Guaranteed cost control for an overhead crane with practical constraints:
Fuzzy descriptor system approach

Ying-Jen Chen, Wen-June Wang, Chun-Lung Chang

Abstract

This study considers a practical overhead crane control problem of designing a fuzzy control to ensure that the trolley arrives precisely at its destination; the load swing angle approaches to zero, and the energy of input and states is minimized. Furthermore, the constraints for the overhead crane operation are also considered and satisfied. Those constraints are that when the crane is moving, the swing angle has to be small; the swing speed has to be slow, and the moving force is limited. The fuzzy descriptor system is chosen to represent the dynamics of the overhead crane, and a guaranteed cost fuzzy control design is proposed for the fuzzy descriptor system. Moreover, this study also illustrates why the fuzzy descriptor system instead of traditional T-S fuzzy system is selected here as the model of the overhead crane. The simulation results demonstrate the effectiveness of the proposed control design scheme.

Keywords:
Guaranteed cost
Fuzzy control
Input constraint
State constraint
Fuzzy descriptor system
Linear matrix inequality (LMI)
Overhead crane

1. Introduction

Overhead cranes are utilized extensively to move heavy cargo in factories and harbors. Many studies have investigated automatic control of overhead cranes. Takagi and Nishimura (2003) presented a centralized control system that considers coupling of up-and-down and rotational directions for a jib-type crane. Input-shaping technique has been adopted to reduce residual vibration (Singhose et al., 2000). Both Takagi and Nishimura (2003) and Singhose et al. (2000) focused on the control of suppressing load swing, but did not consider the problem of position error. Al-Garni et al. (1995) proposed an optimal control method for overhead cranes, however the stability of the control system was not considered in this study. Furthermore, several studies have designed the decoupled control (Lee, 1998) and optimal control (Hamalainen et al., 1995; Goritov and Korikov, 2001; Piazzi and Visioli, 2002) for overhead cranes according to the linearized crane model. However, the difference between the original nonlinear dynamics and the linearized model may reduce the stability and performance of the designed control systems.

Sharkawy et al. (2003) designed a stable fuzzy control for overhead cranes, but they did not consider the control performance and the practical constraints.

Some linguistic fuzzy control methods have also been proposed for overhead cranes (Benhidjeb and Gissinger, 1995; Liang and Koh, 1997; Nailey and Trabia, 2000; Li and Lee, 2001; Cho and Lee, 2002; Chang, 2007). Although linguistic fuzzy control is a simple control method, it is normally designed according to the experience or intuition of designers without considering the system model. Therefore, such fuzzy control cannot guarantee the stability of the control system. Stability and stabilization issues for T-S fuzzy control system have been widely discussed during the last decade (Tanaka et al., 1998; Kim and Lee, 2000; Tuan et al., 2001). A descriptor model is known to present a practical system better than a standard dynamic model (Dai, 1989). Therefore, an extended T-S fuzzy control system described by the descriptor form called fuzzy descriptor system is considered in Taniguchi et al. (2000). Although T-S fuzzy system has been successfully applied to many applications (Tanaka et al., 2001a, 2002, 2004; Li et al., 2004), few applications have been reported based on fuzzy descriptor system.

Guaranteed cost control is another important issue for control systems, and has been investigated for T-S fuzzy systems (Tanaka and Wang, 2001b; Tuan et al., 2001; Choi and Park, 2004; Zhang et al., 2007; Boukas, 2006) and fuzzy descriptor systems (Ma et al., 2005). Furthermore, input/state constraints should be
generally considered in the control of practical systems (Tanaka et al., 2002, 2004). Therefore, guaranteed cost control with input constraint has been proposed for some nonlinear systems (Son et al., 2001; Tong et al., 2006). Unfortunately, so far as we know, we have not found any study considering guaranteed cost control with the input/state constraints for fuzzy descriptor systems.

