Productivity Dispersion and Plant Selection in the Ready-Mix Concrete Industry

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January 20 2010

Inefficient Plants and Economic Performance

- Most studies find considerable cross sectional differences in output produced by plants that use similar inputs.
- Squeezing out this "inefficiency" would result in enormous social gains.
- In the ready-mix concrete industry there is enormous dispersion of productivity: A plant in the 90^{th} percentile of productivity produces 4 times the value added as a plant in the 10^{th} percentile (if both plants use the same bundle of inputs).

Quantitative Study of Productivity Dispersion

The quantitative study of productivity dispersion is key to understand:

1. Quantitative importance of different factors affecting firm efficiency and firm growth.
2. Slow Reallocation: which type of model can generate the type of reallocation we see in many industries following deregulation or trade liberalization.

Why do inefficient plants exist?

Argument: Inefficient producers should exit the industry and be replaced by more productive entrants.
$\Rightarrow$ Selection eliminates Dispersion.

- Exit rate for plants in the bottom 20% is twice the rate as plants in the top 20% (7% versus 3%).
- How to rationalize this tiny effect of productivity on exit?

Wrong: We need a dynamic model to think through this!

1. Frictions make reallocation costly $\Rightarrow$ entry costs and adjustment costs
2. Focus of Cross-Sectional Productivity Dispersion: but productivity bounces around over time:
   $\Rightarrow$ Temporal Productivity Dispersion

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Measurement Error or “Real” Inefficiency?

- Measurement Error could also cause this dispersion!
- How to separate measurement error from productivity dispersion?

1. I use the proxy variable models of Ackerberg, Caves and Frazer (2006) to separate measurement error from productivity differences: “true” productivity is correlated with material demand and investment choices.
2. I estimate a simple entry, exit and investment model with exogenous productivity evolution: parameters (such as the effect of productivity) are estimated to rationalize the exit, entry and investment rates.

Conclusion: Dispersion in the Data

- The median plant has value added of about \$ 550 000 dollars a year.
- OLS yields an estimated dispersion of about \$ 1 000 000 between the 10^{th} and the 90^{th} percentile, , if these two plants use the median level of inputs (in the data) of capital and labor.
- Using the methods of Ackerberg, Frazer and Caves I find that a plant in the 10^{th} percentile produces \$ 490 000 less value added than a plant in the 90^{th} percentile.

Conclusion: Dispersion from an Entry and Exit Model

- I estimate a multi-agent dynamic model of entry and exit to find the effect of low productivity on profits (using a CCP approach).
- Plants in the bottom quintile of productivity have profits which are $ 220 000 lower than plants in the top quintile.
- Produces about the substantial dispersion, somewhat less than what we observe in the data.
- Why so little effect on exit rates?
  1. High Sunk Costs.
  2. Productivity is Volatile.

Roadmap

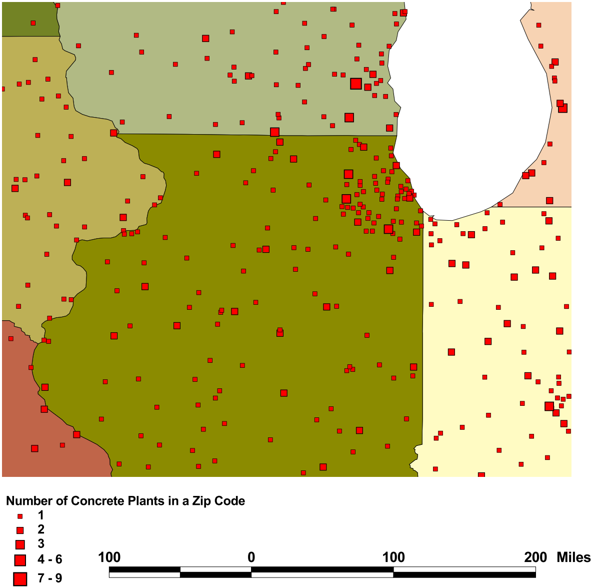
1. Industry Background.
2. Model.
3. Data.
4. Estimating Productivity Dispersion.
5. Productivity Dispersion Generated by a dynamic Entry and Exit model.
6. Counterfactuals Experiments on the role of Adjustment Costs and Productivity Volatility (In Progress).

