The effect of phase matching factor on four wave mixing in optical communication systems: fuzzy and analytical analysis

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Abstract: In this paper, a modified analysis of Four Wave Mixing (FWM) by incorporating the third-order dispersion parameters is reported. With the help of a modified formula for FWM crosstalk, the effect of second- and third-order dispersion parameters at different input channel powers has been analysed. Under the combined effect of second- and third-order dispersion parameters, the crosstalk introduced by FWM is reduced. Further, we propose a fuzzy-based approach using Adaptive-Network-Based Fuzzy Inference System (ANFIS) to calculate FWM power by varying input channel powers at different intensity-dependent phase-matching factors, and observed that the results are very similar to analytical results.

Keywords: FWM; four-wave mixing; dispersion; channel spacing; adaptive neuro fuzzy inference system; ANFIS; adaptive-network-based fuzzy inference system.


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1 Introduction

The latest trend in long-distance light-wave communication systems is towards larger information transmission capacity and longer repeater-less distance. The former requires high bit rates and Wavelength Division Multiplexing (WDM) for increasing the bit rate. When a high-power optical signal is launched into a fibre, the linearity of the optical response is lost. Four Wave Mixing (FWM) is due to changes in the refractive index with optical power, known as the optical Kerr effect. FWM is a third-order non-linearity in silica fibres that is analogous to inter-modulation distortion in electrical systems. When three electromagnetic waves with optical frequencies co-propagate through one fibre, they mix to produce a fourth inter-modulation product. In the FWM effect, three co-propagating waves produce nine new optical sideband waves at different frequencies. When this new frequency falls in the transmission window of the original frequencies, it causes severe cross talk between the channels propagating through an optical fibre. FWM occurs when light of three different wavelengths is launched into a fibre; it gives rise to a new wave. This newly generated wave, as a result of FWM, co-propagates with the originally transmitted signals and interferes with them. It causes severe degradation of the WDM channels (Agrawal, 1995; Tkach et al., 1995a, 1995b; Song et al., 1995; Sharma et al., 2004; Inoue and Toba, 1995; Stolen and Bjorkholm, 1982; Yamamotto and Nakazawa, 1997; Zeiler et al., 1996).

In this paper, we derive a FWM intensity-dependent phase matching factor that includes second- and third-order dispersion parameters. We analyse the effect of the intensity-dependent phase-matching factor over FWM crosstalk under the individual and combined effect of second- and third-order dispersion parameters and different input channel powers. Section 2 discusses the applied fuzzy logic approach, and Section 3 discusses the combined effect of second- and third-order dispersion parameters. The results and discussion are given in Section 4.

2 Analytical analysis

The propagation constant in terms of Taylor series can be expanded as

\[
\beta = \beta_0 + (\omega - \omega_0) \frac{d\beta}{d\omega} + \frac{1}{2} (\omega - \omega_0)^2 \frac{d^2\beta}{d\omega^2} + \frac{1}{6} (\omega - \omega_0)^3 \frac{d^3\beta}{d\omega^3} + \ldots
\]

(1)

where, \(d\beta/d\omega = \tau\) = the propagation delay per optical length. Now

\[
\beta = \beta_0 + (\omega - \omega_0)\tau + \frac{1}{2} (\omega - \omega_0)^2 \frac{d\tau}{d\omega} + \frac{1}{6} (\omega - \omega_0)^3 \frac{d^2\tau}{d\omega^2} + \ldots
\]

(2)

\[
\Delta \beta = \beta - \beta_0 = 2\pi \left[ (f - f_c)\tau + \pi (f - f_c)^2 \frac{d\tau}{d\omega} + \frac{2\pi^2}{3} (f - f_c)^3 \frac{d^2\tau}{d\omega^2} + \ldots \right]
\]

(3)

where the dispersion parameters are defined (Sharma et al., 2004; Tkach et al., 1995b) as

\[
\beta_2 = \frac{d\tau}{d\omega} = \frac{\lambda^2}{2\pi c} \frac{\partial \tau}{\partial \lambda} \text{ is second order dispersion parameter.}
\]

