On the Impact of Uplink Power Control in Network MIMO Systems with MMSE and SIC Receivers

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Abstract—Network multiple input, multiple output (MIMO) systems are built around a broadband backbone network that allows for the fast communication of channel state information (CSI) as well as user data between different base stations. Previous works have shown that multicell channel adaptive (opportunistic) power control can minimize the sum power or maximize the sum rate when the backbone is used for the exchange of CSI in network MIMO systems. In this work we investigate the gains of multicell opportunistic power control under per user fairness constraints when both CSI and user data are shared between multiple sites. We find that multicell opportunistic power control working in concert with uplink joint signal detection is an efficient means both for the capacity and the power control problems that not only minimizes sum power or maximizes overall capacity, but is also able to provide arbitrary level of fairness.

I. INTRODUCTION

Network multiple input multiple output (MIMO) systems built around a broadband backbone network have been demonstrated to increase the overall capacity of multicell systems [1], [2], [3]. Fast coordination allows joint transmitter algorithms for the downlink [4] and joint receiver algorithms for the uplink [5]. Along a bit different line, some works have explored the gains of network MIMO due to multicell coordinated scheduling, power control and precoding [6], [7], [8] rather than exploiting the potential of joint signal processing techniques.

Recently, opportunistic multicell SINR target setting and power control have been proposed as a means to minimize the sum power subject to some capacity constraint (the power control problem) or to maximize the sum rate subject to a sum power constraint (the capacity problem) [9], [10]. These findings are in line with earlier results on opportunistic power control that suggest that under a sum power constrained most power should be used for the strongest user link [11].

Utilizing a fast high capacity backbone both for multicell radio resource management and in particular power allocation and for coordinated signal processing (in particular for joint detection for the uplink) appears to be an appealing approach to maximize system capacity. Intuitively, if multicell joint processing effectively converts the multicell system to a (virtual) single cell (multiuser MIMO) system, then the core idea of single cell opportunistic power control becomes applicable also in the network (virtual) MIMO system.

Therefore, in this paper our objective is to gain an insight into the potential gains of multicell uplink power control without and with uplink joint detection. The gains we are interested in include the sum power saving stemming from power minimization over multiple cells and capacity enhancement by means of sum rate maximization. Our basic methodology is to compare these types of gains for systems without and with joint signal processing and thereby to gain an insight into the benefit of joint power control on top of joint processing.

The paper is structured as follows. First we describe our system model, where we put focus on how we capture the key system characteristics for joint channel state information and data sharing that are necessary for multicell resource management and signal processing. The key modeling element here is the mapping of the overall channel matrix of the physical system to a channel matrix structure that reflects the relationships of users and the uplink receive antennas in the network MIMO system. The problem of optimum target SINR selection and power allocation (for the power control as well as the capacity problem) is formulated in the subsequent section (Section III). Next, in Section IV we propose a solution approach based on the augmented Lagrangian penalty function (ALPF). Section V is dedicated to numerical results and Section VI highlights our conclusions.

II. MODEL DESCRIPTION

We consider a system of $N_{BS}$ physical base stations\(^1\), each base station having for simplicity $n_r$ receive antennas; with a total number of receive antennas of $N_r \equiv \sum n_r = N_{BS} \cdot n_r$, where the summation runs through all physical base stations. The system complies of $K$ users, each of them is provided with $n_t$ transmit antennas and multiplexes in the spatial domain $n_t$ streams by use of a linear spatial precoder $T_k$. We focus on a narrowband flat channel thus modeling, e.g., a generic subcarrier for an OFDM system. In our generalized framework we allow multiple base stations to cooperate in the decoding of the users in a configurable way. On one hand we introduce the concept of virtual base station as a cluster of base stations that cooperate in the reception of a specific user. Therefore,

\(^1\)Could also be sectors; for ease of presentation, we use the term base station.
each user is associated with a single virtual base station. On the other hand, we develop a common framework that models different receiver algorithms. Assume that user $k$ is associated with a virtual base station comprising of $V_k$ base stations. Thus, the signal model corresponding to the virtual base station associated with the $k$th user reads as:

$$\mathbf{y}_k = \alpha_k \cdot \Psi_{k,k} \cdot \mathbf{T}_k \cdot \mathbf{x}_k + \sum_{j \neq k} \alpha_j \cdot \Psi_{k,j} \cdot \mathbf{T}_j \cdot \mathbf{x}_j + \mathbf{n}_k, \quad (1)$$

where

- $\alpha_j = \sqrt{P_j/\sigma^2}$ is a scalar coefficient depending on the total transmit power $P_j$ of user $j$;
- $\Psi_{k,k}$ and $\Psi_{k,j}$ are the $V_k n_r \times n_t$ dimensional channel coefficient matrices, as they will be discussed later;
- $\mathbf{x}_k$ and $\mathbf{x}_j$ are the $n_t$ dimensional transmitted own and interfering signal vectors respectively, and
- $\mathbf{T}_k$ is the MS-$k$ ($n_t \times n_t$) diagonal power loading matrix.

