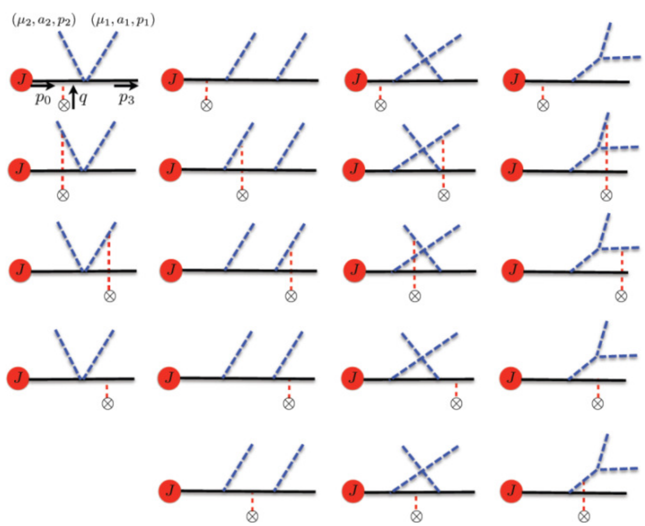
Angular distributions of higher order splitting functions in vacuum and in dense QCD matter

Michael Fickinger,

Johannes-Gutenberg Universität, Mainz



In collaboration with
Grigory Ovanesyan
and Ivan Vitev
arXiv:1304.3497

Theory Seminar, DESY Zeuthen, June 13th 2013

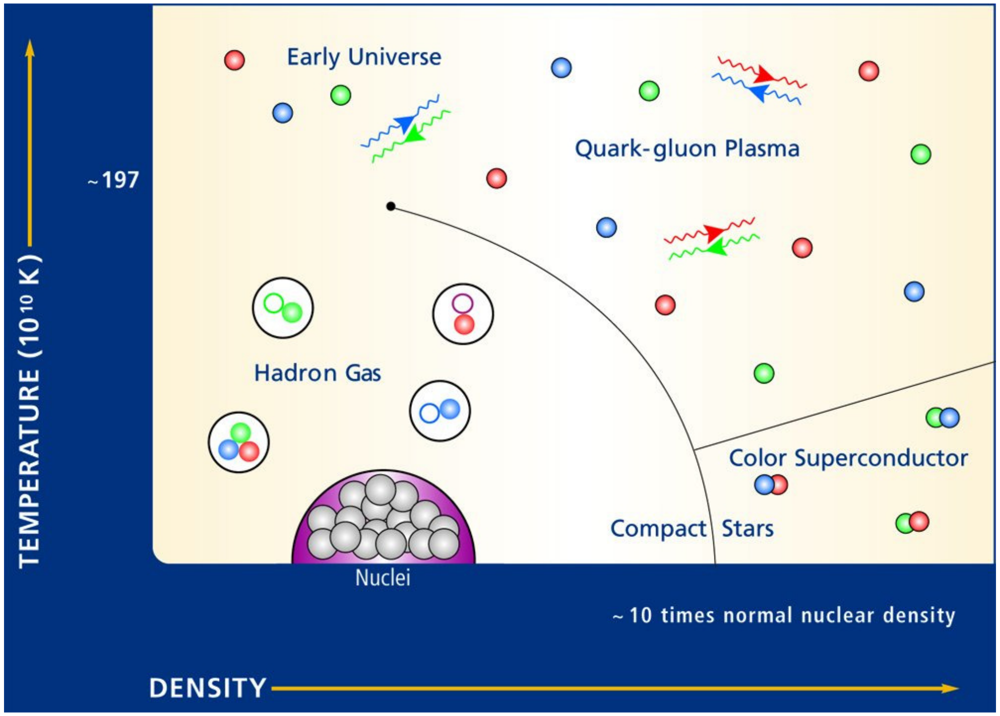
Outline

- Motivation
- Angular Distribution in Vacuum
- Angular Distribution in Medium
- Conclusion

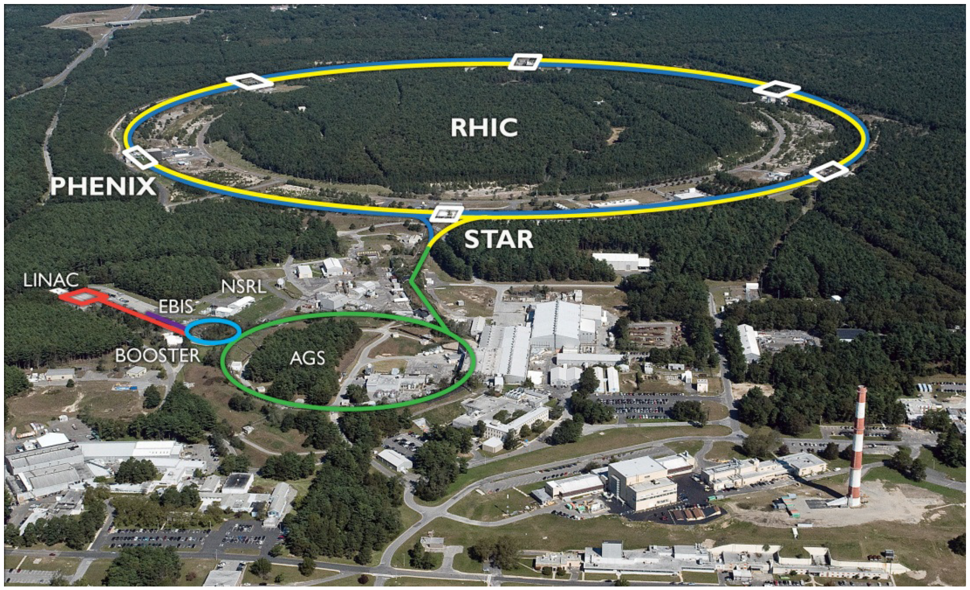
MOTIVATION

Quark Gluon Plasma (QGP)

- phase of QCD, where quarks and gluons are deconfined
- connection to early universe
  (existed a few microseconds after the Big Bang)



Heavy-Ion Experiments



RHIC:

started 2000

p+p and Au+Au collisions

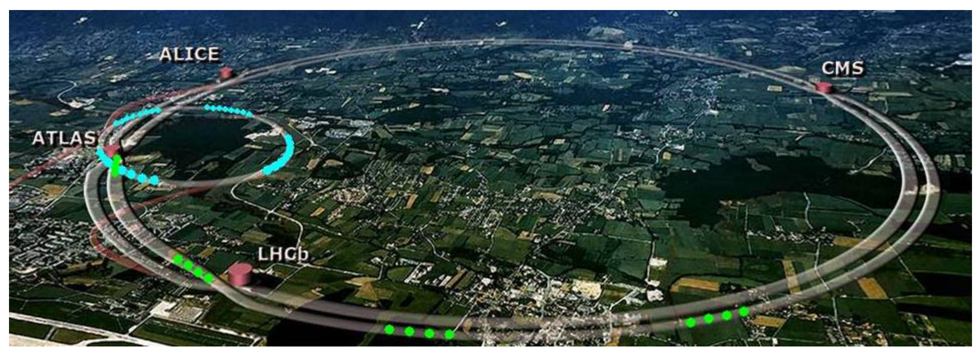
at E_{NN} = 200 GeV

LHC:

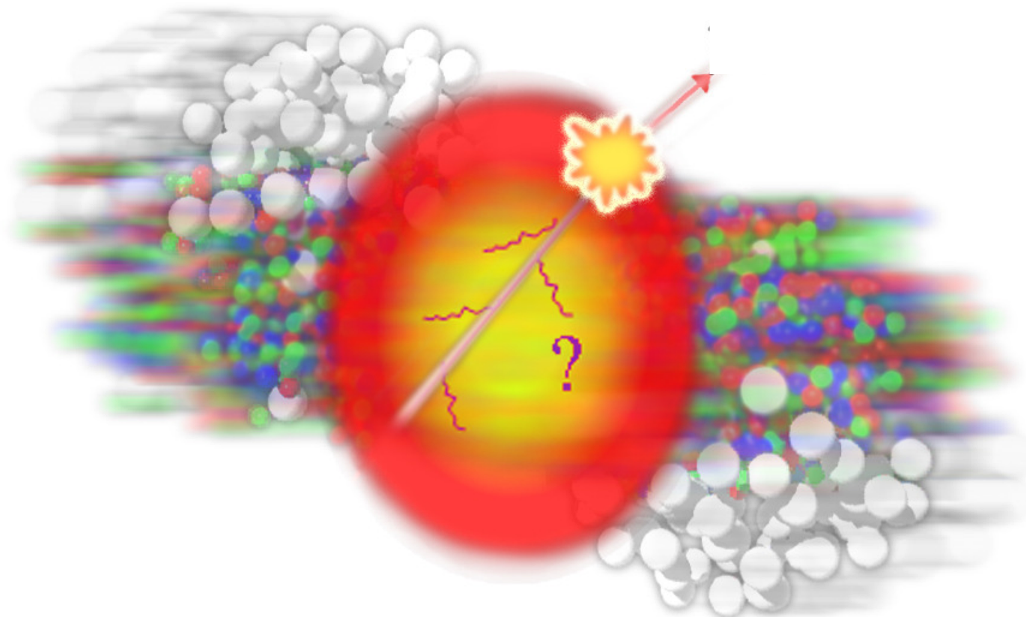
started 2009

p+p and Pb+Pb collisions

at E_{NN} = 2.76 TeV (5.5 TeV)



Jets in Heavy Ion Collisions



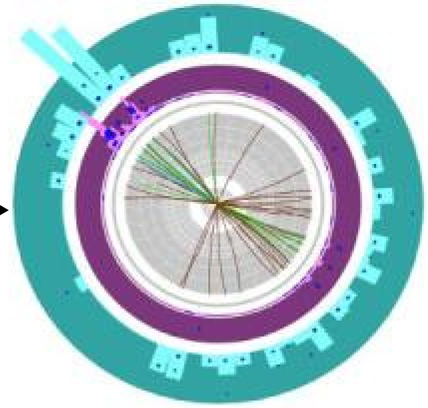
- short lifetime ~ 2-10 fm/c
- jets can be used to study the properties of QGP
- main effects of the medium on the jet:
  jet broadening,
  radiative energy loss
- kinematics:
  dijet events with high p_T jet (RHIC/LHC: p_T~O(10)/O(100) GeV) → medium modifies parton shower

