpha_\omega L)], \qquad (7)$$

where L is the total path length traversed by the ray in the plasma. We see that when $\alpha_\omega L \gg 1$, the plasma emits like a blackbody, and when $\alpha_\omega L \ll 1$, $I_\omega \to B(\omega,T)\alpha_\omega L$, which is the emission in the absence of self-absorption. A convenient dividing line between the two forms of radiation can be set at $\alpha_\omega L \approx 1$.

In our derivation of Eqs. (3), (6), and (7) we neglected internal reflections of the ray at the plasma boundaries. This is justified for tenuous plasmas of sufficiently low charge concentration for which $(\omega_p/\omega)^2 \ll 1$, where ω_p is the plasma radian frequency, $\omega_p^2 = Ne^2/m\epsilon_0$. Since reflection and absorption are interdependent, a small value of ω_p/ω often implies also a small value of $\alpha_\omega\lambda$, where λ is the free-space wavelength. However, for large charge concentrations, $(\omega_p/\omega)^2 \geqslant 1$, reflections can be very large, leading to greatly reduced emission intensities. We take as an example the emission at right angles to the surface of a uniform plasma slab of thickness L. We sum over the intensities of the various rays that bounce back and forth between the surfaces of the slab, and obtain

$$I_\omega = (1-\Gamma)[1-\Gamma \exp(-\alpha_\omega L)]^{-1} \times B(\omega,T)[1-\exp(-\alpha_\omega L)], \qquad (8)$$

where Γ is the power reflection coefficient (the fraction of the power incident on the boundary that is reflected from it). Figure 2 illustrates the variation of the intensity with optical depth, for different values of Γ.

When the plasma is opaque, $\alpha_\omega L \gg 1$, Eq. (8) leads to

$$I_\omega = (1-\Gamma)B(\omega,T), \qquad (9)$$

which in fact is Kirchhoff's second radiation law, and which can be proved on the basis of thermodynamic equilibrium.

A good approximation to Eq. (8) is obtained by setting the term $\Gamma \exp(-\alpha_\omega L)$ equal to zero, with the result,

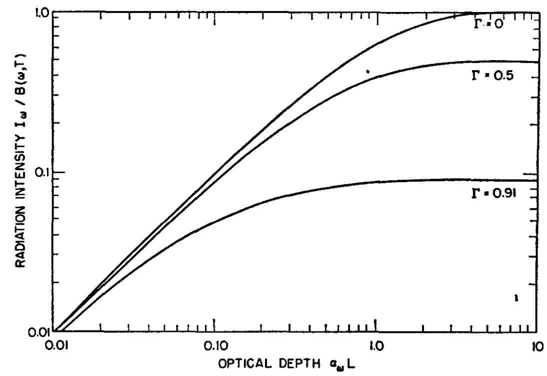

FIG. 2. Intensity of radiation as a function of the optical depth, for different reflections Γ at the surface of a plasma slab of thickness L.

$$I_\omega = (1-\Gamma)B(\omega,T)[1-\exp(-\alpha_\omega L)]. \qquad (10)$$

This is a good approximation because when α_ω becomes small, Γ is also small and Γ approaches zero more rapidly than does α_ω. When $\alpha_\omega L$ is large, Eq. (10) tends to the correct limit given by Eq. (9).

The results of this section constitute a thermodynamic model for the thermal emission of radiofrequency noise. To put this model to experimental test, we must examine the details of the various emission mechanisms from plasmas.

B. Bremsstrahlung

Bremsstrahlung arises from the acceleration of an electron in its hyperbolic or parabolic orbit around an ion or atom. When the free electron, with initial energy $mv_i^2/2$, makes a transition between two states of the continuum in the field of an ion or atom, the radian frequency of the emitted quantum is given by

$$(m/2)[v_i^2 - v_f^2] = \hbar\omega, \qquad (11)$$

where v_f is the final velocity of the electron. Since the process involves transitions between continuous states, the electron can lose any fraction of its initial energy, and the radiation forms a continuum. An electron can also radiate[4] on falling into a bound state of an atom or a positive

[4] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, England, 1952).

ion. At radio frequencies, this emission process is very small compared with bremsstrahlung.

The radiation that results from a collision between two electrons is negligible.[5] During this encounter, the equal and opposite accelerations of the two electrons lead to a zero over-all displacement of charge, and hence to zero dipole radiation. Contributions from higher-order emissions are small compared with bremsstrahlung for electron temperatures below approximately 10^9 °K.

Classical calculations of bremsstrahlung[6] for electron-ion encounters, based on the orbital acceleration of an electron, are satisfactory in the radio-frequency range. The energy radiated in the frequency interval $d\omega$ by one electron is $[2e^2/3c^3\epsilon_0][a(\omega)]^2$, where $a(\omega)=(1/2\pi)\int_{-\infty}^{\infty}a(t)\times\exp(-j\omega t)dt$, is the Fourier component of the electron's acceleration $a(t)$. Since the duration of the collision is short compared with the period of the emitted wave, the power spectrum of the fast impulse gives the characteristic "white" noise of bremsstrahlung. The total emission in the frequency interval $d\omega$ is obtained by summing the contributions from electrons that approach the ion at all possible distances (at all possible collision parameters p of Fig. 3). The emitted power, in all directions and in both polarizations, is,

$$dP_\omega=1.09\times10^{-51}NN_iZ^2T^{-\frac{1}{2}}Gd\omega \quad \mathrm{wm}^{-3}, \qquad (12)$$

and the corresponding absorption coefficient [obtained from Eq. (12) and from Kirchhoff's law] is

$$\alpha_\omega=7.0\times10^{-11}NN_iZ^2T^{-\frac{3}{2}}\omega^{-2}G \quad \mathrm{m}^{-1}. \qquad (13)$$

Here N and N_i are the electron and ion concentrations per cubic meter, Z is the ionic charge, and T the electron temperature in degrees Kelvin.

The parameter G is a slowly varying function of T, ω, and ω_p, and is the result of more refined calculations that take account of quantum mechanical effects, modifications of the Coulomb field by neighboring charges, etc. The magnitude of G varies between approximately 1 and 20. Some physical insight as to the meaning of G is obtained if we write it as $G\approx\ln(p_{\max}/p_{\min})$,

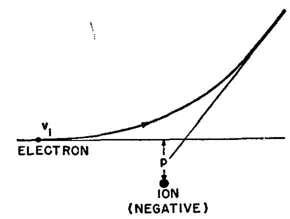

FIG. 3. The geometry of a collision (p is the collision parameter).

where $p_{\max}$ and $p_{\min}$ are the upper and lower cutoff distances of the collision parameter. $p_{\min}$ is generally taken as that distance for which the electron undergoes a 90° deflection, and it is given by $p_{\min}=Ze^2/(4\pi\epsilon_0 mv^2)$; $p_{\max}=v/\omega$ is a statement of our previous assumption that the collision time is short compared with the period of the wave. However, when the plasma is dense, the ion is shielded by neighboring charges, in which case $p_{\max}$ is set equal to the Debye distance, $l_D=(kT/m)^{\frac{1}{2}}\omega_p^{-1}$. Thus we obtain the following useful approximations for G, valid for moderately energetic electrons radiating at microwave and radio frequencies[7,8]:

$$G\approx(3^{\frac{1}{2}}/\pi)\ln([4\pi\epsilon_0/Ze^2m^{\frac{1}{2}}][3kT]^{\frac{3}{2}}\omega^{-1}) \quad \text{for} \quad (\omega_p/\omega)^2\ll1, \qquad (14)$$

$$G\approx(3^{\frac{1}{2}}/\pi)\ln(12\pi Nl_D^3/Z) \quad \text{for} \quad (\omega_p/\omega)^2>1.$$

We need to know the total power radiated by bremsstrahlung over all frequencies to estimate the energy loss from proposed thermonuclear reactors. With

$$G=(3^{\frac{1}{2}}/\pi)\exp(-\hbar\omega/2kT)K_0(\hbar\omega/2kT),$$

where K_0 is the modified Hankel function, we integrate Eq. (12) over all frequencies and obtain[7,8]:

$$P=1.6\times10^{-40}NN_iZ^2T^{\frac{1}{2}}\,\mathrm{wm}^{-3}. \qquad (15)$$

Figure 4 compares[9,10] the yield of power by thermonuclear reactions with the loss of power by bremsstrahlung from an equal volume of plasma. We see that the lowest electron temperature

[5] N. L. G. Redhead, Proc. Phys. Soc. (London) **A66**, 196 (1953); see also W. B. Thompson, Atomic Energy Research Establ. Rept. AERE No. T/M 73 (1957).

[6] H. A. Kramers, Phil. Mag. **46**, 836 (1923).

[7] J. Greene, Princeton University, Project Matterhorn, Rept. No. PM-S-41, Nov. 1958.

[8] H. de Witt, Univ. Calif. Radiation Laboratory Rept. No. UCRL-5377, Oct. 1958.

[9] B. A. Trubnikov and V. S. Kudryavtsev, *Proceedings of the Second United Nations Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958) Vol. **31**, p. 93.

[10] S. Glasstone and R. H. Lovberg, *Controlled Thermonuclear Reactions* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1960).

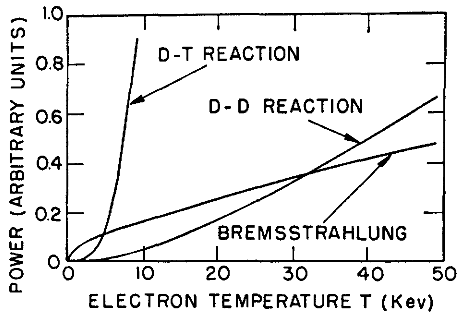

FIG. 4. A comparison between the production of energy by thermonuclear reactions and the loss of energy by bremsstrahlung,[9] as a function of the electron temperature (1 kev $=1.16\times10^7$ °K).

at which a *D-D* reactor will produce net power is approximately 32 kev ($T\approx3.7\times10^8$ °K). The dependence of the power loss of Eq. (15) on Z^2 is also very important. Thus, small amounts of impurity of high ionic charge (oxygen, carbon) enhance bremsstrahlung loss, and the influx of these impurities into the plasma must be kept to a minimum.

At low electron energies the plasma may be only partially ionized, and bremsstrahlung that results from electron-atom or electron-molecule collisions is significant. The calculations[11,12] are not easy and they require a fairly detailed knowledge of the potential distribution of the bound electrons. For monatomic atoms that undergo hard-sphere type of collisions with electrons (for example, a helium atom colliding with an electron that has an energy less than 4 ev), the power radiated is

$$dP_\omega=3.9\times10^{-62}NN_aT^{3/2}F(A)d\omega \text{ wm}^{-3}. \quad (16)$$

Here N_a is the concentration of atoms and A is the screening distance of the bound electrons. The function $F(A)$ is approximately one for helium.

A comparison of Eq. (12) with Eq. (16) tells us the relative importance of bremsstrahlung in a partially ionized gas from electron-atom $(e-a)$ and electron-ion $(e-i)$ collisions. Taking the ratio of the two emissions, we obtain

$$[dP_\omega]_{e-a}/(dP_\omega]_{e-i}\approx10^{-9}T^2(N_a/N_i). \quad (17)$$

A typical glow discharge produced in the laboratory has the following characteristics: T lies between 10^4 and 10^5 °K and $N_i/N_a<10^{-2}$. In these discharges, bremsstrahlung from electron-atom collisions greatly exceeds that from electron-ion collisions. However, if we turn off the discharge, the electrons cool rapidly to room temperature, and Eq. (17) shows that now bremsstrahlung from electron-ion encounters can exceed the radiation from electron-atom collisions.

So far we have neglected self-absorption of the radiation in its passage through the plasma. Let us examine with the aid of Eqs. (7) and (13) how self-absorption modifies the emission spectrum.