To keep the total transmit power constant, $\mathbf{T}_k$ must satisfy:

$$\text{trace}(\mathbf{T}_k \mathbf{T}_k^H) = \sum_{i=1}^{N_t} \left| \mathbf{T}_k^{(i,i)} \right|^2 = N_t \quad \forall k;$$

- $\mathbf{n}_k$ is a $V_k n_r \times 1$ additive white Gaussian noise vector at the $k$th virtual base station with zero mean and covariance matrix $\mathbf{R}_{n_k} \triangleq \mathbb{E}(\mathbf{n}_k \mathbf{n}_k^H) = \sigma^2 \mathbf{I}_{n_r}; \quad \forall k.$

We note that the underlying assumption of the last bullet item on equal noise covariance for all base stations is reasonable for a set of base stations with the same antenna configuration and other physical and hardware characteristics within a limited geographical region and is often used both in the literature and in standardization [12], [13], [14].

Let $\gamma_k$ be the SINR measured on the equalized signal given a specific equalization scheme. Under certain idealized conditions (e.g., ideal channel estimation, asymptotically long codewords, error-free link adaptation, etc.) the spectral efficiency achieved by the $k$th user with vanishing error rate is

$$c_k(\gamma_k) = \log_2(1 + \gamma_k). \quad (2)$$

To allow closed form analysis we assume that each user can reach its theoretical spectral efficiency $c_k(\gamma_k)$.

The "virtual channel matrix" $\Psi_{k,j}$ in (1) is a tool that allows us to model different equalization schemes such as linear MMSE and iterative SIC by simply redefining an auxiliary joint detection matrix $\mathbf{U}_j$, which will be introduced in the following. Let the $(n_r \times n_t)$ channel between user $j$ and a generic base station $n$ be $\mathbf{H}_{n,j}$. The composite channel between user $j$ and virtual base station $k$ is thus obtained by stacking the $V_k$ channels corresponding to the base stations belonging to the selected virtual base station:

$$[\mathbf{H}_{1,j}^H \cdots \mathbf{H}_{k,j}^H \cdots \mathbf{H}_{V_k,j}^H]^H \quad (3)$$

where for simplicity and with a slight abuse of notation the physical base stations belonging to virtual basestation $k$ have been indexed from 1 to $V_k$.

The "virtual channel matrix" $\Psi_{k,j}$ in (1) is obtained by mapping the physical channel matrix $\mathbf{H}_{k,j}$ according to the ($V_k \times K$) joint detection matrix $\mathbf{U}_j$:

$${\Psi}_{k,j} = \begin{pmatrix} \mathbf{U}_k(1,j) \cdot \mathbf{H}_{1,j} \\ \vdots \\ \mathbf{U}_k(n,j) \cdot \mathbf{H}_{n,j} \\ \vdots \\ \mathbf{U}_k(V_k,j) \cdot \mathbf{H}_{V_k,j} \end{pmatrix}. \quad (4)$$

Specifically, the matrix $\mathbf{U}_k$ contains a 1 in the positions corresponding to users that contribute to the signal to be equalized, and it contains a 0 in the positions corresponding to user signals that have been perfectly canceled from the received signal before equalization of user $j$, e.g., because of some previous iteration of a SIC receiver. Some examples of the definition of $\mathbf{U}_k$ for various equalization strategies are reported in the following subsections.

A. Example 1: Single cell detection

When traditional single cell detection is performed, each base station decodes the signal belonging to the subset of the users which is associated with that base station, without any information flow between different base stations. Therefore, each user is decoded by a single base station, and all the other users in the system contribute to interference. This is easily modeled by setting all the joint detection matrices $\mathbf{U}_k$ to row vectors containing a 1 in all the elements. It implicitly follows that $V_k = 1$ for all the users in the system.

B. Example 2: Multi-cell linear joint detection (network MIMO)

As a second example, consider a system where groups of $V_k > 1$ base stations cooperate for the detection of a generic $k$th user. We assume here that a common linear equalization (MMSE) algorithm is performed by joining the signals received by all the $V_k$ base stations. This implies either a common baseband processing unit for the whole virtual base station or real-time baseband signals flow between different base stations. This setting is modeled by defining joint detection matrices as matrices of ones having $V_k$ rows and $K$ columns. Of course the specific channels that are stacked in the $\Psi_{k,j}$ matrix define which base stations are included in the virtual base station that performs detection of user $k$. As an example, the matrix

$${\Psi}_{k,j} = [\mathbf{H}_{b_1,j}^H \cdots \mathbf{H}_{b_2,j}^H \cdots \mathbf{H}_{b_3,j}^H]^H \quad (5)$$

defines that base stations $b_1$, $b_2$ and $b_3$ jointly cooperate for the detection of the $k$th user. The joint detection matrix associated with virtual base station $k$ is now a $(V_k \times K)$ ones matrix.