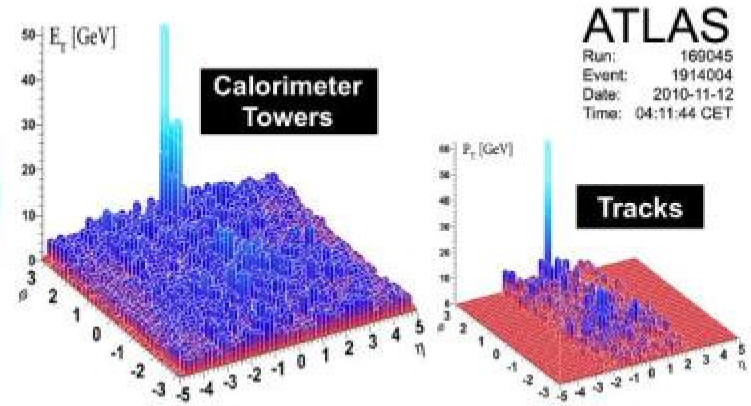
LHC data

one jet disappears →

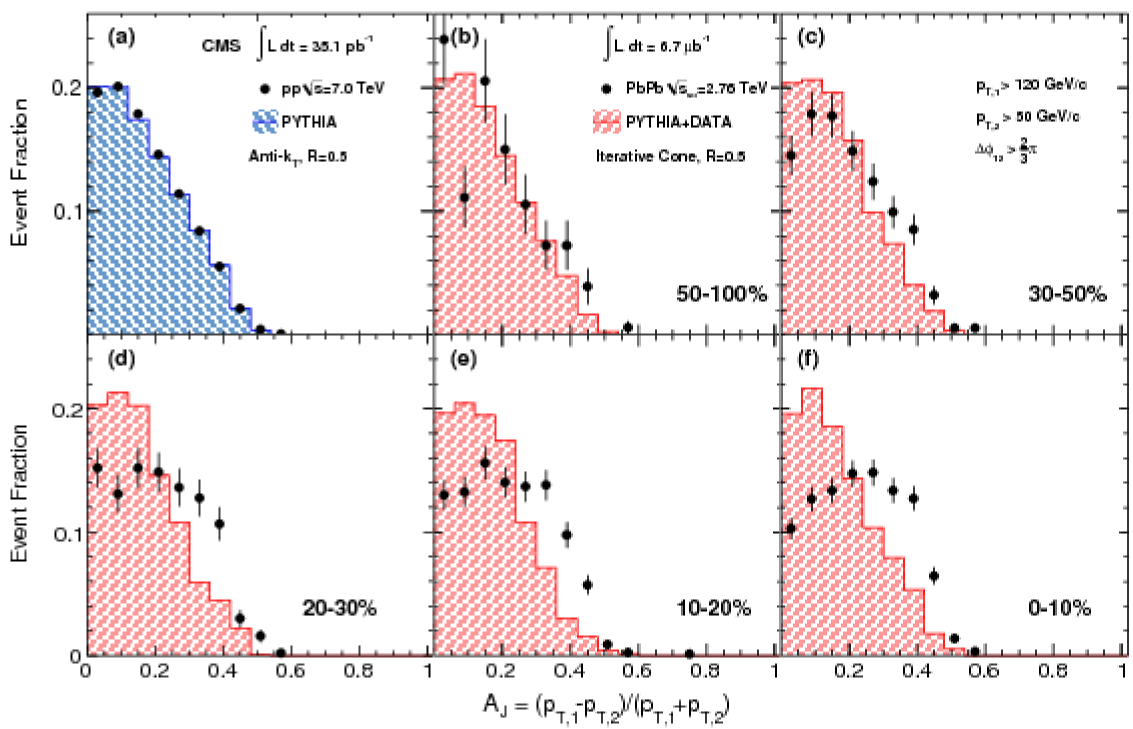
large dijet assymetry
for central collisions!

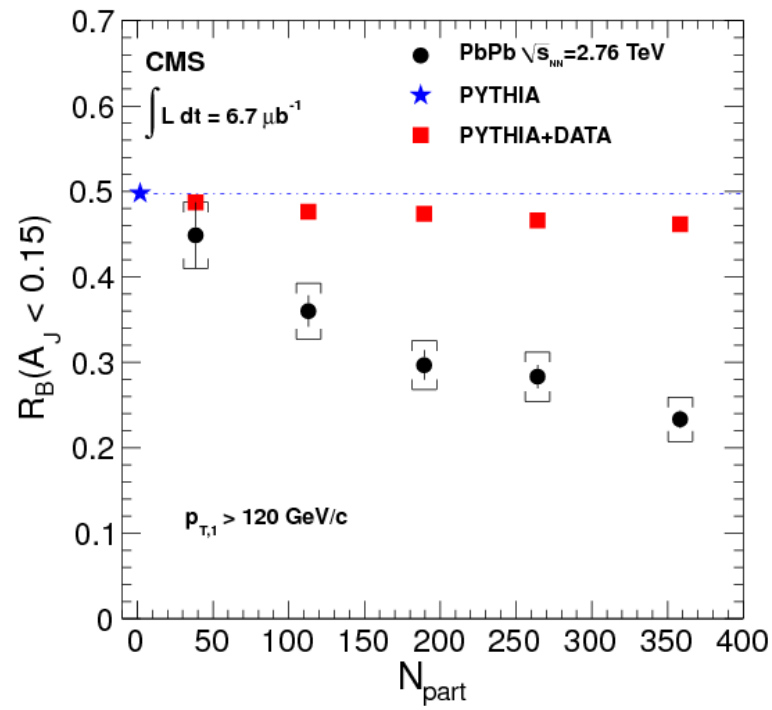
ATLAS





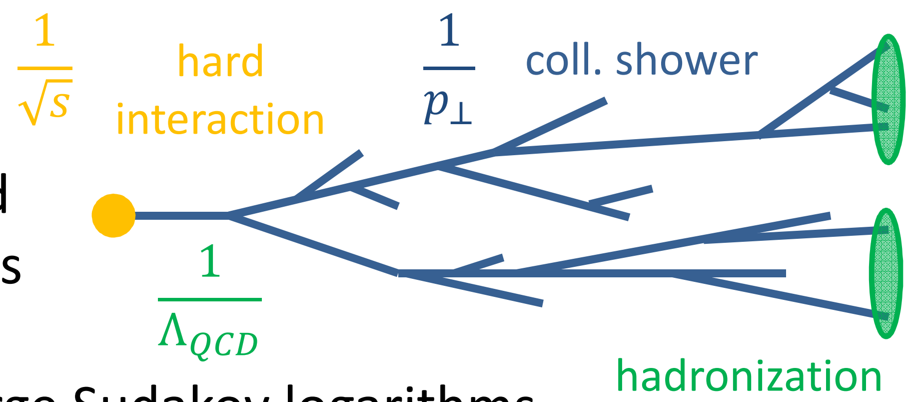
CMS





Parton Showers

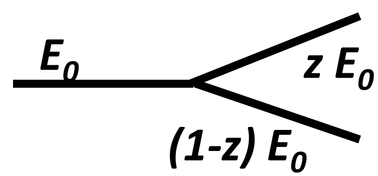
A parton shower is a Monte Carlo generator which allows to calculate exclusive cross sections .



- adds **collinear splittings** and hadronization to calculations of hard processes, includes resummation of large Sudakov logarithms

- uses probabilistic picture of **Altarelli-Parisi splitting functions** to generate **collinear radiation**

$$\mathrm{d}\sigma = \Delta(\tau)P(\tau, z)$$

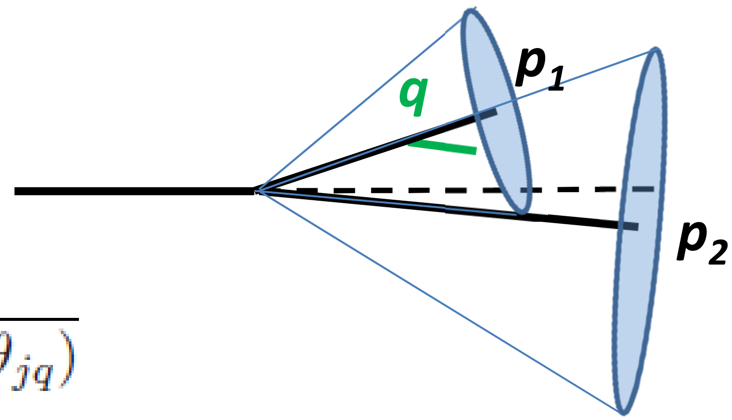


- incorporates coherent branching into **splitting functions**

Marchesini, Webber

Coherent Branching

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{i,j} C_{ij} W_{ij}$$



the antenna function

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q \, p_j \cdot q} = \frac{1 - \cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{jq})}$$

can be split $W_{ij} = W_{ij}^{[i]} + W_{ij}^{[j]}$

and integrating around the axis p_i leads to **angular ordering**

$$\int \frac{\mathrm{d}\phi_{iq}}{2\pi} W_{ij}^{[i]} = \frac{1}{1-\cos\theta_{iq}} \Theta(\theta_{ij} - \theta_{iq})$$

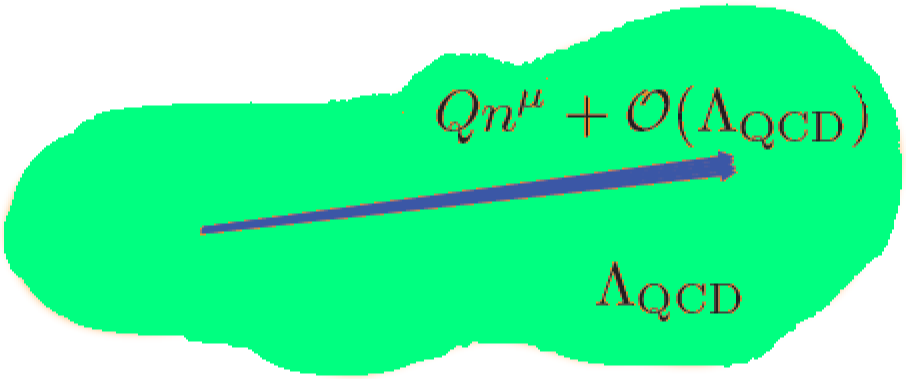
Note: **angular ordering** is an effect due to soft physics but is usually incorporated into the collinear splitting functions

Soft Collinear Effective Theory (SCET)

Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart

- Describes light-like particles (collinear) interacting with a low energetic background (soft)

- describes same kinematic regime as parton shower



- Expansion in $\lambda \approx \sqrt{\frac{\Lambda_{QCD}}{Q}}$

- Power counting:

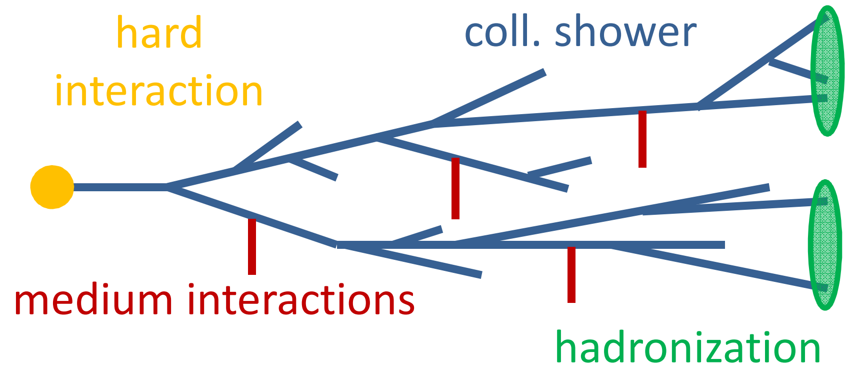
  soft: $p_\mu = (p_+, p_-, p_\perp) \propto Q(\lambda^2, \lambda^2, \lambda^2)$

  collinear: $p_\mu = (p_+, p_-, p_\perp) \propto Q(\lambda^2, 1, \lambda)$

Parton Shower in Medium

very energetic jets compared to average energy of the medium

- hard interaction is not altered by medium
- jet hadronizes outside the medium



- interactions with the medium effect the perpendicular momentum component of the collinear partons but not the longitudinal one
- few interactions with the medium

Goal

this work:

- study angular distributions of collinear radiation in vacuum
- calculate 3-splitting function in medium
- investigate if collinear radiation in medium exhibits angular ordering/anti-ordering

longterm:

- extend Monte Carlo event generator to be applicable to parton showers in medium

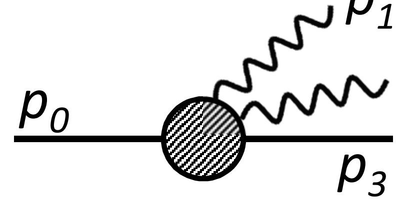
ANGULAR DISTRIBUTION IN VACUUM

Calculation in Vacuum

we want to study angular distributions of collinear splittings

three parton splitting functions

$$
\begin{aligned}
&q \to \bar{q}'_1 + q'_2 + q_3 \ , \quad (\bar{q} \to \bar{q}'_1 + q'_2 + \bar{q}_3) \ , \\
&q \to \bar{q}_1 + q_2 + q_3 \ , \quad (\bar{q} \to \bar{q}_1 + q_2 + \bar{q}_3) \ , \\
&q \to g_1 + g_2 + q_3 \ , \quad (\bar{q} \to g_1 + g_2 + \bar{q}_3) \ , \\
&g \to g_1 + q_2 + \bar{q}_3 \ , \\
&g \to g_1 + g_2 + g_3 \ .
\end{aligned}
$$

