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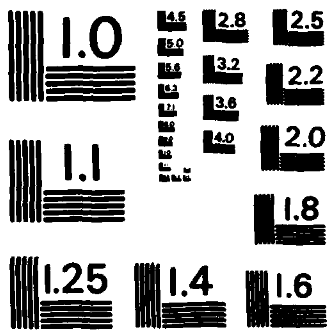

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ON STRUCTURE DETERMINATION FOR POLYNOMIAL INPUT-OUTPUT DIFFERENTIAL SYSTEMS[1]

by

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ABSTRACT

The problem of structure determination for a deterministic class of polynomial input-output differential systems is formulated as a minimum norm-discrete time optimal control problem. The order of the differential equation and the degrees of the polynomials involving the input-output variables play the role of multiple discrete-times while the coefficient parameters play the role of a discrete control variable. The basis of the parameter identification technique is Shinbrot's method of moment functionals using linear combinations of commensurable sinusoids as the modulating functions. Given the system input-output data on a finite time interval, the underlying computations involve calculating a finite set of Fourier series coefficients of moments formed from the data, which can be efficiently carried out via an FFT algorithm, followed by a sequence of singularity tests performed on a controllability type Gram determinant that arises from the formulation.

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1. INTRODUCTION

The problem of structure determination has been considered for the class of linear systems, both stochastic and deterministic, in determining such structural parameters as the controllability/observability indexes [1-3]. However, within the context of system identification, relatively little attention appears to have been given to any class of nonlinear systems. One such class is considered here, namely those single input-single output deterministic systems that can be described by a polynomial input-output differential equation of finite order involving finite degree polynomials of the input-output variables. Examples of this class would include linear feedback control systems with zero memory polynomial type nonlinearities inserted in the forward and feedback loops, and certain classical nonlinear differential equation models such as the Van der Pol and Duffing equations. The major assumptions are that the input-output data is noise-free, or can be made essentially noise-free by appropriate filtering, and that upper bounds are known for the degrees of the polynomials and the order of the differential equation. Although the assumption of noise-free data is idealistic, the technique utilizes Shinbrot's method of moment functionals [4] with linear combinations of commensurable sinusoids as the modulating function basis, thus facilitating a certain measure of selective filtering on the data as a result of the need to calculate only a finite set of Fourier series coefficients while avoiding the necessity to estimate unknown initial or boundary conditions. Previous developments of this approach have focused on linear differential systems [5,6], a class of bilinear systems [7], and polynomial input-output differential systems of the type to be considered here [8], all of presumed known order and structure as far as the differential equation model is concerned.

Following the formulation of the problem as a triple indexed discrete system, a solution is proposed using minimum norm-time optimal control ideas, and some discussion is included regarding noise and bandwidth considerations.

2. FORMULATION

Let $[u(t),y(t)]$ denote the input-output pair observed free of significant measurement noise on a finite time interval $[0,T]$, and let p denote the differential operator d/dt so that $p^2=d^2/dt^2$, etc. The class of polynomial differential equations relating $u(t)$ and $y(t)$ is taken of the form

$$\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} a_i(j,k)p^i[u(t)]^j\,[y(t)]^k = 0 \tag{1}$$

$$0\leq t\leq T, \quad a_i(0,0)=0, \quad i=0,1,2..n$$

where the $a_i(j,k)$ represent parameters which are to be determined along with the structural parameters (n,m,l), the latter consisting of the order of the differential equation n and the highest degrees (m,l) of the polynomials involving the input-output variables.[2] The choice of the time interval length T will be discussed in Section 4.

The actual system order n_0 and the degrees of the polynomials (m_0,l_0) are assumed to satisfy the set membership expressions

$$n_0\in\{1,2..\bar{n}\}, \quad m_0\in\{1,2..\bar{m}\}, \quad l_0\in\{1,2..\bar{l}\} \tag{2}$$

where $(\bar{n},\bar{m},\bar{l})$ represent the previously mentioned *a priori* upper bounds on the structural

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[2] Existence and uniqueness of solutions to (1) on $[0,T]$ is tacitly assumed for whatever values are assigned to the $a_i(j,k)$, given that the data is bounded and piecewise continuous.