Part I:
Industry


Wille
Brothers Co.


932
Wille
Brothers Company
OAK FOREST, IL
MONEE, IL
687-4000
USDOT 401061

Transportation Costs



Demand for Ready-Mix Concrete

- Demand is from the construction sector.
- Large Fluctuations in construction sector employment in the county: 30% per year at the county level.
- Concrete is a small fraction of construction costs: about 6%.

Sunk Costs: (Interviews of Ready-Mix Producers in Illinois)

- New Plant: 2-4 million dollars (2005).
- Plant is Sunk Inactive Plants are often left standing.
- Local Oligopoly due to transportation costs: use the county as a market.

Technological Change in Concrete

Are productivity difference caused by the presence of different plant vintages?

Survey Year	Median Employees	Median Cubic Yards Per Plant	Median Cubic Yards Per Worker	Median Cubic Yards Per Worker Hour
1963	8	15000	1900	1.4
1967	14	26000	2100	1.6
1972	15	35000	2200	1.6
1977	13	33000	2300	1.7
1982	13	25000	2000	1.4
1987	15	36000	2700	1.7
1992	13	32000	2600	1.7
1997	13	40000	3000	1.7

Part II:
Model

Ericson-Pakes Model

Firm Level State:

$$s_{it} = \{ \underbrace{\omega_{it}}_{\text{Productivity}} , \underbrace{k_{it}}_{\text{Capital Stock}} \}$$

(and the state of potential entrants $s_{it}^E = \emptyset$)
Market Level State is the collection of all firm states for N firms and Aggregate State M:

$$s_t = \{s_1, \cdots, s_N, M\}$$

Timing

1. Incumbents privately observe their scrap value $\psi + \epsilon_i$, entrants observe their entry cost $\psi^E + \epsilon_i^E$.
2. Firms simultaneously choose action entry/exit χ^E,χ and investment i.
3. Demand Evolves to its new level from $D[M'|M]$.
4. Productivity evolves according to:

$$P^\omega[\omega_i'|s_i] = \begin{cases} P^\omega[\omega_i'|\omega_i] \text{ if incumbent.} \\ P^\omega[\omega_i'|\text{out}] \text{ if entrant.} \end{cases}$$

5. Profits are realized: $\pi(s')$.

Values and Policies

Value function for entrants:

$$V^E(s) = \max_{\chi^E \in \{0,1\}} \chi^E \left(\psi^E + \epsilon_i^E + V(s) \right)$$

Value function for incumbents:

$$\begin{aligned} V(s) = & \max_{\chi \in \{0,1\}} \chi(\psi + \epsilon_i) \\ & + (1-\chi)\beta \int_{s'} [\max_i \pi(s') - c(i) + \beta V(s')] f(s'|s,i) ds' \end{aligned}$$

Entry, Exit and Investment policies

$$\begin{aligned} \chi^E(s) = & 1\left(V(s) > \phi^E + \epsilon_i^E \right) \\ \chi(s) = & 1\left(\int_{s'} [\pi(s') - c(i^*) + V(s')] f(s'|s,i^*) ds' \leq \phi + \epsilon_i \right) \\ i^*(s) = & \arg\max_i \pi(s') - c(i) + \beta \int_{s'} V(s') f(s'|s,i) ds' \end{aligned}$$

Equilibrium

A Nash Equilibrium is a set of policies χ^*, χ^{E^*}, i^* and value function V^* such that:

- Policies χ^{E^*}, χ^*, i^*, which generate the transition density $f^*(s'|s, i)$, are optimal given V^*.
- Value function V is generated by policies χ^{E^*}, χ^*, i^* and associated transition density $f^*(s'|s, i)$.

Part II:
Data

Dataset Construction

- **Longitudinal Business Database** (IRS Tax Data): Employment and Activity Data for all plants (1976-1999).
- Longitudinal Matches constructed by **Jarmin and Miranda (2002)**: Employer ID, Name and Address Matching.