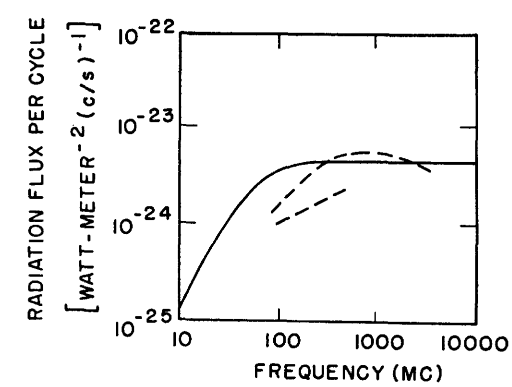

FIG. 5. Radio-frequency spectrum of bremsstrahlung. —— calculated for an imaginary nebula. – – – observations from two gaseous nebulas.[13,14]

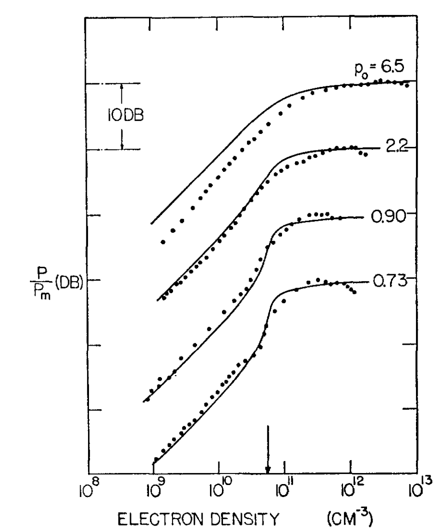

FIG. 6. Intensity of bremsstrahlung from the positive column of a dc discharge as a function of the electron density. • • measurements; —— theory. The gas pressure is in mm-Hg. The curves are displaced by 10 db relative to each other. $P_m=(1-\Gamma)B(\omega,T)$.

[11] L. Nedelsky, Phys. Rev. **42**, 641 (1932).
[12] S. Chandrasekhar and F. H. Breen, Ap. J. **104**, 430 (1946).

As an example,[13] we calculate the flux of radiation $\mathfrak{F}_\omega$ ($\mathfrak{F}_\omega = \int 2I_\omega d\Omega$, where Ω is the solid angle subtended by the plasma at the position of the observer) from an imaginary extraterrestrial object. We take $T=1.2\times10^4$ °K, $\Omega=3.4\times10^{-4}$ sr, and the product $NN_iL=3.9\times10^{32}$ m^{-5} where L is the length of the ray through the plasma. The ionic charge $Z=1$.

The solid line of Fig. 5 illustrates the emission spectrum computed for the extraterrestrial object. At high frequencies self-absorption is negligible, and radiation is just that obtained by summing over the individual binary collisions. As the frequency decreases, self-absorption becomes more and more prominent and the emission approaches the blackbody limit. This trend of events is found by observation.[14] The dashed lines of Fig. 5 show the spectra of two gaseous nebulas. If the sizes and distances of the sources are known, their average electron densities and temperatures can be inferred.

Figure 6 shows[15] the variation of the emission intensity with electron density, as measured at a constant frequency of 3000 Mc. The bremsstrahlung came from a section of the positive column of a dc glow discharge in helium. The measured intensity is normalized to the intensity $P_m=(1-\Gamma)B(\omega,T)$, which is the intensity the plasma would radiate, were it perfectly opaque, $\alpha_\omega L\gg1$ [see Eqs. (9) and (10)]. When the electron density is low, the radiation intensity increases linearly with density, as it should, if self-absorption is negligible, and if the radiation originates from collisions of electrons with atoms. The agreement with theory is very satisfactory. Note the sudden increase of the emission towards the limit P_m at high densities. This increase occurs near densities for which $\omega_p/\omega\approx1$. The vertical arrow of Fig. 6 denotes the value of N where $\omega_p/\omega=1$. We shall defer a discussion of this phenomenon. It is the result of the bulk properties of the plasma, and is not explained on the basis of a simple summation over binary collisions.

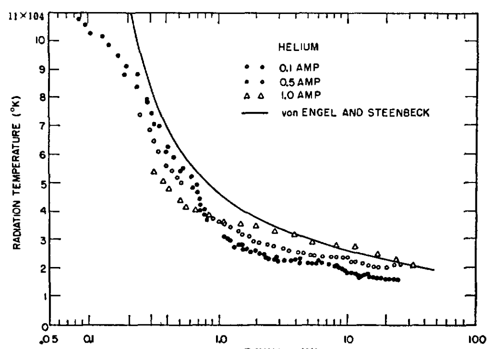

FIG. 7. Comparison between the measured radiation temperature and the calculated electron temperature (——) for three discharge currents in helium. $R=1.25$ cm is the radius of the discharge tube.

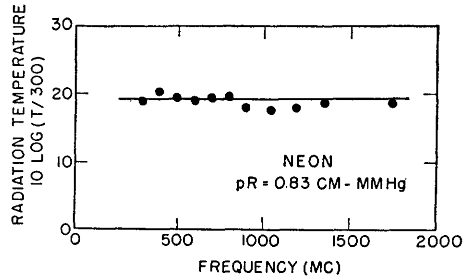

FIG. 8. Radiation temperature as a function of frequency of measurement.[18] (Neon).

The assumption of thermal equilibrium that was introduced in Sec. II. A allowed us to equate the "radiation" temperature, as defined by Planck's law, with the electron temperature. Figures 7 and 8 illustrate attempts to justify this assumption. In Fig. 7 we compare[16] the measured radiation temperature with the predicted electron temperature for a section of the positive column of a helium discharge. The measurements are also in good agreement with Langmuir probe measurements[17] of the electron temperature. Figure 8 demonstrates[18] that the radiation temperature is independent of frequency, a fact which supports the correctness of the emission model.

C. Cyclotron Emission

While bremsstrahlung is a process that takes place within the short time an electron spends in

[13] I. S. Shklovsky, *Cosmic Radio Waves* (Harvard University Press, Cambridge, Massachusetts, 1960).

[14] R. X. McGee, O. B. Slee, and G. J. Stanley, Australian J. Phys. **8**, 347 (1955).

[15] G. Bekefi, J. L. Hirshfield, and S. C. Brown, Phys. Rev. **116**. 1051 (1959).

[16] G. Bekefi and S. C. Brown, J. Appl. Phys. **32**, 25 (1961); see also K. S. Knol, Philips Research Repts. **6**, 288 (1951); L. W. Davis and E. Cowcher, Australian J. Phys. **8**, 108 (1955).

[17] R. J. Bickerton and A. von Engel, Proc. Phys. Soc. (London) **B69**, 468 (1956).

[18] T. D. McLaughlin and H. J. Oskam, University of Minnesota (private communication, November, 1960).

the vicinity of an ion or atom, cyclotron emission is a noncollisional mechanism that occurs in the interval between collisions. It originates from the orbital acceleration of an electron in a magnetic field. Cyclotron emission by ions is insignificant because of their large mass.

For electrons with energies less than several hundred volts, cyclotron emission is concentrated in a single spectral line, whose radian frequency ω equals the electron orbital frequency, $\omega_b = eB/m$ (B is the strength of the magnetic field). In contrast to bremsstrahlung, cyclotron emission is anisotropic, and in general, it is elliptically polarized. If an observer views the total radiation[19] at various angles θ relative to the direction of the magnetic field, he notes that the emission is maximum along B and that it falls off to half this value in a direction perpendicualr to B (see Fig. 9). No cyclotron emission is observed when the observer's antenna, instead of receiving the total radiation, is oriented to receive only that component of the electric vector that lies parallel to B. The emission $P(\theta)$ is

$$dP_\omega(\theta) = (e^2\omega_b^2/16\pi^2\epsilon_0 c][NkT/mc^2] [1+\cos^2\theta] \text{ wm}^{-3}\text{ sr}^{-1}. \quad (18)$$

The total power radiated in all directions is obtained from Eq. (18) or from the classical formula $P = [e^2/6\pi\epsilon_0 c^3]a^2$, where a is the acceleration of the electron in its circular orbit. It is given by

$$P = 5.3\times10^{-32} NB^2T \text{ wm}^{-3}, \quad (19)$$

where B is in gauss.

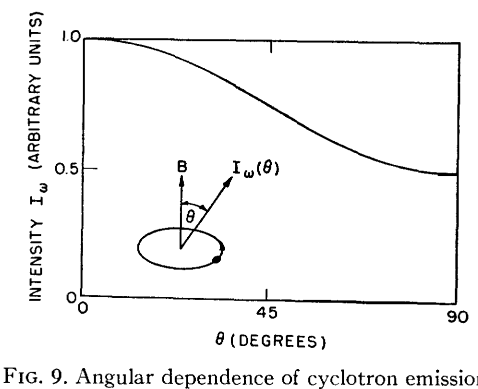

FIG. 9. Angular dependence of cyclotron emission from a cold electron.

From the point of view of the loss of energy from a thermonuclear reactor, the cyclotron emission predicted by Eq. (19) is very large. If the charged particles of the plasma are confined by magnetic forces, we require a magnetic field strength given by $B^2/2\mu_0 \approx 2NkT$. Substituting this value of magnetic field in Eq. (19) ,we obtain

$$P \approx 3.8\times10^{-52} N^2T^2 \text{ wm}^{-3}. \quad (20)$$

The dependence of P on the square of the charged particle density as given by Eq. (20) is the same as for bremsstrahlung [see Eq. (15)], and is also the same as for the production of thermonuclear power. For electron temperatures in excess of 5 kev, cyclotron emission exceeds bremsstrahlung, and it exceeds the D-D power production at all temperatures (see Fig. 4). If this were the

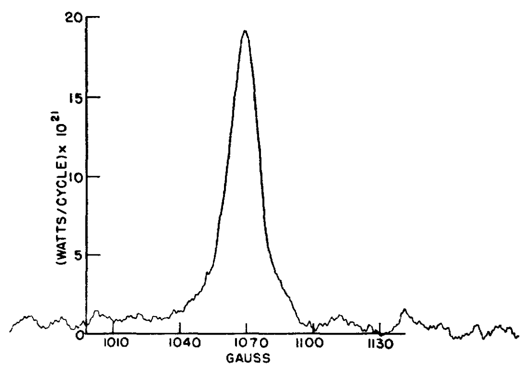

FIG. 10. Measurement of a cyclotron emission line[20] in the absence of self-absorption. (The irregularities in the trace are the results of receiver noise.)

whole story, a thermonuclear power generator that uses deuterium as a fuel could not operate. Fortunately, only a fraction of the radiant power that is predicted by Eq. (20) escapes out of the plasma. This point will be discussed in Sec. II. D.

Figure 10 shows a typical measurement[20] of a cyclotron emission line superimposed on the bremsstrahlung continuum. It was obtained by subjecting the positive column of a weak helium discharge to a uniform axial magnetic field of approximately 1000 gauss; the radiation was received at a frequency of 3000 Mc, under conditions where self-absorption of the radiation was negligible.

[19] H. Rosner, Rept. AFSWC-TR-58-47, Republic Aviation Corp., Farmingdale, Long Island, New York, (1958).

[20] J. L. Hirshfield and S. C. Brown, Phys. Rev. **122**, 719 (1961); also J. L. H. Ph.D. thesis, Massachusetts Institute of Technology, February (1960).

As in optical spectroscopy, a study of the cyclotron emission line affords much information. The total energy density under the line gives the electron pressure, NkT. The line is generally broadened by two effects: (a) collisions of electrons with atoms or ions, as a result of which, the spectral distribution has the form

$$dP_\omega(\theta) = (kT\omega^2 d\omega/16\pi^3c^3)(\omega_p^2\nu) \times[(\omega-\omega_b)^2+\nu^2]^{-1}[1+\cos^2\theta], \quad (21)$$

where ν is the collision frequency for momentum transfer of an electron colliding with an atom or ion; (b) first-order Doppler broadening that results from the thermal motion of the electrons. This effect leads to a Gaussian line shape

$$dP_\omega \propto \exp\left[-(mc^2/2kT)\left\{\frac{\omega-\omega_b}{\omega_b\cos\theta}\right\}^2\right]. \quad (22)$$

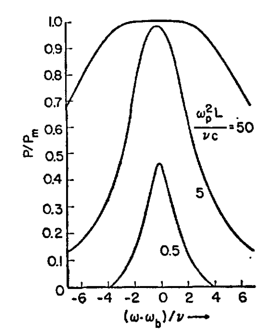

FIG. 11. Effect of self-absorption on the cyclotron emission from a low-density plasma, for different values of $\omega_p^2L/c\nu$.

Hence if (a) predominates, the width of the line gives us the collision frequency, and if (b) predominates, it gives us the electron temperature.

Self-absorption of cyclotron emission is important to our understanding of the role played by this radiation in the over-all energy loss from plasmas. For very tenuous plasmas ($\omega_p/\omega_b \ll 1$), the absorption coefficient α_ω is found from Kirchhoff's law and from Eqs. (21) or (22), and the radiation intensity I_ω, from Eq. (7). As the optical depth $\alpha_\omega L$ increases as a result of increasing L or ω_p, the peak intensity of the cyclotron line grows until it reaches the blackbody intensity $B(\omega,T)$. A further increase in $\alpha_\omega L$ causes the line to spread. This is illustrated in Fig. 11 for the case where the spectrum of the line is determined by collisions [Eq. (21)].

