Simultaneous Time-Frequency Channel Estimation Based on Compressive Sensing for OFDM System

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Abstract—Conventional time domain synchronous orthogonal frequency division multiplexing (TDS-OFDM) systems have the difficulty to support 256QAM or higher-order modulations and suffer from performance loss, especially under the severe fading channels with long delays. In this letter, a simultaneous time-frequency channel estimation method based on compressive sensing (CS) is proposed to solve this problem. First, the auxiliary channel information is obtained by exploiting the signal structure of TDS-OFDM. Then, a very few frequency-domain pilots in the OFDM block are used to acquire the accurate channel impulse response under the framework of CS. Besides, the auxiliary channel information is utilized to further reduce the complexity of the classical CS algorithm. Simulation results show that the proposed scheme can well support 256QAM under fading channels with long delays and have better performance than the conventional schemes.

Keywords—Simultaneous time-frequency, channel estimation (CE), compressive sensing (CS), orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

Time domain synchronous OFDM (TDS-OFDM) is the core technique of the digital television/terrestrial multimedia broadcasting (DTMB) standard, which has been successfully deployed in many countries, such as China, Laos, Cambodia, etc. [1]. TDS-OFDM outperforms classical cyclic prefix OFDM (CP-OFDM) in spectral efficiency by using a pseudo noise (PN) sequence as the guard interval (GI) as well as the training sequence (TS) for synchronization [2] and channel estimation (CE) [3]. However, the PN sequence and the OFDM block in TDS-OFDM systems would cause the mutual inter-block interference (IBI) to each other under multipath channels. Thus, the high-complexity iterative CE is needed to remove the mutual interference, but it still couldn’t avoid the non-negligible performance loss under selective fading channels due to the residual IBI [4]. Consequently, TDS-OFDM is difficult to accommodate the high-order modulations like 256QAM and to well support the emerging ultra-high definition TV (UHDTV) service [5].

Some methods have been proposed to solve this problem, but still have specific shortcomings. The dual PN padded OFDM (DPN-OFDM) is a simple and effective solution, while significant loss of spectral efficiency will be caused due to the duplicated PN sequence [6]. Without any loss of the spectral efficiency, the emerging theory of compressive sensing (CS) is explored to address the problem, whereby the IBI-free region of small size within the received PN sequence is utilized to reconstruct the wireless channel with high dimension [7]. However, when the fading of wireless channels with long delays becomes severe, i.e., the channel delay spread becomes larger, which is common in the single frequency network environments [8], [9], the performance of the CS-based scheme deteriorates rapidly due to the reduced size of the required IBI-free region.

To enhance the performance of TDS-OFDM under severely fading channels with long delays, we propose a simultaneous time-frequency CE scheme under the framework of CS. The specific contributions of this paper can be summarized as follows: 1) Unlike the very recently proposed CS-based scheme [7], which completely relies on the PN sequence for channel recovery, the proposed scheme merely uses the time-domain PN sequence to acquire the auxiliary channel information, while the accurate CE depends on a small number of frequency-domain pilots in the OFDM block based on CS. In this way, the proposed scheme can work well as long as the maximum channel delay is not larger than the GI length. 2) Moreover, the auxiliary channel information is used to further reduce the complexity of the classical CS algorithm, making it applicable for practical systems.

The remainder of this paper is organized as follows. The system model is introduced in Section II. The proposed simultaneous time-frequency CE based on CS is addressed in Section III. The simulation results are shown in Section IV. Finally, the conclusions are provided in Section V.

II. SYSTEM MODEL

Fig. 1 compares the system model of conventional schemes and the proposed solution. As shown in Fig. 1(a), the conventional DPN-OFDM uses the second received PN sequence immune from IBI for CE. Thus, the iterative IBI removal can be avoided. However, this will cause significant spectral efficiency loss [6]. In Fig. 1(b), the recent CS-based scheme uses the IBI-free region of size $G$ within the received PN sequence for CE, but the maximum channel length is limited to $L = M - G + 1$, where $M$ is the PN sequence length [7]. When
the channel length becomes larger, the size of the required IBI-free region becomes smaller, resulting in severe performance deterioration due to the reduced number of observations.

