Learning Regular Sets from Queries and Counterexamples

Deepak D'Souza

Department of Computer Science and Automation
Indian Institute of Science, Bangalore.

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Outline

Dana Angluin

Learning Regular Sets from Queries and Counterexamples, in *Information & Computation*, 1987.

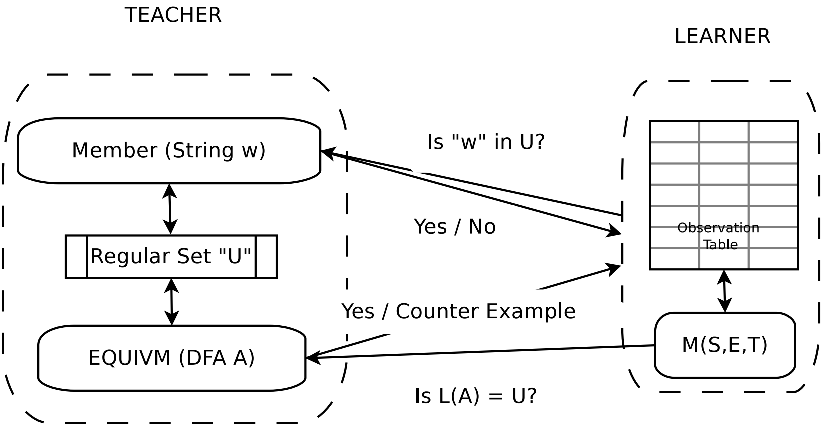
Angluin's L^* algorithm

Teacher has a regular language U in mind.
The Learner can ask two types of queries:

- Is a given string w in U? Teacher answers "Yes" or "No".
- Does a given DFA $\mathcal{A}$ accept the language U? Teacher answers "Yes" or gives a counterexample x.

Angluin's algorithm for the Learner finds the canonical DFA for U, in a number of steps polynomial in the number of states of the canonical DFA for U and the length of the longest counterexample returned by the teacher.

Working of L* Algorithm



Angluin's Algorithm by Example

Suppose the Teacher has in mind the language

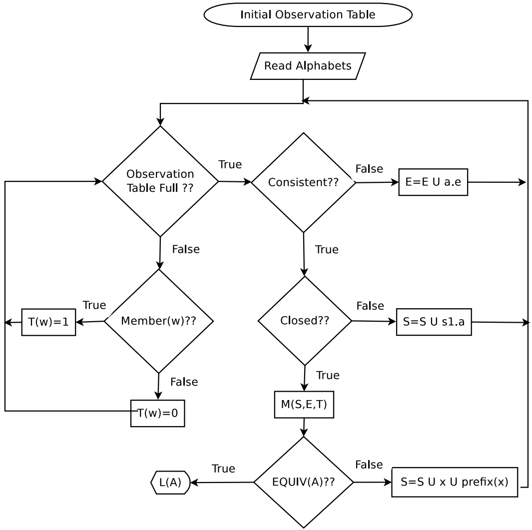
$U = \{w \in \{a, b\}^* \mid$ number of a's is even and number of b's is even$\}$

The Learner asks the Teacher if ϵ, a, and b belong to U, and obtains the following Observation Table:

		ϵ
S	ϵ	1
$S.\{a, b\}$	a	0
	b	0

The set of strings S represents the states of the automaton constructed by the Learner.
Entry (s, e) of the table represents the fact that from state s the automaton accepts/rejects the string $s \cdot e$.

Flow Chart of L* Algorithm - 0

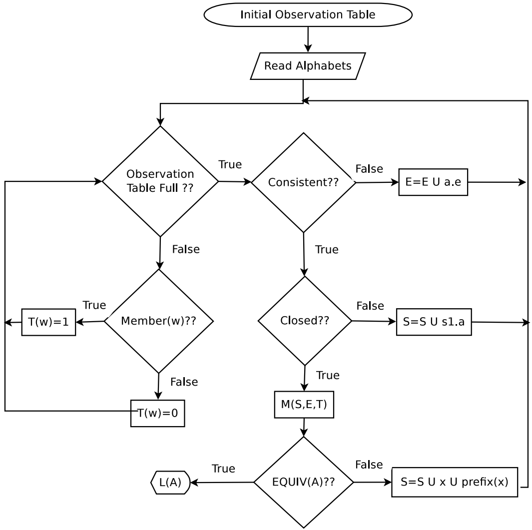


Initial OT

S

	ϵ
ϵ	?