When the plasma is not tenuous, Eqs. (21) and (22) are not applicable, and the absorption coefficient must be found from a careful analysis[20–22] of the propagation characteristics of the two modes (sometimes called the ordinary and the extraordinary) by which the radiation is carried out of the plasma. Figure 12 illustrates the polarization of the characteristic waves for propagation along the magnetic field ($\theta=0$) and for propagation perpendicular to the magnetic field ($\theta=90°$). When $\theta=0$, the two characteristic waves are circularly polarized; one wave rotates in the direction of the orbiting electron and thus exhibits cyclotron resonance, and the other wave rotates in the opposite direction and is not greatly affected by the presence of the magnetic field. When $\theta=90°$ there are again two waves; the first has its electric vector parallel to the magnetic field and is thus completely unaffected by its presence. The second wave has its electric vector rotating in the plane perpendicular to the magnetic field, and it is both longitudinal and transverse to the direction of propagation. Since this wave has a longitudinal component, it can modulate the electron density. The electron experiences an ac space-charge force in addition to the magnetic force, and this results in a frequency displacement of the cyclotron emission line from the frequency $\omega=\omega_b$ to a frequency given approximately by $\omega^2=\omega_p^2+\omega_b^2$.

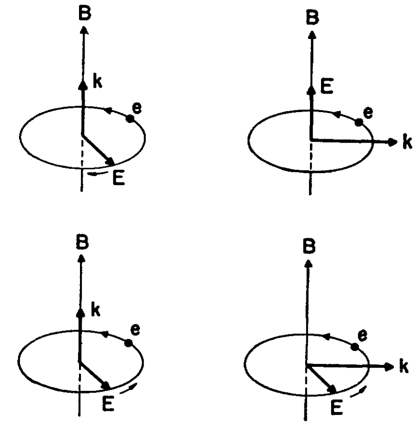

FIG. 12. Polarization of the two characteristic waves that propagate parallel and perpendicular to B; **k** is the propagation constant.

Figure 13 illustrates the fate of the cyclotron

[21] L. Mower, Sylvania Electric Products, Inc., Mountainview, California, Rept. No. MPL-1 (1956).

[22] S. J. Buchsbaum and W. P. Allis, notes on "Plasma Dynamics," Summer Session, Massachusetts Institute of Technology, (1959).

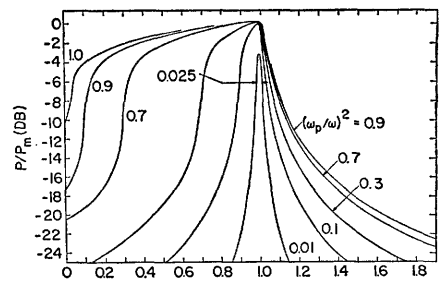

FIG. 13. Spectrum of cyclotron emission as a function of electron density, for propagation along B. The thickness of the plasma slab $L=0.11\lambda$; $\nu/\omega=0.01$; λ is the free-space wavelength.

emission line with increasing electron density for propagation along the magnetic field. The calculations are for a uniform slab of plasma of thickness L, with the magnetic field oriented along L. Reflections from boundaries are neglected. As ω_p/ω increases, the line widens out asymmetrically (compare with Fig. 11), until it finally disappears in the blackbody continuum for $\omega_p/\omega \approx 1$. At high densities, the cyclotron radiation is in a sense trapped, and at best it provides an additional mechanism of energy mixing within the plasma.

Measurements[20] made on the positive column of a dc discharge, shown in Fig. 14, are in qualitative agreement with the calculations of Fig. 15. Figure 15 shows calculations of the radiation intensity for propagation perpendicular to the magnetic field. As ω_p/ω increases, the spectral line shifts to higher and higher frequencies, for reasons mentioned previously, while at the same time it broadens out and disappears in the blackbody continuum.

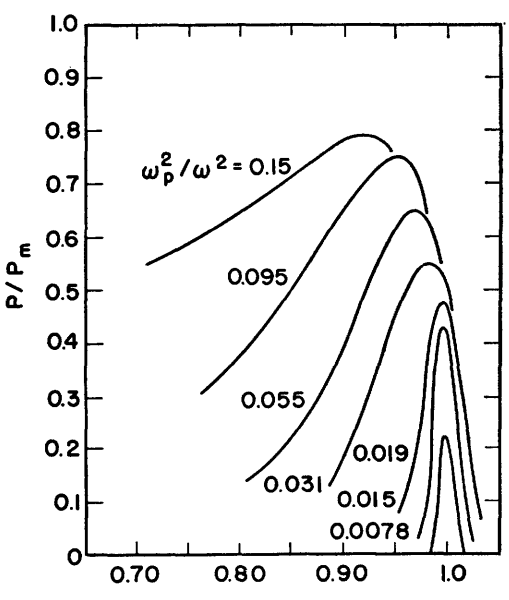

FIG. 14. Measurements of cyclotron emission for different electron densities.[20] Propagation perpendicular to B. $P_0=0.05$ mm Hg, and $L=0.1\lambda$.

D. Cyclotron Emission by Energetic Electrons

If cyclotron emission from energetic electrons were confined to frequencies near the cyclotron (or plasma) frequency, as was found to be the case for cold electrons, the large amounts of this energy produced in thermonuclear reactors [see Eq. (20)] would cause little worry as regards the

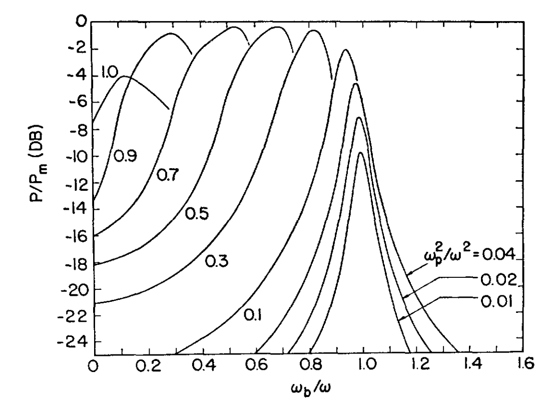

FIG. 15. Cyclotron emission for propagation perpendicular to B. $L/\lambda=0.11$; $\nu/\omega=0.03$.

balance between the rate of production of energy and the rate of loss by radiation. These reactors are intended to operate at high electron densities ($\omega_p/\omega_b > 1$), in which case, as was shown in Sec. II. C, the radiation cannot escape freely out of the plasma. The total radiation would then consist of blackbody radiation from zero frequency to $\omega \approx \omega_p$, and bremsstrahlung from $\omega \approx \omega_p$ to $\omega = \infty$. However, even for mildly relativistic electrons, most of the energy resides in the higher harmonics of the electron orbital frequency. For electron temperatures of 25 kev, (3×10^8 °K), 50% of the cyclotron energy is in the higher harmonics; for 50-kev electrons, approximately 94%. This fact greatly modifies our earlier thinking about energy losses from thermonuclear devices. The balance between bremsstrahlung loss and nuclear energy production (see Fig. 4) is in itself somewhat precarious. Now, the cyclotron emission in the higher harmonics constitutes a major problem.[9,23–26]

A single electron of speed v radiates in an infinite set of harmonics n of its orbital frequency. The rate of emission at the radian frequency ω is

$$\eta_\omega^{(o,x)}(\beta,\theta)=(e^2\omega^2/8\pi^2\epsilon_0 c)$$
$$\times\sum_{n=1}^{\infty} A_n^{(o,x)}(\beta,\theta)\delta[n\omega_b(1-\beta^2)^{\frac{1}{2}}-\omega]. \quad (23)$$

Here $\beta=v/c$, A_n is a dimensionless parameter which determines the strength of the emission, and $\omega_b=eB/m$, where m is now the rest mass of the electron. The emission in the higher harmonics decreases very rapidly with increasing harmonic number n (see Fig. 16). The emission is also concentrated in a narrow cone oriented at 90° to the magnetic field, and the cone becomes narrower the higher the harmonic number. For electron energies not exceeding several kev, and for $\theta=90°$, the emission per electron can be calculated from Eq. (23), using the following approximations for A_n: $A_n^{(o)}=0$; $A_n^{(x)}=[n^{2n}/(2n+1)!]\beta^{2n}$.

The characteristics of the spectrum of cyclotron emission is obtained by summing Eq. (23) over all the plasma electrons. The spectrum illustrated in Fig. 17 was computed for the following properties of the plasma: a uniform slab of thickness $L=1$ m, immersed in a magnetic field of 10 000 gauss oriented along the faces of the slab. The electron density is approximately 10^{14} cm^{-3} and the electron temperature is 50 kev. These figures apply approximately to a reactor in which the outward kinetic pressure of the charged particles $2NkT$ is counteracted by the inward magnetic pressure $B^2/2\mu_0$.

The various curves of Fig. 17 labeled 1, 2, 3··· represent the emission from the individual harmonics. Their widths are the result of second-order Doppler broadening that arises from the relativistic change of mass of the electron. The total emission at a given frequency is obtained by adding the contributions from all the harmonics, and is shown by the upper curve of Fig. 17. At frequencies $\omega/\omega_b>3$, the harmonics overlap so strongly that the net emission forms a monotonically decreasing function of frequency.

We have neglected self-absorption which prevents the total emitted intensity from exceeding the blackbody limit [$I_\omega/B(\omega,T)=1$ on the ordinate of Fig. 17]. Thus the plasma radiates almost

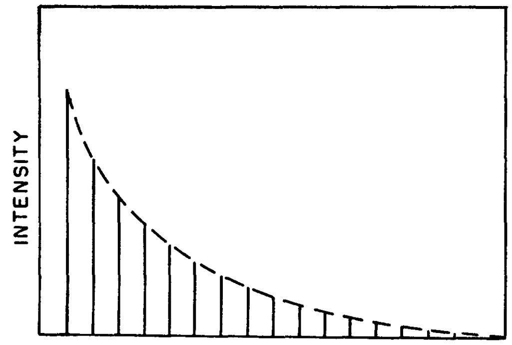

FIG. 16. Sketch of the intensity of the harmonics of cyclotron emission from one electron of low energy ($v^2/c^2\ll 1$). The vertical lines are the harmonics.

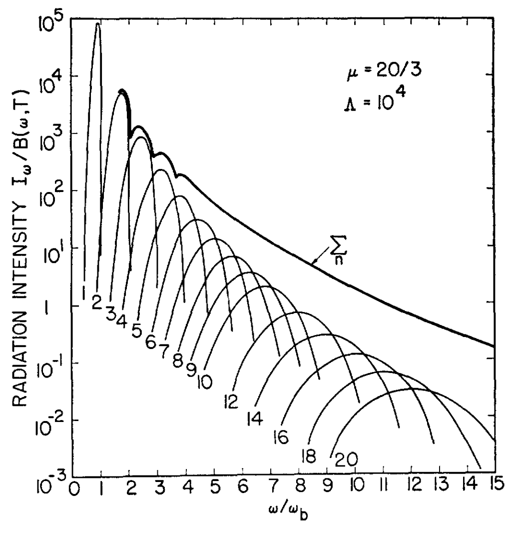

FIG. 17. Cyclotron emission spectrum from a plasma with average energy of 75 kev ($\mu\equiv mc^2/\bar{u}=20/3$, $T=50$ kev). The parameter $\Lambda\equiv\omega_p^2L/\omega_b c=10^4$ specifies the electron density and strength of the magnetic field. Self-absorption is neglected.

[23] B. A. Trubnikov, "Magnetic Emission of a High-Temperature Plasma," Dissertation, Moscow, (1958); English translation by the U. S. Atomic Energy Commission, Rept. No. AEC-tr-4073, June, (1960); also, B. A. Trubnikov and A. E. Bazhanova, *Plasma Physics and the Problem of Controlled Thermonuclear Reactions* (Pergamon Press, London, 1959), Vol. III, p. 141.

[24] W. E. Drummond and M. N. Rosenbluth, Phys. Fluids **3**, 45 (1960).

[25] D. B. Beard, Phys. Fluids **2**, 379 (1959); **3**, 324 (1960).

[26] J. L. Hirshfield, D. E. Baldwin, and S. C. Brown, Phys. Fluids **4**, 198 (1961).