The proposed simultaneous time-frequency CE based on CS can be illustrated in Fig. 1(c). The ith TDS-OFDM symbol $s^i = [e^T, (x^i)^T]^T$ is composed of the known PN sequence $e = [e_0, e_1, \ldots, e_{M-1}]^T$ of length $M$ and the OFDM block $x^i = [x^i_0, x^i_1, \ldots, x^i_{N-1}]^T$ of length $N$ in the time domain, which is the inverse discrete Fourier transform (IDFT) output of the OFDM block $X^i = [X^i_0, X^i_1, \ldots, X^i_{N-1}]^T$ in the frequency domain. Unlike the conventional TDS-OFDM schemes [4], [6], [10], the OFDM block in the frequency domain contains not only the traffic data, but also a small number of pilots, whose location set can be denoted as $\Gamma = \{P_0, P_1, \ldots, P_{J-1}\}$ ($0 \leq P_0 < P_1 < \cdots < P_{J-1} \leq N-1$) can be assumed without loss of generality according to [11]).

Note that by exploiting the sparse nature of wireless channels and CS theory, the pilot number $J$ could be reduced significantly (far less than $L$, only around 1% of $N$) compared to conventional pilot-based OFDM schemes [12]. Thus, the spectral efficiency loss due to the pilot overhead is negligible. Furthermore, the pilot positions are randomized and the pilot power could be boosted to get better estimation performance.

**Notation:** We use boldface letters to denote matrices and column vectors; $\otimes$ represents the circular correlation; $(-)^T$, $(-)^H$, $(-)^{-1}$, $(\cdot)^*$ denote the transpose, conjugate transpose, matrix inversion, Moore-Penrose matrix inversion, diagonal matrix, and $\|\cdot\|_p$ norm operations, respectively; $x|_S$ is generated by restricting the vector $x$ to its $S$ largest components, and $\sup(x|_S)$ denotes the location set of the $S$ largest components of $x$; $x|_{\Gamma}$ denotes the entries of the vector $x$ on the set of $\Gamma$; $\Phi_{\Gamma}$ represents the sub-matrix comprising the $\Gamma$ columns of $\Phi$.

### III. Simultaneous Time-Frequency CE Based on CS

#### A. The Application Background of CS

In wireless communications, the discrete-time CIR $h^i = [h^i_0, h^i_1, \ldots, h^i_{L-1}]^T$ comprising $S$ taps can be modeled as [13]

$$h^i_l = \sum_{s=0}^{S-1} \alpha_s \delta [l - \tau_s], \quad 0 \leq l \leq L - 1,$$

where $\alpha_s$ and $\tau_s$ are the path gain and the normalized path delay of the $s$th path, respectively, while the channel length $L$ is usually assumed be not larger than the PN sequence length $M$, i.e., $L \leq M$.

At the receiver, the received pilots $Y^i|_r = [Y^i_{P_0}, Y^i_{P_1}, \ldots, Y^i_{P_{J-1}}]^T$ after cyclic reconstruction [4] can be presented as

$$Y^i|_r = \text{diag}(X^i) \Phi h^i + N|_r,$$

where $N|_r$ denotes the noise vector, while $\Phi = [\varphi_{m,n}]_{J \times L}$ is extracted from the DFT matrix $W = [w_{m,n}]_{N \times L}$ by selecting the row according to the randomized pilot positions, i.e.,

$$\varphi_{m,n} = w_{P_{m-1}, n} = \frac{1}{\sqrt{N}} \exp [-j2\pi(n-1)P_{m-1}/N],$$

for $1 \leq m \leq J, 1 \leq n \leq L$.