C. Example 3: Multi-cell full SIC joint detection (network MIMO)

Hybrid successive interference cancellation (within a virtual base station) implies that (with proper numbering of the users), User-$j$ does not cause interference to User-$k$ if $k > j$ and if both users are decoded by the same virtual base station. The first detection step is based on conventional MMSE, therefore the first decoded user experiences exactly the same SINR as
in example 2. If the detection of the first user succeeds, then
– under the assumption of ideal channel estimation – it is
possible to remove the contribution of the decoded user from
the signal received by the correspondent virtual base station.
Thus, the detection of the second user associated to the same
virtual base station does not suffer from interference from the
previously decoded user. This process is iterated until all the
users belonging to the virtual base station have been decoded.
The performance of SIC can be enhanced by optimization of
the order of cancelation. However, since the optimal detection
order is not easily estimated in most practical applications,
we neglect detection order optimization in this work. SIC is
readily modelled by use of the joint detection matrices \( U_{k,j} \).
Assume, e.g., a system with 3 users (labelled from 0 to 2) and
a single virtual base station including 2 base stations (labelled
0 and 1). The joint detection matrix for the 3 users reads as:

\[
U_0 = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
0 & 1 & 1
\end{bmatrix},
U_1 = \begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 1 \\
1 & 0 & 1
\end{bmatrix},
U_2 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

D. Example 4: Multi-cell hybrid SIC joint detection (network
MIMO)

In this subsection we model a simplified SIC algorithm for
virtual base stations, where each user is decoded by a single
base station, but communication of the decoded user bits is
allowed between base stations belonging to the same virtual
base station. To understand the algorithm consider a system
with one virtual base station composed by 3 base stations
(indexed 0 to 2) and 3 users (indexed 0 to 2). User 0 is decoded
by base station 0, which successively sends its information bits
to the other physical sites within the virtual base station. Base
station 1 and 2 are so able to cancel the contribution of user 0
from their received signal (given that they are able to estimate
the channel providing from that user). User 1 is then decoded
by base station 1 and its information bits are sent to base
station 2. Finally, base station 2 decodes user 2 after having
removed both the interference from user 0 and 1.

The scenario described in this subsection is modeled by the
following joint decoding matrices:

\[
U_0 = \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
0 & 1 & 1
\end{bmatrix},
U_1 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1
\end{bmatrix},
U_2 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Compared to a full SIC algorithm as in Example 3, the hybrid
SIC shows significant implementation advantages at the likely
cost of reduced performance. In particular, hybrid SIC does
not require communication of the baseband signals between
different base stations (which need accurate quantization) but
only of the decoded binary sequences. Furthermore, only the
antennas belonging to a given base station are employed in the
equalization process, thus simplifying the equalization process.

III. PROBLEM FORMULATION
A. Calculating the SINR in the Network MIMO System

To calculate the SINR, we now recall the following key rela-
tions from [14] on multi-cell systems without joint processing
and apply it to the network MIMO system of the present study.
The key observation is that by replacing the physical channel
matrix \( H \) by the "virtual channel matrix" \( \hat{\Psi} \) we can make
use of the results of [10]. We rewrite the signal model (1) in
a compact form as

\[
y_k = \alpha_{k,k} \Psi_{k,k}^T x_k + z_k + n_k,
\]

where \( z_k = \sum_{j \neq k} \alpha_{k,j} \Psi_{k,j}^T x_j \) denotes the \((V_k \times 1)\)
interference vector from users in other cells, with covariance
matrix

\[
R_{z_k} \triangleq \mathbb{E} \left( z_k z_k^H \right) = \sum_{j \neq k} \alpha_{k,j}^2 \Psi_{k,j}^T T_j \Psi_{j,k}^H.
\]

For ease of notation, we define an equivalent noise vector that
accounts for both inter-cell interference and background noise

\[
v_k = z_k + n_k.
\]

It is easy to show that \( v_k \) is zero-mean with covariance \( R_{vk} = R_{z_k} + R_{nk} \), see the Appendix of [10].

In this work we assume that the received signal is filtered
through a linear MMSE receiver with weighting matrix \( G_k \) to
obtain the \((n_t \times 1)\) dimensional estimate

\[
\hat{x}_k = G_k y_k,
\]

where the \((n_t \times V_k)\) linear MMSE weighting matrix \( G_k \) can
be expressed as:

\[
G_k = \frac{1}{\alpha_{k,k}} T_k \Psi_{k,k}^T \left( \Psi_{k,k} T_k \Psi_{k,k}^T + \frac{1}{\alpha_{k,k}} R_{z_k} \right)^{-1}
= \left( I + T_k \Psi_{k,k} R_{z_k} T_k \Psi_{k,k}^T \right)^{-1} \alpha_{k,k} T_k \Psi_{k,k} R_{v_k}^{-1},
\]

where \( R_{v_k} \triangleq \alpha_{k,k}^2 \Psi_{k,k}^T R_{z_k} \Psi_{k,k} \), see e.g. [15, Chapter 12].