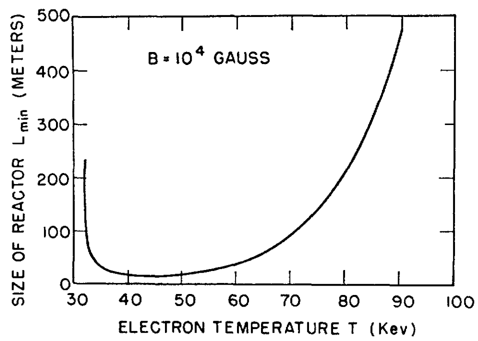

FIG. 18. Minimum size of reactor that uses deuterium fuel as a function of the electron temperature ($B=10^4$ gauss and N is given by $2NkT=B^2/2\mu_0$).

as a blackbody from $\omega=0$ to some characteristic frequency ω^*. This characteristic frequency (equal to $\omega^*=11.2\ \omega_b$ in our case) is a function of the electron energy, density, and the plasma dimensions. At frequencies greater than ω^*, the cyclotron emission can escape almost freely from the plasma. The dividing line between the two regimes of emission is conveniently defined by $\alpha(\omega=\omega^*)L=1$. Thus the total cyclotron emission from unit area of plasma surface is

$$I=\int_0^\infty B(\omega,T)[1-\exp(-\alpha_\omega L)]d\omega \approx \int_0^{\omega^*} B(\omega,T)d\omega + L\int_{\omega^*}^\infty B(\omega,T)\alpha_\omega d\omega. \tag{24}$$

The second term of Eq. (24) represents the free emission of cyclotron radiation for frequencies $\omega>\omega^*$. This radiation decreases nearly exponentially with increasing frequency and we shall neglect it. Performing the integration in Eq. (10) leads to an energy loss of $kT(\omega^*)^3/24\pi^3c^2$ w m^{-2}. In thermonuclear reactors, this energy loss, when added to the bremsstrahlung loss, should not exceed the power produced by the thermonuclear reactions.

We see that cyclotron emission establishes a minimum size for a thermonuclear power generator, since the cyclotron emission is largely proportional to the surface area, while nuclear energy production and bremsstrahlung are volume effects. For the plasma we have chosen and for a *D-D* thermonuclear reaction, the minimum plasma size $L_{\min}$ is several meters.[9] Figure 18 shows a plot of $L_{\min}$ as a function of the electron temperature for a fixed magnetic field of 10^4 gauss, and an electron density given by the relation $2NkT=B^2/2\mu_0$.

The outrageously large sizes of reactors which these calculations predict are indeed discouraging. Mirrors, to reflect the radiation back into the plasma, have been suggested.[24] These may provide only a partial cure since they are effective only over frequencies where the plasma is a sufficiently good absorber.

Several recent reappraisals of the magnitude of $L_{\min}$ show large divergencies. The most recent calculation[27] predicts the following result for the reactor size containing a plasma with 40-kev electrons:

$$L_{\min}\approx 10^4(1-\Gamma_m)/\beta B, \tag{25}$$

where L is in centimeters, B is the magnetic field in gauss, Γ_m is the power reflection coefficient of the mirrors, and $\beta=4NkT\mu_0/B^2$. The minimum size of the plasma given by Eq. (25) is several hundred times smaller than the one quoted in Fig. 18 (for $\beta=1$, $\Gamma_m=0$). The main reason for this discrepancy comes from using larger thermonuclear cross sections in the calculation of Eq. (25). Whether this last, and more encouraging, calculation is the final estimate, remains to be seen.

E. Macroscopic Model of Noise Emission

Our model for calculating the thermal emission from a plasma is restricted in two ways: (1) the plasma must be sufficiently tenuous to permit us to neglect dispersion and other bulk properties of the medium; (2) the plasma must be large enough compared with the wavelength to permit use of geometrical optics. Although we quoted theoretical results for dense plasmas ($\omega_p/\omega>1$), and although we described measurements of noise emission from plasmas a fraction of a wavelength in size, we neglected to say how the computations are made.

Assumption (1) implies that the electrons of the plasma which undergo oscillations in the electromagnetic field of their own making do not react back on the field. However, the electrons oscillate out of phase with the electromag-

[27] W. E. Drummond and M. N. Rosenbluth, Paper presented at the A. P. S. Meeting of the Division of Plasma Physics, Gatlinburg, Tennessee, November, 1960; Bull. Am. Phys. Soc. 6, 205 (1961); Phys. Fluids 4, 277 (1961).

netic waves, and the fields they create can react strongly with the original radiation, particularly when the ac conduction currents of the charges become comparable in magnitude with the displacement currents of the waves. This interaction becomes appreciable at frequencies smaller or equal to the plasma frequency, ω_p. In other words, assumption (1) requires that the refractive index of the plasma does not depart appreciably from unity.

Our former basic Eq. (4) is only valid[28] for tenuous media with sufficiently low absorption coefficients. Specifically, one finds that if the complex refractive index is written as $(\mu+j\chi)$, Eq. (4) is valid if we can discard terms of order $(\chi/\mu)^2$ and higher. Near $\omega=\omega_p$, $\mu\rightarrow 0$ and this condition is not met.

Assumption (2) and the technique of tracing rays [Eqs. (1), (2), (3)], though useful for a plasma of astronomical dimensions, is too crude when the plasma is only a few wavelengths across, when it exhibits large temperature and density gradients, and when diffraction at boundaries becomes appreciable. Ray tracing, being a geometrical optics concept, is applicable when[29]

$$|(\lambda/8\pi\mu^2)(\partial\mu/\partial r)|^2\ll 1, \tag{26}$$

where λ is the free-space wavelength. Hence, we see that gradients in μ should be small, and μ itself should not approach zero. In the presence of a dc magnetic field, there are two transport equations of the form of Eq. (2), one for each characteristic wave, and there are two[30,31] Kirchhoff's equations of the form of Eq. (4). The two characteristic waves propagate independently of each other when electron density gradients are small and when the refractive indices of the two waves are sufficiently different from one another. When this is not so, coupling between the waves leads to great complications.[29]

In the remainder of this section we shall outline attempts to circumvent the above limitations of our emission model by approaching the whole problem from the macroscopic,[32] rather than from the microscopic or single particle, point of view.

Probably the first systematic approach towards taken by Rytov.[28] The plasma is assumed to contain microscopic, fluctuating, current elements embedded in a medium whose time-average physical properties are specified by its electric and magnetic permeabilities. The fluctuating current elements are the sources of the incoherent electromagnetic radiation that reaches the observer. This radiation is computed from Maxwell's field equations in accordance with the boundary conditions which the field vectors must obey. Maxwell's equations for the electric and magnetic fields **E** and **H**, anywhere inside an isotropic medium, for radiation at a frequency ω, are written in the following way:

$$\nabla\times\mathbf{H}=j\omega\epsilon_0\mathbf{E}+\sigma\mathbf{E}+\mathbf{J}, \tag{27}$$

and

$$\nabla\times\mathbf{E}=-j\omega\mu_0\mathbf{H}. \tag{28}$$

Here ϵ_0 and μ_0 are the electric and magnetic permeabilities of empty space, and σ is the radiofrequency conductivity (generally complex) of the plasma. The term $(\sigma\mathbf{E})$ represents the conduction current resulting from the motion of the free charges of the plasma. The quantity **J** is the impressed electric current density of the elementary noise sources. Since the magnetic properties of an ionized gas can be often neglected, the magnetic permeability of Eq. (28) is assumed to be permeability of free space, μ_0. We note in Eqs. (27) and (28) the absence of time-dependent quantities and the appearance of the frequency ω. The presence of a single frequency must not be interpreted as a periodic variation of the field variables. We deal with fluctuating quantities, and **E**, **H**, and **J** are Fourier components of time-dependent functions.

We must now assume something for **J**. In the determination of noise power we are not really interested in **J** itself. What we need in computing Poynting's vector is the space average of the product of **J** in one volume element of plasma, with the complex conjugate of **J** in a neighboring

[28] S. M. Rytov, *Theory of Electrical Fluctuations and Thermal Radiation* (U.S.S.R. Academy of Sciences, Moscow, 1953); English translation by the Air Force Cambridge Research Center, Bedford, Massachusetts, Rept. No. AFCRC-TR-59-162, July, 1959.

[29] J. A. Ratcliffe, *The Magneto-Ionic Theory and Its Applications to the Ionosphere* (Cambridge University Press, New York, 1959).

[30] F. V. Bunkin, Soviet Phys.—J. E. T. P. **5**, 227 (1957); **5**, 665 (1957).

[31] D. F. Martyn, Proc. Roy. Soc. (London) **A193**, 44 (1948).

[32] P. Parzen and L. Goldstein, Phys. Rev. **82**, 724 (1951).

element. This space average is given by

$$\langle J_\alpha(\mathbf{r}')J_\beta^*(\mathbf{r}'')\rangle = 4kT\,\mathrm{Re}(\sigma)(d\omega/2\pi)\delta_{\alpha\beta}\delta(\mathbf{r}'-\mathbf{r}''). \quad (29)$$

The delta function correlation $\delta(\mathbf{r}'-\mathbf{r}'')$ is a statement of the randomness of the fluctuations of the currents in adjacent volume elements, whose position vectors are $\mathbf{r}'$ and $\mathbf{r}''$. In other words, the current in one volume element depends only on the field in the same volume element. The factor $\delta_{\alpha\beta}$ is a unit tensor which says that there is no correlation between the orthogonal components α, β, of the current density vectors.

The temperature T in Eq. (29) comes from uniting Maxwell's equations with the principles of thermodynamic equilibrium [to achieve this union, Rytov used Eq. (9)]. Equation (29) is a three-dimensional form of Nyquist's theorem that relates the current fluctuations to the resistance of a current-carrying wire. We also see that when there is no dissipation (σ pure imaginary) there is no radiation, which is another statement of Kirchhoff's law. Furthermore, since there is no correlation between neighboring current elements of the plasma, σ and T can have almost arbitrarily large spatial gradients.

A computation of Poynting's vector from Eqs. (27), (28), and (29), together with appropriate boundary conditions at the plasma surfaces, in principle leads to a complete solution of the noise emission from the medium. The mathematics is difficult because one must sum over the waves from the elementary currents and the waves diffracted at boundaries, traveling in a dissipative medium. However, when σ and T are independent of position, the theory leads to the following elegant generalization[33] of Kirchhoff's law [that now replaces the geometrical optics Eq. (7)] applicable to a plasma of arbitrary size and of arbitrarily large diffraction at its boundaries: the noise power P_ω (for one polarization) received at some remote distance from the plasma (called the Fraunhoffer region) is

$$P_\omega = B(\omega,T)SA_\omega \text{ w sr}^{-1}. \quad (30)$$

Here S is the projection of the cross-sectional area of the plasma onto a plane perpendicular to

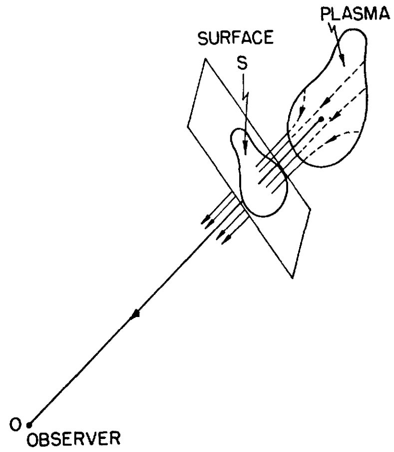

FIG. 19. Relationship between the position of the observer, the plasma, and the surface S.

the direction of observation (see Fig. 19). A_ω is the fraction of the power absorbed by the plasma from a plane polarized test wave $\mathbf{E}_i$, $\mathbf{H}_i$, launched from the position of the observer. The determination of A_ω involves an experiment or a boundary value solution of a plane wave incident on a lossy dielectric. A_ω is given by,

$$A_\omega = \mathrm{Re}(\sigma)\int_V |\mathbf{E}|^2 dV \Big/ \int_S \mathrm{Re}(\mathbf{E}_i\times\mathbf{H}_i^*)\cdot d\mathbf{S}, \quad (31)$$

where V is the volume of the plasma and $\mathbf{E}$ the electric field strength at some point in V. Therefore, if we know the rf conductivity, the calculation of the emission from the plasma reduces to finding $\mathbf{E}$ within the plasma. Often $\mathbf{E}$ can be written in terms of one or more plane waves, $E=E_0\exp(j\omega t - j\boldsymbol{\gamma}\cdot\mathbf{r})$, where γ the complex propagation constant, is related to the complex conductivity σ, the complex refractive index μ, and the complex dielectric coefficient K through

$$(\lambda/2\pi)\gamma = \mu = K^{\frac{1}{2}} = (1+\sigma/j\omega\epsilon_0)^{\frac{1}{2}}. \quad (32)$$

When the plasma is sufficiently large, so that multiple internal reflections can be neglected, E can be represented by one plane wave, and Eqs. (30), (31), and (32) give

$$P_\omega = (1-\Gamma)B(\omega,T)S[1-\exp(-\alpha_\omega L)]. \quad (33)$$

Here Γ is the power reflection coefficient, and α_ω is

$$\alpha_\omega = (4\pi/\lambda)\,\mathrm{Im}K^{\frac{1}{2}} = (4\pi/\lambda)\,\mathrm{Im}[1+\sigma/j\omega\epsilon_0]^{\frac{1}{2}}. \quad (34)$$

[33] M. L. Levin, Soviet Phys.—J. E. T. P. 4, 225 (1957); Doklady Akad. Nauk S.S.S.R. 102, 53 (1955).