The dynamics of an overhead crane with position error and load swing angle are exactly transformed into a fuzzy descriptor system. A guaranteed cost fuzzy control method is then proposed for the fuzzy descriptor system. Since a real overhead crane has some practical constraints, the control design also considers the input/state constraints. To our knowledge, this is the first study of guaranteed cost control with input/state constraints for the fuzzy descriptor system. Notably, there is no difference between non-guaranteed cost control with input/state constraints for the fuzzy input/state constraints. To our knowledge, this is the first study of some practical constraints, the control design also considers the load swing angle are exactly transformed into a fuzzy descriptor with input constraint has been proposed for some nonlinear (Tanaka et al., 2002, 2004). Therefore, guaranteed cost control can be modeled by two nonlinear second-order-differential mass, and parameters, where

\[
(m_L + m_c) \ddot{x}_1(t) + m_L \dot{y}(t) \cos(\theta(t)) = u(t),
\]

\[
m_L \ddot{y}(t)(\cos(\theta(t)) + m_L \ddot{\theta}(t) = -m_L g \sin(\theta(t)),
\]

where \( g \) is the gravitational acceleration.

Let \( x_0 \) be the desired trolley position, and \( x_0(t) = x_1(t) - x_0 \) be the position error. The nonlinear differential equations (1) and (2) can be rewritten as (3) with states \( \theta(t) \) and \( x_4(t) \):

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & (m_L + m_c)/m_L & 0 & l \cos(\theta(t)) \\
0 & 0 & 1 & 0 \\
0 & \cos(\theta(t)) & 0 & l
\end{bmatrix}
\begin{bmatrix}
x_4(t) \\
\dot{x}_4(t) \\
\dot{\theta}(t) \\
\dot{\theta}(t)
\end{bmatrix}
= \begin{bmatrix}
0 \\
\beta^2 (\dot{\theta}(t) \sin(\theta(t))) \\
0 \\
-g \sin(\theta(t))
\end{bmatrix}
+ \begin{bmatrix}
1/m_L \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{y}(t) \\
\dot{\theta}(t)
\end{bmatrix}
+ \begin{bmatrix}
u_1(t) \\
u_2(t)
\end{bmatrix}.
\]

For safety, the overhead crane considered in this study operates under constraints \( \|\dot{\theta}(t)\| < \lambda_0 = \pi/12, \|\ddot{\theta}(t)\| < \lambda_0 = \pi/4 \) and \( \|u(t)\| < \lambda_u \|\cdot\|_2 \) (denotes the 2-norm operation). These constraints indicate that the swing angle has to be small; the swing speed has to be slow, and the moving force is limited.

### 3. Fuzzy descriptor system for the overhead crane

In this section, the nonlinear dynamics (3) of the overhead crane is exactly transformed into the fuzzy descriptor. By the sector nonlinearity concept (Tanaka and Wang, 2001b), \( \sin(\theta(t)) \) and \( \cos(\theta(t)) \) under the constraint \( \|\theta(t)\| < \pi/12 \) can be exactly represented as

\[
\sin(\theta(t)) = \sum_{i=1}^{2} \mu_{i1}(\theta(t)) a_1 \theta(t), \quad \quad (4)
\]

\[
\cos(\theta(t)) = \sum_{k=1}^{2} v_k(\theta(t)) b_k, \quad \quad (5)
\]

where \( a_1 = 1, a_2 = \sin(\pi/12)/\pi(\pi/12), b_1 = 1, b_2 = \cos(\pi/12) \), and

\[
\mu_{i1}(\theta(t)) = \begin{cases} 
1, & \text{if } \theta(t) = 0, \\
\frac{\sin(\theta(t)) - a_2 \theta(t)}{\theta(t) - a_2}, & \text{otherwise},
\end{cases}
\]

\[
\mu_{i2}(\theta(t)) = 1 - \mu_{i1}(\theta(t)),
\]

\[
\nu_1(\theta(t)) = \frac{\cos(\theta(t)) - b_2}{1 - b_2}, \quad \quad \nu_2(\theta(t)) = 1 - \nu_1(\theta(t)).
\]