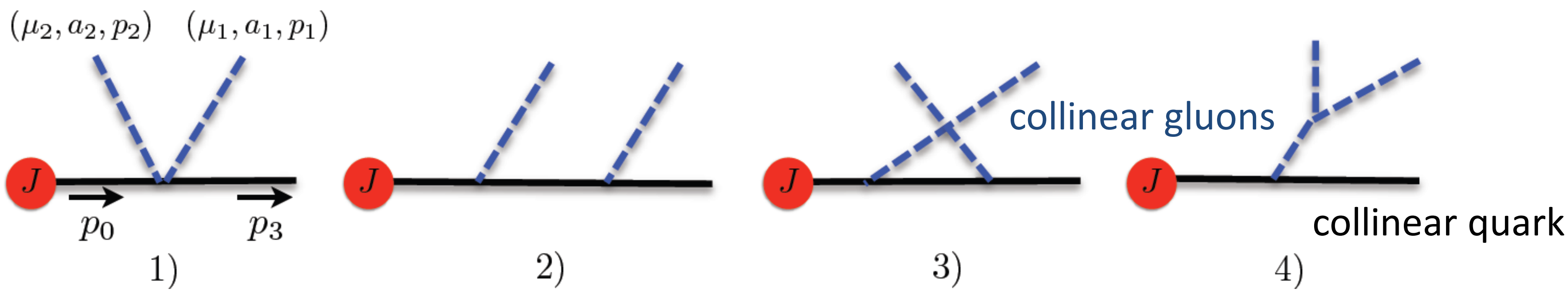


spin averaged matrix elements $\sum \left| \mathcal{M}^{(0)}_{n+2} \right|^2 = \frac{4g^4}{s^2_{123}} \langle \hat{P}_{q\to ggq} \rangle \sum \left| \mathcal{M}^{(0)}_{n} \right|^2$

where

$$s_{ij} \equiv (p_i + p_j)^2 \qquad z_i = \bar{n}\cdot p_i / \bar{n}\cdot(p_1 + p_2 + p_3) = \bar{n}\cdot p_i / \bar{n}\cdot p_0$$

our calculation in SCET



reproduces well know QCD result (for ϵ = 0):

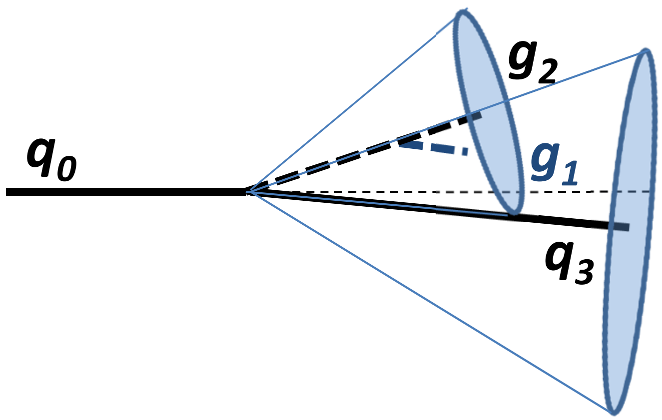
Catani, Grazzini

$$\langle \hat{P}_{g_1 g_2 q_3} \rangle = C_F^2 \, \langle \hat{P}^{(\mathrm{ab})}_{g_1 g_2 q_3} \rangle + C_F C_A \, \langle \hat{P}^{(\mathrm{nab})}_{g_1 g_2 q_3} \rangle$$

$$\langle \hat{P}^{(\mathrm{ab})}_{g_1 g_2 q_3} \rangle = \frac{s_{123}^2}{2 s_{13} s_{23}} \frac{z_3(1+z_3^2)}{z_1 z_2} + \frac{s_{123}}{s_{13}} \frac{z_3(1-z_1)+(1-z_2)^3}{z_1 z_2} - \frac{s_{23}}{s_{13}} + (1 \leftrightarrow 2)$$

$$\langle \hat{P}^{(\mathrm{nab})}_{g_1 g_2 q_3} \rangle = \frac{[2(z_1 s_{23} - z_2 s_{13}) + (z_1 - z_2) s_{12}]^2}{4(z_1+z_2)^2 s_{12}^2} + \frac{1}{4} + \frac{s_{123}^2}{2 s_{12} s_{13}} \left(\frac{1+z_3^2}{z_2} + \frac{1+(1-z_2)^2}{1-z_3} \right) - \frac{s_{123}^2}{4 s_{13} s_{23}} \frac{z_3(1+z_3^2)}{z_1 z_2} + \frac{s_{123}}{2 s_{12}} \left(\frac{z_1(2-2z_1+z_1^2) - z_2(6-6z_2+z_2^2)}{z_2(1-z_3)} \right) + \frac{s_{123}}{2 s_{13}} \left(\frac{(1-z_2)^3 + z_3^2 - z_2}{z_2(1-z_3)} - \frac{z_3(1-z_1)+(1-z_2)^3}{z_1 z_2} \right) + (1 \leftrightarrow 2).$$

Coherent Branching in Vacuum



$$z_1 \ll z_2, z_3$$

needed to define the order of the splittings

define angular anti-ordering
$$\int \frac{\mathrm{d}\phi_{iq}}{2\pi} X_{ij} = \frac{1}{1-\cos\theta_{iq}} \Theta(\theta_{iq} - \theta_{ij})$$

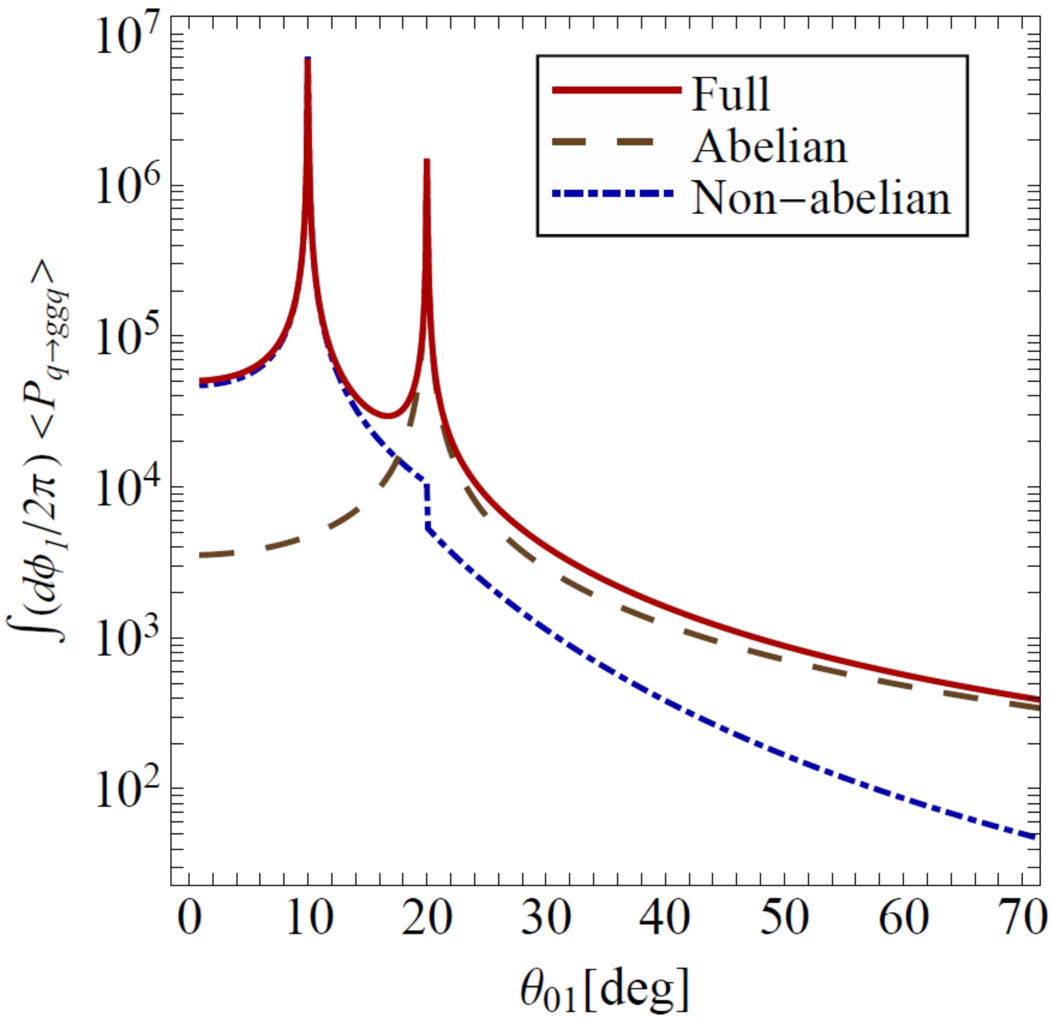
$$\langle P_{q_0 \to g_1 g_2 q_3} \rangle = \frac{4C_F(1-c_{23})}{z_1^2} z_2(1-z_2) \frac{1-z_2+z_2^2/2}{z_2} \left(C_F \left(W_{23}^{[3]} + X_{23} \right) + C_A \left(W_{23}^{[2]} \right) \right)$$

If gluon 1 is radiated at a large angle it sees only the charge of the parent quark since it cannot resolve the daughter partons