- **Annual Survey of Manufacturing** (1972-1999): 30% sample of plants each year.
- **Census of Manufacturing** (1963-2000): all plants each five year. Detailed Input and Output Information (Products and Material Trailer).

Dataset Construction

	CMF	ASM	LBD
Collection	Questionnaire	Questionnaire	IRS Tax Data
Years	Every 5 years	1972-2000	1976-1999
Entry/Exit/Payroll	70% (Not-AR)	30%	All
Input and Output Data	70% (Not-AR)	30%	All

Table: Description of Census Data Sources

Imputed Data

Census Imputes data in 3 important ways (Syverson (2004)).

- **Administrative Records**: Plants with fewer than 5 employees have their data imputed (% 30 of the sample).
- **Cold Deck Imputes**: plants are given the same capital/labor ratio or shipments/labor ratio and the median plant.
- **Hot Deck Imputes**: plant data is imputed using information from another plant with similar employment.

$\Rightarrow$ I need to strip out the imputed data since it will bias:

1. Productivity Dispersion (upward for cold imputes).
2. Correlation Productivity and Exit (attenuates them).
3. Autocorrelation of Productivity (increases this).

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County Summary Statistics

	Mean	Standard Deviation	5th Percentile	95th Percentile
Total Value of Shipment (in 000's)	3181	12010	0	14000
Value Added (in 000's)	1408	5289	0	6500
Total Assets Ending (in 000's)	1090	14134	0	4700
Concrete Plants	1.86	3.24	0	6
Employment	27	79.03	0	110
Payroll (in 000's)	4238	74396	0	3600
0-5 Employee Plants	0.52	1.07	0	2
5-20 Employee Plants	0.78	1.34	0	3
more than 20 Employee Plants	0.86	1.49	0	3
Employment in Construction	1495	5390	11	6800

Note: N=24677 when imputes are counted in, 7403 when imputes tossed out.

Part IV:
Measuring Productivity Dispersion

Ackerberg, Caves and Frazer Technique

$$y_{it}(\text{value added}) = f(l_{it}, k_{it}) + \overbrace{\underbrace{\omega_{it}}_{\text{True Productivity}} + \underbrace{\epsilon_{it}}_{\text{Measurement Error}}}^{\rho_{it}:\ \text{TFP}}$$

Assume that the firm's state is:

$$s_{it} = \{k_{it}, \omega_{it}, x_{it}\}$$

In many models of industry dynamics investment is strictly increasing in ω_{it}, and thus we get:

$$\omega_{it} = h(i_{it}, k_{it}, x_{it})$$

Put this into equation above to get:

$$\begin{aligned} y_{it} &= f(l_{it}, k_{it}) + h(i_{it}, k_{it}, x_{it}) + \epsilon_{it} \\ &= \phi(i_{it}, l_{it}, k_{it}, x_{it}) + \epsilon_{it} \end{aligned}$$

Estimate the $\hat{\phi}(\cdot)$ function nonparametrically, and recover the ϵ_{it}.

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Ackerberg, Caves and Frazer Technique II

- Note that x includes the number of firms but not productivity for other firms.

Under the assumption that ω_{it} follows a first order markov process:

$$\omega_{it+1} = g(\omega_{it}, \hat{\chi}_{it}) + \xi_{it}$$

where ξ_{it} is the innovation in productivity and $\hat{\chi}_{it}$ is the exit propensity score.

Since capital in chosen in the last period and materials and labor are choosen today then we get the following orthogonality conditions:

$$\mathbf{E}\xi_{it}\begin{pmatrix} l_{it} \\ k_{it+1} \end{pmatrix} = 0$$

Estimate β_l and β_k using GMM.