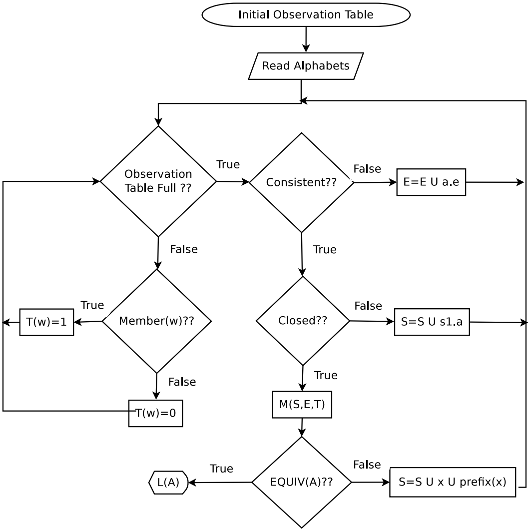
Flow Chart of L* Algorithm - 1



Observation Table

		ϵ
S	ϵ	?
$S.\{a, b\}$	a	?
	b	?

Flow Chart of L* Algorithm - 2



Observation Table

		ϵ
S	ϵ	1
$S.\{a,b\}$	a	0
	b	0

Closure

Closure of Observation Table:- An observation table is called closed provided that for each t in S.A there exist an s in S such that row(t) = row(s)

Not Closed means:-

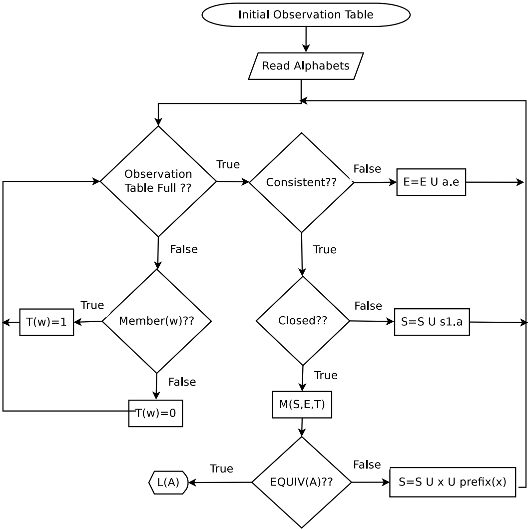
There is a row(s_1·a) in S.A is different from all s in S

To resolve this add s_1·a to S

Observation Table:

		ϵ
S	ϵ	1
$S.\{a, b\}$	a	0
	b	0

Flow Chart of L* Algorithm - 3



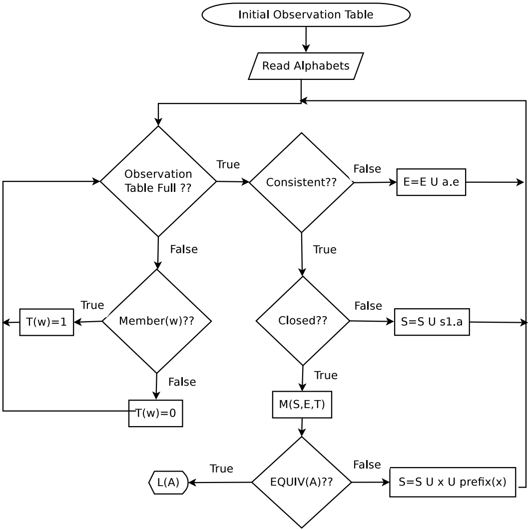
Observation Table

	ϵ
ϵ	1
a	0
b	0
aa	?
ab	?

S: rows ϵ, a

$S.\{a, b\}$: rows b, aa, ab

Flow Chart of L* Algorithm - 4



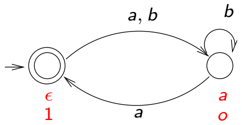
Observation Table

		ϵ
S	ϵ	1
	a	0
$S.\{a, b\}$	b	0
	aa	1
	ab	0

Constructing DFA from Observation Table: M(S,E,T)

Rules to construct Automata:

- Q = {row(s): s∈S}
- q_0 = row(ϵ)
- F = {row(s): s∈S and T(s)=1}
- δ(row(s), a) = row(s·a)