(4)

\[
\beta_3 = \frac{d^2\tau}{d\lambda^2} = \left( \frac{\lambda^2}{2\pi c} \right)^2 \left[ \lambda^2 \frac{\partial^2 \tau}{\partial^2 \lambda} + 2\lambda \frac{\partial \tau}{\partial \lambda} \right] = \frac{\lambda^2}{2\pi c} \left[ \frac{\partial D}{\partial \lambda} + 2\lambda D \right]
\]

(5)

is third order dispersion parameter.

\[
\frac{\partial \tau}{\partial \lambda} = D, \quad \frac{\partial^2 \tau}{\partial \lambda^2} = D_1 \quad \text{and} \quad \frac{\partial^3 \tau}{\partial \lambda^3} = D_2.
\]

We can neglect the second term in equation (6) because it produces only phase delay of the carrier signal and has no influence on the distortion of the signal (Tkach et al., 1995b). Now, putting the values of \(\beta_2\) and \(\beta_3\) in equation (3) and considering the values only up to the third-order derivatives of propagation delay, we get
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\[ \Delta \beta = 2 \pi \left[ \frac{\lambda^2}{2 \pi c} \frac{\partial \tau}{\partial \lambda} \right] \]
\[ + \frac{2 \pi^2}{3} \left( \frac{\lambda^2}{2 \pi c} \right) \left[ \frac{\lambda^2}{\partial D} + 2 \lambda \Delta \right] \]
\[ \Delta \beta = \frac{\pi \lambda^2}{c} \Delta f \left[ D + \frac{\lambda \Delta f}{3 c} \left( 2 D + \lambda \frac{\partial D}{\partial \lambda} \right) \right] \]  
(6)

FWM induced cross talk in WDM systems (Inoue and Toba, 1995; Stolen and Bjorkholm, 1982) and (Yamamoto and Nakazawa, 1997; Zeiler et al., 1996) is given as

\[ P_{\text{FWM}} = \frac{\eta}{\eta - \gamma} \frac{\gamma^2}{|P_1, P_2, P_3|} \exp(-\alpha L) \left[ \frac{1 - \exp(-\alpha L)}{\alpha^2} \right] \sqrt{a^2 + b^2} \]

and the FWM efficiency is expressed as

\[ \eta = \frac{\alpha^2}{\alpha^2 + \Delta \beta^2} \left[ 1 + \frac{4 \exp(-\alpha L) \sin^2(\Delta \beta L / 2)}{(1 - \exp(-\alpha L))^2} \right] \]  
(7)

where \( \Delta \) is channel spacing, \( D \) is fibre chromatic dispersion, \( \frac{\partial D}{\partial \lambda} \) is dispersion slope, \( \alpha \) is the attenuation factor, \( c \) is velocity of light and \( \Delta \beta \) is linear phase matching factor.

The intensity dependent phase matching factor can be defined (Song et al., 1995) as

\[ \Delta \beta' = \Delta \beta - \gamma m (P_1 + P_2 - P_3) \left[ \frac{(1 - \exp(-\alpha L))}{\alpha L_{\text{eff}}} \right] \]  
(8)

where \( m \) is an integer and taken here as 0.63. The phase matching factor changes as the wave propagates along the fibre due to fibre loss. Hence, for long fibres (\( L >> L_{\text{eff}} \)) the effective fibre length is calculated as

\[ L_{\text{eff}} = \frac{1 - \exp(-\alpha L)}{\alpha} = \frac{1}{\alpha} \]

FWM power can be expressed as

\[ P_{\text{FWM}}' = \frac{\eta'}{\eta' - \gamma} \frac{\gamma^2}{|P_1, P_2, P_3|} \exp(-\alpha L) \left[ \frac{1 - \exp(-\alpha L)^2}{\alpha^2} \right] \]  
(9)

FWM efficiency

\[ \eta' = \frac{\alpha^2}{\alpha^2 + \Delta \beta'^2} \left[ 1 + \frac{4 \exp(-\alpha L) \sin^2(\Delta \beta' L / 2)}{(1 - \exp(-\alpha L))^2} \right] \]  
(10)

where

\( \gamma \): Non-linear coefficient = \( (2 \pi n_2)/(\lambda A_{\text{eff}}) \)
\( n_2 \): Fibre non-linear refractive index = 2.68 \times 10^{-20} \text{ m}^2/\text{W} \)
\( P_1, P_2, P_3 \): Input channel powers
\( A_{\text{eff}} \): Effective area of fibre core
\( L_{\text{eff}} \): Effective fibre length.