Then "true productivity" can be computed as:

$$\omega_{it} = \hat{\phi}_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it}$$

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Log Value Added	Production Function Census Years	First-Stage ACF Census Years
Constant	1.148	1.021
	(0.05)	(0.07
Log Salaries	0.675	0.659
	(0.01)	(0.03)
Log Assets	0.258	0.243
	(0.01)	(0.03)
Log Investment		0.039
		(0.01)
Zero Investment		0.179
		(0.04)
1st Competitor		-0.139
		(0.06)
Multi-Unit Firm		0.162
		(0.02)
Squared and Cube		X
Observations	11097	8499
R^2 Adjusted	0.83	0.89

Table: ACF-First Stage and Production Function Estimates

Structural Estimates of Productivity

Dependent Variable: Log Value Added	Census Year
Log Salaries ($\hat{\beta}_l$)	0.682
	(0.059)
Log Assets ($\hat{\beta}_k$)	0.260
	(0.038)
Observations	8521

Table: Ackerberg, Caves and Frazer estimates of productivity.

Dispersion

Variable	Mean	Std.
Log Value Added	6.36	1.39
Predicted Output $\hat{\phi}$	6.33	1.28
TFP (ρ)	0.13	0.71
Productivity (ω)	1.06	0.46
Measurement Error (ϵ)	0.03	0.60

Dispersion

Dispersion if all plants used the same bundle of inputs, but brought their own productivity residual:

$$\hat{VA}^{\rho_q} = \exp(\beta_l l_{50} + \beta_k k_{50} + \rho_q)$$
$$\hat{VA}^{\omega_q} = \exp(\beta_l l_{50} + \beta_k k_{50} + \omega_q + \epsilon_{50})$$
$$\hat{VA}^{\epsilon_q} = \exp(\beta_l l_{50} + \beta_k k_{50} + \omega_{50} + \epsilon_q)$$

	Dispersion in thousands \$ due to		
	Productivity (ω_q)	Measurement (ϵ_q)	TFP (ρ_q)
10%	440	310	330
25%	490	440	440
50%	550	550	550
75%	630	710	790
90%	930	950	1400

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Serial Correlation of Productivity

	1 Year Lag	5 Year Lag	Exit Propensity $\hat{\chi}$
Lagged TFP ρ	0.547	0.129	
	(0.012)	(0.013)	
Lagged ϵ	0.459	0.097	
(Measurement Error)	(0.016)	(0.014)	
Lagged ω	0.581	0.383	
(True Productivity)	(0.014)	(0.018)	
Lagged ω	0.554		0.663
(True Productivity)	(0.018)		(0.291)

Table: Autocorrelation of Productivity, TFP and Measurement Error

Part V:
Dynamic Model of Entry and Exit

Dependent Variable: Jarmin-Miranda Exit	I		II	
Size				
Medium (7-17 Employees)	-4.4%	(0.2)	-5.3%	(0.8)
Large (Over 17 Employees)	-5.1%	(0.2)	-5.4%	(0.8)
Productivity				
2nd Quintile			0.2%	(1.2)
3rd Quintile			-2.2%	(1.1)
4th Quintile			-3.3%	(1.0)
5th Quintile			-3.7%	(1.0)
Log Construction Employment	-7.8%	(1.5)	-6.0%	(0.3)
1st Competitor	3.2%	(0.2)	2.0%	(0.7)
Log of more than one competitor	0.9%	(0.1)	0.5%	(0.3)
Observations	64482		4627	
χ^2	1089		109	
Log Likelihood ($\mathcal{L}$)	-12656		-826	

Table: The relationship between productivity and exit.

From		To		
		Out	Small	Large
Out		99.1%◇	0.9%	0.0%
Small^{+}	Low Productivity*	8.5%	86.2%	5.3%
	High Productivity**	3.8%	89.9%	6.3%
Large^{++}	Low Productivity	2.3%	15.2%	82.4%
	High Productivity	1.8%	13.2%	84.9%

$^{+}$ Small: Plant with fewer than 15 employees.
$^{++}$ Big: Plant with at least 15 employees.
*Low Productivity: Productivity below the median for the year.
**High Productivity: Productivity above the median for the year.

Table: Low productivity plants are less likely to grow than high productivity plants.