To proceed, we also recall the following result from [14].

\[
\gamma_k = \min_{s \in [1, n_s]} \gamma_{k,s} \geq \gamma_{k} (p)
\]

where \( p = (P_1 \ldots P_K)^T \) is the power allocation vector, and

\[
\gamma_k (p) = \frac{P_k d_{k,k}^H \chi_{k,k}}{\sum_{j \neq k} P_j d_{j,j}^H \chi_{j,j} + \mu_{\text{max}}(\Omega_{k,j,1} + \Gamma n_t \chi_{k,k}) + \mu_{\text{max}}(\Omega_{k,j,2})}.
\]

Here, \( \mu_{\text{max}}(\cdot) \) is the maximum eigenvalue operator for a
Hermitian matrix, while \( \Omega_{k,j,1} \) and \( \Omega_{k,j,2} \) are defined as

\[
\Omega_{k,j,1} = \left( \Psi_{k,k}^T \Psi_{k,k} \right)^{-1} \Psi_{k,k}^T \Psi_{k,j} \Psi_{k,j}^T \Psi_{k,k} \Psi_{k,j}^T \Psi_{k,k} \Psi_{k,k}^{-1},
\]

\[
\Omega_{k,j,2} = \left( \Psi_{k,k}^T \Psi_{k,k} \right)^{-1}.
\]

This bound allows to associate a single SINR value

\[
\gamma_k (p) \triangleq \min_{s \in [1, n_s]} \gamma_{k,s}
\]

with each MS-\( k \). In what follows, we will search for SINR
targets \( \gamma_k^{\text{tgt}} \) which are feasible for the lower-bound (and hence
for each individual stream) and let \( \Gamma = \text{diag}(\gamma_1^{\text{tgt}}, \ldots, \gamma_K^{\text{tgt}}) \).
B. Minimizing Sum Power Under Fixed SINR Target

The above result was used in [14] to design power control schemes which maintain a fixed minimum SINR target $\gamma_{k}^{\text{tgt}}$ for every stream $s$ by enforcing $\bar{\gamma}_k(p) \geq \gamma_{k}^{\text{tgt}}$ for each user. As shown in [14], the transmit power of MS-$k$ must satisfy:

$$P_k \geq \gamma_{k}^{\text{tgt}} \cdot \left( \sum_{j \neq k} P_j \cdot d_{k,j}^{-2} \right) \cdot \left( \frac{\sigma_n^2 N_0}{d_{k,k}^{-2} \chi_{k,k}} \right) + + \frac{\sigma_n^2 N_0 \chi_{k,k}}{d_{k,k}^{-2} \chi_{k,k}}. $$

Moreover, the power vector that satisfies this requirement and minimizes the sum power is:

$$p^* = (I - \Gamma F)^{-1} \Gamma n, \quad (11)$$

where $n$ is a $K$-dimensional effective noise variance vector whose $k^\text{th}$ element is $\eta = \frac{N_0 \sigma_n^2 \chi_{k,k}}{d_{k,k}^{-2} \chi_{k,k}}$, and

$$F_{k,j} = \begin{cases} \frac{d_{k,j}^{-2} \chi_{k,j} \chi_{j,k}}{d_{k,k}^{-2} \chi_{k,k}} & k \neq j; \\ 0 & k = j. \end{cases} \quad (12)$$

Still, Equation (11) requires feasibility of the SINR targets, which in practice can not be guaranteed à priori. Precoding optimization was showed to be effective to balance the conservativeness of the bound (8) and increase feasibility, see [14].

C. Optimal SINR Target Selection: The Power Control and the Capacity Problems

In this paper we take a step further and explore a key observation, not fully exploited in [14]. Since the minimum user-stream SINR bound (8) allows to associate a single SINR target per user, one can regard each MS-BS connection as an equivalent SISO system and model the minimum user-stream capacity as function of the power allocation with a Shannon-like expression (normalized to the bandwidth) as

$$c_k(\gamma_{k}^{\text{tgt}}) = \log_2(1 + \gamma_{k}^{\text{tgt}}) \quad \forall k, \quad (13)$$

where we enforce

$$\bar{\gamma}_k(p) = \frac{P_k}{\eta_k + \sum_{j \neq k} G_{kj} P_j} \geq \gamma_{k}^{\text{tgt}} \quad \forall k, \quad (14)$$

with $G = I + F$. This observation is the basis for optimizing the minimum user-stream SINR targets.