In this paper, we consider the following cases by varying the value of \( m \):

- If all input channels have equal powers i.e., \( P_1, P_2, P_3 = P_0 \), then, including all pump-induced phase modulation contributions and fibre losses, the intensity dependent phase matching factor is given by Song et al. (1995) as

\[ \Delta \beta' = \Delta \beta - 0.63 \gamma P_0 \]  
(10)

- If we have one input channel, i.e., \( P_1 \), transmitted over a loss less fibre, then the phase modulations induced by stokes wave are negligible and given by Stolen and Bjorkholm (1982) and Yamamoto and Nakazawa (1997) as

\[ \Delta \beta' = \Delta \beta - 2 \gamma P_0 \]  
(11)

- If all input channels have equal powers i.e., \( P_1, P_2, P_3 = P_0 \), and we neglect the stokes wave induced phase modulation and fibre loss, then from Inoue and Toba (1995),

\[ \Delta \beta' = \Delta \beta - \gamma P_0 \]  
(12)

- If two or three input channels are transmitted without considering the all input channels induced phase modulation contributions, then from Tkach et al. (1995a, 1995b), Sharma et al. (2004), Jang (1993) and Takagi and Sugeno (1983)

\[ \Delta \beta' = \Delta \beta \]  
(13)

3 Fuzzy Inference System (FIS)

Fuzzy logic technique contains the potential to give a simplified control for various engineering and non-engineering applications. The rule-based character of fuzzy models allows for a model interpretation in a way that is similar to the one that humans use to describe reality. Conventional methods for statistical validation, based on numerical data, can be complemented by the human expertise that often involves heuristic knowledge and intuition. Despite a number of successful applications, fuzzy modelling is still new in the area of WDM optical communication systems.

A Fuzzy Logic Controller (FLC) consists of fuzzifier, inference engine, fuzzy rule base, and de-fuzzifier. Fuzzy logic techniques have been applied to non-linear, time varying, or non-stationary systems. A fuzzy logic system has the advantage of lower development costs, superior features and better end product performance (Jang, 1993; Takagi and Sugeno, 1983).

The FLC membership functions are defined over the range of input and output variable values and linguistically describe the variables in the Universe of Discourse (UOD). Selection of the number of membership functions and their initial values is based on process knowledge and intuition.
The output from each rule can be treated as a fuzzy singleton. The FLC control action is the combination of the output of each rule using the weighted average defuzzification method and can be viewed as the centre of gravity of the fuzzy set of output singletons.

UOD of the various input and output variables are as follows:

Input variables:
- Channel Power = 5–40 mW.
- Phase matching value = 0–2.

Output variable:
FWM Power and is trained using ANFIS.

3.1 Architecture of the ANFIS

The Adaptive-Network-Based Fuzzy Inference System (ANFIS) (Jang, 1993) can simulate and analyse the mapping relation between the input and output data through a hybrid learning to determine the optimal distribution of membership function. It is mainly based on the fuzzy ‘if-then’ rules from the Takagi and Sugeno type (Takagi and Sugeno, 1983). The equivalent ANFIS architecture of the type from Takagi and Sugeno is shown in Figure 1. It comprises five layers in this inference system. Each layer involves several nodes, which are described by the node function. The output signals from nodes in the previous layers will be accepted as the input signals in the present layer. After manipulation by the node function in the present layer, the output will be served as input signals for the next layer. To explain the procedure of the ANFIS simply, we consider two inputs \( x \) and \( y \) and one output \( f \) in the fuzzy inference system. Hence, the rule base will contain two fuzzy ‘if-then’ rules as follows:

**Rule 1:** If \( x \) is \( A_1 \) and \( y \) is \( B_1 \), then \( f = p_1 x + q_1 y + r_1 \).

**Rule 2:** If \( x \) is \( A_2 \) and \( y \) is \( B_2 \), then \( f = p_2 x + q_2 y + r_2 \).

Nine fuzzy rules obtained from the membership function shown in Figure 1(a) and (b) without subclustering are given below:

If Channel power is \( \text{in} 1 \text{mf1} \) and Phase matching value is \( \text{in} 2 \text{mf1} \) then FWM power is \( \text{mf1} \).
If Channel power is \( \text{in} 1 \text{mf1} \) and Phase matching value is \( \text{in} 2 \text{mf2} \) then FWM power is \( \text{mf2} \).
If Channel power is \( \text{in} 1 \text{mf1} \) and Phase matching value is \( \text{in} 2 \text{mf3} \) then FWM power is \( \text{mf3} \).
If Channel power is \( \text{in} 1 \text{mf2} \) and Phase matching value is \( \text{in} 2 \text{mf1} \) then FWM power is \( \text{mf4} \).
If Channel power is \( \text{in} 1 \text{mf2} \) and Phase matching value is \( \text{in} 2 \text{mf2} \) then FWM power is \( \text{mf5} \).
If Channel power is \( \text{in} 1 \text{mf2} \) and Phase matching value is \( \text{in} 2 \text{mf3} \) then FWM power is \( \text{mf6} \).
If Channel power is \( \text{in} 1 \text{mf3} \) and Phase matching value is \( \text{in} 2 \text{mf1} \) then FWM power is \( \text{mf7} \).

If Channel power is \( \text{in} 1 \text{mf3} \) and Phase matching value is \( \text{in} 2 \text{mf2} \) then FWM power is \( \text{mf8} \).
If Channel power is \( \text{in} 1 \text{mf3} \) and Phase matching value is \( \text{in} 2 \text{mf3} \) then FWM power is \( \text{mf9} \).

Figure 1(a) Input membership function (channel power) of fuzzy logic without subclustering

Figure 1(b) Input membership functions (phase matching value) of fuzzy logic without subclustering

Figure 1(c) Input membership function (channel power) of fuzzy logic with subclustering

Figure 1(d) Input membership function (phase matching value) of fuzzy logic with subclustering

Nine fuzzy rules obtained from the membership function shown in Figure 1(c) and (d) with subclustering are given below:
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If Channel power is in1mf1 and Phase matching value is in2mf1 then FWM power is mf1.
If Channel power is in1mf2 and Phase matching value is in2mf2 then FWM power is mf2.
If Channel power is in1mf3 and Phase matching value is in2mf3 then FWM power is mf3.
If Channel power is in1mf4 and Phase matching value is in2mf4 then FWM power is mf4.
If Channel power is in1mf5 and Phase matching value is in2mf5 then FWM power is mf5.
If Channel power is in1mf6 and Phase matching value is in2mf6 then FWM power is mf6.
If Channel power is in1mf7 and Phase matching value is in2mf7 then FWM power is mf7.
If Channel power is in1mf8 and the Phase matching value is in2mf8 then FWM power is mf8.
If Channel power is in1mf9 and the Phase matching value is in2mf9 then FWM power is mf9.

4 Results and discussion

In our analytical calculations, we assume the case of equal input channel power of the three input channels such that \( P_1 = P_2 = P_3 = P_0 \).

\[ D = \text{Fibre chromatic dispersion} = 0.5 \text{ ps/km-nm} \]
\[ \frac{\partial D}{\partial \lambda} = \text{Dispersion slope} = 0.08 \text{ ps/km-nm}^2 \]
\[ \alpha = \text{Attenuation factor} = 0.25 \text{ dB/km} \]
\[ \lambda = 1550 \text{ nm} \]