Solving Dynamic Game

- Discretize the State Space into 3 capital states and 5 productivity states, 10 demand states.
- Max of $N = 6$ plants per market.
- Large State Space quickly becomes an issue: this model has 2.5 million states.
- Use a discrete action version of a Stochastic Algorithm to solve the model.
- Termination Criterion: simulation based one in Fershmann-Pakes several million times faster to compute than the original termination criteria.
- Can solve this model with in 3-4 hours, the limitation here is mainly due to the storage of the state space.

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Dynamic Choice

- Action $a_i^t = k_i^{t+1}$: Pick size or exit in the next period.
- "Reduced Form" Profit Function:

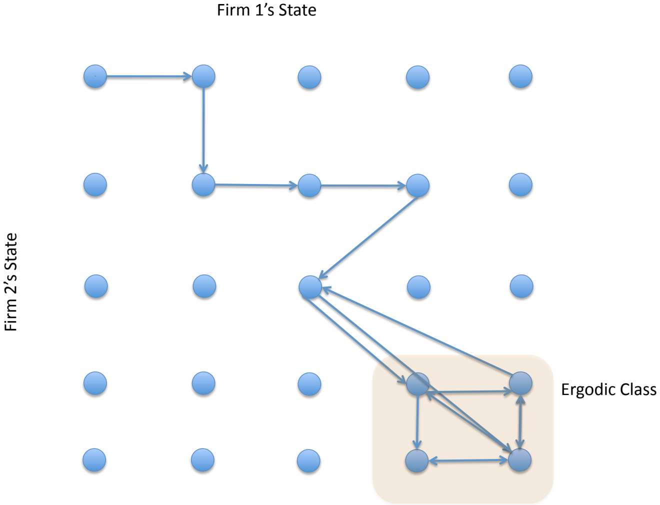
$$
\begin{aligned}
r(a^{t+1}, s^{t+1}|\theta) = \; & \textstyle\sum_{j\in A} \mathcal{I}(k_i^{t+1} = j)\theta_{1j}\text{Fixed Cost} \\
& + \textstyle\sum_{j\in A} \theta_{2j}\mathcal{I}(k_i^{t+1} = j)\omega_i^{t+1}\text{Productivity Effect} \\
& + \textstyle\sum_{j\in A} \theta_{4j}\mathcal{I}(k_i^{t+1} = j)M^{t+1}\text{(Demand)} \\
& + \theta_{5j}\log(\textstyle\sum_{-i} k_i^{t+1} \neq \emptyset)\text{(Competition)} \\
& + \epsilon_{a_{it}}
\end{aligned}
$$

where ϵ_{ait} is i.i.d. private information logit.

- Denote the observable state x_i as:

$$
s_i = \{ \underbrace{k_i, \omega_i}_{x_i\text{: observable state}}, \epsilon_{a_{it}} \}
$$

Stochastic Algorithm



Discrete Action Stochastic Algorithm

1. $W(a_i, x)$ Choice-Specific Value Function: Value of taking action a_i in observable state x (before ϵ_{it} observed).
2. The optimal strategies are just:

$$\Psi(a_i | x, W, \theta) = \frac{\exp\left(W(a_i, x)\right)}{\sum_{j \in A} \exp\left(W(j, x)\right)}$$

3. Hit Counter: $h(a, x)$ keeps track of how often you have visited the state.

Discrete Action Stochastic Algorithm

1. Draw a state next period x':

$$x'|a_i \sim D[M'|M]1(k' = a_i)\prod_{j\neq i} P^{\omega}(\omega_j'|x_j)\Psi(k_j'|x) \quad (1)$$

2. Increment the hit counter $h(a_i, x) = h(a_i, x) + 1$.

3. Compute the payoffs R of the action as:

$$\begin{aligned} R =& r(a_i, x') - \tau(a_i, x_i') \\ &+ \beta \sum_{j\in A} W(j, x')P[j|x'] + \beta E(\varepsilon|x', P) \end{aligned} \quad (2)$$