For Eqn. (2), the CS theory implies that the original signal $h^i$ can be exactly recovered from a very small number of observations $J$ which is far less than the signal dimension $L$, as long as $h^i$ is sparse [14]. Fortunately, numerous studies and experiments have verified that wireless channels are sparse, especially in wireless broadcasting systems like DMB [7], [15], which indicates that the sparse channel can be recovered by a very few frequency-domain pilots in the proposed scheme under the framework of CS.

#### B. The Simultaneous Time-Frequency CE Based on CS

Unlike the conventional CS-based or pilot-based CE schemes which depend only on either time- or frequency-domain information [4], [6], [12], [16], the proposed simultaneous time-frequency CE based on CS firstly utilizes the PN-based correlation in the time domain to acquire the auxiliary channel information, and then the frequency-domain pilots are used for the final accurate CE under the framework of CS. In addition, among the CS algorithms, compressive sampling matching pursuit (CoSaMP) algorithm is widely used due to its robustness to noise, where the most significant components of the original sparse signal are identified in an iterative manner [17]. However, it requires the priori knowledge of the sparsity level of the original signal, which is usually unknown in practice, and it has relatively high computational complexity. We further propose the auxiliary information based CoSaMP (A-CoSaMP) to solve these problems of the conventional CoSaMP. In this way, better performance as well as lower complexity could be achieved. The proposed method consists of 3 steps and can be detailed as follows:

1. **Step 1.** Auxiliary channel information acquisition based on time-domain PN correlation: The received time-domain PN
sequence \(d^i\) without IBI removal is directly correlated with the local known PN sequence \(c\) to obtain the coarse estimate of the channel \(\tilde{h}^i\) as
\[
\tilde{h}^i = \frac{1}{M}c \otimes d^i = h^i + v,
\]
where \(v\) denotes the additive white Gaussian noise (AWGN) as well as the effect of interference caused by the preceding OFDM block. As shown in Fig. 2, where the ITU Vehicle B (ITU-VB) channel model [18] with the signal-to-noise ratio (SNR) of 10 dB is considered, although the coarse path delay estimation is not accurate due to the existence of IBI, the near-optimal auto-correlation property of the PN sequence ensures that the auxiliary channel information necessary for the following A-SP algorithm could be acquired from such coarse estimation with low complexity. The auxiliary channel information includes the channel length, the locations of the most significant taps and the approximate channel sparsity level.

In practice, the path gains in \(\tilde{h}^i\) are discarded directly, and only the path delays of the most significant taps are retained in the initial coarse path delay set
\[
T_0 = \{l : \|\tilde{h}^i\|_2 \geq E_{th}\}_{l=0}^{L-1},
\]
where \(E_{th}\) is the power threshold configured according to [19].

Then, the channel sparsity level \(S\) can be approximated by the initial channel sparsity level \(S_0 = \|T_0\|_0\) and a compensation number \(a\) to combat the interference effect,
\[
S = S_0 + a = \|T_0\|_0 + a.
\]

2) Step 2. Cyclic reconstruction of the OFDM block: The cyclic reconstruction of the OFDM block is an essential process for TDS-OFDM to restore the CP property and can be implemented by the simple add-and-subtraction operation [4], which is a relatively mature technique widely used in conventional TDS-OFDM systems. After that the pilots in the received OFDM block could be used for CE.

3) Step 3. Accurate CE based on A-CoSaMP: The received pilots extracted from the cyclically reconstructed OFDM block can be used for the accurate CE by solving Eqn. (2) via the proposed A-CoSaMP algorithm. The A-CoSaMP algorithm is proposed based on the principle of the classical CoSaMP [17] algorithm and is summarized in Algorithm 1.

**Algorithm 1** Auxiliary information based CoSaMP (A-CoSaMP).

**Inputs:**
1) Initial coarse path delay set \(T_0\), channel sparsity level \(S\), initial channel sparsity level \(S_0\);
2) Noisy measurements \(m \triangleq Y|_r\);
3) Observation matrix \(\Theta \triangleq \text{diag}(X|_r)\Phi\).