In Figure 2, we described the combined effect of first- and second-order dispersion parameters \( \beta_2 \) an \( \beta_3 \) over FWM crosstalk introduced in the optical fibre of effective core area 50 \( \mu \text{m}^2 \) and fibre length of 20 Km at different values of phase matching factors, when three channels, each having equal channel power i.e., \( P_0 \) are transmitted through the fibre. It is clear from Figure 3 that the FWM has the same effect under the combined effect of first- and second-order dispersion parameters i.e., \( (\beta_2 + \beta_3) \) for all the values of \( m \) up to channel power of 10 mw, but as the channel powers increases towards 40 mW, the FWM effect for \( m = 2 \) becomes comparatively more effective. All these values are calculated with fibre loss \( \alpha' = 0.25 \text{ dB} \) and the fibre non-linear refractive index \( n_2' = 2.68 \times 10^{-20} \text{ m}^2/\text{W} \). The pump wavelengths are taken as 1558 nm and 1558.8 nm, resulting in channel spacing of 0.8 nm. For the linear phase matching factor \( m = 0 \), FWM power is calculated as \(-22.8721 \text{ dB} \) at 40 mW while for the intensity dependent phase matching factor \( m = 0.63 \), this value increases to about \(-22.3834 \text{ dB} \). The effect of FWM power at different values of input channel powers and \( 'm' \) can be observed from Figure 2.

But, under the effect of first order dispersion parameter i.e., \( (\beta_2) \), the \( P_{FWM} \) reduces for \( m = 0 \) and \( m = 0.63 \) but increases for the \( m = 1 \) and \( m = 2 \) than the \( P_{FWM} \) under
\( \beta_2 + \beta_3 \) and shown in Figure 4. From Figure 5, it is clear that if \( \beta_2 \) is compensated, the \( P_{\text{FWM}} \) increases to large values and especially for \( m = 2 \). This means that if we transmit a single channel over a loss less fibre, the cross talk introduced by FWM is maximum and it is very low, comparatively, if we transmit three channels of equal channel power, including all pump-induced phase modulation contributions and fibre losses, as shown. It is also clear that under the effect of \( \beta_2 + \beta_3 \), the FWM crosstalk is minimum.

**Figure 4** FWM power vs. input channel power at different values of phase matching factors ‘\( m \)’ under the individual effect of \( \beta_2 \) only with \( L = 20 \) km, channel spacing = 0.8 nm and effective core area \( (A_{\text{eff}}) = 50 \mu m^2 \) (see online version for colours)

![Figure 4](image)

**Figure 5** FWM power vs. input channel power at different values of phase matching factors ‘\( m \)’ under the individual effect of \( \beta_3 \) only with \( L = 20 \) km, channel spacing = 0.8 nm and effective core area \( (A_{\text{eff}}) = 50 \mu m^2 \) (see online version for colours)

![Figure 5](image)

The control surfaces by obtained using fuzzy rule based design gives interdependency of input variables and output variables, and are shown in Figures 6 and 7. The control surface shows that the FWM power is reduced by the combined effect of second- and third-order dispersion. It is observed from Figures 5 and 8 that the results obtained using fuzzy logic are similar to analytical results i.e., FWM power is \(-17 \) dBm at channel power 30 mw and \( m = 1 \) as shown in Figure 8.

**Figure 6** Control surface without subclustering (see online version for colours)

![Figure 6](image)

**Figure 7** Control surface with subclustering (see online version for colours)

![Figure 7](image)

**Figure 8** Fuzzy rules using subclustering (see online version for colours)

![Figure 8](image)

5 Conclusion

In this paper, we concluded that the modified formula for FWM power \( (P_{\text{FWM}}) \) provided better values of \( P_{\text{FWM}} \) at longer optical distances and larger values of input channel powers. The calculated values of \( P_{\text{FWM}} \) under the individual effects of second order \( (\beta_2) \), third order \( (\beta_3) \), and the
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combined effect of second- and third-order dispersion parameters ($\beta_2 + \beta_3$), are more accurate. It is also observed that under the effect of $\beta_2 + \beta_3$, the FWM crosstalk is minimum. We further designed a fuzzy controller for the third order ($\beta_3$) dispersion parameter and observed that the results obtained using fuzzy logic are similar to analytical results.

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