In network MIMO, it is possible (and as he shall see beneficial) to exploit the possibility to set the SINR targets such that the sum power is kept at a minimum level and the overall system capacity (sum rate) target $c_m$ is reached. This problem is formulated as follows:

$$\text{minimize } \Gamma, p \quad \sum_k P_k$$

subject to:

$$\sum_k c_k(\gamma_{k}^{\text{tgt}}) \geq c_m \quad (15)$$

$$\gamma_{k}^{\text{tgt}} = \bar{\gamma}_k(p) \quad \forall k,$$

in the optimization variables $\Gamma$ (SINU targets) and $p$ (power). We are also interested in the dual formulation of problem (15), that is, maximizing the multi-cell capacity (sum rate) subject to a total power budget

$$\text{maximize } \Gamma, p \quad \sum_k c_k(\gamma_{k}^{\text{tgt}})$$

subject to:

$$\sum_k P_k \leq P_{\text{tot}} \quad (16)$$

$$\gamma_{k}^{\text{tgt}} = \bar{\gamma}_k(p) \quad \forall k.$$

D. Enforcing Fairness Constraints

Fairness can be enforced in the above formulations by limiting the ratio between SINR targets, i.e.

$$\gamma_{k}^{\text{tgt}} \leq \Phi_{kj} \gamma_{j}^{\text{tgt}} \quad \forall k, j \neq k. \quad (17)$$

The matrix $\Phi$ collects the fairness ratios. These constraints are written more compactly as $\alpha(\Gamma) \preceq \beta(\Gamma)$, where $\alpha = \text{vec}(a_1 \ldots a_k)$ with $a_k = (1 - e_k) \gamma_{k}^{\text{tgt}}$, and $\beta(\Gamma) = \text{vec}(\Phi^T \gamma_{\text{tgt}})$. To account for fairness constraints, we include the inequalities $\alpha(\Gamma) \preceq \beta(\Gamma)$ in (15)-(16). In what follows, we develop a novel efficient SINR-target optimization procedure and combine it with iterative algorithms for power loading optimization. As we will see, the minimum SINR bound (8) is quite conservative and including the power loading matrix $T_k$ in the optimization is instrumental to enhance the performance.

IV. SOLUTION APPROACHES

In [10] we proposed several methods to solve problems (15) and (16). Since these advanced techniques will be used to investigate the impact of the uplink power control in network MIMO systems, we will review them briefly below.

A. Augmented Lagrangian Method

We first propose to solve the problems formulated in Section III-C through the augmented Lagrangian penalty function method [16]. In this method, the constrained non-linear optimization task is transformed into an unconstrained problem by adding a penalty term to the Lagrangian function as follows:

$$L(\Gamma, p, \nu, \mu, \varepsilon) = \sum_k P_k + \mu \left( \sum_k c_k(\gamma_{k}^{\text{tgt}}) - c_m + \nu^T(a - b) + + \varepsilon \left[ \left( \sum_k c_k(\gamma_{k}^{\text{tgt}}) - c_m \right)^2 + \sum_n (a_n - b_n)^2 \right] \right)$$

The Lagrangian for problem (16) follows similarly. It can be shown that if the optimum Lagrange multipliers are known, the solution to this unconstrained problem corresponds to the solution of the original problem (15) regardless of the value of the penalty parameter $\varepsilon$, see e.g. [16, Chapter 9].

B. Series of convex Approximations

The previous method can yield analytic solution in closed form for small systems, but scales poorly with the system size. To enhance scalability, we propose to use an approximation to "convexify" the problem. Inspired by [17], we use the relation $\theta \log(x) + \beta \leq \log(1 + x)$, with $\theta = \frac{x}{1 + x}$ and $\beta = \log(1 + x_0) - \theta \log(x_0)$ to approximate the link capacity.

2Here, $e_k$ is the vector with 1 in the $k$-th coordinate and 0's elsewhere.
Algorithm 1 Series of convex approximations.

Initialize $p^{(0)}, \theta^{(0)},$ and $\beta^{(0)}$ to some feasible values for the original problem (16).

Start with iteration step $t = 1$.

1) Solve the approximate optimization problem (20). Let \{$(\theta^{(t)}, p^{(t)})$\} denote the solution of the $t$-th iteration.

2) Update $\theta^{(t+1)}$, $\beta^{(t+1)}$ at $\gamma^{(t)}_n = 2(p^{(t)})$.

3) Update $t = t + 1$ and repeat until convergence.

The approximation becomes exact for $x = x_0$. We replace the expression (13) with a more conservative one:

$$\tilde{c}_k(p) = \left[\theta_k \log_2(\gamma_k(p)) + \beta_k\right] \leq c_k(p).$$

By applying the approximation (18) to the objective function of problem (16), we obtain the following approximate problem

$$\begin{align*}
\text{maximize} & \quad \sum_k \left[\theta_k \log_2(\gamma^{(t)}_k) + \beta_k^{(t)}\right] \\
\text{subject to} & \quad \gamma_k^{(t)} \leq \frac{P_k}{n_k + \sum_{j \neq k} G_{kj}} \quad \forall k \\
& \quad \sum_k P_k \leq P_{\text{tot}},
\end{align*}$$

which explicitly optimizes the SINR targets $\Gamma$ and the transmit power $p$. Here, the SINR expression (14) has been added to the constraint set to provide an explicit relationship between these variables. Similarly to [17], we propose to solve problem (16) through a sequence of convex approximations according to the iterative Algorithm 1. At the $t$-th iteration of the algorithm, the following problem $P^{(t)}$ is solved:

$$\begin{align*}
\text{maximize} & \quad \sum_k \left[\theta_k^{(t)} \gamma_{k,t} + \beta_k^{(t)}\right] \\
\text{subject to} & \quad \sum_k \hat{\epsilon}_{k,t} \leq \sum_k \hat{P}_k - \log(n_k + \sum_{j \neq k} G_{kj} e^{\hat{\beta}_j}) \quad \forall k \\
& \quad \sum_k \epsilon_{k,t} \hat{P}_k \leq P_{\text{tot}}.
\end{align*}$$