$\mathcal{A}_1$

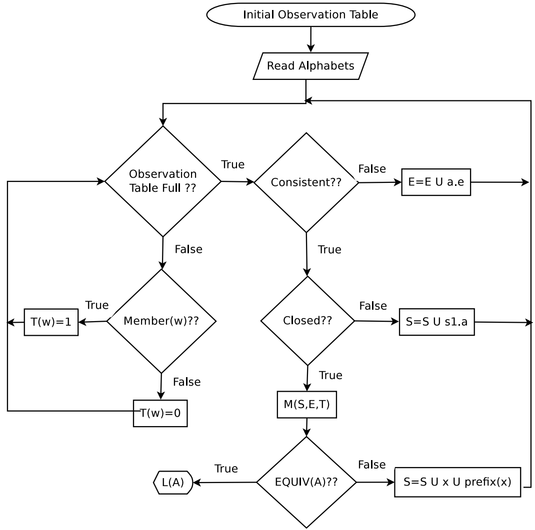
Observation Table:

		ϵ
S	ϵ	1
	a	0
$S.\{a,b\}$	b	0
	aa	1
	ab	0

- A = {Q,S,δ,Σ,F}
- F = $\{q_0\}$
- S = $\{q_0\}$
- Q = $\{q_0, q_1\}$
- Σ = { Alphabets }
- δ =

	a	b
q_0	q_1	q_1
q_1	q_0	q_1

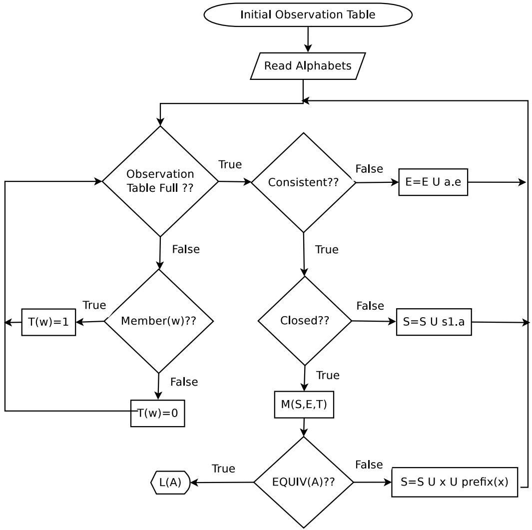
Flow Chart of L* Algorithm - 5



Observation Table

		ϵ
S	ϵ	1
	a	0
	b	0
	bb	?
$S.\{a, b\}$	aa	1
	ab	0
	ba	0
	bba	?
	bbb	?

Flow Chart of L* Algorithm - 6



Observation Table

		ϵ
	ϵ	1
S	a	0
	b	0
	bb	1
	aa	1
	ab	0
$S.\{a, b\}$	ba	0
	bba	0
	bbb	0

Consistency

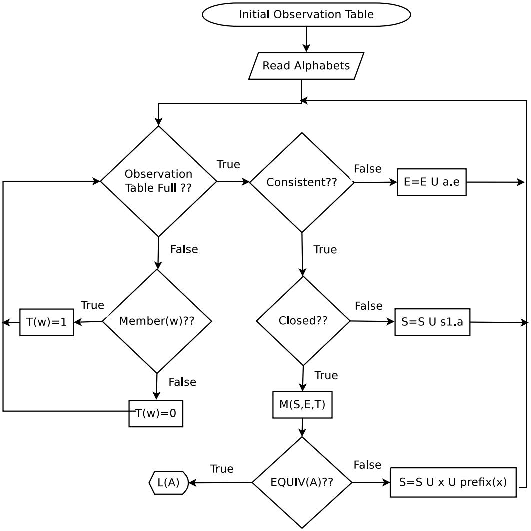
Consistency of Observation Table:-
An observation table is called consistent provided that whenever s_1 and s_2 are elements of S such that row(s_1) = row(s_2) for all a in A, row(s_1·a) = row(s_2·a)

Not Consistent means:-
There exist a certain combination of s_1 and s_2 in S, e in E, and a in A such that row(s_1) = row(s_2) but T(s_1·a·e)≠T(s_2·a·e)
To resolve this add a·e to E

Observation Table:

		ϵ
S	ϵ	1
	a	0
	b	0
	bb	1
$S.\{a,b\}$	aa	1
	ab	0
	ba	0
	bba	0
	bbb	0

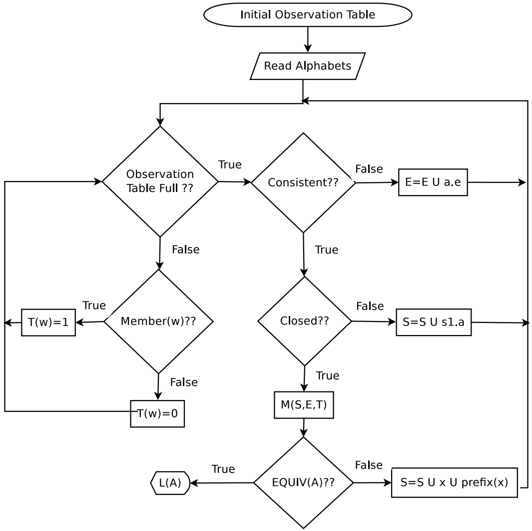
Flow Chart of L* Algorithm - 7



Observation Table

		ϵ	a
S	ϵ	1	?
	a	0	?
	b	0	?
	bb	1	?
$S.\{a,b\}$	aa	1	?
	ab	0	?
	ba	0	?
	bba	0	?
	bbb	0	?

Flow Chart of L* Algorithm - 8



Observation Table

		ϵ	a
S	ϵ	1	0
	a	0	1
	b	0	0
	bb	1	0
$S.\{a,b\}$	aa	1	0
	ab	0	0
	ba	0	0
	bba	0	1
	bbb	0	0

Angluin's Algorithm by Example

Suppose the Teacher has in mind the language

$U = \{w \in \{a, b\}^* \mid$ number of a's is even and number of b's is even$\}$

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Angluin's Algorithm by Example

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The Learner asks the Teacher if ϵ, a, and b belong to U, and obtains the following Observation Table:

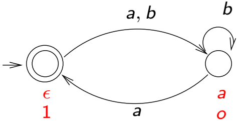
		ϵ
S	ϵ	1
$S.\{a, b\}$	a	0
	b	0

The set of strings S represents the states of the automaton constructed by the Learner.
Entry (s, e) of the table represents the fact that from state s the automaton accepts/rejects the string $s \cdot e$.
This table is not "closed" as there are no states (or "rows") corresponding to $\epsilon \cdot a$ and $\epsilon \cdot b$.

Angluin's Algorithm by Example: 2

Learner closes table by adding string a to S, and asking membership queries for aa and ab.
He now gets the observation table:

		ϵ
S	ϵ	1
	a	0
$S.\{a, b\}$	b	0
	aa	1
	ab	0



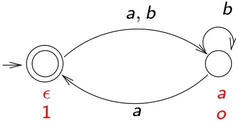
$\mathcal{A}_1$

This table is closed and consistent, and represents the DFA $\mathcal{A}_1$.

Angluin's Algorithm by Example: 2

Learner closes table by adding string *a* to *S*, and asking membership queries for *aa* and *ab*.
He now gets the observation table:

		ϵ
S	ϵ	1
	a	0
$S.\{a, b\}$	b	0
	aa	1
	ab	0



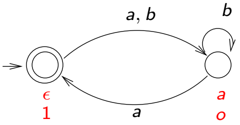
$\mathcal{A}_1$

This table is closed and consistent, and represents the DFA $\mathcal{A}_1$.
Learner now asks the Teacher if $\mathcal{A}_1$ represents the language she has in mind.

Angluin's Algorithm by Example: 2

Learner closes table by adding string a to S, and asking membership queries for aa and ab.
He now gets the observation table:

		ϵ
S	ϵ	1
	a	0
$S.\{a, b\}$	b	0
	aa	1
	ab	0



$\mathcal{A}_1$

This table is closed and consistent, and represents the DFA $\mathcal{A}_1$.
Learner now asks the Teacher if $\mathcal{A}_1$ represents the language she has in mind. Teacher replies with counterexample bb which is in U but is not accepted by $\mathcal{A}_1$.

Angluin's Algorithm by Example: 3

Learner adds *bb* and its prefixes to his set S, makes membership queries for *ba*, *bba*, and *bbb* to obtain the observation table:

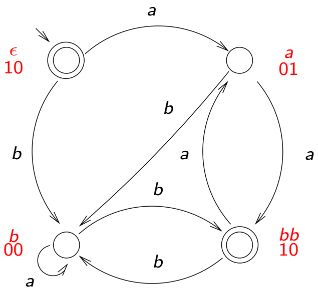
		ϵ
S	ϵ	1
	a	0
	b	0
	bb	1
$S.\{a, b\}$	aa	1
	ab	0
	ba	0
	bba	0
	bbb	0

This table is closed but not consistent. The rows for *a* and *b* are identical, but *aa* and *ba* have different rows.