4. Update the W-function: $W'(a_i, x) = \alpha R + (1-\alpha)W(a_i, x)$, where $\alpha = \frac{1}{h(a_i,x)}$.

5. Update the policy function:

$$\Psi(a_i|x, W) = \frac{\exp\left(W(a_i, x)\right)}{\sum_{j\in A}\exp\left(W(j, x)\right)} \quad (3)$$

6. Repeat (1-6) starting in (a_i', x')

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6. Repeat (1-6) starting in (a'_i, x')

Discrete Action Stochastic Algorithm

1. Draw a state next period x':
$$x'|a_i \sim D[M'|M]1(k' = a_i) \prod_{j \neq i} P^{\omega}(\omega_j'|x_j)\Psi(k_j'|x) \tag{1}$$
2. Increment the hit counter $h(a_i, x) = h(a_i, x) + 1$.
3. Compute the payoffs R of the action as:
$$\begin{aligned} R = & r(a_i, x') - \tau(a_i, x_i') \\ & + \beta \sum_{j \in A} W(j, x')P[j|x'] + \beta E(\varepsilon|x', P) \end{aligned} \tag{2}$$
4. Update the W-function: $W'(a_i, x) = \alpha R + (1 - \alpha)W(a_i, x)$, where $\alpha = \frac{1}{h(a_i, x)}$.
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How to Find the Agent's Decision Rule (Ψ)

- Solving the entire game takes several hours: infeasible at this point.

- **Hotz and Miller**(1993) Conditional Choice Probability, extended to games by Pakes, Ostrovsky, Berry (2007), Bajari, Benkard and Levin (2006), Pesendorfer and Schmidt-Dengler (2008) to games.

Forward Simulation

- I will use a forward simulation technique to approximate the value of taking an action a in state x, i.e. $W(a,x)$, for all actions $j \in A$ in all states in the data:

$$W(a_i, x) = \mathbf{E}_{a_i^0, x_0} \sum_{t=0}^{\infty} \beta^t \left[r(a_i^t, a_{-i}^t, x) - \tau(a_i^{t+1}, x_i | \theta) + \epsilon_{a_i^t} \right]$$

$$\approx \frac{1}{K} \sum_{k=1}^{K} \sum_{t=0}^{T} \beta^t \left(\pi(x_t^k, a_t^k) + E(\varepsilon | P[\cdot | x_t^k]) \right) + \beta^T \xi$$

- Replace Ψ with $\hat{P}$, replace $D[M'|M]$ with $\hat{D}$ and P^{ω} with $\hat{P}^{\omega}$.

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Estimation Criterion

- Estimation via Maximum Likelihood (globally concave).
- Issue with Simulation Error in the W function, which makes ML inconsistent: use an Indirect Inference Criterion to get around this problem.

Indirect Inference

- Linear Probability Regression:

$$1(a_{it} = \text{small}) = Z_n \beta_{\text{small}}$$

  obtain coefficients $\hat{\beta}_{\text{small}}$ (where Z_n is previous state dummies, demand, productivity, log construction employment, competitors, etc...).
- Run this Linear Probability Regression on Simulated Data:

$$\Psi(a_i = \text{small} | W, \theta) = Z_n \beta_{\text{small}}(\theta)$$

- Find the parameter θ that makes $\beta_{\text{small,medium,large}}(\theta)$ as close as possible to $\hat{\beta}_{\text{small,medium,large}}(\theta)$:

$$\mathcal{Q}(\theta) = \left(\hat{\beta} - \tilde{\beta}(\theta))\right)' \mathbf{W} \left(\hat{\beta} - \tilde{\beta}(\theta)\right)$$

Imputed Data

- Dynamic Game: need all plants in a market to have non-imputed data, which is quite a problem for larger markets (remember that more than 70% of the data is missing).

$$x = \{M, k_i, \omega_i, k_1, \omega_1, \cdots, k_N, \omega_N\}$$

- **Incumbent Plants**: Include only plants with productivity data ω_i. Fill in ω_n using multiple imputation software.
- **Potential Entrants**: No potential entrants have a missing data problem (no ω_i for them). Draw potential entrants with a probability equal to the percentage of incumbents that have ω_i (rough way to re-weight the sample to account for missing data).