Moreover, \( \ddot{\theta}(t) \) under the constraint \( \|\ddot{\theta}(t)\| < (\pi/4)^2 \) (since \( \|\ddot{\theta}(t)\| < (\pi/4) \) is exactly represented as

\[
\ddot{\theta}(t) = \sum_{j=1}^{2} \mu_{j2}(\dot{\theta}(t)) a_{2j}, \quad \quad (6)
\]

where \( a_{12} = (\pi/4)^2, a_{22} = 0 \), and

\[
\mu_{j1}(\dot{\theta}(t)) = \frac{\ddot{\theta}(t)}{(\pi/4)^2}, \quad \quad \mu_{j2}(\dot{\theta}(t)) = 1 - \mu_{j1}(\dot{\theta}(t)),
\]

\[
\mu_{j2}(\dot{\theta}(t)) > 0, \quad j = 1, 2.
\]

According to (4)–(6), dynamic equation (3) can be exactly represented as (7) under the constraints \( \|\dot{\theta}(t)\| < \pi/12 \) and \( \|\ddot{\theta}(t)\| < (\pi/4)^2 \):

\[
\sum_{k=1}^{2} \nu_k(\theta(t)) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & (m_L + m_c)/m_L & 0 & l b_k \\
0 & 0 & 1 & 0 \\
0 & b_k & 0 & l
\end{bmatrix}
\begin{bmatrix}
x_4(t) \\
\dot{x}_4(t) \\
\dot{\theta}(t) \\
\dot{\theta}(t)
\end{bmatrix}
= \begin{bmatrix}
x_4(t) \\
\dot{x}_4(t) \\
\dot{\theta}(t) \\
\dot{\theta}(t)
\end{bmatrix}
= \begin{bmatrix}
0 \\
\beta^2 (\ddot{\theta}(t) \sin(\theta(t))) \\
0 \\
-g \sin(\theta(t))
\end{bmatrix}
+ \begin{bmatrix}
1/m_L \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{y}(t) \\
\dot{\theta}(t)
\end{bmatrix}
+ \begin{bmatrix}
\mu_{11}(\theta(t)) a_1 \theta(t) \\
\mu_{12}(\theta(t)) a_{21} \theta(t) \\
\mu_{21}(\theta(t)) a_2 \theta(t) \\
\mu_{22}(\theta(t)) a_{22} \theta(t)
\end{bmatrix}.
\]
where

\[ M \] is asymptotically stable i.e. position error tend asymptotically to zero) under the constraints

This work aims to design a controller such that the crane system

The defuzzification is performed as

The guaranteed cost control aims to design the input

The modified parallel distributed compensation (PDC) is employed (Taniguchi et al., 2000):

where

The optimization problem is to require

\[ \min_{F_1, F_2, M_{ik}} \]

subject to LMIs

Theorem 1. If matrices \( \Gamma_1 \) and \( \Gamma_2 \) are the solutions of the optimization problem: it is required to

where

\( d_0, d_0 \) is for \( \theta(t) = d_0 x(t) \) and \( d_0, d_0 \) is for \( \theta(t) = d_0 x(t) \), then the closed-loop system (13) is asymptotically stable with minimized guaranteed cost value \( \gamma \) (i.e. cost function \( f(x(t), u(t)) \)). Moreover, the input constraint \( \|u(t)\| \leq \lambda_0 \) and the state constraints \( \|\theta(t)\| \leq \lambda_0 \) are enforced at all times

4. Guaranteed cost fuzzy controller design

This section presents a guaranteed cost fuzzy controller design method for the overhead crane with the input/state constraints

\[ u(t) \leq \lambda_0, \quad \|u(t)\| \leq \lambda_0 \quad \text{and} \quad \|\theta(t)\| \leq \lambda_0 \].