Angluin's Algorithm by Example: 4

Learner adds $\epsilon \cdot a$ (that is, a) and its suffixes to the set E, and makes membership queries to obtain the observation table:

		ϵ	a
S	ϵ	1	0
	a	0	1
	b	0	0
	bb	1	0
$S.\{a, b\}$	aa	1	0
	ab	0	0
	ba	0	0
	bba	0	1
	bbb	0	0



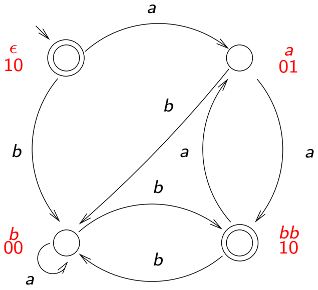
$\mathcal{A}_2$

This table is closed and consistent. So Learner conjectures the automaton $\mathcal{A}_2$.

Angluin's Algorithm by Example: 4

Learner adds $\epsilon \cdot a$ (that is, a) and its suffixes to the set E, and makes membership queries to obtain the observation table:

		ϵ	a
S	ϵ	1	0
	a	0	1
	b	0	0
	bb	1	0
$S.\{a, b\}$	aa	1	0
	ab	0	0
	ba	0	0
	bba	0	1
	bbb	0	0



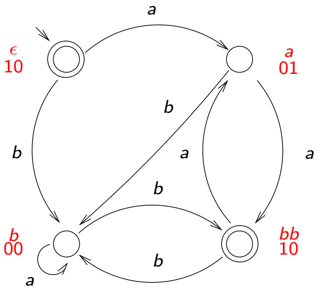
$\mathcal{A}_2$

This table is closed and consistent. So Learner conjectures the automaton $\mathcal{A}_2$. Teacher responds with counterexample abb.

Angluin's Algorithm by Example: 5

Learner adds *abb* and its prefixes to S, makes membership queries to obtain the observation table:

		ϵ	a
S	ϵ	1	0
	a	0	1
	b	0	0
	ab	0	0
	bb	1	0
	abb	0	1
$S.\{a,b\}$	aa	1	0
	ba	0	0
	aba	0	0
	bba	0	1
	bbb	0	0
	$abba$	1	0
	$abbb$	0	0



$\mathcal{A}_2$

Angluin's Algorithm by Example: 6

Learner adds b and its suffixes to E, and makes membership queries to obtain the observation table:

		ϵ	a	b
S	ϵ	1	0	0
	a	0	1	0
	b	0	0	1
	ab	0	0	0
	bb	1	0	0
	abb	0	1	0
$S.\{a,b\}$	aa	1	0	0
	ba	0	0	0
	aba	0	0	1
	bba	0	1	0
	bbb	0	0	1
	$abba$	1	0	0
	$abbb$	0	0	1

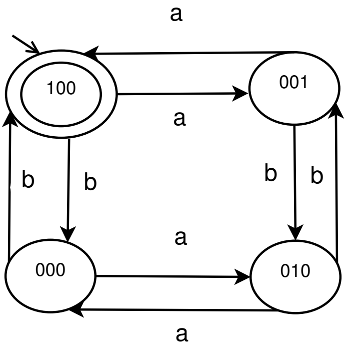


Table is closed and consistent, so Learner conjectures DFA $\mathcal{A}_3$.

Angluin's Algorithm by Example: 6

Learner adds b and its suffixes to E, and makes membership queries to obtain the observation table:

		ϵ	a	b
S	ϵ	1	0	0
	a	0	1	0
	b	0	0	1
	ab	0	0	0
	bb	1	0	0
	abb	0	1	0
$S.\{a,b\}$	aa	1	0	0
	ba	0	0	0
	aba	0	0	1
	bba	0	1	0
	bbb	0	0	1
	$abba$	1	0	0
	$abbb$	0	0	1

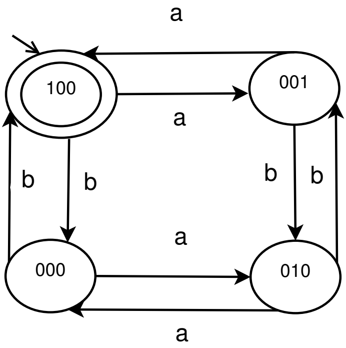


Table is closed and consistent, so Learner conjectures DFA $\mathcal{A}_3$. Teacher responds with "Yes!".

Conclusion

- Angluin's automata learning framework
- Learns the smallest automaton that satisfies given constraints.
- Does it efficiently in polynomial time in smallest automaton that satisfies the given constraints.