→ the nonabilian part is angular ordered, the abilian part is not

Angular Distribution in Vacuum

Splitting function q→ggq



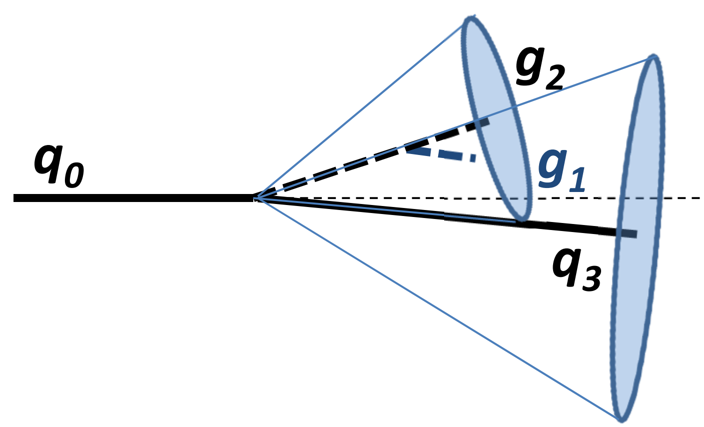
$E_0 = 100$ GeV

$z_1 = 0.03$

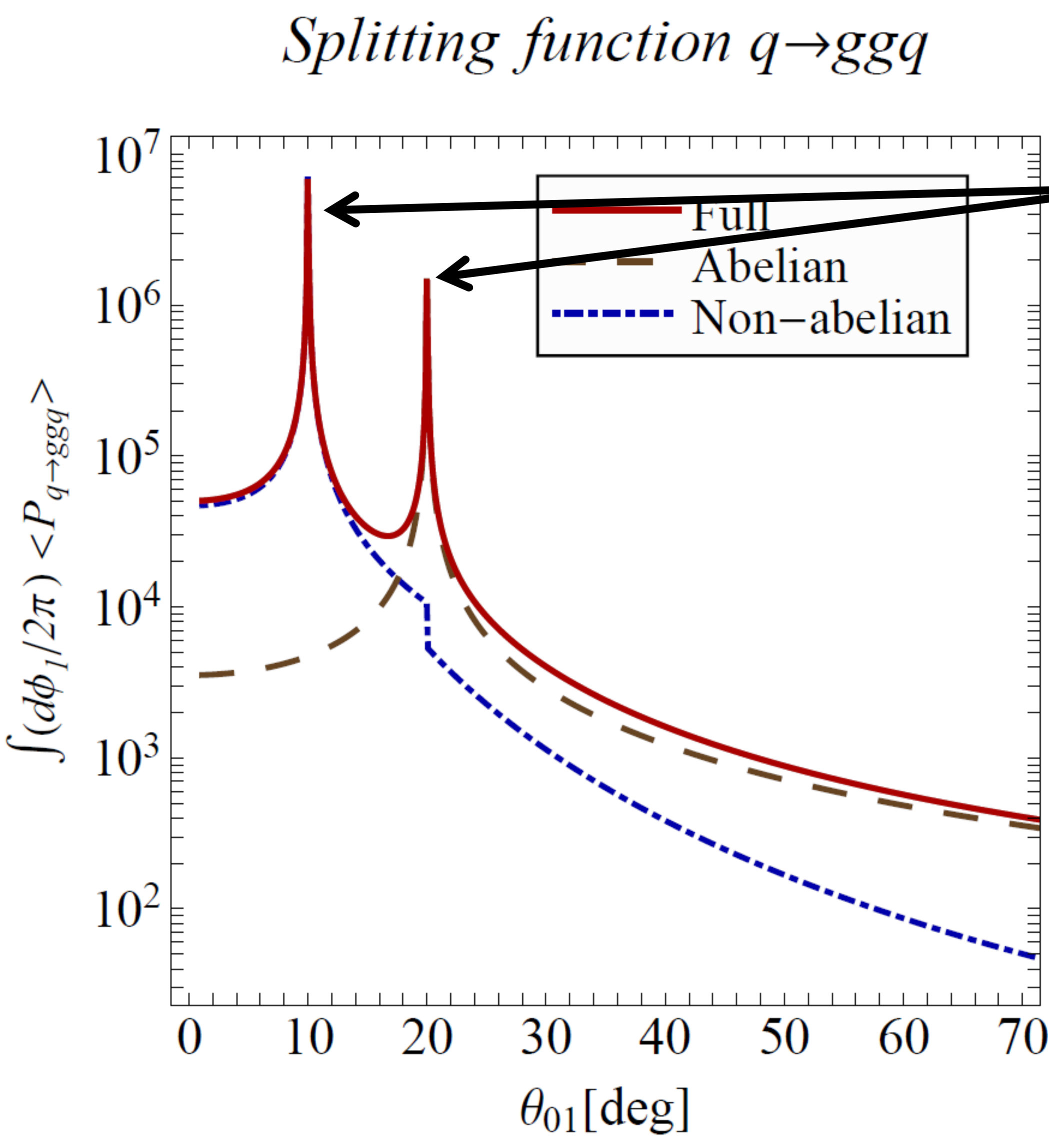
$z_2 = 0.643$

$\theta_{02} = 10°$

$\theta_{03} = 20°$



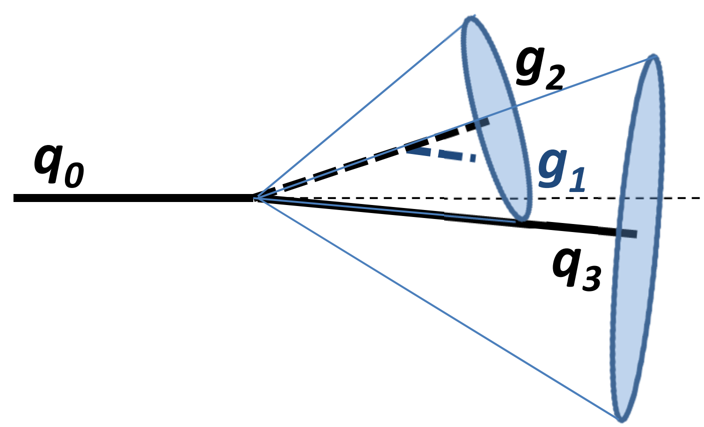
Angular Distribution in Vacuum



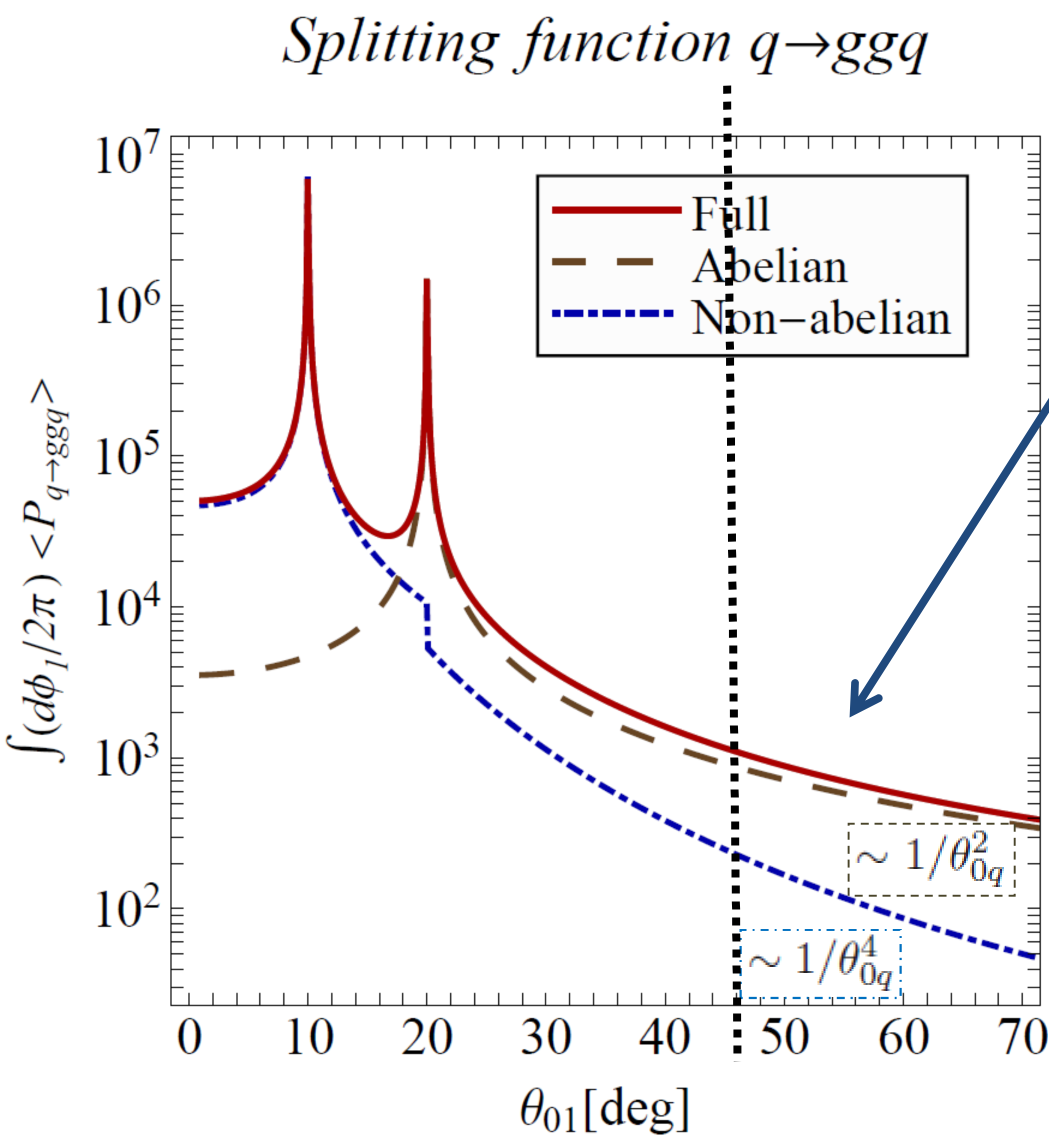
- coll. enhancement

  $\theta_{01} \sim \theta_{02} = 10°$

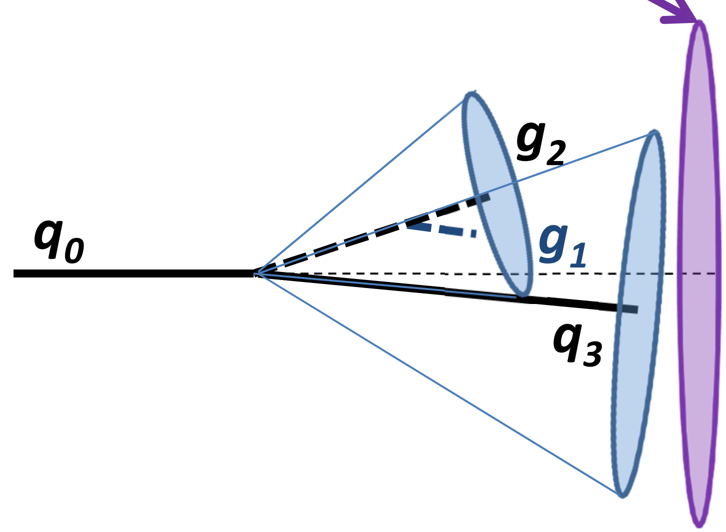
  $\theta_{01} \sim \theta_{03} = 20°$



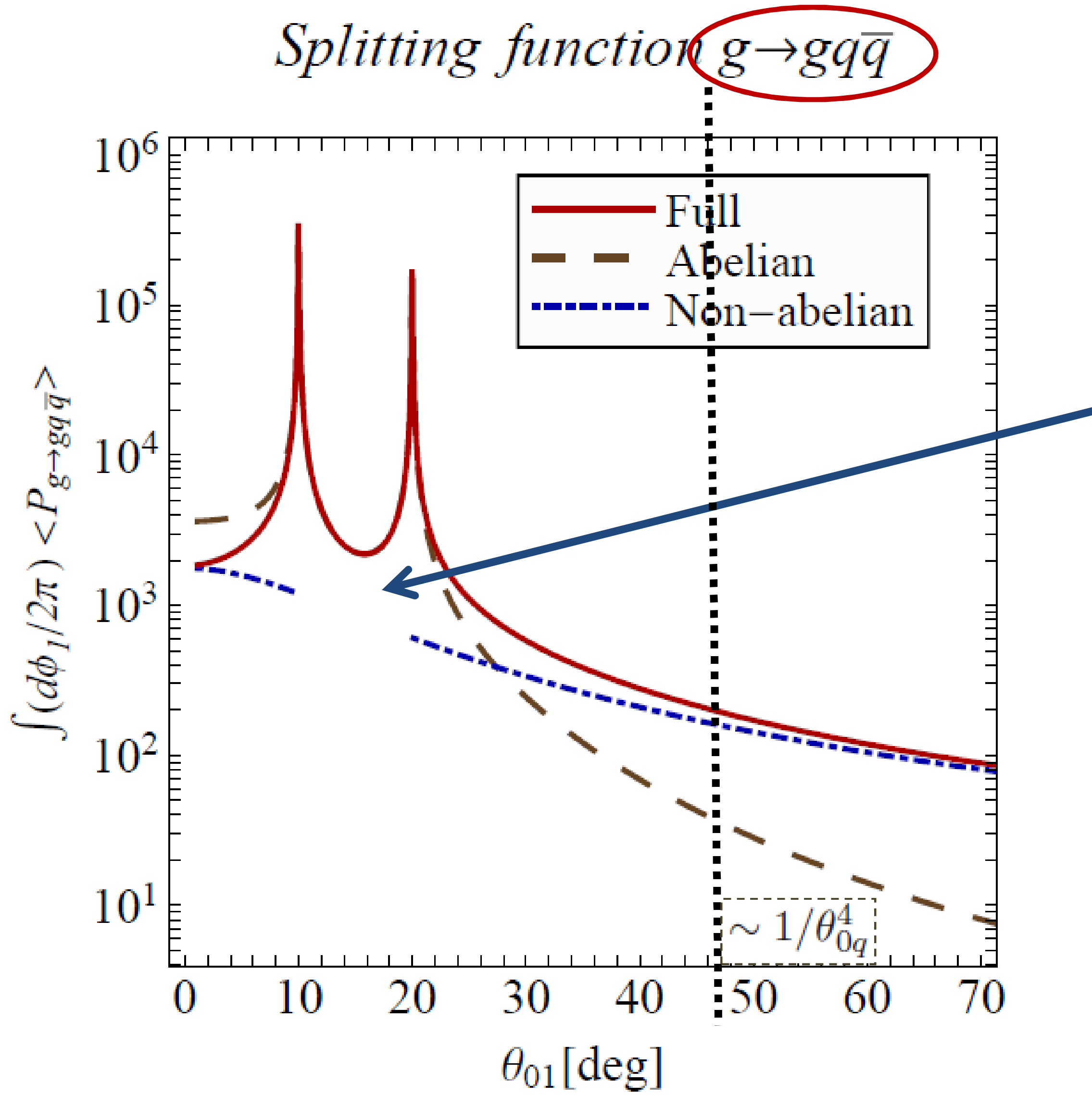
Angular Distribution in Vacuum



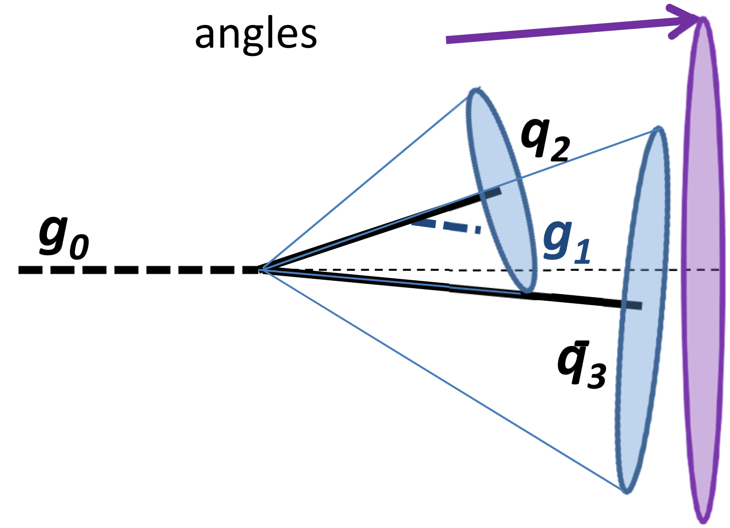
- coll. enhancement
- angular ordering visible at large angles:
  - not exactly zero at large angles



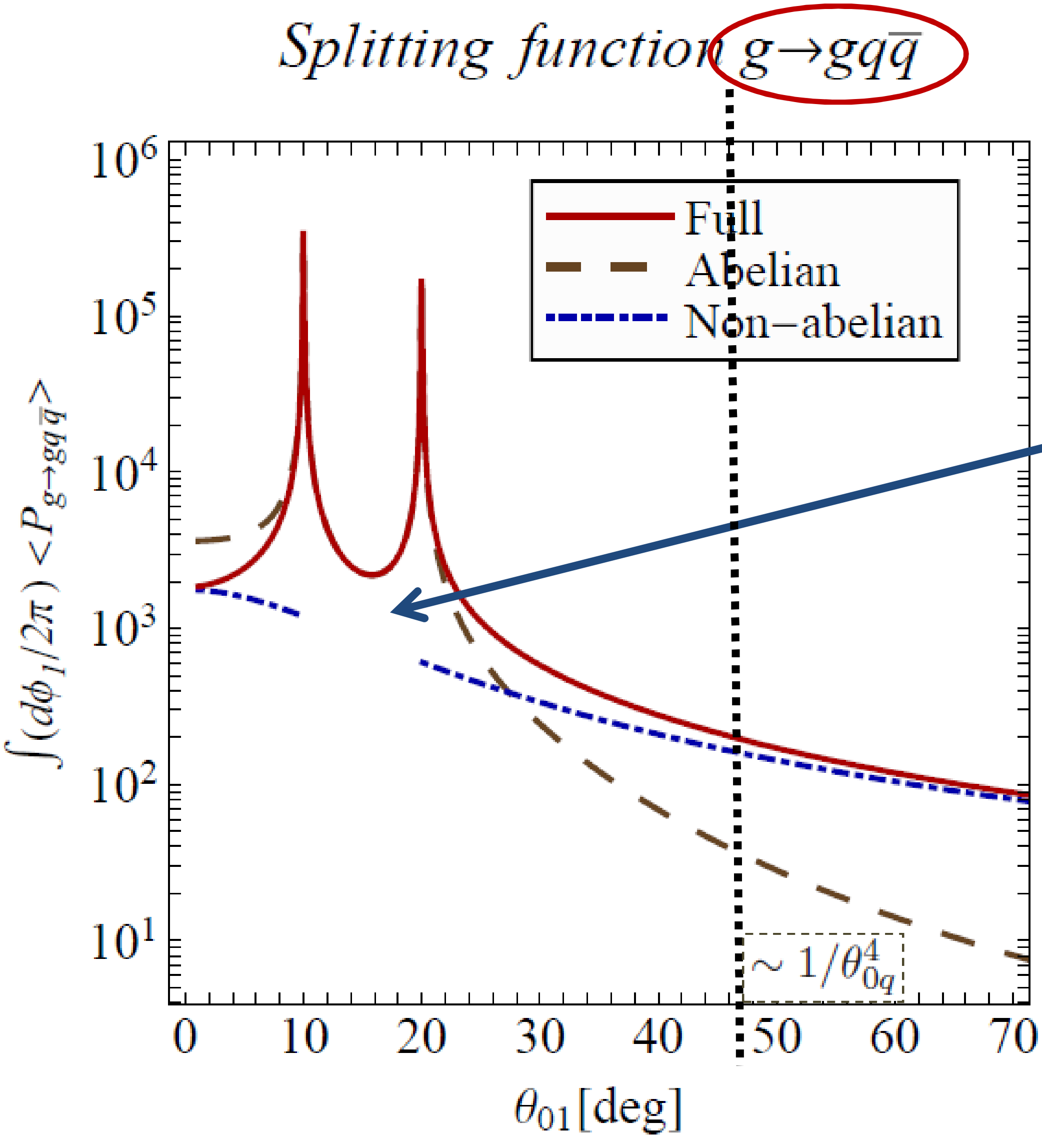