The following cost function of states and input is considered:

\[ J(x(t), u(t)) = \int_0^\infty x^T(t)Qx(t) + u(t)R_u(t) \, dt, \]
In this case, the local feedback gains are obtained by
\[ F_k = M_k \Gamma^{-1}. \]

**Proof.** Consider a Lyapunov function candidate
\[ V(X(t)) = X^T(t)E^T P X(t) \]
with
\[ E^T P = P^2 E^T = \begin{bmatrix} P_1 & 0 \\ 0 & P_1 \end{bmatrix}, \]
where \( P_1 > 0 \) and
\[ P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_1 \end{bmatrix}. \]

Define \( H = [I \ 0] \), hence \( x(t) = HX(t) \). Then
\[
V(X(t)) + x^T(t)Qx(t) + u^T(t)Ru(t)
\]
\[ = \frac{4}{\gamma^2} \sum_{i=1}^{2} h_i(z(t))v_i(\theta(t))X^T(t)\left(G^*_{ik} + P^T G_k \right)X(t)
\]
\[ + X^T(t)H^T Q X(t) + \frac{4}{\gamma^2} \sum_{i=1}^{2} \sum_{j=1}^{2} h_i(z(t))v_i(\theta(t))v_j(\theta(t))X^T(t)F^T_{ik} RF^T_{ik} X(t)
\]
\[ \leq \frac{4}{\gamma^2} \sum_{i=1}^{2} h_i(z(t))v_i(\theta(t))X^T(t)\left(G^*_{ik} + P^T G_k \right)
\]
\[ + \frac{4}{\gamma^2} \sum_{i=1}^{2} \sum_{j=1}^{2} h_i(z(t))v_i(\theta(t))v_j(\theta(t))X^T(t)F^T_{ik} RF^T_{ik} X(t)
\]
\[ = \frac{4}{\gamma^2} \sum_{i=1}^{2} h_i(z(t))v_i(\theta(t))X^T(t)
\]
\[ \times \left(G^*_{ik} + P^T G_k + H^T Q X(t) + F^T_{ik} R F^T_{ik} \right) X(t). \] (20)

Therefore if
\[ G^*_{ik} + P^T G_k + H^T Q + F^T_{ik} R F^T_{ik} < 0, \] (21)
then
\[
V(X(t)) + x^T(t)Qx(t) + u^T(t)Ru(t) < 0. \] (22)

Let \( \Gamma = \gamma^{-1} P^{-1} \), where \( \gamma \) is a positive scalar. Hence
\[
\Gamma = \begin{bmatrix} \Gamma_1 & 0 \\ -\Gamma_2 & \Gamma_1 \end{bmatrix}, \]
where \( \Gamma_1 = \gamma P_1 \) and \( \Gamma_2 = \gamma P_1 P_2 P_1^{-1} \). Multiplying (21) by \( \Gamma^T \) from left side and by \( \Gamma \) from right side, then divided by \( \gamma \), and by Schur complement, the following inequality is obtained:
\[
\begin{bmatrix} \Gamma^T G^*_{ik} + G_k \Gamma & \Gamma^T H^T \\ H \Gamma & -\gamma^{-1} Q^{-1} \end{bmatrix} > 0. \] (23)

Define \( M_k = F_k \Gamma \) and follow the definitions of \( H \) and \( G_{ik} \), (23) is equivalent to (16). Therefore if (16) holds, then (22) holds and \( V(X(t)) < 0 \), which indicates that the closed-loop system is asymptotically stable. Integrating (22) from 0 to \( \infty \) yields
\[
J = \int_{0}^{\infty} x^T(t)Qx(t) + u^T(t)Ru(t) dt = -x^T(0)E^T P X(0) + \int_{0}^{\infty} x^T(t)P \dot{x}(t) dt.
\] (24)

Since the closed-loop system is asymptotically stable that means \( x(\infty) = 0 \),
\[
J = \int_{0}^{\infty} x^T(t)Qx(t) + u^T(t)Ru(t) dt < x^T(0)P_1 x(0). \] (25)