The above formulation is obtained from problem (19) through the exponential change of variables $\gamma_{k,t}^{(t)} \leftarrow e^{\gamma_{k,t}^{(t)}}$, $P_k \leftarrow e^{\hat{P}_k}$, and a log-transform of the constraints. Algorithm 1 iteratively solves the convex approximate problems $\{P^{(t)}\}$ in the variables $P$ and $\theta$, and appropriately tunes $p$ and $\beta$ to improve the objective function until convergence.

C. Power Loading Matrix Optimization

The mathematical framework devised in the previous sections applies to the case of equal power allocation to all streams (i.e., $T_k = I_{n_t}$). Both problems (15) and (16) exploit the minimum per-stream SINR bound (8), i.e.

$$\gamma_k(p) \geq \frac{P_k d_{k,k}^{\rho}}{\sum_{j \neq k} P_j d_{k,j}^{\rho} \mu_{k,j} + N_t \sigma^2_n \mu_{k,j}},$$

to optimally allocate the SINR targets and transmission power. Originally studied in [14, Lemma 1], this bound does not apply when the power loading matrix $T_k$ is also included in the optimization. In what follows, we ask whether optimizing $T_k$ can bring an additional gain in network MIMO systems where the SINR targets are initially optimized based on this bound.

From the signal model (1), when user $k$-th uses a diagonal power loading matrix $T_k \in C^{n_x \times n_t}$ with $\sum_{s=1}^{n_t} |T_k^{(s,s)}|^2 = n_t$, the post-processing SINR of its $s$-th stream becomes

$$\gamma_{k,s} = \frac{P_k |T_k^{(s,s)}|^2}{\zeta_{k,s}} - 1,$$

where

$$\zeta_{k,s} = \left\{ \left( d_{k,k}^{s} \chi_{k,k} \Psi_{k,k} \left( \sum_{j \neq k} P_j d_{j,k}^{s} \chi_{j,k} \Psi_{j,k} T_j T_j^* \Psi_{j,k} \right) \right)^{1/2} + \frac{1}{P_k} \right\}^{1/2}$$

denotes the effective interference after MMSE processing. In [14], a heuristic algorithm for distributing the transmit power over different streams was presented. By inverting equation (21) for fixed SINR targets, the algorithm finds a near optimal (sum power minimizing) power loading matrix for uplink transmission. Optimizing $T_k$ was shown to enhance the feasibility space of a rough SINR targets selection with respect to the equal power allocation case. By applying this algorithm to our optimized SINR targets for problem (15), the total sum power can be reduced further. However, some modifications are necessary for the capacity maximization problem. At the optimal point of problem (16), the SINR targets will consume the entire power budget $P_{\text{tot}}$. In this case, by better distributing the power budget $P_{\text{tot}}$ through an optimized $T_k$ allows to sustain higher SINR targets, thus yielding a throughput gain.

To capitalize on these gains, we propose Algorithm 2. The SINR targets are initialized to the optimal values $\Gamma = \text{diag}(\gamma_{k,t})$ yielded by either problem (15) or (16), i.e. with $T_k = I_{n_t}$, $\forall k$. For sum-rate maximization, Algorithm 2 iteratively tunes the SINR targets, along with the matrix $T_k$ and the transmission powers, until the entire power budget $P_{\text{tot}}$ is spent. At every iteration, the effective interference and the new $T_k$ are computed in steps a) and b), respectively; the control parameter $\epsilon(t)$ is used to update the SINR targets as

$$\gamma_k^{(t)} = \gamma_k^{(t-1)} - \epsilon(t) \quad \forall k,$$

which become the new reference for the power control update in step c). Finally, $\epsilon(t)$ is tuned differently for the two problems in step 3: for problem (15), $\epsilon(t)$ is kept fixed to 1 so that the SINR targets remain unchanged, thus reflecting the original algorithm in [14]; for problem (16), $\epsilon(t)$ is tuned with a subgradient-like step until the power budget constraint is met with equality (at with point $\epsilon(t)$ will not change anymore).