**Output:** The \(S\)-sparse CIR estimate \(\hat{h}\).

1: \(a_0|_{T_0} \leftarrow \Theta|_{T_0} m;\) {Initial configuration}
2: \(u \leftarrow m - \Theta a_0;\) {Initial residual}
3: \(k \leftarrow 0;\) {Initial iteration flag}
4: while \(k < S - S_0\) do
5: \(p \leftarrow \Theta^T u;\) {Generate target CIR proxy}
6: \(\Pi \leftarrow \sup\{p\}_{2(S-S_0)};\) {Significant entry identification}
7: \(\Omega \leftarrow \sup\{a_k\} \cup \Pi;\) {Unite support}
8: \(b_{[3]} \leftarrow \Theta_3^T u;\) {Least square signal estimation}
9: \(b_{[3]} \leftarrow 0;\)
10: \(a_{k+1} \leftarrow b|_{\Omega};\) {Select most significant entries}
11: \(u \leftarrow m - \Theta a_{k+1};\) {Update residual}
12: \(k \leftarrow k + 1;\) {Update iteration flag}
13: end while
14: \(\hat{h} \leftarrow a_k;\)

Compared to the CoSaMP algorithm, the proposed A-CoSaMP algorithm has the following differences. First, the approximate channel sparsity level and SNR obtained in **Step 1** are utilized to further reduce the times of iterations. For example, if the locations of 3 paths out of the total 6 paths have been successfully detected in **Step 1**, the iteration number will be approximately reduced from 6 in CoSaMP to 3 in A-CoSaMP, thus the complexity will be reduced by about 50%.

**C. Computational Complexity**

In the proposed CE method, the \(M\)-point circular correlation in **Step 1** could be efficiently implemented by \(M\)-point FFT, so the corresponding complexity is \(O(M\log_2(M))\). In **Step 2**, it requires the complexity of \(O(M\log_2(3M)/4 + 3M)\) for the cyclic reconstruction operation.

In fact, the main computational burden of the proposed method is the A-CoSaMP algorithm used to acquire the actual path delays in **Step 3**. Each iteration has the complexity of \(O(4JS_i^2 + 8S_i^3)\), and the overall complexity of CoSaMP comprising \(S\) iterations is \(O(4JS_i^3 + 8S_i^3)\) [17]. However, as has been discussed in Section III-B, only \(S - S_0\) iterations are required by A-CoSaMP, since some of the locations of the
TABLE I
THE MULTIPATH CHANNEL PARAMETERS FOR SIMULATION

<table>
<thead>
<tr>
<th>Path Index</th>
<th>ITU-VB</th>
<th>CDT-8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delay (μs)</td>
<td>Gain (dB)</td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>-2.5</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>8.90</td>
<td>-12.8</td>
</tr>
<tr>
<td>4</td>
<td>12.90</td>
<td>-10.0</td>
</tr>
<tr>
<td>5</td>
<td>17.10</td>
<td>-25.2</td>
</tr>
<tr>
<td>6</td>
<td>20.00</td>
<td>-16.0</td>
</tr>
</tbody>
</table>

TABLE II
THE PARAMETERS FOR SIMULATIONS

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Bandwidth</td>
<td>$B$</td>
<td>7.56 MHz</td>
</tr>
<tr>
<td>Central Frequency</td>
<td>$M$</td>
<td>730 MHz</td>
</tr>
<tr>
<td>GI Length</td>
<td>$N$</td>
<td>256</td>
</tr>
<tr>
<td>OFDM Block Length</td>
<td>$O$</td>
<td>3780</td>
</tr>
<tr>
<td>Modulation Schemes</td>
<td>$O$</td>
<td>16QAM, 256QAM</td>
</tr>
<tr>
<td>Observation Length</td>
<td>$S$</td>
<td>36</td>
</tr>
<tr>
<td>Pilot Number</td>
<td>$T$</td>
<td>36</td>
</tr>
<tr>
<td>Boosted Power</td>
<td>$A$</td>
<td>1</td>
</tr>
<tr>
<td>LDPC Code Rate</td>
<td>$L$</td>
<td>0.6</td>
</tr>
<tr>
<td>LDPC Code Length</td>
<td>$L$</td>
<td>7488</td>
</tr>
</tbody>
</table>