Moreover, inequality (15) is equivalent to \( x^T(0)F^{-1}_1 x(0) < 1 \) which indicates that \( x^T(0)P_1 x(0) < 1 \). Therefore,
\[
J = \int_{0}^{\infty} x^T(t)Qx(t) + u^T(t)Ru(t) dt < x^T(0)P_1 x(0) \leq \gamma. \] (26)

Concerning about the input/state constraints, if (15), (17), (18) and (19) hold, the input constraint \( ||u(t)|| \leq \lambda_u \) and the state constraints \( ||\theta(t)|| \leq \lambda_\theta \) and \( ||\dot{\theta}(t)|| \leq \lambda_{\dot{\theta}} \) are guaranteed at all time \( t > 0 \) (Theorem 11 and Theorem 30 of Tanaka and Wang, 2001b). \( \square \)

Based on the analysis above, the design procedure of guaranteed cost fuzzy control for the overhead crane is summarized as follows.

**Design procedure**

**Step 1:** According to the practical constraints \( ||u(t)|| \leq \lambda_u \), \( ||\theta(t)|| \leq \lambda_\theta \) and \( ||\dot{\theta}(t)|| \leq \lambda_{\dot{\theta}} \) of the overhead crane, construct the fuzzy descriptor system (8) that is exactly equivalent to the nonlinear dynamics of the overhead crane under these constraints.

**Step 2:** Set \( Q \) and \( R \) of the cost function (11) according to the performance requirement.

**Step 3:** Solve the optimization problem of Theorem 1 to obtain minimized guaranteed cost value \( \gamma \), matrices \( \Gamma_1 \) and \( M_k \), thus \( F_k = M_k \Gamma^{-1} \) can be obtained.

**Step 4:** Obtain the guaranteed cost fuzzy controller in (12) to assure that the controlled crane system satisfies cost function \( J(X(t), u(t)) < \gamma \) and the practical constraints \( ||u(t)|| \leq \lambda_u \), \( ||\theta(t)|| \leq \lambda_\theta \) and \( ||\dot{\theta}(t)|| \leq \lambda_{\dot{\theta}} \).

**Remark 1.** Optimal control for the overhead crane has been designed according to the linearized crane model without considering input/state constraints (Hamalainen et al., 1995; Goritov and Korikov, 2001; Piazzi and Visioli, 2002) (only Piazzi and Visioli (2002) has the input constraint). Those studies do not consider the difference between nonlinear dynamics and the linearized model. The difference may cause that the performance of the linearized controlled system cannot be guaranteed in the original nonlinear system. In contrast, this study exactly models the nonlinear dynamics of the overhead crane into the fuzzy descriptor system. In other words, the proposed fuzzy controller can guarantee the control performance for the original nonlinear crane system. Furthermore, the fuzzy controller design also considers the input/state constraints.

**Remark 2.** Many studies have dealt with control design problem for overhead cranes by “linguistic” fuzzy control methods (Benhidjeb and Gissinger, 1995; Liang and Koh, 1997; Nalley and Trabia, 2000; Li and Lee, 2001; Cho and Lee, 2002; Chang, 2007). However, such fuzzy controllers are normally designed by experience and intuition without considering the system model and performance analysis. Conversely, the proposed fuzzy controller design is represented in terms of LMI conditions that can be efficiently solved by the existing LMI tools. Moreover, the control performance of the controlled system is also guaranteed.