V. Numerical Results

In this section we consider a three cell system, each of which is serving a single MS. In an OFDM cellular network, for example, this setting corresponds to the situation in which a single MS is served on an OFDM resource block and interference is caused by MS served in other cells (that is assuming perfect intercell orthogonality). The main parameters of this system are summarized in Table I. MS-1 is located at the cell edge,
while MS-2 and MS-3 are close to their respective serving base stations. Table II reports six fairness ratios (fairness differences in dB) between the best and the worst SINR targets, reflecting increasing fairness constraints from unfair allocation (case 1) to almost egalitarian SINR allocation (case 6).

First, we consider the case without joint processing. That is, in our example, each user is detected at its own serving base station (4 receive antennas) and suffers interference from the other users. The serving base station can use an MMSE receiver, in which case each user suffers interference from all the other users. Alternatively, it can use successive interference cancelation (SIC), in which case User-\( j \) is interfered only by User-\( k \), where \( k > j \).

Figure 1 shows the sum uplink power as the function of the SINR targets in Cell-1 and Cell-2 with a sum rate target \( R = 4 \) [bps/Hz] when using SIC. The minimum sum power is used when \( \gamma_1 \) and \( \gamma_2 \) are set to -36 dB and 3 dB respectively, in which case \( \gamma_3 \) must be set to 8 dB to reach the sum rate target. When various levels of fairness are imposed (lower figure), the SINR target of User-1 is dramatically increased, while the User-3 target is decreased. The price for this fairness is depicted in Figure 2, which shows the individual and the sum power for the six fairness cases that we study. Here we observe that the sum power is around 350 mW in Case 1 (no fairness), and it is 1900 mW in Case 6. The dotted line at 250 mW indicates which cases are feasible in a system with individual power constraints at 250 mW. In such systems Cases 1-3 would be feasible, but not Cases 4-6. In Figures 3-4 we increase the sum rate target to 10.5 bps/Hz. We immediately notice on the sum power surface the large increase of the necessary power levels, which in fact renders this target rate infeasible without joint processing, even without any fairness requirement (notice the 1800 mW target levels for User-2 and User-3 in Figure 4). Figures 5-6 refer to network MIMO with joint detection and SIC. Looking at the sum power surface of Figure 5, we notice that it is "brought back" to feasible ranges, which rough

Algorithm 2 Iterative SINR and power loading optimization.

Given \( t = 0 \), \( \epsilon^{(0)} = 1 \), \( P_{\text{tot}}, \epsilon_{\text{gap}} \) and \( T_k^{(0)} = I_n \) \( \forall k \).

Initialize SINR targets \( \Gamma_k^{(0)} = \text{diag}(\gamma_k^{(0)}) \) and transmission powers \( p_k^{(0)} \) solving either problem (15) or (16).

Repeat:

1) \( t = t + 1 \).
2) For \( k = 1 \) to \( K \)
   a) Given \( \{T_k^{(t-1)}, P_k^{(t-1)}\} \), compute the interference \( \xi_{k,s} = \nu(T_k^{(t-1)}, P_k^{(t-1)}) \) as in (22).
   b) Calculate the optimum loading matrix \( T_k^{(t)} \) as

\[
(T_k^{(t)})_s,s = \sqrt{\frac{\xi_{k,s} n_t}{\sum_{j=1}^{n_s} \xi_{k,j}}} \quad \forall s \in [1,n_t].
\]

c) Calculate the new SINR targets \( \Gamma_k^{(t)} \) and update the optimum transmit power \( P_k^{(t)} \) as

\[
\gamma_k^{(t)}(\epsilon^{(t-1)}) = \frac{\epsilon^{(t-1)}}{\epsilon^{(t-1)} - 1} \quad \forall k
\]

\[
P_k^{(t)} = \frac{\epsilon^{(t-1)} \xi_{k,s}}{[T_k^{(t)}]_{s,s}} (\gamma_k^{(t)} + 1) \quad \forall k, s
\]

3) Update the control parameter \( \epsilon \):
   - If objective is power minimization: \( \epsilon^{(t)} = \epsilon^{(t-1)} \).
   - If objective is throughput maximization:

\[
\epsilon^{(t)} = \left\{ \epsilon^{(t-1)} - \kappa \left( \sum_k p_k^{(t)} - P_{\text{tot}} \right) \right\}^+. 
\]

Until \( |P_k^{(t)} - P_k^{(t-1)}| \leq \epsilon_{\text{gap}}, \forall k \).

Table I

INPUT PARAMETERS OF THE 3-CELL OFDMA SYSTEM

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter Site Distance [m]</td>
<td>500</td>
</tr>
<tr>
<td>Distance of the MS-( i )'s from their serving BS (( i = 1 \ldots 3 ))</td>
<td>0.45, 0.15, and 0.1 [ISD] respectively</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>5.07</td>
</tr>
<tr>
<td>Shadow fading</td>
<td>Lognormal; st. dev: 10 dB</td>
</tr>
<tr>
<td>Fast fading model</td>
<td>Rayleigh flat</td>
</tr>
<tr>
<td>AWGN noise variance</td>
<td>( \sigma_n^2 = 0.01 )</td>
</tr>
<tr>
<td>Antenna configurations</td>
<td>2\times4 and 2\times12 (virtual) MIMO</td>
</tr>
</tbody>
</table>

Figure 1. The upper figure shows the sum uplink power in the 3-cell system without network MIMO joint processing and employing SIC receiver at each base station. The minimum sum power without fairness constraint is provided at point SINR1=-36 dB, SINR2=3 dB and SINR3=8 dB. The lower figure shows the optimum SINR settings for the six fairness cases that we study.

