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# Large Right Handed Rotations, Neutrino Oscillations and Proton Decay*

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**Abstract**

Right Handed (RH) rotations are not observable in the standard model (SM). The freedom is used to give the phenomenologically correct mass matrices all kind of different forms and this is one of the reasons for the proliferation of models for the fermionic masses.

The SM must be however extended and in most extensions, RH currents appear at higher energy scales. At those energies the RH rotations are not irrelevant any more, they can affect neutrino oscillations, proton decay, baryon assymetry, R-parity violating interactions etc.

We study possible implications of large RH mixing in GUTs. Those are interesting not only because large mixing induce large effects. They are intimately related to large lepton mixing in GUTs, via a relation between LH mixing of the leptons and the RH ones of the d-quarks ,("$d - \ell$ duality") and can change considerably the branching ratios of proton decay. Observation of proton decay channels as well as neutrino oscillations will teach us about RH rotations and will reduce, therefore, considerably the freedom in the fermionic mass matrices. Some interesting examples are studied in detail. In particular a new $E_6$ model which realizes naturally a $d$ - $\ell$ duality by mixing with exotic $E_6$ fermions.

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Superkamiokande [1] confirmed in this conference once again the observation that large neutrino mixing is responsible for the atmospheric neutrino anomaly. This leads to a puzzle: how can leptons have large left-handed (LH) mixing in clear contradiction with the small LH mixing of the quarks. I am emphasizing "LH mixing" because the fermions acquire also right-handed (RH) mixing. Those are unobservable in the standard model (SM) but may play an important role in its extensions [2] [3]. The aim of this talk is to emphasize the importance of RH rotations and the fact that they may be the solution of the mixing puzzle. In particular in terms of the *duality* observed in certain GUTs between d-quarks and charged leptons [4] [5] [6]:

*RH rotations of d-quarks* $\longleftrightarrow$ *LH rotations of charged leptons*
*LH rotations of d-quarks* $\longleftrightarrow$ *RH rotations of charged leptons.*

This means that the observed large LH mixing of the leptons corresponds not to the LH mixing of the quarks but to their large RH mixing which is not observable in the SM.
I would like to present in this talk some examples of this "duality" but before that let me explain what are the RH rotations and why they are important and "observable".

*What are RH rotations?*

reiTo diagonalize a general complex (mass) matrix M one needs a bi-unitary transformation, i.e. two unitary matrices $U_{L,R}$ , such that

$$U_L{}^{\dagger} M U_R = M_{diagonal} \tag{1}$$

or

$$U_L{}^{\dagger} M M^{\dagger} U_L = (M_{diag.})^2 = U_R{}^{\dagger} M^{\dagger} M U_R \quad . \tag{2}$$

Only in the case of hermitian (symmetric) matrices is $U_R$ related to $U_L$

$$M = M^{\dagger}(M^T) \implies U_R = U_L(U_L{}^*) \quad . \tag{3}$$

RH fermions are singlets in the SM and only LH charged currents are involved in the weak interactions

$$\mathcal{L}_W = W_\mu \overline{u_L} \gamma^\mu V_{CKM} d_L \; + \; h.c. \tag{4}$$

where

$$V_{CKM} = U_L^{u\dagger} U_L^d \quad .$$

The neutral currents are not affected by RH mixing (as long as the $U_R's$ are unitary). However RH rotations are involved in many extensions of the SM, especially in GUTs. They affect therefore phenomena like: [1]

(i) Proton decay,
(ii) Neutrino oscillations,
(iii) Leptogenesis via decays of RH neutrinos as the origin of baryon asymmetry [9],
(iv) R-parity violating or leptoquark induced contributions,
(v) Radiative corrections etc.

The SM must be extended (at least) to explain the origin of the fermionic mass matrices, hence: *one cannot simply neglect the RH rotations.*

[1] Note, that the observation of RH contributions to B decay is still an open possibility [7]. Also, the $\Lambda_b$ polarization observed in ALEPH [8] may be an indication for a RH effect.

Large RH mixing angles are not unnatural and can be induced via a symmetry. E.g. using $P_{LR}$ invariance [10] – a generalization of Parity for gauge theories. One can show that a strong RH rotations of the light families will force the proton to decay mainly into kaons [3]

$$P \longrightarrow \bar{\nu}_\mu K^+ \qquad \text{and} \qquad P \longrightarrow \mu^+ K^0.$$

even without SUSY. But also in SUSY-GUTs the branching ratios of the proton decay will be drastically changed [11] [12].