We recognize Eq. (33) as our former geometrical optics result that was derived in Sec. II. A. However, the absorption coefficient given by Eq. (34) is derived from the dielectric coefficient (or conductivity) of the plasma rather than from elementary absorption processes. Since we know how to evaluate the conductivity for a plasma of arbitrarily large charge concentration, we need no longer limit ourselves to tenuous plasmas with small dispersions.

The conductivity of a plasma is calculated from Boltzmann's equation.[22,34] For cold electrons

$$\sigma = -(Ne^2/m)\int_0^\infty [(\nu - j\omega)/(\nu^2+\omega^2)] \times (4\pi/3)(\partial f/\partial v)v^3 dv, \quad (35)$$

where ν is the collision frequency for momentum transfer[35] between an electron and an atom or an ion, and $f(v)$ is the distribution of electron velocities, which we must assume to be Maxwellian in our present considerations of the thermal emission of noise. When ν is independent of velocity, Eq. (35) becomes

$$\sigma = (Ne^2/m)[(\nu - j\omega)/(\nu^2+\omega^2)]. \quad (36)$$

This equation is used extensively in radio-astronomy in computing the emission of radio noise.[2,13] Although it is valid only when ν is a constant, one nevertheless uses Eq. (36) by allowing ν to have an appropriate dependence on the electron temperature.

We shall now illustrate the kind of results that are obtained from the macroscopic theory by calculating the emission perpendicular to a uniform plasma slab of thickness L. Figure 20 shows a plot of $P_\omega/(1-\Gamma)B(\omega,T)$ as a function of frequency for various collision rates ν. This spectrum is almost identical with that of Fig. 5, except for the pronounced peaks near the plasma frequency $\omega_p/\omega \approx 1$. At frequencies near the plasma frequency, the medium is highly dispersive; the group velocity of the radiation tends to zero, the absorption coefficient becomes large, and the emission quickly tends to the blackbody limit. In fact, this narrow range in frequencies close to ω_p proves to be useful for the determination of the electron temperature of high-temperature fully ionized plasmas produced in the laboratory. The reason for this is the following: when $\omega > \omega_p$, the bremsstrahlung is very weak [see Eq. (12)] and is difficult to measure. Even if it can be measured (in the short lifetime of today's thermonuclear plasmas), the electron density must be known before T can be calculated. When $\omega < \omega_p$, reflections from boundaries are very large and the blackbody emission intensity is greatly reduced [see Eq. (33)]. The reflection coefficient is generally not known. However, when $\omega \approx \omega_p$, Γ is not excessively large and the temperature can be estimated from a single measurement of the noise power, $dP_\omega = kT\omega^2 d\omega/8\pi^3c^2$.

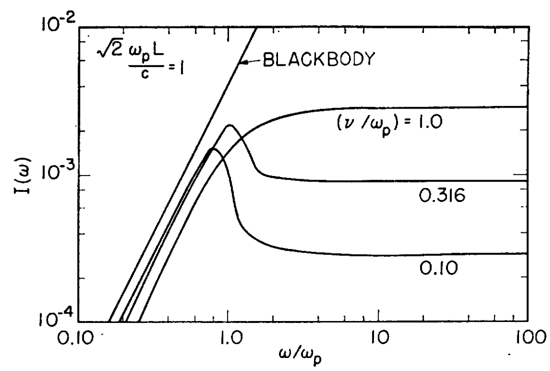

FIG. 20. Bremsstrahlung spectrum from a plasma slab as calculated from the macroscopic theory; ν is independent of the electron velocity. I_ω is in units of kT/L^2. The thickness of the plasma is $L = c/(2^{\frac{1}{2}}\omega_p)$.

Figure 21 shows the emission from the plasma slab as a function of the optical depth $\alpha_\omega L$ for various values of L and ν. Curve (a) was calculated from the geometrical-optics approximation 33; the remaining curves were calculated from the exact Eqs. (30) and (31). The oscillations in the curves are the result of the mutual interference of waves reflected internally, between the faces of the slab. The greater the length of the plasma and the greater ν/ω (the greater the absorption per unit length of path), the more closely does the exact calculation approach the geometrical-optics solution. Note that the more accurate geometrical-optics calculations given by Eq. (8) and shown in Fig. 2 do not take account of interference, and likewise do not exhibit oscillations.

[34] W. P. Allis, *Handbuch der Physik*, Vol. 21 (Springer-Verlag, Berlin, Germany, 1956), pp. 383–444.

[35] S. C. Brown, *Basic Data of Plasma Physics* (Technology Press of M. I. T. and John Wiley & Sons, Inc., New York, 1959).

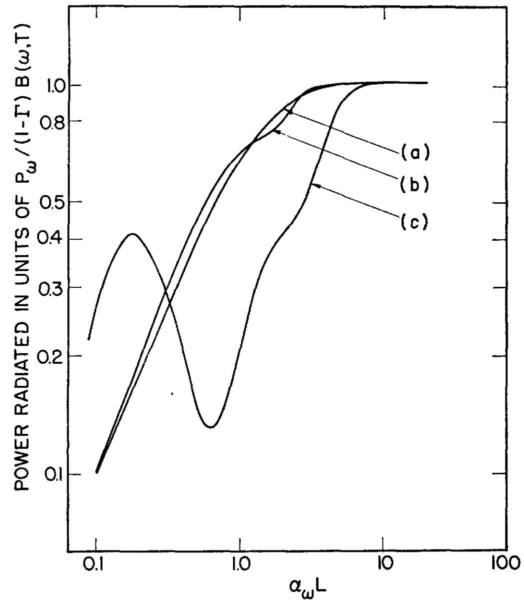

FIG. 21. Emission as a function of the optical depth: Curve (a) calculated from geometrical optics. Curves (b) and (c) calculated from Maxwell's equations. In (b) $\nu/\omega=0.3$, $L/\lambda=1$; in (c) $\nu/\omega=0.06$, $L/\lambda=0.07$.

However, the results of Eq. (8) and Fig. 2 are useful. They apply to plasmas with jagged boundaries, whose nonuniformities are greater than approximately one wavelength, which thus tends to destroy interference effects. The results of Fig. 21 (b) and (c) refer to smooth boundaries.

In the macroscopic model for the emission of noise from an isotropic plasma, the collision frequency $\nu(v)$ is the principal physical parameter that determines the magnitude and the temperature dependence of bremsstrahlung. For electron-atom encounters, $\nu(v)$ is difficult to obtain theoretically,[37] as difficult as are the bremsstrahlung calculations given by Eq. (16). However, measurements of $\nu(v)$, or of the related cross section $Q(v)=\nu(v)/N_a v$, have been made for thirty years.[35] Although there are no good measurements of $\nu(v)$ for electron-ion collisions, several calculations are available.[38,39]

The theory that has been outlined here for the case of emission from isotropic media has also been worked out partially for anisotropic plasmas acted upon by external magnetic fields. In the correlation function given by Eq. (29), the scalar conductivity is replaced by appropriate elements of the tensor conductivity.[40] Unfortunately it does not seem possible to reduce the resulting equations to a generalized Kirchhoff's law for the emitted power, as was done for isotropic plasmas [Eq. (30)]. Hence, each probelm must be evaluated by adding contributions to the emission from the individual current elements (which has not been done yet), or use must be made of the geometrical optics Eq. (10) together with absorption coefficients $\alpha_\omega^{(o,x)}$ derived from elements of the tensor conductivity (as was done in the calculations for Figs. 13 and 15).

F. Radiation from Landau Damping

Landau damping[41,42] is a mechanism whereby electrons whose thermal velocity is near the phase velocity of a plasma wave can abstract energy from it. These electrons are in a sense trapped in the potential wells of the wave. Electrons that travel somewhat slower than the phase velocity are given energy and are thus speeded up. Electrons that travel faster give up some of their energy and their speed is thus reduced. Since in the tail of a Maxwellian distribution of velocities there are more slow than fast electrons, the wave will, on an average, lose energy. By Kirchhoff's law, damping implies radiation, and the question then arises whether or not Landau damping is built into the formulation presented in Sec. II. E. The answer is no, since Landau damping is a cooperative phenomenon that takes place over characteristic distances of one or more wavelengths. A delta function correlation assumed in Eq. (29) excludes such large-scale interactions.

In Eq. (29) the assumption of a delta-function correlation implies that the ac current at a point is only a function of the ac field at that point. It is not difficult to visualize exceptions of this assumption. Consider an energetic electron orbiting in a magnetic field. The size of its orbit may be comparable with the wavelength, particularly

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[37] W. P. Allis and P. M. Morse, Z. Physik **70**, 567 (1931).
[38] V. L. Ginzburg, *Theory of the Distribution of Radio Waves in the Ionosphere* (Gostekhizdat, Moscow, 1949); J. Phys. (U. S. S. R.) **8**, 253 (1944).
[39] E. Everhardt, G. Stone, and R. J. Carbone, Phys. Rev. **99**, 1287 (1955); see also D. C. Kelly, Phys. Rev. **119**, 27 (1960).
[40] S. M. Rytov, Soviet Phys.—Doklady **1**, 555 (1956).
[41] L. Landau, J. Phys. (U. S. S. R.) **10**, 25 (1946).
[42] G. Francis, *Ionization Phenomena in Gases* (Academic Press, Inc., New York, 1960).

at a frequency equal to the cyclotron frequency, where the phase velocity of the wave is much smaller than the velocity of light in free space. Under these conditions, the current at a point becomes a function of both the ac field at that point and of its gradient. To take account of these effects, Eq. (29) is modified[43] to read

$$\langle J_\alpha(\mathbf{r}')J_\beta^*(\mathbf{r}'')\rangle = 4kT(d\omega/2\pi)(2\pi)^{-3}$$
$$\times\delta_{\alpha\beta}\int \mathrm{Re}\sigma(\omega,\boldsymbol{k})\exp[-j\boldsymbol{k}\cdot(\mathbf{r}'-\mathbf{r}'')]d\boldsymbol{k}, \quad (37)$$

where $\boldsymbol{k}$ is the propagation constant of the waves in the medium.

We note that when the radio-frequency conductivity becomes independent of the wavelength in the plasma, Eq. (37) reduces to Eq. (29). However, our solution of Boltzmann's equation, given by Eq. (35), does not depend on wavelength in the medium, and, therefore, this solution cannot exhibit phenomena like Landau damping. Wavelength-dependent terms in the expression for the conductivity are absent because we assumed for simplicity that the electric and magnetic field components vary as $\exp(j\omega t)$ instead of as $\exp(j\omega t - j\boldsymbol{k}\cdot\mathbf{r})$. This is equivalent to the assumption that the displacement of an electron, because of its thermal motion, is small compared with the wavelength in the plasma. Computations of the conductivity of an electron gas that do not make this assumption will be found elsewhere.[22,44]

Radiation from Landau damping has not been calculated or measured. In the absence of a magnetic field, Landau damping is an absorption from a purely longitudinal, electrical, plasma wave. Emission of transverse electromagnetic waves that comes about as a result of this absorption process can occur only in the presence of some mechanism that transforms the energy in a longitudinal wave into a propagating electromagnetic wave. Such mechanisms are[45–48]: gradients of electron density or temperature and density fluctuations. The efficiency of this transformation is very small and calculations point to the fact that bremsstrahlung almost always exceeds radiation from Landau damping.

In the presence of a magnetic field, Landau damping of propagating electromagnetic waves can occur,[49] because as we recall (see Fig. 12), the waves are partly longitudinal with respect to the direction of propagation. At boundaries, the transformation of these waves into purely transverse electromagnetic waves is efficient, and now, radiation from Landau damping may be significant.[50]

III. NONTHERMAL RADIATION

A great deal is understood concerning the origin of thermal emission and a sound theoretical background is available for its interpretation. The reverse is true with regard to nonthermal emission. There are several conjectures about the origin of the various forms of nonthermal emission which are observed to come from extraterrestrial plasmas and from plasmas produced in the laboratory. Measurements and most of the interpretations are being made by radioastronomers. No controlled laboratory experiments are made despite the fact that present day thermonuclear research can provide us with the plasmas we need to study this very facinating subject.

A. Radiation from Plasma Oscillations

It was mentioned in Sec. I that a certain type of nonthermal emission is characterized by large bursts of farily short duration. Figure 22 serves as an example of this type of radiation. Here are shown[51] a number of identical scans across the sun's disk taken at consecutive times, at a frequency of 1420 Mc. The large peaks from individual bright areas of the sun represent nonequilibrium emission processes; the well-marked lower envelope of the curves is the bremsstrahlung noise of the "quiet" sun. Bursts of radio noise are not confined to extraterrestrial plasmas.