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Discount Rate	$\beta = 0.95$
Number of Firms per Market	$N = 6$
Number of Demand States	$D = 10$
Number of Productivity States	$\#\Omega = 5$
Number of Size States	$\#(a_i) = 3$
Number of Firm States	$\#(x_i) = 16$
Number of Encoded States	$\#S^e = 2.5$ million
Entry Cost of a Large Firm	$\tau(\text{large}, \emptyset) = 2M$

Table: Baseline Parameters for the Dynamic Model of Entry/Exit and Productivity.

		ML		II
Fixed Costs	Small†	-338	($51^{\diamond}$)	-340
	Medium††	-476	(60)	-467
	Large†††	-398	(69)	-388
Log Construction	Small	4	(4)	-15
Employment (M)	Medium	48	(9)	30
	Large	75	(11)	58
Productivity (ω)	Small	180	(47)	207
	Medium	225	(54)	242
	Large	100	(62)	116
Log Competitors	Small	10	(4)	36
($\log(N)$)	Medium	-57	(9)	-29
	Large	-49	(13)	-22
Adjustment Costs	Out $\rightarrow$ Small	-1392	(40)	-1392
$\tau(a_i, x)$	Out $\rightarrow$ Medium	-1977	(67)	-1976
	Out $\rightarrow$ Large	-2000	(78)	-2000
	Small $\rightarrow$ Medium	-594	(46)	-607
	Small $\rightarrow$ Large	-919	(59)	-953
	Medium $\rightarrow$ Small	-57	(46)	-34
	Medium $\rightarrow$ Large	-312	(58)	-333
	Large $\rightarrow$ Small	-281	(48)	-231
	Large $\rightarrow$ Medium	-416	(59)	-386
Variance of ϵ		195		193

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Productivity Effect

- How big is the productivity effect: $\omega = 0.68$ for the lowest quintile and $\omega = 1.58$ for the top quintile.

- So a change from the bottom to the top quintile raises profits by \$220,000 for a medium firm.

- Why is the effect of productivity so much larger than competition or demand? (versus the static exit regression) Let's look at NPV regressions.

Productivity Persistence

Dependent Variable:	NPV Activity Years	NPV Productivity	NPV Log Construction Employmen
	I	II	IV
Medium	0.82		
(7-17 employees)	(0.06)		
Large	1.09		
(18 + employees)	(0.07)		
Productivity (ω)		1.77	
		(0.10)	
Log Construction			6.35
Employment			(0.06)
Constant	10.77	10.5	16.25

Counterfactuals IN PROGRESS

The goal of this paper is not just the measurement of productivity dispersion and dynamics, but also understanding what sustains it.

Two counterfactuals:

1. Reduce the volatility of productivity by 50%. Replace the productivity transition matrix $\hat{P}^{\omega}$ by:

$$\underline{P}^{\omega} = \frac{1}{2}\hat{P}^{\omega} + \frac{1}{2}I$$

2. Reduce Entry and Adjustment Costs by 20%.

Conclusion

- Measurement error is responsible for a large fraction of productivity dispersion.
- Yet a plant in the 90^{th} has 2 times the value added as a plant in the 10^{th} percentile in this industry.
- A dynamic entry and exit model shows that the value added of a 90^{th} percentile plant must be 1.5 times those of a 10^{th} percentile plant to rationalize entry, exit and size choices.
- A industry dynamic perspective implies large dispersion of productivity in many industries: large enough to rationalize observed productivity dispersion.
- Sunk Costs increase the dispersion in the industry.
- Unpredictable productivity also increase dispersion.

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Measuring Productivity I: P's and Q's

Weak Correlation between these:

Productivity Residuals	VA	Revenue	Quantity
Value Added	1		
Shipments	0.88	1	
Cubic Yards of Concrete	0.25	0.18	1

But entry and exit is governed by profitability per unit of input not productivity per unit of input: I use sales based measures of productivity.

Productivity and Age