$$\langle P_{g_0\to g_1 q_2 \bar{q}_3}\rangle = \frac{2T_R(1-c_{23})}{z_1^2} z_2(1-z_2)(z_2^2+(1-z_2)^2)\left(C_F\left(W_{23}^{[2]}+W_{23}^{[3]}\right)+C_A\left(X_{23}\right)\right)$$



- coll. enhancement
- angular ordering visible at large angles
- angular anti-ordering visible at small angles:
  - not exactly zero at small angles

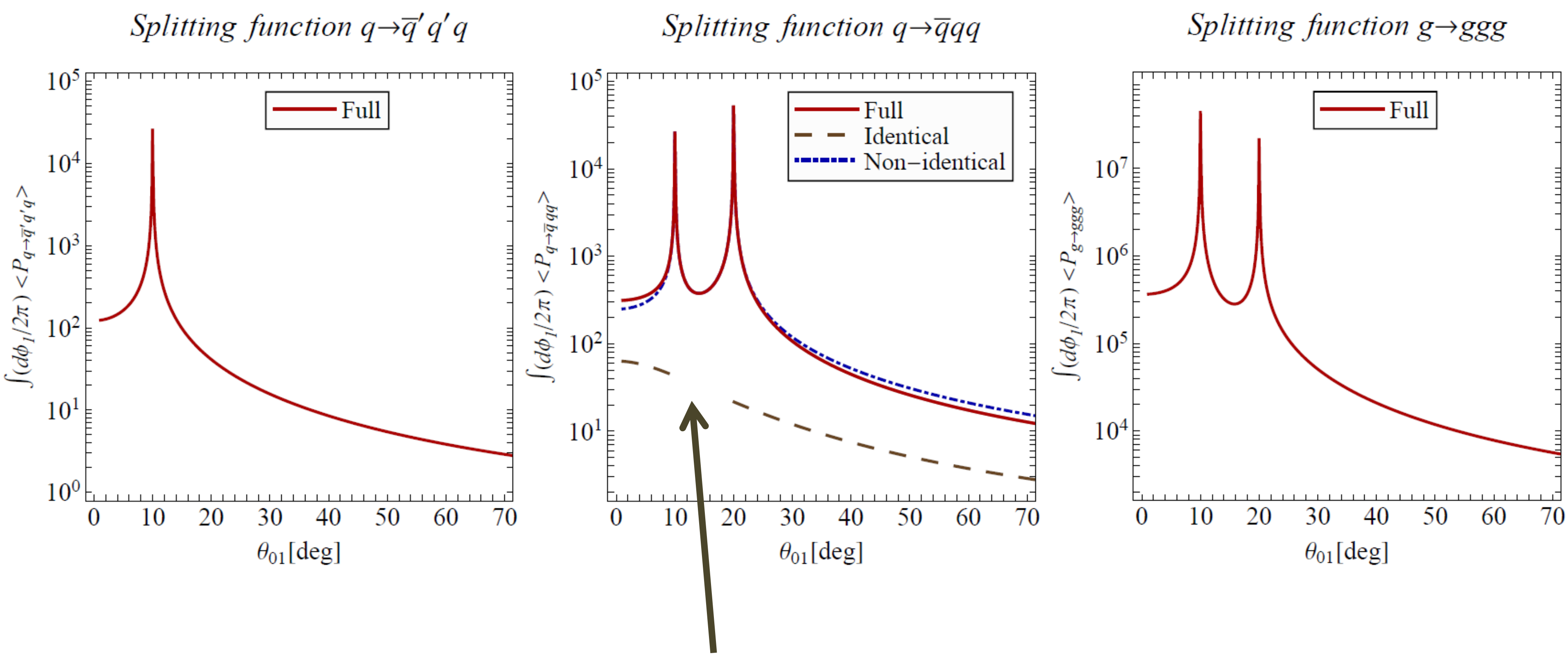


$$\langle P_{g_0\to g_1 q_2 \bar{q}_3}\rangle = \frac{2T_R(1-c_{23})}{z_1^2} z_2(1-z_2)(z_2^2+(1-z_2)^2)\left(C_F\left(W_{23}^{[2]}+W_{23}^{[3]}\right)+C_A\left(X_{23}\right)\right)$$



- coll. enhancement
- angular ordering visible at large angles
- angular anti-ordering visible at small angles:
  - absolute value of non-abilian term
  - zero between 10° and 20°

Angular Distribution in Vacuum



anti-ordered

all other graphs are neither angular ordered nor anti-ordered!