[43] H. A. Haus, J. Appl. Phys. **32**, 493 (1961).
[44] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).
[45] G. B. Field, Astrophys. J. **124**, 555 (1956).
[46] R. W. Gould, Calif. Inst. Technol. Electron Tube and Microwave Laboratory, Rept. No. 4, November, 1955; see also K. G. Emeleus and A. Garscadden, Naturwissenschaften **21**, 491 (1960).
[47] D. A. Tidman, Phys. Rev. **117**, 366 (1960); E. N. Parker and D. A. Tidman, Phys. Fluids **3**, 369 (1960).
[48] J. Dawson and C. Oberman, Princeton University, Project Matterhorn, Rept. No. PM-S-39, October, 1958.
[49] J. E. Drummond, Phys. Rev. **110**, 293 (1958); see also L. Mower, Phys. Rev. **116**, 16 (1959).
[50] H. W. Wyld, Jr., Phys. Fluids **3**, 408 (1960).
[51] G. P. Kuiper, *The Sun* (University of Chicago Press, Chicago, Illinois, 1953).

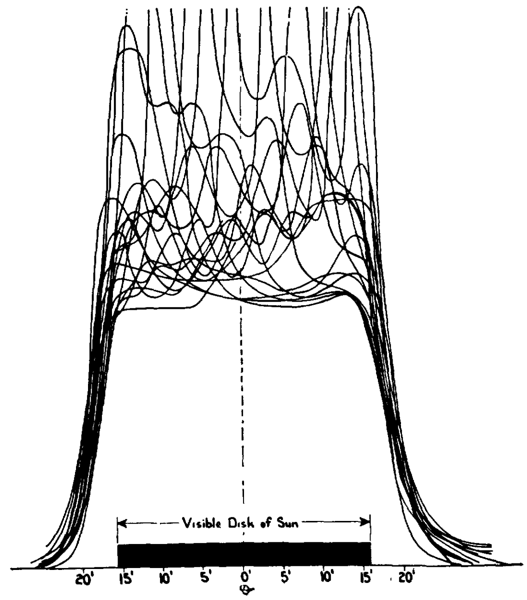

FIG. 22. Superimposed records of the emission from the sun[51] as measured at a frequency of 1420 Mc. The large peaks represent nonthermal emission. The lower envelope is the thermal emission from the "quiet" sun.

They have been also observed[52] to emanate from high-energy thermonuclear devices.

Figure 23 illustrates the frequency spectrum[51] of one type of burst observed at times of strong flare activity on the sun. The dark areas are areas of constant noise power. We note that at any one time emission occurs in two frequency bands which differ in frequency by about a factor of two. The great similarity of the spectra in the two bands suggests that the noise comes from the same source, possibly a nonlinear oscillation of a section of plasma, in which the fundamental and second harmonic frequencies have appreciable amplitudes.

It is known that longitudinal electron oscillations of a frequency equal to the plasma frequency can be excited in a plasma.[53] These longitudinal oscillations can give rise to emission of electromagnetic waves of the same frequency, (see Sec. II. F). The shift of the radiation spectrum from higher to lower frequencies with increasing time of observation (Fig. 23) suggests that the disturbance travels away from the sun (from regions of higher electron density, to regions of lower density). We can calculate the apparent speed with which the disturbance travels from the known distribution of the electron density in the corona, if we assume that the signal originates at a point in the corona where the frequency of the disturbance equals the local plasma frequency. The velocities are of the same order as the velocities of the corpuscular streams of matter from solar prominences, a fact that supports the interpretation of this phenomenon.

Quantitative interpretations are lacking be-

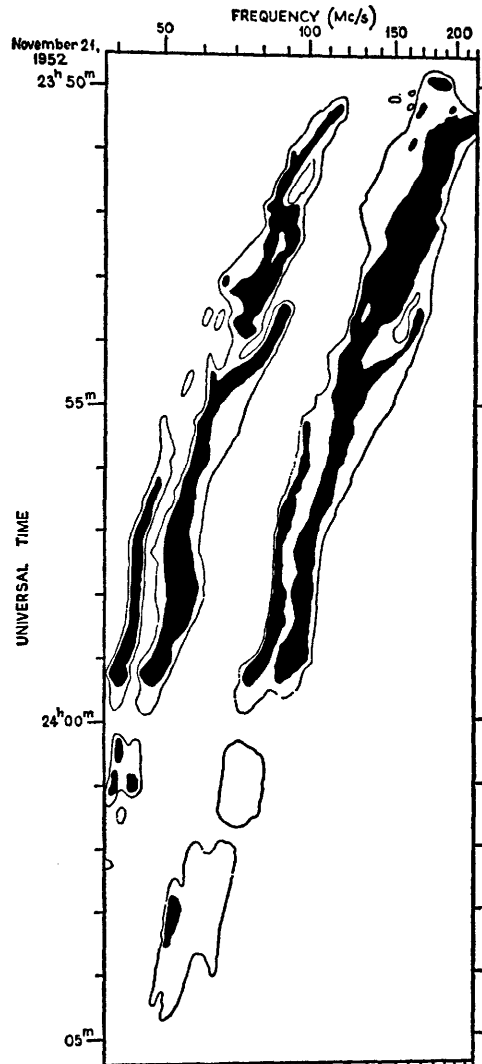

FIG. 23. Dynamic spectrum of a burst from the sun, showing contours of constant intensity.[51] (The flux shown by the dark areas is approximately 10^{-20} w/m^2 per cps.)

[52] M. A. Heald, Proc. Conf. on Controlled Thermonuclear Reactions, USAEC-Rept. T1D-7520, September, 1956, p. 202.

[53] D. H. Looney and S. C. Brown, Phys. Rev. **93**, 965 (1954).

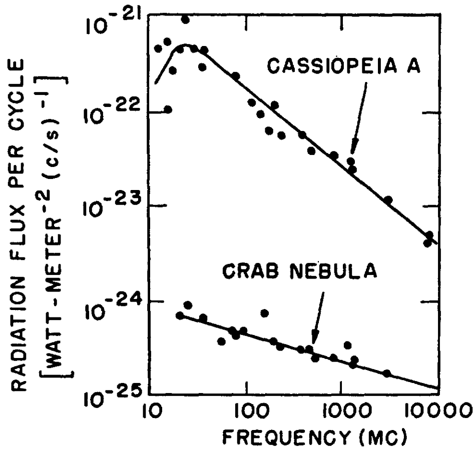

FIG. 24. Spectrum of nonthermal emission thought to originate from relativistic electrons orbiting in a magnetic field.[13]

cause: (1) the agency that causes the plasma oscillations is not known; (2) the magnitude of the energy stored in the plasma oscillation is not known; (3) the efficiency with which the longitudinal waves transform into electromagnetic waves is not known.

B. Cyclotron Emission from Electrons with Non-Maxwellian Distributions

A very widespread form of nonthermal emission, both from our and other galaxies, has a spectrum as illustrated[13] in Fig. 24. The intensity (or the flux) falls off monotonically with increasing frequency (except at very low frequencies), approximately as

$$I_\omega \propto \omega^{-x}. \tag{38}$$

The exponent x lies between 0.6 and 1.2 for most of the radio sources that have been studied.

It is thought[13] that this form of radio noise originates from cyclotron emission of highly relativistic electrons (of energy $\epsilon \gg mc^2$), orbiting in weak interstellar magnetic fields, whose intensities are of the order of 10^{-5} gauss. High electron energies are a prerequisite for this hypothesis. Cold electrons, $(v^2/c^2 \ll 1)$, radiate at their orbital frequency $\omega = \omega_b = eB/m$ (see Sec. II. C) and when $B = 10^{-5}$ gauss, $\omega/2\pi \approx 30$ cps, while the noise is observed at tens to thousands of megacycles. Likewise, emission by mildly relativistic electrons ($\epsilon \approx mc^2$) cannot explain the observed spectra. Once again the emission intensity is greatest at the fundamental frequency $\omega = \omega_b$, and falls off quickly with increasing harmonic number (see Fig. 16).

Matters are very different in the case of cyclotron emission by highly relativistic electrons, as is shown in Fig. 25. The intensity in the successive harmonics first increases with increasing harmonic number, reaches a maximum value, and then falls off rapidly. The harmonics are so closely spaced that the spectrum is almost continuous, [the separation between harmonics is $\omega_b(1-v^2/c^2)^{\frac{1}{2}}$]. Hence, in the calculation of the rate of emission by an electron, we can replace the summation over individual harmonics [Eq. (23)] by an integration with the result that[13,23]

$$\eta_\omega^{(o,x)} = (3^{\frac{1}{2}} e^2 \omega_b^2 / 64\pi^3 \epsilon_0 c) \times y\left[\int_y^\infty K_{5/3}(t)dt \mp K_{2/3}(y)\right]. \tag{39}$$

Here $y = (2/3)(\omega/\omega_b)[mc^2/\epsilon]^2$, ϵ is the total energy of an electron (rest plus kinetic), $\epsilon = mc^2(1-\beta^2)^{-\frac{1}{2}}$, and K_q is the modified Hankel function of order q. The peak in the intensity, as obtained from Eq. (39), occurs at a frequency,

$$f_{\max} = 10.7 B\epsilon^2 \text{ Mc}, \tag{40}$$

where B is the magnetic field intensity in gauss, and ϵ is the electron energy in Mev. The total intensity of radiation over all frequencies is

$$P = 6.1 \times 10^{-22} B^2 \epsilon^2 \quad \text{w per electron}. \tag{41}$$

The emission is concentrated within a very narrow cone of width $\Delta\theta \approx mc^2/\epsilon$, whose axis is oriented perpendicular to the direction of the

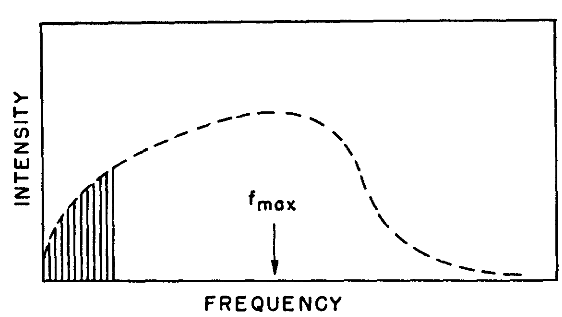

FIG. 25. Sketch of the intensity of cyclotron emission from one electron of relativistic energy ($\epsilon \gg mc^2$).

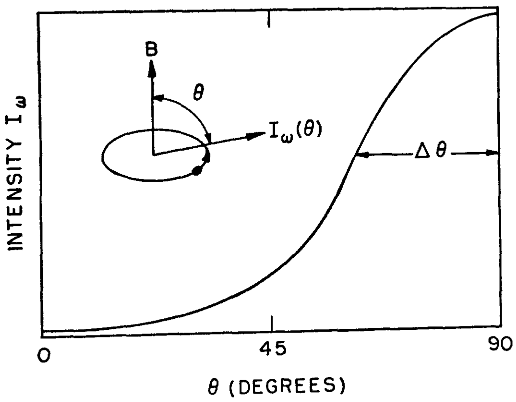

FIG. 26. Angular dependence of cyclotron emission by a relativistic electron. ($\Delta\theta \approx [1-(v/c)^2]^{\frac{1}{2}}$).

magnetic field (see Fig. 26). The intensity of the emission that has its electric vector along the magnetic field [denoted in Eq. (39) by superscript o] differs considerably from the emission that has its electric vector perpendicular to B (superscript x), and the radiation is therefore strongly polarized.

In accordance with Eq. (40), an electron of 1000 Mev energy, rotating in a magnetic field of 10^{-5} gauss, radiates its maximum energy at a frequency $f_{\max} \approx 100$ Mc, a frequency that lies well within the range of the observed spectrum. To fit the theory to the measured spectrum, a highly non-Maxwellian distribution of electron energies $f(\epsilon)$ must be postulated. The emission is obtained by summing Eq. (39) over the electron energy distribution:

$$j_\omega^{(o,x)} = 4\pi N \int \eta_\omega^{(o,x)}(\epsilon) f(\epsilon) p^2 dp, \qquad (42)$$

where p is the momentum of the electron, $p = \epsilon\beta/c$, and $f(\epsilon)$ is normalized so that $\int f(\epsilon) 4\pi p^2 dp = 1$. We assume that $f(\epsilon) = \epsilon^{-\gamma}$, where γ is an arbitrary positive number, and we substitute Eq. (39) into Eq. (42). The emission in both polarizations ($I_\omega = I_\omega^{(o)} + I_\omega^{(x)}$) is calculated to be

$$I_\omega \propto \omega^{-(\gamma-3)/2}, \qquad (43)$$

and the degree of polarization is calculated to be,

$$(I_\omega^{(o)} - I_\omega^{(x)})/(I_\omega^{(o)} + I_\omega^{(x)}) = 3(\gamma-1)/(3\gamma+1). \qquad (44)$$

Equation (43) demonstrates that cyclotron emission from energetic electrons with a non-Maxwellian distribution has a spectrum of the same form as the measured spectrum given by Eq. (38). Equating Eq. (38) to Eq. (43), we find that γ lies between 4.2 and 5.4, when x lies between 0.6 and 1.2; and Eq. (44) shows that the degree of polarization of the radiation lies between 71 and 77%.