**Remark 3.** From the crane dynamics (1) and (2), \( \ddot{x}(t) \) and \( \ddot{\theta}(t) \) can be solved as
\[
\ddot{x}(t) = -m_l \cos(\theta(t)) \sin(\theta(t))g + m_l \dot{\theta}(t)^2 \sin(\theta(t)) + u(t),
\]
\[
\ddot{\theta}(t) = \frac{\cos(\theta(t))m_l \dot{\theta}(t)^2 \sin(\theta(t)) + m_l g \sin(\theta(t)) + m_l g \sin(\theta(t))}{(\dddot{\theta}(t)g + m_l \cos^2(\theta(t)))}. \] (27)
There are four nonlinear terms \( \cos(\theta(t)) \), \( \sin(\theta(t)) \), \( \dot{\theta} \) and \( 1/(m_c - m - m_\theta \cos^2(\theta(t))) \) in (27) and (28). Therefore it needs \( 2^4 \) rules for traditional T–S fuzzy system to represent the crane dynamics. Moreover, the input \( u(t) \) is concerned with the nonlinear term \( 1/(m_c - m - m_\theta \cos^2(\theta(t))) \), the T–S fuzzy system has different input matrices \( B \). Hence, by using Theorem 26 of Tanaka and Wang (2001b), 139 LMI conditions are needed to obtain the guaranteed cost controller for the T–S fuzzy system without input/state constraints. Conversely, it needs only 8 rules in the proposed method for the fuzzy descriptor system to represent the crane dynamics. Moreover, in Theorem 1, only 10 LMI conditions are needed to obtain the guaranteed cost controller for the fuzzy descriptor system without input/state constraints. The comparison between the fuzzy descriptor system and the T–S fuzzy system is summarized in Table 1, which clearly indicates why this study adopts the fuzzy descriptor system instead of the traditional T–S fuzzy system to represent the nonlinear dynamics. The detail issue of the advantages for adopting the fuzzy descriptor system instead of the traditional T–S fuzzy system to represent the nonlinear dynamics can be found in Section 4 of Taniguchi et al. (2000). Notably, to our knowledge, the guaranteed cost control methods for the T–S fuzzy system in the previous studies such as Tanaka and Wang (2001b) and Tuan et al. (2001) have no input/state constraints. Hence, in the comparison, we only consider the guaranteed cost control conditions of the fuzzy descriptor system and of the T–S fuzzy system without the input/state constraints.

### Table 1
Comparison between fuzzy descriptor system and T–S fuzzy system.

<table>
<thead>
<tr>
<th>Item</th>
<th>Fuzzy descriptor system</th>
<th>T–S fuzzy system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rules for the modeling</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Number of LMI conditions</td>
<td>10</td>
<td>139</td>
</tr>
</tbody>
</table>

### Table 2
Overhead crane parameters.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_c )</td>
<td>1000</td>
<td>kg</td>
</tr>
<tr>
<td>( m_u )</td>
<td>1500</td>
<td>kg</td>
</tr>
<tr>
<td>( l )</td>
<td>8</td>
<td>m</td>
</tr>
<tr>
<td>( g )</td>
<td>9.8</td>
<td>m/s²</td>
</tr>
<tr>
<td>( \lambda_u )</td>
<td>2000</td>
<td>N</td>
</tr>
</tbody>
</table>

Consider an overhead crane as shown Fig. 1. Table 2 lists the constant parameters of its nonlinear dynamics (1) and (2) and the input force constraint \( \lambda_u \). The constraints \( \|\dot{\theta}(t)\| \leq \lambda_\theta = \pi/12 \), \( \|\theta(t)\| \leq \lambda_\theta = \pi/4 \) and \( \|u(t)\| \leq \lambda_u = 2000 \) are given. The initial state is \( x(0) = [0 0 0 0]^T \), and the desired trolley position is \( x_d = 30 \). \( Q = \text{diag}(5,1,1,1) \) and \( R = 0.01 \) are set. As shown in step 3 of the design procedure, by solving the optimization problem of Theorem 1, the minimized guaranteed cost value \( \gamma = 99.618 \) and the local feedback gains \( F_{14} = [16.1 340.9 3997.6 -1122.9] \), \( F_{21} = [16.5 358.9 -2239.2 -840.7] \), \( F_{21} = [16.8 370.7 3308.2 -982.4] \), \( F_{31} = [16.4 383.8 -2291.4 -859.9] \), \( F_{12} = [17.2 387.6 1694.1 -1088.9] \), \( F_{22} = [16.0 339.8 -4271.8 -1481.2] \), \( F_{32} = [17.3 391.0 1589.4 -1050.4] \) and \( F_{23} = [15.4 315.0 -4652.8 -1638.3] \) are obtained. Then the guaranteed cost fuzzy controller to satisfy \( J(x(t), u(t)) < \gamma \) and \( \|\dot{\theta}(t)\| \leq \lambda_\theta \), \( \|\theta(t)\| \leq \lambda_\theta \) and \( \|u(t)\| \leq \lambda_u \) is constructed as (12). Fig. 2 shows the simulation result obtained by the above fuzzy controller for the crane dynamics (1) and (2). The system is asymptotically stable, and the input/state constraints are satisfied.