Cascade

$$\langle P^{\rm casc}_{q\to ggq}[k_1,k_2,p]\rangle^{(0)} = s_{123}\left(\frac{\langle P^{(0)}_{q\to gq}[k_2,p+k_1]\rangle\langle P^{(0)}_{q\to gq}[k_1,p]\rangle}{s_{13}} + \frac{\langle P^{(0)}_{q\to gq}[k_1,p+k_2]\rangle\langle P^{(0)}_{q\to gq}[k_2,p]\rangle}{s_{23}} + \frac{\langle P^{(0)}_{q\to gq}[k_1+k_2,p]\rangle\langle P^{(0)}_{g\to gg}[k_1,k_2]\rangle}{s_{12}}\right)$$

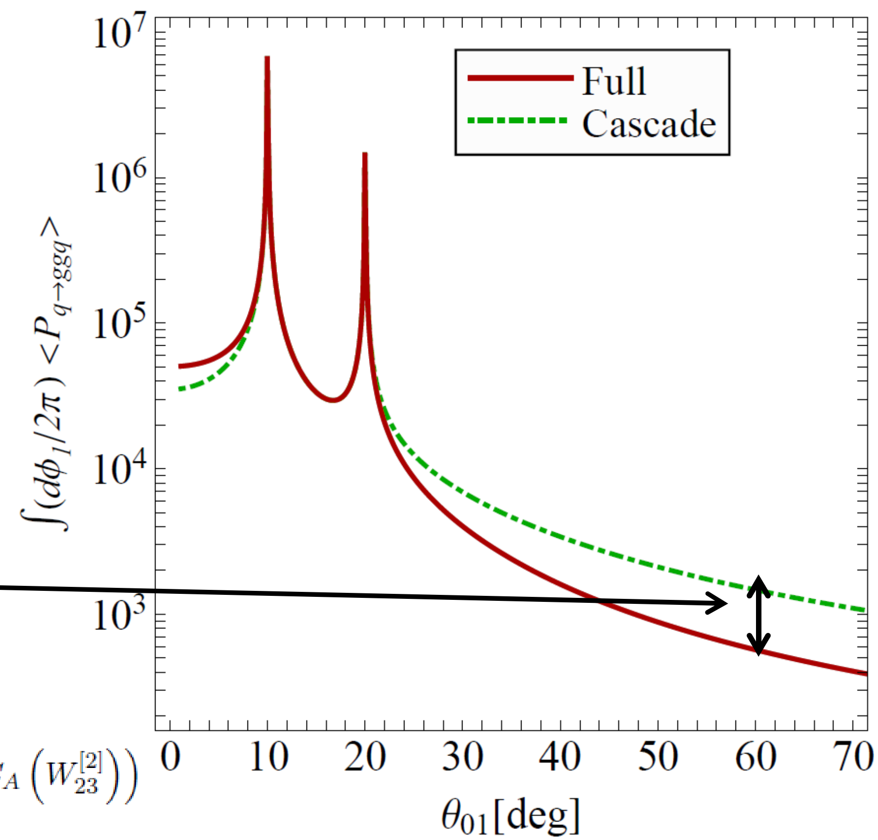
approximation of the full 3-parton splitting using 2-parton splittings

$$\langle P^{\rm cascade}_{q_0\to g_1 g_2 q_3}\rangle = (1-c_{23})\frac{4C_F}{z_1^2}z_2(1-z_2)\frac{1-z_2+z_2^2/2}{z_2} \times\left(C_F\left(W^{[3]}_{23}+X_{23}\right)+C_A\left(W^{[2]}_{23}+X_{23}\right)\right)$$

factor $(C_F+C_A)/C_F$

$$\langle P_{q_0\to g_1 g_2 q_3}\rangle = \frac{4C_F(1-c_{23})}{z_1^2}z_2(1-z_2)\frac{1-z_2+z_2^2/2}{z_2}\left(C_F\left(W^{[3]}_{23}+X_{23}\right)+C_A\left(W^{[2]}_{23}\right)\right)$$

Splitting function q→ggq



ANGULAR DISTRIBUTION IN MEDIUM

Calculation in Medium

- SCET_G, extension of SCET which includes *Ovanesyan, Vitev*
  Glauber modes $p_G \sim Q\,(\lambda^2, \lambda^2, \lambda)$, in addition to
  collinear $p_C \sim Q\,(\lambda^2, 1, \lambda)$ and soft $p_S \sim Q\,(\lambda^2, \lambda^2, \lambda^2)$ modes

- medium is modeled with a finite number of scattering centers with static Debye-screened potential *Gyulassy, Wang*

$$H = \sum_{n=1}^{N} H(q; x_n) = 2\pi\delta(q^0)\, v(q) \sum_{n=1}^{N} \mathrm{e}^{iqx_n}\, T^a(R) \otimes T^a(n)$$

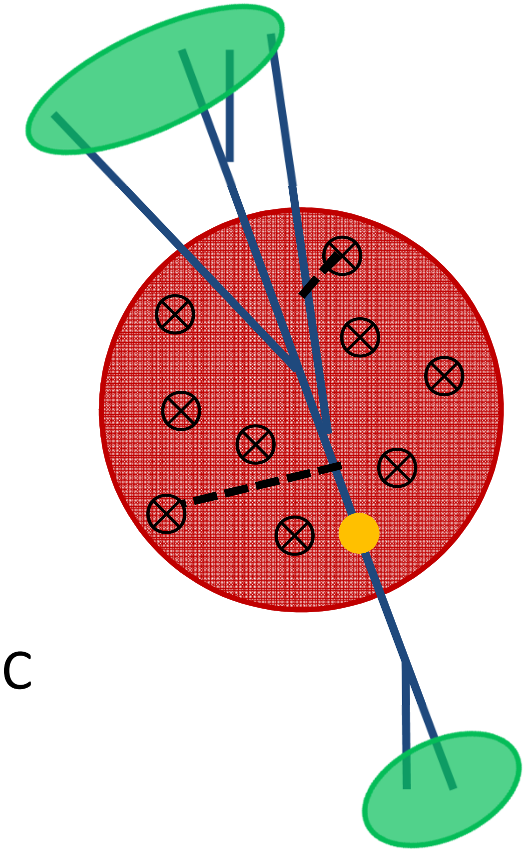
$$v(q) = \frac{4\pi\alpha_s}{q_z^2 + \mathbf{q^2} + \mu^2}$$

medium parameters:

- Debye screening scale: $\mu = 0.75\ GeV$
- medium size: $L = 5\ fm$
- gluon scattering length: $\lambda_g = 1\ fm$

Calculation in Medium

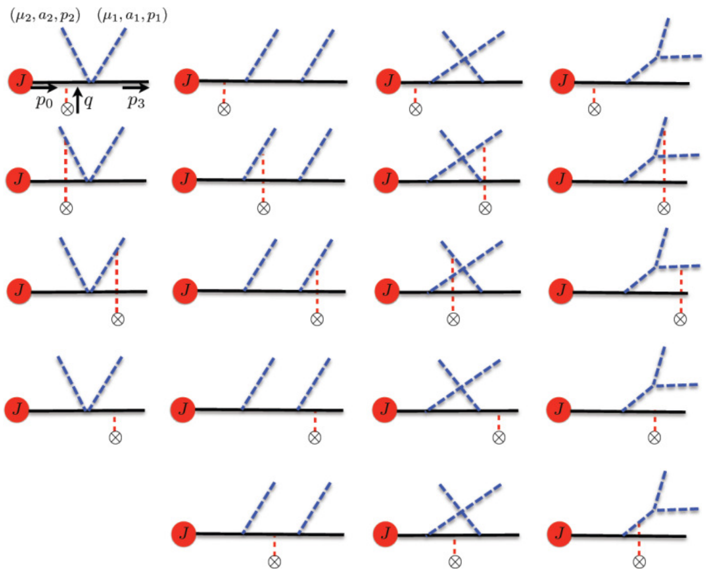
Gyulassy, Levai, Vitev



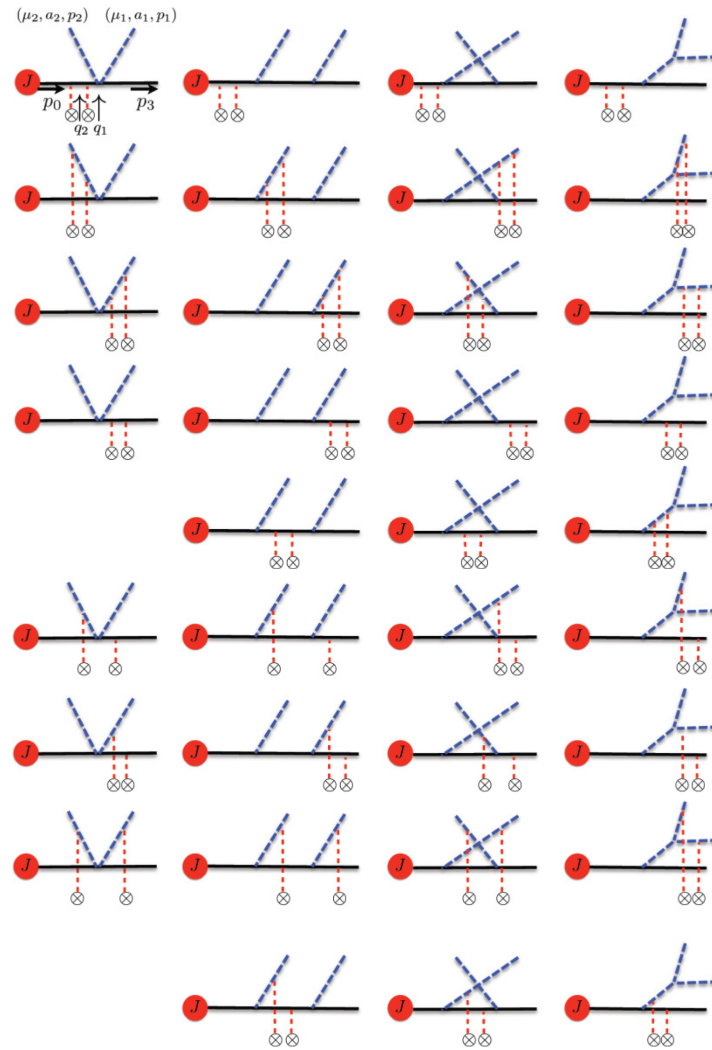
- expansion in opacity (# of interactions):
  single born = one medium interaction
  double born = two medium interactions
- Important to include double born contact limit for first order in opacity
- average over scattering centers
- first order of opacity valid approximation at RHIC

Gyulassy, Levai, Vitev

19 single born diagrams:



34 double born diagrams:



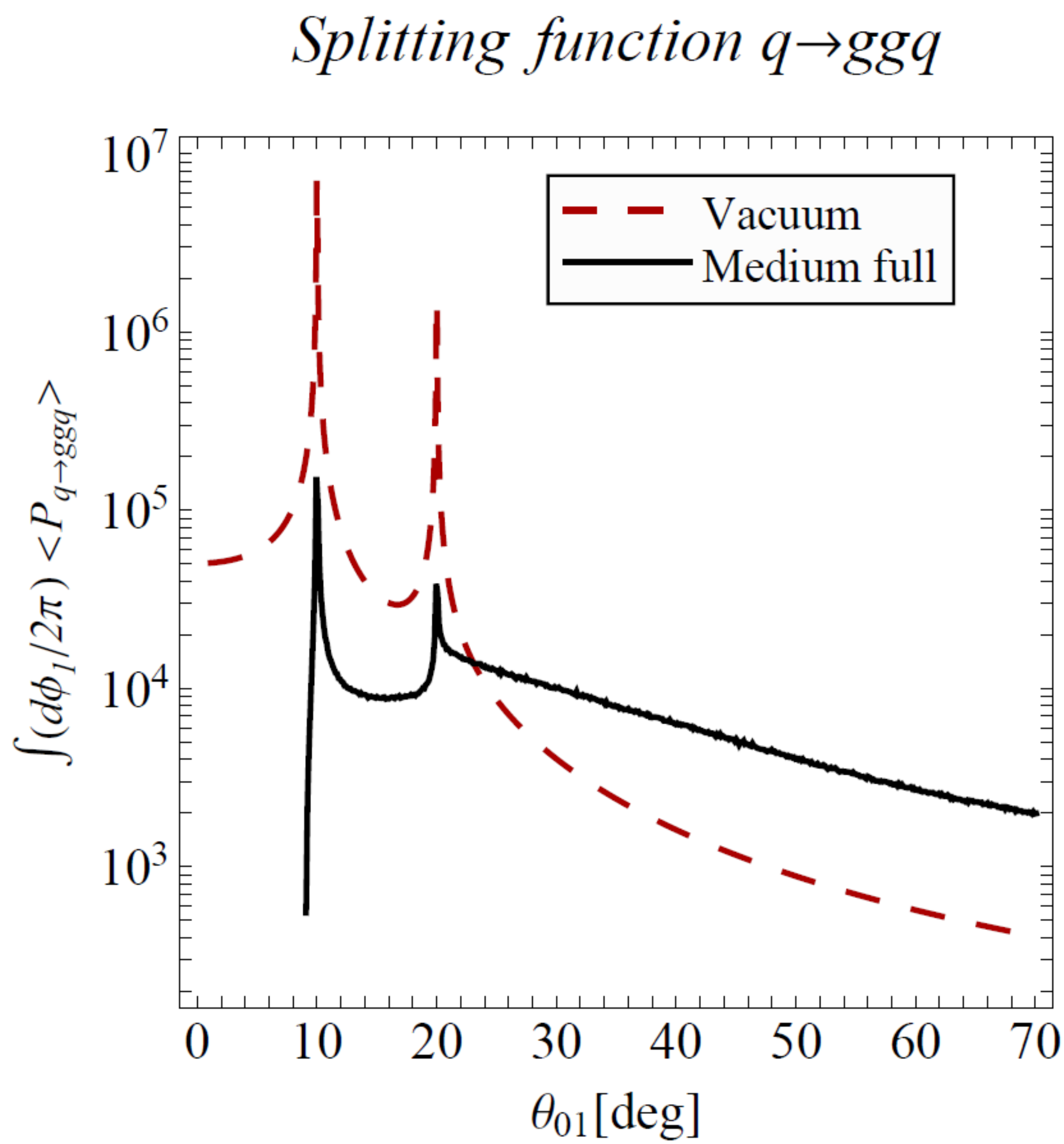
$$\mathcal{M}_k^{(1)} = -g^2\, \varepsilon_1^{i_1}\, \varepsilon_2^{i_2}\, \bar{\chi}_{n,p} \left(\int d\Phi\, C_k\, \Gamma_k^{i_1 i_2} I_k^{(1)} \right) J,$$