Two further items support the hypothesis of the cyclotron emission mechanism: (1) the distribution of electron energies $f(\epsilon) \approx \epsilon^{-5}$, that we derived, agrees with the form of the energy distribution of primary cosmic particles; (2) the concentration of relativistic electrons required to explain the observed radiation intensities is of the order of 10^{-11} electrons per cc, for radiation from our galaxy, and this figure agrees with densities of primary cosmic particles. A crucial test of the theory would be the measurement of the polarization of the radiation. While the Crab nebula shows high degrees of polarization of its optical radiation, no polarization has been detected at radio frequencies, although our calculations indicate large effects. The lack of easily measurable polarization is thought to be the result of two depolarizing phenomena[13]: (1) the twisted, irregular, magnetic fields in neighboring volumes of space have different directions, and we receive the radiation from electrons orbiting about randomly oriented axes; (2) the radiation from an electron, in passing through interstellar, ionized matter, suffers Faraday rotation.

Can anything be said concerning the sharp decrease of intensity at low frequencies (see Fig. 24) observed in some radio sources? One of the suggestions is that self-absorption of the radiation on its way out of the radio source may be the cause of this effect.[23] The absorption coefficient $\alpha_\omega^{(o,x)}$ of a low-density plasma ($\omega_p/\omega < 1$) is

$$\alpha_\omega^{(o,x)} = -[32\pi^4 c^2 N/\omega^2] \int \eta_\omega^{(o,x)}(\epsilon) \times [\partial f(\epsilon)/\partial\epsilon] p^2 dp, \qquad (45)$$

where N is the electron concentration. The intensity of emission is given by our well-known formula,

$$I_\omega^{(o,x)} = B^{(o,x)}(\omega, T_r)[1-\exp(-\alpha_\omega^{(o,x)} L)]. \qquad (46)$$

Here, however, $B(\omega, T_r)$ is not the Planck formula [Eq. (5)] since this formula is valid only for a Maxwellian distribution of electron energies. When $f(\epsilon)$ is not Maxwellian, the appropriate equation, obtained by taking the ratio of Eq. (42) and Eq. (45), is

$$B^{(o,x)}(\omega,T_r)=\frac{\omega^2}{8\pi^3c^2}\times\left[-\frac{\int\eta_\omega^{(o,x)}(\epsilon)f(\epsilon)p^2dp}{\int\eta_\omega^{(o,x)}(\epsilon)[\partial f(\epsilon)/\partial\epsilon]p^2dp}\right]. \quad (47)$$

The bracketed term of Eq. (47) defines the radiation temperature T_r, which for a Maxwellian distribution becomes the electron temperature T.

The effect of self-absorption is now computed from Eqs. (45), (46), and (47), with the result that,

$$I_\omega \propto \omega^{5/2}[1-\exp(-A\omega^{-1-(\gamma/2)})], \quad (48)$$

where A is a parameter that depends on the electron density, size of the radio source, etc. We note that Eq. (48) predicts the general characteristics of the spectrum shown in Fig. 24: at low frequencies the intensity increases sharply with ω, reaches a peak, and then falls off monotonically. At high frequencies it falls off as $\omega^{-(\gamma-3)/2}$, in agreement with Eq. (43).

Whether the radiation can come into the kind of energy equilibrium assumed above may be somewhat doubtful, in view of the very low density of relativistic electrons. Electron scattering of the radiation may be the dominant process of energy loss from the ray (see Sec. II.A).

C. Amplification of Nonthermal Radiation

Although the previous interpretaion of the origin of the nonthermal component of radio noise is fairly convincing for many sources, including our own galaxy, it appears to be somewhat less convincing when applied to some very intense sources. Centaurus A has an emitting power approximately 100 times that of our galaxy; Cygnus A, 10^5 times. The theory would then require magnetic fields, electron densities, or both to be that much greater than in our interstellar space. The stored energy density tends to become uncomfortably large in these radio sources. The possibility of amplification of the radiation in its passage through the plasma has been suggested[54,55] as a means of circumventing these difficulties. Such an amplification would also have important consequences[56] on the total radiant energy loss from thermonuclear devices (see Sec. II. D). We shall discuss one amplification process, which though plausible, has not been yet confirmed experimentally.

A detailed look at self-absorption of radiation α in its passage through a medium shows that it is made up of two competing processes: the true, stimulated, absorption α_{sa} of a photon by, for instance, an electron; less the stimulated emission α_{se} in which a photon, interacting with an electron, has a certain probability of creating a second photon of the same frequency. The absorption as measured in the laboratory, or as computed from Maxwell's and Boltzmann's equations, is the sum total of these effects:

$$\alpha_\omega=\alpha_{sa}-\alpha_{se}. \quad (49)$$

The rate of production of photons by stimulated emission bears a simple relationship to the rate of production by the various spontaneous processes we discussed in Secs. II. B, C, and D. The ratio of stimulated emission to spontaneous emission, for a plasma in thermal equilibrium, is $[8\pi^3c^2/\hbar\omega^3]B(\omega,T)$. At radio frequencies this ratio is very large, and stimulated emission greatly exceeds spontaneous emission. Is it, therefore, possible for α_{se} to exceed α_{sa}, giving a negative value of α in Eq. (49), and thus causing an exponential growth of radiation, rather than an exponential decrease? This can happen in special circumstances, and what the circumstances are can be answered best with the aid of Eq. (45).

In Eq. (45), $\eta(\epsilon)$, the rate of emission by a single electron is a positive quantity, and when the distribution function $f(\epsilon)$ is Maxwellian, α_ω is always positive, and the radiation is attenuated [it can be shown that $\alpha_{se}/\alpha_{sa}=\exp(-\hbar\omega/kT)$ and hence by Eq. (49), $\alpha_\omega\geqslant 0$]. In fact, whenever the distribution function is a monotonically decreasing function of energy, the wave is attenuated, irrespective of the nature of the emission process.

[54] R. Q. Twiss, Australian J. Phys. 11, 564 (1958).
[55] G. Bekefi, J. L. Hirshfield, and S. C. Brown, Phys. Fluids 4, 173 (1961).
[56] G. Bekefi, J. L. Hirshfield, and S. C. Brown, Phys. Rev. (to be published).

However, if in some energy range there is an excess of energetic electrons compared with the population in neighboring energy ranges $(\partial f/\partial\epsilon>0)$ amplification may take place. The two conditions for negative absorption are:

$$\partial f(\epsilon)/\partial\epsilon>0,$$
$$\partial\{\eta(\epsilon)\epsilon[\epsilon^2-(mc^2)^2]^{\frac{1}{2}}\}/\partial\epsilon<0, \quad (50)$$

somewhere in the energy range ϵ.

As an example, we shall calculate the amplification of cyclotron emission from mildly relativistic electrons $\epsilon\approx mc^2$ (see Sec. II. D). We assume a distribution function of the form

$$f(\beta)\propto\beta^p\exp(-b\beta^2);\quad p\geqslant 0, \quad (51)$$

where β is v/c. When $p=0$, the distribution is Maxwellian; when $p\neq 0$, the distribution function is peaked at some electron velocity, $v\neq 0$. For a fixed mean electron energy $\bar{u}$, the spike in the distribution function becomes narrower, the larger the value of p.

Substituting Eqs. (23) and (51) into Eq. (45), we calculate the absorption coefficient of radiation emitted perpendicular to a plasma slab of thickness L, immersed in a magnetic field B oriented along the faces of the slab[56]:

$$\alpha_\omega L=-2\pi\Lambda\mu^{2-n}\frac{[(p+3)/4]^{2-n}}{[(p+1)/2]!}\cdot\sum_{n\geq\Omega}^{\infty}\frac{n^{2n-1}}{(2n+1)!}(1-Q)^{-2}X^{(2n+p-1)/2}[p-2X]\exp(-X). \quad (52)$$

Here n is the order of the harmonic, and Ω is ω/ω_b. The parameter $\mu=mc^2/\bar{u}$ specifies the mean electron energy, and the parameter $\Lambda=\omega_p{}^2L/\omega_bc$ specifies the electron concentration and the strength of the magnetic field. $X=[(p+3)/4]\times\mu[1-(\Omega/n)^2]$ is the frequency variable, and $Q=X\mu^{-1}[(p+3)/4]^{-1}$ is a parameter which is generally small compared with unity. When $X<p/2$, α_ω is negative and when $X>p/2$, it is positive. Thus, in narrow frequency ranges near the maxima of the harmonics, α_ω becomes negative (the radiation is amplified), and outside these ranges the radiation attenuates in the normal way. Figure 27 illustrates this effect for the first two harmonics. The calculations are for

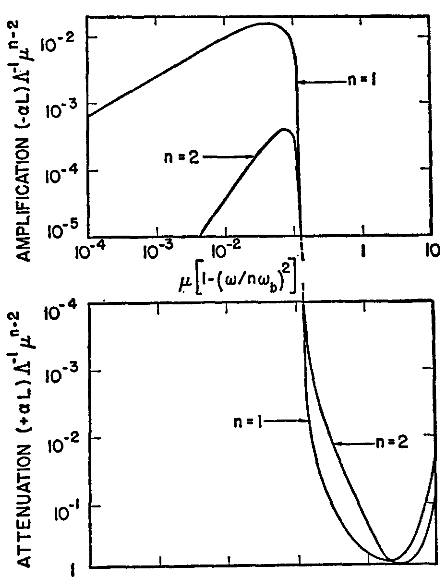

FIG. 27. The transition from a positive to a negative absorption coefficient, with varying frequency, for the first two harmonics of cyclotron emission. The plot is for a distribution function $f\propto\beta^p\exp(-b\beta^2)$, with $p=0.2$. $\mu\equiv mc^2/\bar{u}$ and $\Lambda=\omega_p{}^2L/\omega_bc$.

a distribution function with $p=0.2$. Despite the fact that this represents a relatively small perturbation of the Maxwellian distribution, the amplification of the radiation $(-\alpha_\omega L)$ is appreciable. For a magnetic field of 10^4 gauss, a mean electron energy $\bar{u}$ of 75 kev and an electron density of 2×10^{12} cm^{-3}, the peak value of $(-\alpha_\omega)$ for the first harmonic, as found from Fig. 4, is $\alpha_\omega\approx 0.11$ cm^{-1}, and the wave amplifies at a rate of 0.5 db per centimeter path length. When $\bar{u}=7.5$ kev, the amplification is 5 db/cm.

Figure 28 shows a complete view of the emission spectrum. Except for the peak of nonthermal radiation at $\omega=\omega_b$, the emission follows closely the trend of events of the Maxwellian plasma discussed in connection with Fig. 17: the radiation intensity is blackbody up to a frequency $\omega^*=4.3\omega_b$, and at frequencies greater than ω^*, the cyclotron emission leaves the plasma with negligible self-absorption.

Despite the narrowness of the emission spike at $\omega=\omega_b$ (the half-power width $\Delta\omega$ is $3\times10^{-3}\ \omega_b$), the total emission under the spike exceeds the blackbody emission summed between frequencies $\omega=0$ and $\omega=\omega^*$. Therefore, the amplification process can play an important part in the total radiation loss from thermonuclear reactors, since in these devices we have no reason to assume that the electron distribution is strictly Maxwellian. In fact, the form of the perturbation of the Maxwell distribution we have been considering $[f \propto \beta^{0.2} \exp(-b\beta^2)]$ is observed in existing mirror devices in which scattering of electrons by ions depletes the plasma of slow electrons.