*Down-Lepton Duality*

**SU(5)**: The duality comes about naturally already in this minimal GUT.
This is because the fermions are distributed in the families $(\bar{\mathbf{5}}+\mathbf{10})^i, \quad i = 1,2,3.$, in such a way that $e_L$ and ${d_L}^c$ are always in the same representation, but not $e_L$ and $d_L$ etc.

$$\psi(\bar{\mathbf{5}}) = \begin{pmatrix} \vec{d^c} \\ e^- \\ \nu \end{pmatrix}_L \qquad \chi(\mathbf{10}) = (\vec{u^c}, \begin{pmatrix} \vec{u} \\ \vec{d} \end{pmatrix}, e^c)_L \quad .$$

Hence, if only $H_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ are used to give the fermions masses, as e.g. in

$$y_{ij}\ \psi^i_{\bar{\mathbf{5}}}\ {\chi_{\mathbf{10}}}^j < H_{\bar{\mathbf{5}}} >$$

one obtains the mass relation

$$m^\ell = (m^d)^T$$

which realizes the *$d$ - $\ell$ duality* [4]. However, $H_{\bar{\mathbf{5}}}$ and $H_{\mathbf{5}}$ alone cannot account for the masses of the light families and adding other Higgs representations will break this duality.

**SO(10)**: The whole family (with $\nu_R$) is here in one irrep. $\Psi_{\mathbf{16}}$, but there is a "memory" of $SU(5)$ in the coupling. It is possible, therefore, to use combinations of symmetric and antisymmetric couplings to generate asymmetric mass matrices which give approximately the $d$ - $\ell$ duality. This is usually done in terms of a broken $U(1)$ family group and the use of non-renormalizable contributions [5]. However, the matrix elements are fixed in this framework only up to $O(1)$ factors. This leads to factors $[O(1)]^3$ in the see-saw neutrino mass matrix, which means that this method is, in general, not suitable for predictions in the neutrino sector.
It is possible also to use models without non-renormalizable contributions and a conserved $U(1)$ or $Z_n$. This symmetry dictates the couplings and the mass matrices textures such that the experimentally known parameters, i.e. masses and mixing of the quarks in addition to the masses of the charged leptons, dictate (up to one parameter) the properties of the neutrinos [13]. $SO(10)$ is broken in this model, as follows:

$$SO(10) \xrightarrow{M_U} SU_C(4) \times SU_L(2) \times SU_R(2) \xrightarrow{M_I} SM, \tag{5}$$

where $M_U$ and $M_I$ are fixed by the requirement of unification to be:

$$M_U = 1.31 \times 10^{16} \qquad M_I = 6.14 \times 10^{10} \tag{6}$$

in terms of the required Higgs representations.
One can choose then the free parameter such that the solution will account for the observed atmospheric neutrino anomaly together with either large-angle MSW or small-angle

MSW. Also, specific predictions for the Proton (and Neutron) decay branching ratios are obtained, which differ from the conventional $SO(10)$ prediction [14].

| *channel* | *conventional* (Kane + Karl) [15] | *LA MSW* (Achiman+Merten) | *SA MSW* (Achiman+Merten) |
|---|---|---|---|
| $e^+\pi^0$ | 38% | 21% | 25% |
| $\nu\pi$ | 15% | 35% | 36% |
| $\frac{e^+\pi^0}{\nu\pi}$ | 25 | 0.6 | 0.7 |
| $\mu^+\pi^0$ | $\sim 0$ | 8.5% | 6% |
| $\mu^+K^0$ | 18% | 2.6% | 1% |
| $\nu K^+$ | $\sim 0$ | 3.5% | 2.3% |

We see that the neutrino channels are enhanced with respect to the charged lepton ones.

$$\tau(e^+\pi^0) = 10^{34\pm 1.7} yrs, \tag{7}$$

i.e. in the range of possible observation by s-kamiokande.

*A new kind of model for the mass matrices in $E_6$ GUT* [16][17].

$\mathbf{E_6}$: Under $E_6 \supset SO(10) \times U(1)$ one family decomposes as

$$\mathbf{27}_{E_6} = (\mathbf{16} + \mathbf{10} + \mathbf{1})_{SO(10)} \quad .$$

The "exotic fermions" $(\mathbf{10} + \mathbf{1})_{SO(10)}$ are denoted as follows:

$$\mathbf{10}_{\mathbf{5}+\bar{\mathbf{5}}} = (\vec{D}, E^c, N^c) + (\vec{D}^c, N, E) \qquad ; \qquad \mathbf{1_1} = L$$

(where $D(E)$ are $d(e)$-like quarks(leptons)).

*The main property of this model* [18] *is obtained by mixing the light fermions with these exotic ones*:
One starts with all light mass matrices $m_u$,$m_d$,$m_\ell$ and $m_{\nu_D}$ diagonal and equal (via a light $< H_{\mathbf{27}} >$). The heavy Higgs representations generate VEVs in a different direction. These give the exotic fermions heavy masses and mix them with the light ones. The mixing leads then to the "observed" mass matrices. In particular, $m_d$ obtains off-diagonal contributions due to mixing with the $D_i$ but $m_u$ remains diagonal as there are no $u$-like exotic fermions. (Rosner [19] suggests that this is the reason why $m_{d_i} < m_{u_i}$). The special thing about this scenario is that it realizes naturally the $d - \ell$ duality. This is due to the opposite sign of the charges of the leptons ($e(-1)$) and the d-quarks ($d(2/3)$). In the Yukawa coupling $d^c D$ goes therefore with $E^c e$ and $D^c d$ with $e^c E$ [19][6]. I.e. the mixing of $(d_i)^c$ corresponds to the mixing of $e_i$ etc.
Hence:

$$\textit{large RH mixing of } d_i \longleftrightarrow \textit{large LH mixing of } e_i \quad .$$

In contrast with $SU(5)$ [4] or $SO(10)$ [5] models the duality here is independent of the details of the Higgs representations.