$$\mathrm{d}\mathbf{\Phi}_i = \frac{\mathrm{d}^2\mathbf{q}_i}{(2\pi)^2}\, \mathrm{e}^{-i\mathbf{q}_i \delta \mathbf{x}_i}\, \tilde{v}(\mathbf{q}_i)$$

$$\tilde{v}(\mathbf{q}) = 4\pi\alpha_s/(\mu^2 + \mathbf{q}^2)$$

$$\mathcal{M}_k^{(2c)} = g^2\, \varepsilon_1^{i_1}\, \varepsilon_2^{i_2}\, \bar{\chi}_{n,p} \left(\int d\Phi_{1\perp}\, d\Phi_{2\perp}\, C_k\, \Gamma_k^{i_1 i_2} I_k^{(2c)} \right) J,$$

Angular Distribution in Medium



<u>scenario 1:</u>

$E_0 = 100$ GeV

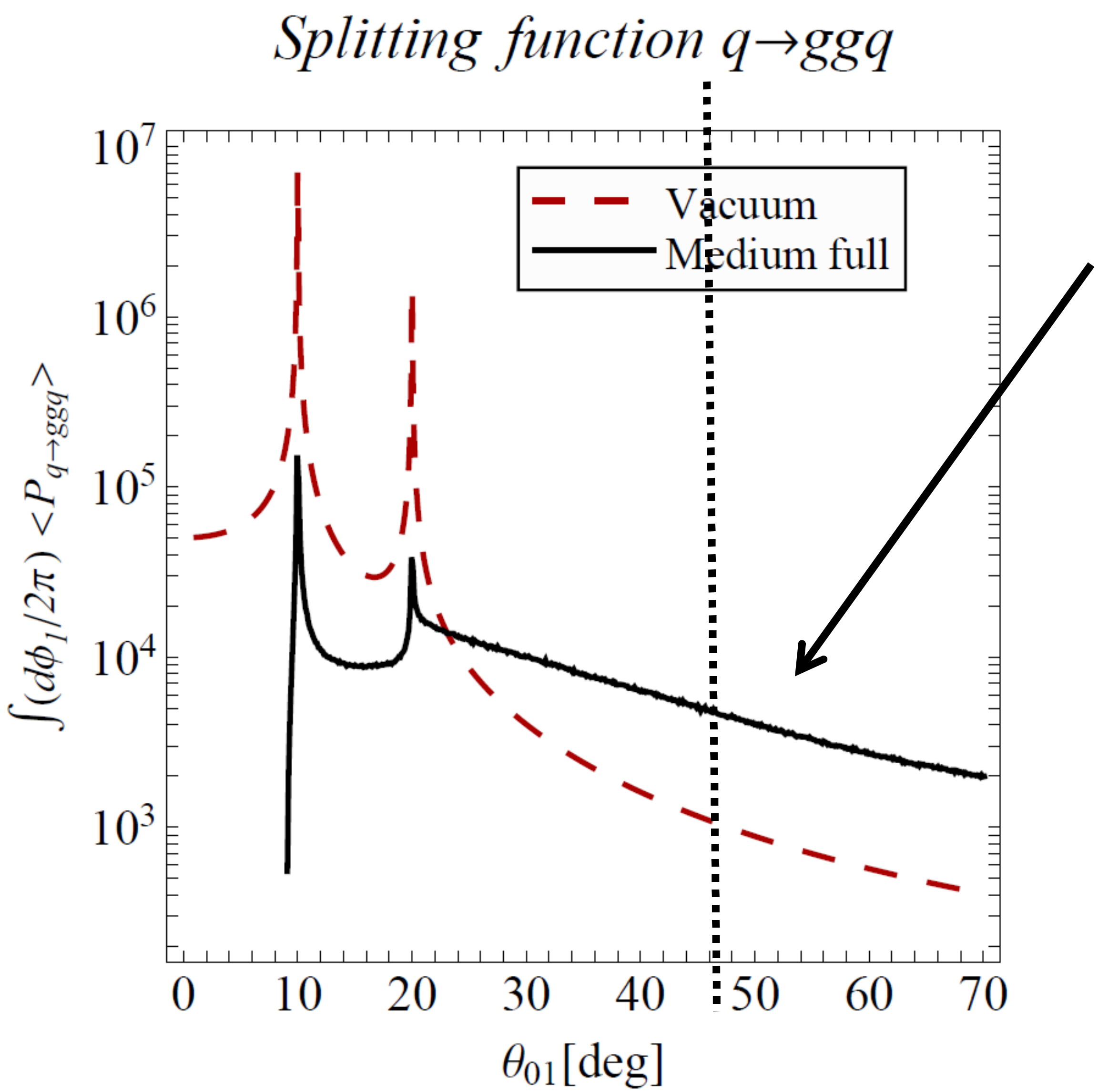
$z_1 = 0.03$

$z_2 = 0.643$

$\theta_{02} = 10°$

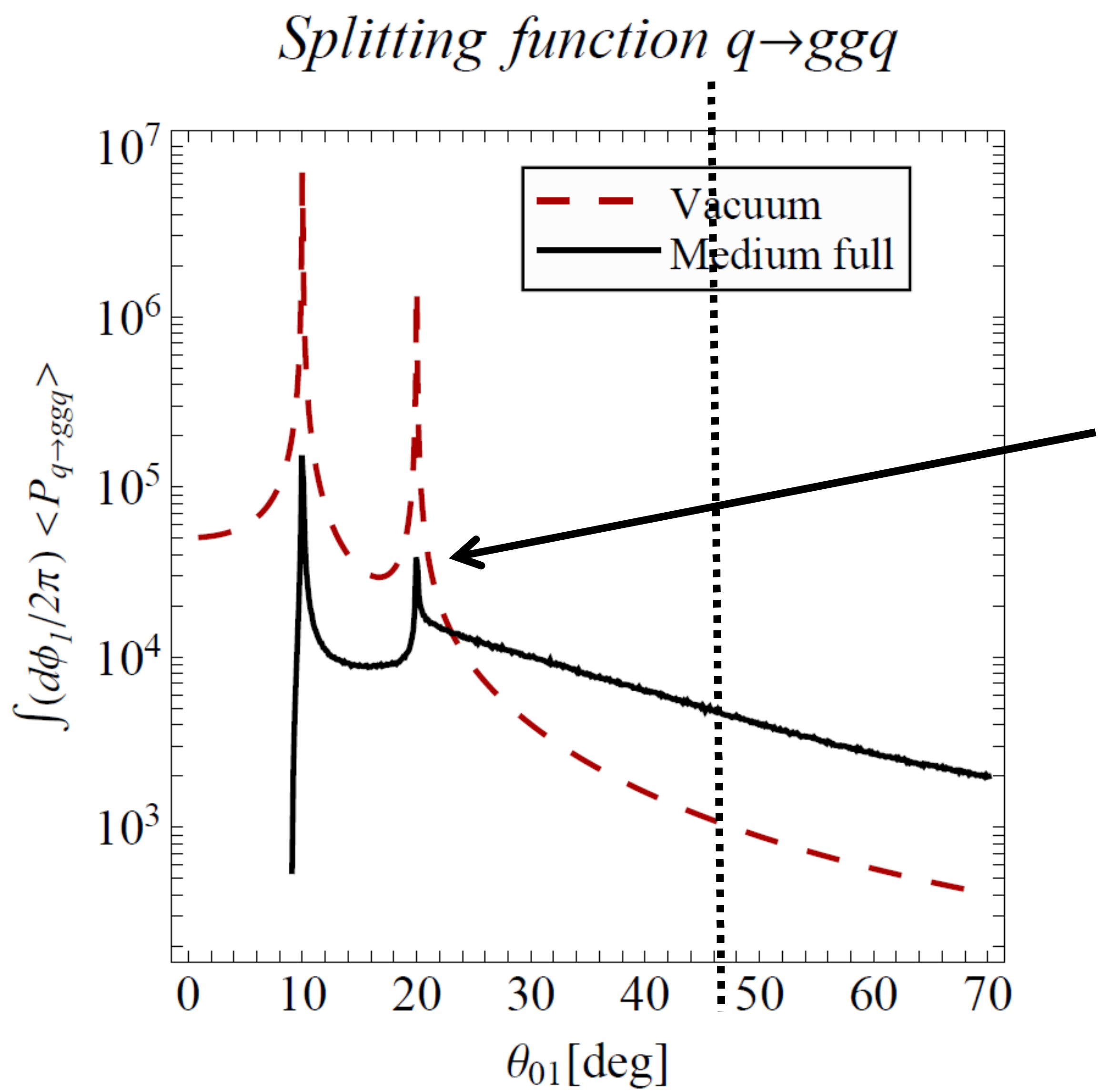
$\theta_{03} = 20°$

Angular Distribution in Medium



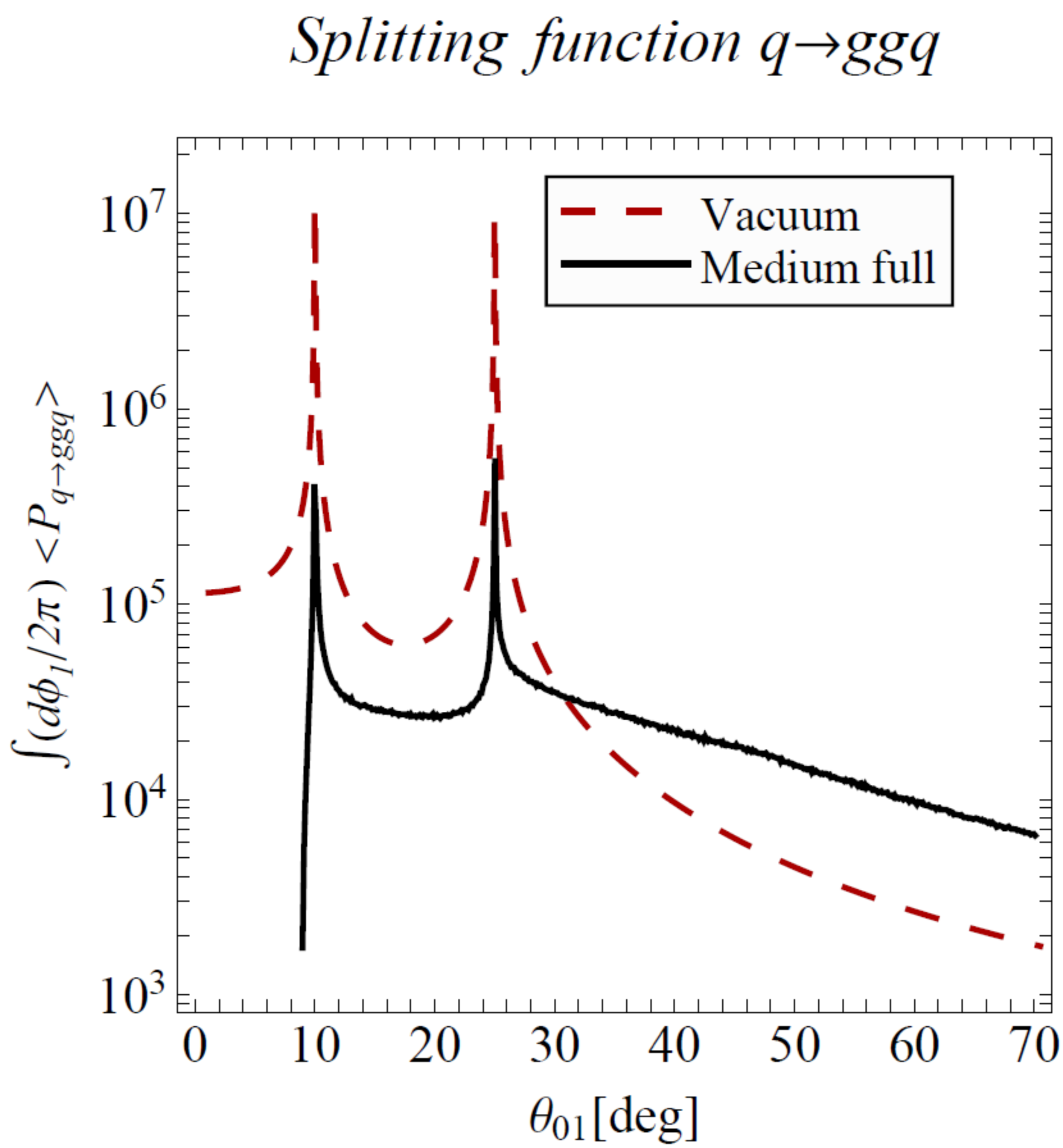
- coll. enhancement
- NO angular ordering
- jet broadening