Table II

FAIRNESS RATIO BETWEEN THE BEST AND WORST SINR TARGET.

<table>
<thead>
<tr>
<th>Case</th>
<th>20dB</th>
<th>10dB</th>
<th>3dB</th>
<th>1.76dB</th>
<th>0.41dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
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</tbody>
</table>
observation is confirmed by the individual sum power curves of Figure 6. Nevertheless, individual SINR target optimization is obviously necessary to reach this high sum rate, which is only feasible for fairness cases 1-2. Thus, opportunistic SINR target setting and joint detection together can dramatically increase the theoretically reachable sum rate target.

Figures 7-10 reports results for sum rate maximization under sum power and fairness constraints. Without joint processing over multiple base stations, the maximum achievable sum rate is around 5.3 bps/Hz (assuming perfect SIC). This "unfair" sum is reduced to around 2 bps/Hz (see upper plot of Figure 9) when imposing near equal rate allocations. In contrast, Figure 8 shows results for the same sum power budget but with joint
target setting can significantly reduce the overall power usage of perfect channel knowledge at the transmitter, optimal SINR to obtain numerical results. The results show that in the case under a power budget constraint and used the ALPF method under a sum rate target constraint and maximizing the sum rate of target SINR optimization as minimizing the sum powerportunistic SINR target setting. We formulated the problem purpose of gaining insight into the potential benefits of op-
system employing MMSE and MMSE SIC receivers for the significant.

Figure 6. This figure, similarly to Figure 4, shows the individual and sum power (upper) and rate (lower) allocations, but now for the network MIMO case for the 10.5 bps/Hz sum rate case. With network MIMO and perfect SIC, this sum rate is feasible for Cases 1-2. For Case 3, the required power for User 1 would be too high in a practical system.

Figure 7. Sum rate maximization for the 6 fairness cases that we study for a sum power budget 724.5 mW. Again we see that the maximum sum rate is reached at drastically different SINR targets in Case 1, whereas Case 6 enforces near equal SINR target setting. The maximum sum rate without network MIMO that we can reach is 5.3 bps/Hz.

processing over all three base stations. Now the maximum (i.e. without fairness) sum rate is 11.5 bps/Hz which decreases to around 7 bps/Hz with the equal rate constraint (Figure 9).

Figure 10 compares the power allocation without and with joint multicell processing under the same power budget. Interestingly, the individual power levels are actually quite similar (being around the “border” or around practically feasible per-user power levels depending on the fairness setting).

Finally, Figure 11 shows the results of iterative channel inversion with successive SINR target adjustment. Here we set the sum rate target to 11.5 bps/Hz and reach a minimum total power budget of 550 mW. For all mobile stations there is a SINR target gain (here only MS-1 is shown) due to the adaptive power loading algorithm. This gain may not appear large, but considering that this increase of the target SINR is available for all mobile stations in the scheduling time interval (that is in every millisecond), the overall sum rate gain can be significant.

VI. CONCLUSIONS

In this paper we developed a model of network MIMO system employing MMSE and MMSE SIC receivers for the purpose of gaining insight into the potential benefits of op-
portunistic SINR target setting. We formulated the problem of target SINR optimization as minimizing the sum power under a sum rate target constraint and maximizing the sum rate under a power budget constraint and used the ALPF method to obtain numerical results. The results show that in the case of perfect channel knowledge at the transmitter, optimal SINR target setting can significantly reduce the overall power usage or increase the overall capacity. It is also an efficient means to balance between fairness and minimum power or maximum rate. An important future work is to evaluate this model with imperfect channel knowledge.

REFERENCES

Figure 9. The maximum sum rates for the six fairness cases that we study without (upper) and with (lower) network MIMO processing and assuming perfect SIC in both cases. We notice that the sum rate is twice as much for network MIMO without fairness and more than 3 times as much with the equal rate allocation constraint ($\approx 7$ bps/Hz with network MIMO and 2 bps/Hz without network MIMO).

Figure 10. The individual optimum power allocations for the three users for the 3 cell system without (upper) and with (lower) network MIMO assuming the same sum power budget (724.5 mW). We notice that the per-user power allocations are actually quite similar in the two systems.

Figure 11. Iterative channel allocation with power loading and SINR target adjustment for sum power minimization in the network MIMO system, for which we have seen results in Figure 5 and 6. The upper figure shows the individual and sum power allocations for 50 iterations reaching the minimum sum power of 550 mW. The lower figure is the SINR target evolution for User-1, which is now higher than the original target without precoding.