significant taps have been detected already, the complexity of the proposed A-SP is reduced to $O(4J S^2 + 8S^3)(S - S_0)$. So the complexity of A-CoSaMP is lower than that of CoSaMP.

IV. SIMULATION RESULTS

This section investigates the performance of the proposed scheme based on CS under typical broadcasting channels. The low density parity check coded (LDPC-coded) TDS-OFDM adopting the well-known iterative decoding algorithm called belief propagation (BP) with the maximum iteration number of 30 [20]–[22] is used for simulation.

The typical six-path ITU-VB channel model ($L = 152$) [7] and the China digital television test 8th channel model (CDT-8) [23], [24] which has a very strong echo path ($L = 241$) close to the PN sequence length ($M = 256$) are adopted for evaluation, of which the channel parameters are listed in Table I. 256QAM for the static channel and 16QAM with a receiver velocity of 40 km/h are used to evaluate the support for UHDTV and mobile services, respectively. In the CS-based scheme, the last $G = 36$ samples of the IBI-free region are used for CE, so the number of frequency-domain pilots $T = 36$ (around 1% of the subcarrier number $N = 3780$) is utilized in the proposed scheme for fair comparison. All the parameters for simulations are listed in Table II for clear illustration.

Figs. 3 and 4 present the mean square error (MSE) performance comparison of the proposed scheme with the conventional CS-based and DPN-OFDM schemes under the two different channels. It can be seen from Fig. 3 that under the ITU-VB channel, the MSE performance of the proposed scheme enjoys a significant SNR gain of 0.5 dB and 7 dB compared to those of CS-based and DPN-OFDM schemes, respectively, when the target MSE of $10^{-3}$ is considered. If the channel length $L$ is fairly close to the GI length $M$, e.g., the CDT-8 channel considered in Fig. 4, the MSE performance of the proposed scheme is 6 dB better than that of DPN-OFDM, while the recent CS-based TDS-OFDM cannot work due to the reduced size of the IBI-free region. Moreover, the MSE of the proposed scheme is quite close to the Cramér-Rao lower bound (CRLB) which indicates the good performance as well.

Figs. 5 and 6 compare the LDPC-coded bit error rate (BER) performance of the proposed scheme with the conventional CS-based and DPN-OFDM schemes under the two considered channels. The BER performance with the ideal CE is also presented as the benchmark for comparison. As shown in Fig. 5, under the ITU-VB channel, the BER performance
of the proposed scheme enjoys a significant SNR gain of about 0.5 dB and 2.5 dB compared with CS-based and DPN-OFDM schemes, respectively, when the target BER of $10^{-4}$ is considered. It can be observed from Fig. 6 that under the CDT-8 channel, the BER performance of the proposed scheme is about 2 dB better than that of the DPN-OFDM scheme, while the CS-based scheme cannot work due to the IBI-free region is severely contaminated by the channel with long delay spread. In addition, the actual BER curve is very close to that of the ideal case of perfect channel state information, which indicates the good performance of the proposed scheme.

V. CONCLUSIONS

The proposed simultaneous time-frequency CE method based on CS for TDS-OFDM can support the high-order modulations like 256QAM under severe fading channels when the maximum delay is fairly close to the GI length. It also has better performance than the conventional DPN-OFDM and CS-based schemes in both static and mobile scenarios. Thus, the proposed method is expected to extend TDS-OFDM in the emerging UHDTV applications. Moreover, it can be also directly applied to other unique word padded multicarrier systems and CP-OFDM systems with time-domain preamble or TS.

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