Angular Distribution in Medium



- coll. enhancement
- NO angular ordering
- jet broadening
- NO angular anti-ordering
- large cancelations between single and double born contributions

Angular Distribution in Medium



<u>scenario 2:</u>

E_0 = 100 GeV

z_1 = 0.03

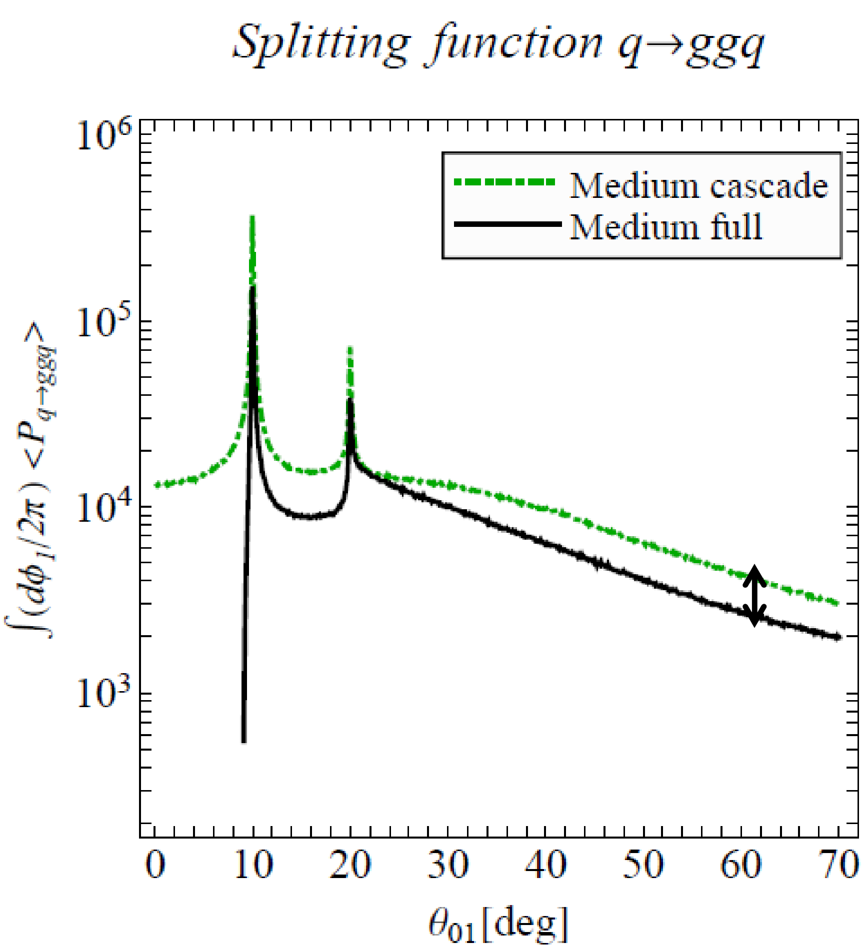
z_2 = 0.282

θ_{02} = 25°

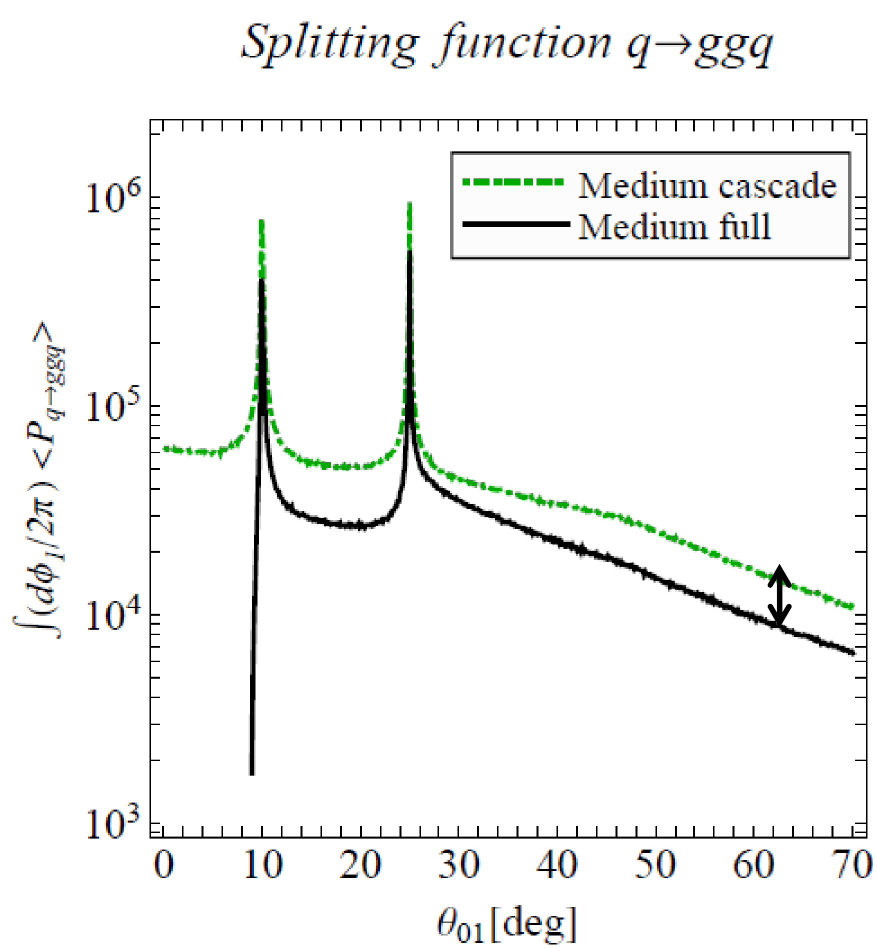
θ_{03} = 10°

Angular Distribution in Medium

scenario1:



scenario 2:

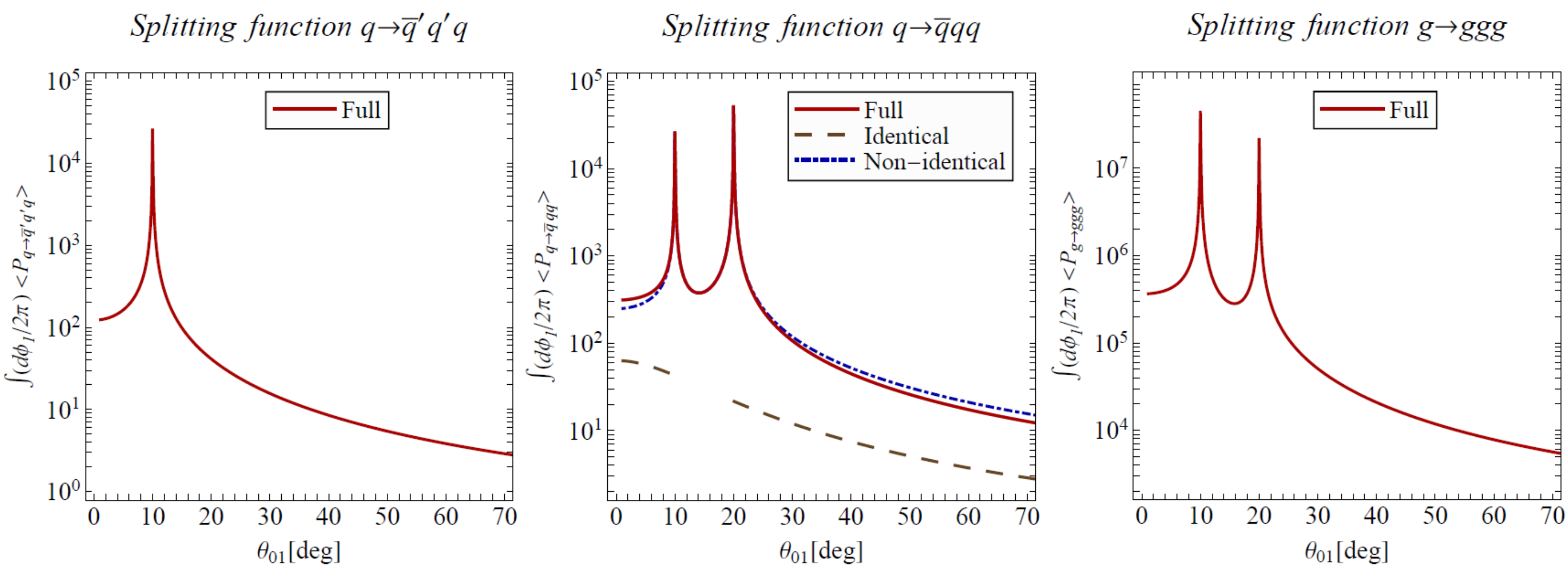


Conclusion

- We calculated the q->qgg splitting function in medium
- We studied the angular distributions of **collinear radiation** in vacuum and dense QCD matter
- The full splitting functions are neither angular ordered nor anti-ordered
- The radiation in medium falls off less step with large angles than in vacuum

BACKUP SLIDES

Angular Distribution in Vacuum



Angular Distribution in Vacuum

$$\langle P_{q_0\to\bar{q}'_1 q'_2 q_3}\rangle = \frac{C_F T_R\,(1-c_{23})}{z_1}\frac{(1-z_2)\,(2(1-z_2)+z_2^2)}{z_2}\left(W_{23}^{[2]}+X_{23}\right)$$

$$\langle P_{q_0\to\bar{q}_1 q_2 q_3}\rangle = \frac{C_F(1-c_{23})}{z_1}\Bigg[T_R\left(\frac{(1-z_2)(1+(1-z_2)^2)}{z_2}\left(W_{23}^{[2]}+X_{23}\right)\right.$$
$$\left.+\frac{z_2(1+z_2^2)}{(1-z_2)}\left(W_{23}^{[3]}+X_{23}\right)\right)+2(C_F-C_A/2)\,X_{23}\Bigg]$$

$$\langle P_{g_0\to g_1 g_2 g_3}\rangle = \frac{4C_A^2(1-c_{23})}{z_1^2}z_2(1-z_2)\left(\frac{z_2}{1-z_2}+\frac{1-z_2}{z_2}+z_2(1-z_2)\right)\left(W_{23}^{[2]}+W_{23}^{[3]}+X_{23}\right)$$