**Fig. 2.** Simulation result of the overhead crane for \( Q = \text{diag}(5,1,1,1) \) and \( R = 0.01 \).
(i.e. $\max ||u(t)|| = 495.05 < 2000$, $\max ||\dot{\theta}(t)|| = 0.029 < \pi/12$ and $\max ||\ddot{\theta}(t)|| = 0.028 < \pi/4$) with the designed controller. In this case, the settling time (2%) of $x_1$ is $t_s = 61.68$, and the cost function is $J = 75897 < \gamma$. Notably, the (1,1) elements of $Q$ is concerned with $x_1(t)$; the (3,3) elements of $Q$ is concerned with $\dot{\theta}(t)$; the (4,4) elements of $Q$ is concerned with $\ddot{\theta}(t)$, and $R$ is concerned with $u(t)$. Therefore if $Q = \text{diag}(5,1,0.1,0.1)$ and $R = 0.005$ are set, which means that the relative weights of $\dot{\theta}(t)$, $\ddot{\theta}(t)$ and $u(t)$ in the cost

Fig. 3. Simulation result of the overhead crane for $Q = \text{diag}(5,1,0.1,0.1)$ and $R = 0.005$.

Fig. 4. Simulation result of the overhead crane for $Q = \text{diag}(3,1,1,1)$ and $R = 0.01$. 
function (11) degrade, then the faster response of $x_1(t)$ can be obtained with larger $\|\dot{x}(t)\|$, $\|\dot{y}(t)\|$ and $\|u(t)\|$. The simulation result is shown in Fig. 3. In this case, the setting time (2%) of $x_1$ is $t_s = 43.25$, and $\max \|u(t)\| = 590.45$, $\max \|\dot{y}(t)\| = 0.035$ and $\max \|\dot{x}(t)\| = 0.034$ (the input/state constraints are satisfied). Conversely, if $Q = \text{diag}(3,1,1)$ and $R = 0.01$ are set, which means that the relative weight of $x_1(t)$ in the cost function (11) degrade, then smaller $\|\dot{x}(t)\|$, $\|\dot{y}(t)\|$ and $\|u(t)\|$ can be obtained with slower response of $x_1(t)$. The simulation result is shown in Fig. 4. In this case, the settling time (2%) of $x_1$ is $t_s = 88.07$, $\max \|u(t)\| = 430.40$, $\max \|\dot{y}(t)\| = 0.025$ and $\max \|\dot{x}(t)\| = 0.024$. The comparison of control responses for different $Q$ and $R$ is given in Table 3 to illustrate how to choose $Q$ and $R$ according to the performance requirement.

6. Conclusion

This study presents a guaranteed cost fuzzy control method for an overhead crane. The dynamics of an overhead crane is exactly transformed into a fuzzy descriptor system, and a guaranteed cost fuzzy controller with input/state constraints is then designed for the transformed fuzzy descriptor system. To our knowledge, this is the first study of guaranteed cost control with input/state constraints for the fuzzy descriptor system. The reason for adopting the fuzzy descriptor system, instead of the ordinary T–S fuzzy system, to represent the crane dynamics has also been presented. Finally, the simulation results show the effectiveness of the proposed fuzzy control method.

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