Is the Hofstadter energy spectrum observable in far-infrared absorption?

Vidar Gudmundsson
Science Institute, University of Iceland, Dunhaga 3, IS-107 Reykjavik, Iceland.

Rolf R. Gerhardts
Max-Planck-Institut für Festkörperforschung, Heisenbergstraße 1, D-70569 Stuttgart, Federal Republic of Germany

The far-infrared absorption of a periodically modulated two-dimensional electron gas in a perpendicular constant magnetic field is calculated self-consistently within the Hartree approximation. For vanishing modulation the magnetoplasmon dispersion shows simple anticrossings with harmonics of the cyclotron resonance, as expected. For increasing modulation we identify intra- and intersubband magnetoplasmon modes charaterized by the number of flux quanta through each unit cell of the periodic potential.

The Hofstadter energy spectrum[1,2] indirectly determines the transport properties[3] of a two-dimensional electron gas (2DEG) with a square lattice modulation. Recently it has been claimed that even the direct effects of the splitting of the Landau bands into subbands has been observed.[4] Here we discuss the possibility to directly observe the subband splitting in optical measurements, for example, by far-infrared (FIR) absorption, or in Raman scattering. Since screening effects have been shown to be very important in the ground state[5] we shall treat the Coulomb interaction between the electrons on equal footing in the excited and the ground state by utilizing the time-dependent Hartree approximation. Results of such calculations have been published for isolated quantum dots[6,7] and for unidirectionally modulated 2DEG, but to our knowledge, corresponding results are not available for the square lattice modulation.[a]

The technical details of the ground state calculation have been published elsewhere.[5] The square lateral superlattice with period L is spanned by the lattice vectors $\mathbf{R} = m\mathbf{l}_1 + n\mathbf{l}_2$, where $\mathbf{l}_1 = L\hat{\mathbf{x}}$, and $\mathbf{l}_2 = L\hat{\mathbf{y}}$ are the primitive translations of the Bravais lattice $\mathcal{B}$; $n, m \in Z$. The reciprocal lattice $\mathcal{R}$ is spanned by $\mathbf{G} = G_1\mathbf{g}_1 + G_2\mathbf{g}_2$, with $\mathbf{g}_1 = 2\pi\hat{\mathbf{y}}/l_1$, $\mathbf{g}_2 = 2\pi\hat{\mathbf{x}}/l_2$, and $G_1, G_2 \in Z$. The ground-state properties of the interacting 2DEG in a perpendicular homogeneous magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ and the periodic potential

[a] The collective excitations of a 2DEG with a two-dimensional magnetic-field modulation have been studied by X. Wu and S. E. Ulloa in Phys. Rev. B **47**, 10028 (1993), but without considering interaction effects in the ground state.

$V(\mathbf{r}) = V\{\cos(g_1 x) + \cos(g_2 y)\}$ are calculated in the Hartree approximation. The Hartree single-electron states $|\alpha)$ and their energies ε_α are labelled by the quantum numbers $\{n_l, \mu, \nu\} = \alpha$, where $n_l \geq 0$ is a Landau band index, $\mu = (\theta_1 + 2\pi n_1)/p$, $\nu = (\theta_2 + 2\pi n_2)/q$, with $n_1 \in I_1 = \{0, \cdots p-1\}$, $n_2 \in I_2 = \{0, \cdots q-1\}$, $\theta_i \in [-\pi, \pi]$, and $pq \in N$ is the number of magnetic flux quanta through the lattice unit cell. The Hartree potential "felt" by each electron and caused by the total electronic charge density of the 2DEG, $-en_s(\mathbf{r})$, and the positive neutralizing background charge, $+en_b$, together with the external periodic potential $V(\mathbf{r})$ do not couple states at different points in the quasi-Brillouin zone $\theta = (\theta_1, \theta_2) \in \{[-\pi, \pi] \times [-\pi, \pi]\}$. However, states at a given point depend, in a self-consistent way, on states in the whole zone through $n_s(\mathbf{r})$. The periodic potential broadens the Landau levels into Landau bands which due to the comensurability conditions between L and the magnetic length $l = (c\hbar/eB)^{1/2}$, are split into pq subbands. The resulting energy spectrum, the Hofstadter butterfly[1,2], retains essentially it's complicated gap structure but is strongly reduced in symmetry by the electron-electron interaction.[5] The symmetry depends on the mean electron density, or, equivalently, the chemical potential.

In order to calculate the FIR absorption of the 2DEG we perturb it by a monochromatic external electric field without restricting it's dispersion relation to that of a free propagating field. The power absorption is calculated from the Joule heating of the self-consistent electric field consisting of the external and the induced field.[8] The peaks in the absorption spectra represent absorption of the collective modes of the 2DEG. Single-electron excitations are strongly damped and do not show up in the spectra.[8]

In the calculation we use GaAs parameters, $m^* = 0.067 m_0$, and $\kappa = 12.4$. The power dissipation is made possible by retaining a small but finite imaginary part of the frequency in the electron susceptibility. The absorption of a homogeneous 2DEG ($V = 0$) is compared in Fig. 1 with the first-order dispersion in $\mathbf{k}$ of a magnetoplasmon, $\omega^2 = \omega_c^2 + 2\pi e^2 n_s k/(\kappa m^*)$. Kohn's theorem manifests itself by the fact that only one peak is visible for $\mathbf{k} \to 0$.[9] For finite wave vectors $\mathbf{k}$ the magnetoplasmon interacts with harmonics of the cylcotron resonance, resulting in so the called Bernstein modes.[10] To make the Hofstadter subband structure of the Landau bands discernible, we have to resort to short periods $L = 50$ nm and strong modulation, as is seen in Fig. 2. Here we use $\mathbf{k}L = k_1 L = 0.2$, and the filling factor $\nu = 1/2$. For strong modulation and $pq = 3$ the dispersion of the magnetoplasmon is relatively flat for small values of $\mathbf{k}$. The structures in the absorption are very sensitive to the location of the chemical potential with respect to the subbands as can be confirmed by comparing the absorption for $\nu = 1/2$ in Fig. 2 with the absorption for $\nu = 5/6$ in Fig. 3. The inset in Fig. 2 shows the absorption for