Let me give now some more details of the model. The most general mass matrix of $d, d^c, D, D^c$ [19] ($d(I_L = 1/2)$ and $d^c, D, D^c$ all have $I_L = 0$)

$$M_d = \begin{array}{c} \\ d^c \\ D^c \end{array} \begin{array}{c} \begin{array}{cc} d & D \end{array} \\ \left( \begin{array}{cc} m & \bar{M} \\ \bar{m} & M \end{array} \right) \end{array},$$

$m$ – the pure "light" $3 \times 3$ mass matrix (EW scale)
$\bar{m}$ – the "light" $3 \times 3$ mixed matrix ($\Delta I_L = 1/2$ , EW scale)
$M$ – the pure "'heavy" $3 \times 3$ mass matrix ($\Delta I_L = 0$ , heavy exotic scale)
$\bar{M}$ – the mixed "heavy" $3 \times 3$ mass matrix ($\Delta I_L = 0$ , heavy exotic scale).
For the charged leptons we have (note that there are here different Clebsch-Gordan factors):

$$M_e = \begin{array}{c} \\ e^c \\ E^c \end{array} \begin{array}{c} \begin{array}{cc} e & E \end{array} \\ \left( \begin{array}{cc} m_1 & \bar{m}_1 \\ \bar{M}_1 & M_1 \end{array} \right) \end{array},$$

To find the physical states, one must diagonalize the mass matrices. To get the LH rotations one uses :

$$M_d^\dagger M_d = \left( \begin{array}{cc} m^\dagger m + \bar{m}^\dagger \bar{m} & m^\dagger \bar{M} + \bar{m}^\dagger M \\ m^\dagger \bar{M} + \bar{m}^\dagger M & \bar{m}^\dagger \bar{M} + M^\dagger M \end{array} \right) \tag{8}$$

and the RH rotations are obtained by diagonalization of:

$$M_d M_d^\dagger = \left( \begin{array}{cc} m m^\dagger + \bar{M}\bar{M}^\dagger & m\bar{m}^\dagger + \bar{M}M^\dagger \\ m\bar{m}^\dagger + \bar{M}M^\dagger & \bar{m}\bar{m}^\dagger + MM^\dagger \end{array} \right) \tag{9}$$

If $\quad m, \bar{m} << M, \bar{M}$ , $\quad M_d^\dagger M_d \quad$ is similar to the see-saw matrix, so that

$$d \approx d_0 - \frac{\bar{m}^2}{M^2} D_0 \qquad D \approx \frac{\bar{m}^2}{M^2} d_0 + D_0,$$

i.e very small heavy mixing.
For $M_d M_d^\dagger$ it is possible to use the same approximation only if $\quad m, \bar{m} << \bar{M} << M$.
In this case one obtains for the RH part:

$$d^c \approx d_0^c - \frac{\bar{M}^2}{M^2} D_0^c \qquad D \approx \frac{\bar{M}^2}{M^2} d_0^c + D_0^c,$$

i.e. the heavy-light RH mixing is in general larger.

The $3 \times 3$ light-light mixing matrices are in general not diagonal even if $m$ was diagonal. They are also not unitary but for the LH mixing the deviation is very small $O(\frac{m^2}{M^2})$. For the RH rotations however the deviations from unitarity are of $O(\frac{\bar{M}^2}{M^2})$. Those must be also small to avoid inconsistency with experimental limits on the neutral current and in particular from $\quad e^+e^- \longrightarrow Z \longrightarrow b\bar{b}$. [2] Very heavy $M$ and/or small heavy-light mixing

[2]This is an indication that the scale of $M$ should be quite high. Also, note that already $\frac{\bar{M}}{M} = \frac{1}{10}$ gives $g_{bR}\delta g_{bR} \approx 10^{-3}$, hence the heavy scale difference must not be very large.

does not exclude large RH rotations. The Family mixing in the light-light matrix is more dependent on the light mixed matrix $\bar{m}$.
One can show [18] that it is possible to choose the heavy mass matrices in such a way that the known masses and mixing are obtained (at least for two families).

*Conclusions about RH rotations*

$d - \ell$ duality, especially in $E_6$, is a natural explanation for the contradiction between the large LH mixing angles of the leptons and the small ones of the quarks.

Many models are known to be able to give the fermionic masses, $V_{CKM}$, $\nu$- properties etc. (within the experimental errors). This is only an indication that the mass question is far from from being solved. Part of the problem is related to the fact that those models disregard the RH rotations.
The hope is that correlations between neutrino physics, proton decay, leptogenesis, baryon assymetry etc. will tell us something about the RH rotations. This will limit considerably the freedom in the fermionic mass matrices.

Many recent models use asymmetric mass matrices which induce large RH mixing. One cannot simply neglect this fact and must consider possible implication of the RH rotations to have a complete model.

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