Whether this process of amplification can occur also in the case of cyclotron emission from highly relativistic electrons ($\epsilon \gg mc^2$), depends very much on the energy spread $\Delta\epsilon$ of the peaked distribution function $f(\epsilon)$. The reason is that when the radiation from successive harmonics overlaps strongly (see Fig. 17 for emission at $\omega/\omega_b > 3$), the second inequality of Eq. (50) is not satisfied and amplification does not occur. The condition for no overlapping is approximately $n\Delta\epsilon/\epsilon < 1$, where n is the harmonic number. The limitation on $\Delta\epsilon$ becomes particularly stringent for highly relativistic electrons, for which the emission is most intense near the harmonic number $n_{\max} = [\epsilon/mc^2]^3$ [$n_{\max}$ is the harmonic that corresponds to $f_{\max}$ of Eq. (40)]. Assuming $\epsilon = 500$ Mev, $n_{\max} = 10^9$, and $\Delta\epsilon/\epsilon < 10^{-9}$. Hence, amplification of the extraterrestrial radiation discussed in Sec. III. B would require almost monoenergetic streams of electrons. It is not known if such streams exist in the interstellar space.

IV. MEASUREMENT OF RADIO-FREQUENCY NOISE FROM PLASMAS

The radiation from plasmas generally manifests itself to the observer as a succession of rapid, irregular fluctuations of the electromagnetic field. The radiation has a noise-like character, and is similar to the thermal and shot noise of the receivers and amplifiers that are used in the detection of this radiation. The intensity of radiation at the position of the observer is often several orders of magnitude smaller than the receiver noise, and the problem is to detect the very small change in the noise level at the output of the receiver, caused by the radiation from the plasma falling on the measuring antenna.

The signal from the antenna is amplified (by as much as 120 db), rectified, and displayed on a recording device. Suppose the signal gives a voltage reading ΔSi on the recorder. In the absence of the signal, the statistical noise fluctuations of the whole receiver cause recorder fluctuations of

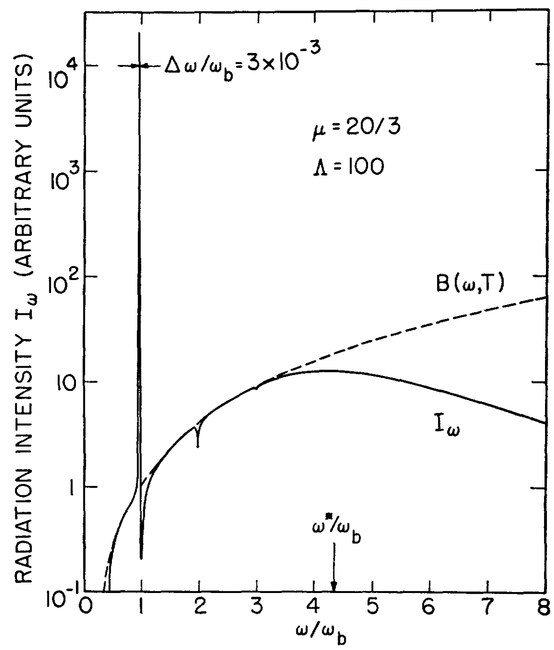

FIG. 28. The emission spectrum, with the large peak of nonthermal radiations superimposed on the blackbody continuum ($p=0.2$).

amplitude ΔR, centered about a level R. In order that the signal may be detected, we must satisfy the condition

$$\Delta Si > \Delta R. \tag{53}$$

Theoretically, the fluctuations ΔR can be reduced to arbitrarily small values by increasing the time t (the time constant of the recording device), during which the pulses of noise are analyzed. The reason for this is qualitatively as follows: the amplifier with a bandwidth $\Delta(\omega/2\pi)$ responds in amplitude and phase only to pulses of noise whose length is of the order of or greater than $2\pi/\Delta\omega$. If the recorder has a time constant t, it averages $n \approx t\Delta(\omega/2\pi)$ pulses, and the voltage fluctuations ΔR decrease as $n^{-\frac{1}{2}}$. It can be shown that now the condition for detectability of the signal is

$$\Delta Si > \Delta RK[(\Delta\omega/2\pi)t]^{-\frac{1}{2}}, \tag{54}$$

where K is a dimensionless parameter that lies between approximately 1.5 and 7, depending on the construction of the receiver.[57]

The above ideas can be put into a form that is more useful for numerical calculations. Consider

[57] P. D. Strum, Proc. Inst. Radio Engr. 46, 43 (1958).

an antenna completely surrounded by a plasma that radiates as a blackbody at a temperature T. The antenna absorbs the power $kT\Delta(\omega/2\pi)$ and delivers this power to the amplifiers. Suppose the blackbody temperature is now changed by a small amount ΔT. The minimum detectable change in T predicted by theory is[57,58]

$$\Delta T > K(F-1)T_0[(\Delta\omega/2\pi)t]^{-\frac{1}{2}}. \tag{55}$$

Here T_0 is the temperature of the receiver taken as 290°K, and F is the noise figure of the system. The parameter $(F-1)$ represents the ratio of the noise power generated in the receiver to the noise power that is generated by a pure resistance maintained at a temperature of 290°K. The value of F depends on the frequency of the radiation and on the design of the receiver.[59] For frequencies between 10 and 10^4 Mc, $F\approx 3$ to 10, and for frequencies between 10^4 and 10^5 Mc, $F\approx 10$ to 200, where the lower value of F corresponds to the lower frequency in the given range. For example, when $F=10$, $\Delta(\omega/2\pi)=10$ Mc, and $t=100$ sec, Eq. (55) predicts a minimum detectable change in the blackbody temperature of approximately 0.1°K.

The sensitivity predicted by Eq. (55) is not achieved in a practical device in which the signal falling on the antenna is merely amplified, rectified, and then displayed on a recorder with a long time constant. The reason is that we neglected an important contribution to the noise fluctuations which comes about as a result of fluctuations in the gain of the amplifiers. Even if the amplifiers are carefully stabilized against changes in the voltage, temperature, etc., there remains a residue of gain fluctuations that is the result of spontaneous changes in the circuit elements. The spectrum of the gain fluctuations is a rapidly decreasing function of frequency. Therefore, the effects of fluctuations in gain can be minimized[58] by modulating the signal from the antenna at a sufficiently high frequency (where the gain fluctuations are small), and by detecting only those signals that are in synchronism with the modulation frequency. Modulation frequencies used vary between several tens and several hundreds of cycles per second. With the use of modulation techniques, sensitivities are achieved that approach closely the sensitivity predicted by Eq. (55).

[58] R. H. Dicke, Rev. Sci. Instr. **17**, 268 (1946).
[59] C. T. McCoy, Proc. Inst. Radio Engr. **46**, 61 (1958).

Figure 29 shows[15] the principal components of a 3000-Mc receiver that incorporates the modulation technique described above. The radiation from the plasma (here the plasma is situated in a waveguide) is compared periodically with the radiation from a calibrated noise source. The radiation is mixed with a signal from a local oscillator, and the difference frequency (30 Mc) is amplified by an i.f. amplifier that has a bandwidth of 2 Mc. The output from the amplifier is rectified and amplified further by a narrow-band (1 cps) 30-cycle amplifier, tuned in phase synchronism to the switching frequency (30 cps). The output current is observed on a recorder with a 1–100-sec time constant. By adjusting the precision-calibrated attenuators for a null reading on the recorder, the measurements of the absolute noise power radiated by the plasma are rendered independent of the magnitude of the amplifier gain and of its response characteristics to the input signal (such as its linearity).

Placing the plasma directly into the waveguide structure (if this is possible) has two advantages.[15,16] Reflections from the boundaries of the plasma can be minimized by impedance-matching devices, and there is no antenna being used whose receiving characteristics must be known. When the plasma is situated outside the receiver as it is in radioastronomy or in the case of very energetic plasmas produced in the laboratory, two parameters of the antenna are required: its directionality (that is, the width of its principal lobe), and its gain,[60] G. For instance, if the plasma subtends a very much smaller angle at the position of the observer, than is the angle which the antenna lobe subtends, then the power received is

$$P_r = PG\lambda^2/4\pi R^2. \tag{56}$$

Here P is the power radiated by the plasma per unit solid angle in the direction of the observer, within the bandwidth of the receiver. R is the distance (much greater than the wavelength) between the plasma and the observer. If, on the other hand, a uniform plasma subtends an angle

[60] S. Silver, *Microwave Antenna Theory and Design* (McGraw-Hill Book Company, Inc., 1949), Radiation Laboratory Series, Vol. 12.

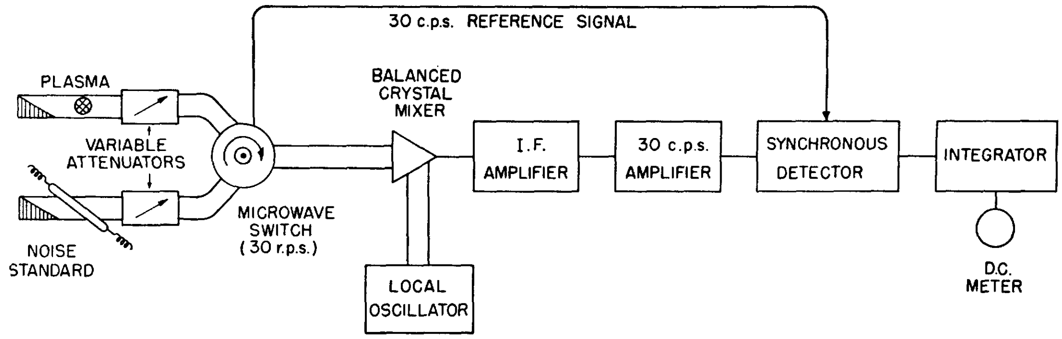

FIG. 29. Block diagram of a microwave radiometer

that is much greater than the angular width of antenna lobe, the received power is independent of the gain of the antenna, and

$$P_r = P\lambda^2/S, \tag{57}$$

where S is the projection area of the plasma onto a plane perpendicular to the direction of observation (see Fig. 19).

The great sensitivity of radio telescopes is the result of combining sensitive receivers [ΔT of Eq. (55) as low as 0.01°K has been achieved[61]] with large receiving antennas (that is, antennas with gains G between 10^3 and 10^4). At present, the large radio telescopes are capable of detecting a radiation flux per cycle bandwidth of approximately 10^{-26} w m^{-2} (cps)$^{-1}$. Taking a bandwidth of 1 Mc, the minimum measurable flux is 10^{-20} w m^{-2}, which exceeds greatly the sensitivity of the largest optical telescopes. The resolving power (the beam width of the antenna lobe) is, however, inferior to that of an optical telescope.

The techniques of detection described above are useful in the study of plasmas that remain unchanged in time for periods of at least one millisecond (or are repetitive, as in pulsed discharges), in which case integration and synchronous detection can be used. The techniques are not useful for most high-energy plasmas of thermonuclear temperatures presently produced in the laboratory. These plasmas are "one-shot" flashes of short duration. However, since they have high electron temperatures, they often emit sufficiently to permit straight amplification of the signal by means of a low-noise amplifier. Assuming that the amplifier has a noise figure $F=4$, and that the plasma radiates as a blackbody, electron temperatures in excess of several thousand degrees Kelvin are measured easily.[62]

If the sole purpose of measuring the radio-frequency noise is to determine the electron temperature T of a terrestrial plasma, a direct measurement of the noise power is not the most efficient procedure. The reason is that a determination of T from the radiation intensity requires a knowledge of absorptivity A_ω [see Eqs. (10), (13), and (30)]. The absorptivity contains all the details of the individual radiation processes: bremsstrahlung, cyclotron emission, etc. Generally, for a given plasma, A_ω is not known *a priori*, and is difficult to measure accurately. The following method[16] of obtaining T, that does not require A_ω to be known, can be adopted.

The plasma that is being studied (denoted X) is illuminated by a source of blackbody radiation (denoted S) of known variable temperature T_S. An observer views the blackbody radiation through plasma X and compares the total intensity of the radiation along this path with the intensity of the blackbody radiation traveling along a path that has not traversed the plasma. When the radiation intensity along the two paths is made the same (by adjusting T_S), then $T_X = T_S$, and plasma X absorbs as much power from S as it radiates in the direction of the observer. The reason is as follows:

The emission from S is $B(\omega, T_S)$. A fraction A_ω of this radiation is absorbed in its passage through X. Therefore, from Eq. (30), the total intensity

[61] F. D. Drake and H. I. Ewen, Proc. Inst. Radio Engr. **46**, 53 (1958).

[62] M. A. Heald, Princeton University, Project Matterhorn, Rept. MATT-17, August, 1959.

P_T of radiation along this path is

$$P_T = (1 - A_\omega)B(\omega, T_S) + A_\omega B(\omega, T_X). \quad (58)$$

The difference ΔP_ω between this intensity and the intensity of blackbody radiation along the path that does not traverse X is

$$\Delta P_\omega = A_\omega[B(\omega, T_X) - B(\omega, T_S)]. \quad (59)$$

When ΔP_ω is zero, $T_X = T_S$, irrespective of the value of A_ω.