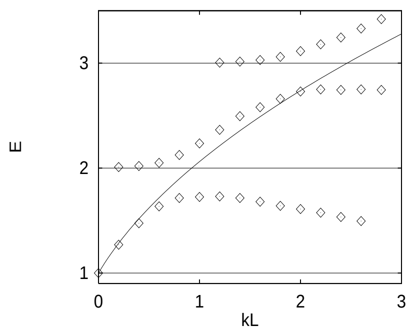

Figure 1: The dispersion of the magnetoplasmon in a homogeneous 2DEG (◇). The energy E is scaled in $\hbar\omega_c$ and the wavevector $\mathbf{k} = k_1$ is scaled with the period length $L = 200$ nm. $V = 0$, $\hbar\omega_c = 0.179$ meV, $pq = 1$, and $T = 1$ K.

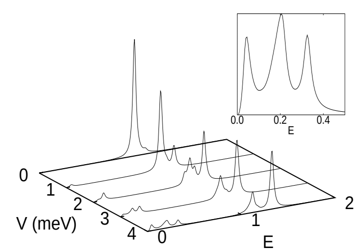

Figure 2: The absorption $P(E = \hbar\omega/\hbar\omega_c)$ in arbitrary units as a function of the modulation strength V for $\nu = 1/2$. The inset shows the intraband absorption peaks for $V = 4$ meV. $k_1L = 0.2$, $L = 50$ nm, $\hbar\omega_c = 8.57$ meV, $pq = 3$, and $T = 1$ K.

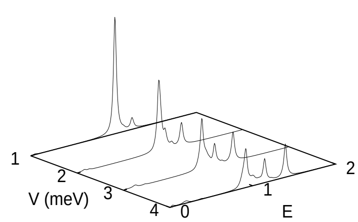

Figure 3: The absorption $P(E = \hbar\omega/\hbar\omega_c)$ in arbitrary units as a function of the modulation strength V for $\nu = 5/6$. $k_1L = 0.2$, $L = 50$ nm, $\hbar\omega_c = 8.57$ meV, $pq = 3$, and $T = 1$ K.

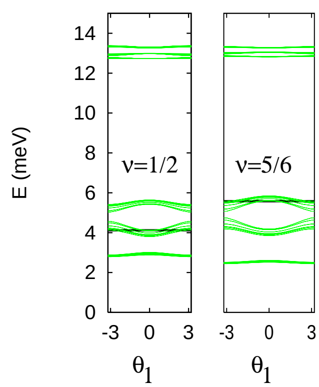

Figure 4: The Hartree dispersion of the two lowest Landau bands in the quasi-Brillouin zone (each band is split into three subbands). The chemical potential is indicated by a solid horizontal line. $L = 50$ nm, $V = 4$ meV, $\hbar\omega_c = 8.57$ meV, $pq = 3$, and $T = 1$ K.

three intra-Landau band magnetoplasmon modes. For $\nu = 1/2$ in Fig. 2 only two subbands are occupied, or partially occupied, as is illustrated in Fig. 4. We have thus *two* absorption peaks for inter-Landau band magnetoplasmons. Fig. 4 also shows that in the case of $\nu = 5/6$ we can have *three* absorption peaks for interband magnetoplasmons. The strong screening effects of partially filled subbands weakens any selections rules that would apply in the case of a noninteracting 2DEG. This is especially evident by comparing the results here to the case of $\nu = 1$, where only one absorption peak is present in the spectrum due to the very weak screening of filled Landau bands.

In summary, the FIR-absorption of a 2DEG in a short-period strongly modulated square lateral superlattice reflects the underlying Hofstadter subband structure. The structure can be seen in the absorption due to either intra- or inter-Landau band magnetoplasmons by tuning the density or the filling factor within the lowest Landau band. Due to the depolarization shift of the peaks an exact information about the Hofstadter butterfly can not be extracted from the absorption spectra. The intra-Landau band magnetoplasmon peaks in the absorption have weak oscillator strengths compared to the interband absorption and are situated in a region on the energy scale that is difficult to detect with present day FIR technology.

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References

1. R.D. Hofstadter, *Phys. Rev.* B **14**, 2239 (1976).
2. D. Pfannkuche and R. Gerhardts, *Phys. Rev.* B **46**, 12606 (1992).
3. R.R. Gerhardts *et. al.*, *Phys. Rev.* B **43**, 5192 (1991).
4. T. Schloesser *et. al.*, *Europhys. Lett.* **33**, 683 (1996).
5. V. Gudmundsson and R.R. Gerhardts, *Phys. Rev.* B **52**, 16744 (1995).
6. D. Pfannkuche and R.R. Gerhardts, *Phys. Rev.* B **44**, 13132 (1991).
7. V. Gudmundsson and R.R. Gerhardts, *Phys. Rev.* B **43**, 12098 (1991).
8. V. Gudmundsson and R.R. Gerhardts, *Phys. Rev.* B, submitted (1996).
9. W. Kohn, *Phys. Rev.* **123**, 1242 (1961).
10. I.B. Bernstein, *Phys. Rev.* **109**, 10 (1958).