parameters (n,m,l). In order to focus on determining (n_0,m_0,l_0), the model (1) will be reparametrized as follows:

$$0 = y(t) + \sum_{i=1}^{n}\sum_{k=1}^{l}\alpha_i(k)\,p^{\,i-1+\delta_{k1}}[y(t)]^k + \sum_{i=1}^{n}\sum_{j=1}^{m}\beta_i(j)\,p^{\,i-1}[u(t)]^j \tag{3}$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\gamma_i(j,k)\,p^{\,i-1}[u(t)]^j\,[y(t)]^k$$

where δ_{k1} represents the Kronecker delta.[3] The presence of δ_{k1} serves the dual purpose of assuring that the highest derivative term appears linearly as $p^n y(t)$ and that causality is preserved in the input-output relation. In turn, this assures that (3) has a normal form state space realization of the form

$$\dot{x}(t) = g(x(t),u(t))$$

$$y(t) = c'x(t)$$

where c is a column vector, prime denotes transpose, and as pointed out in [8] the nonlinear terms in $g(x,u)$ are polynomials in the pair $(c'x,u)$. Normalizing the coefficient $a_0(0,1)$ to unity in passing from (1) to (3) is somewhat arbitrary but can be seen to be valid if the transfer function for the linearized differential equation, i.e., linearized about $u=y=0$, does not contain a pole at the origin.

Shinbrot's method of moment functionals facilitates the conversion of a linear differential relation, such as (1) or (3), to an algebraic equation in the coefficient parameters via the use of so-called modulating functions. The resulting algebraic equation is characterized by functionals on the data. A motivating factor for the developments in [5-8] has been the specification of modulating functions comprised of linear combinations of the commensurable sinusoids $\{\sin k\omega_0 t, \cos k\omega_0 t\}$, $k=0,1..L$, $\omega_0=2\pi/T$, because the functionals on the data that result from the conversion are linearly related to the first L Fourier series coefficients of moments formed from the data which can be calculated to any desired degree of accuracy by a sufficiently high order discrete Fourier transform (DFT). In turn, these DFT's can be efficiently evaluated by a Fast Fourier Transform (FFT) algorithm. In order to apply these relations to the problem at hand, let $f(t)$ denote the $(2L+1)$ dimensional column vector of sinusoids:

$$f(t) = \text{Col}\left[1;\cos\omega_0 t,\sin\omega_0 t;\cos 2\omega_0 t,\sin 2\omega_0 t;\ldots;\cos L\omega_0 t,\sin L\omega_0 t\right] \tag{4}$$

$$0 \le t \le T, \quad \omega_0 = 2\pi/T.$$

Following the procedure outlined in [5,6] and given the upper bound integer $\bar{n}$ for the system differential equation order, let C denote the $(2L+1-\bar{n})\times(2L+1)$ matrix constructed (offline) such that $\Phi(t)$ defined by

$$\Phi(t) = Cf(t) \tag{5}$$

satisfies the end point conditions[4]

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[3] That is, $\delta_{k1}=1$ if $k=1$, while $\delta_{k1}=0$ if $k\neq 1$.

[4] It is easily shown that the matrix C has full rank and that each of its $(2L+1-\bar{n})$ row vectors is determined by the solution to a Vandermonde type matrix equation [5,6].

$$\Phi^{(i)}(0) = \Phi^{(i)}(T) = 0, \quad i=0,1\,..\,(\bar{n}-1). \tag{6}$$

This construction results in a $(2L+1-\bar{n})$ dimensional vector valued modulating function of order $\bar{n}$ in which[5] the derivatives $p^i\Phi(t)=\Phi^{(i)}(t)$ have the representation (cf. (4))

$$(-1)^i p^i \Phi(t) = CD^i \mathbf{f}(t), \quad i=0,1,2\,.. \tag{7}$$

where D is a block diagonal matrix defined by

$$D = \omega_0 \,\mathrm{Diag}\left[0, \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \begin{pmatrix}0 & 2\\ -2 & 0\end{pmatrix} \cdots \begin{pmatrix}0 & L\\ -L & 0\end{pmatrix}\right] \tag{8}$$

and D^0 is defined as the identity matrix.

In accordance with the modulating function approach, (3) is multiplied by $\Phi(t)$ and integrated by parts n times while noting (6) and (7), thereby obtaining the vector equation

$$0 = CY + \sum_{i=1}^{n}\sum_{k=1}^{l} CD^{i-1+\delta_{k1}} Z(0,k)\alpha_i(k) + \sum_{i=1}^{n}\sum_{j=1}^{m} CD^{i-1}Z(j,0)\beta_i(j) \tag{9}$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} CD^{i-1}Z(j,k)\gamma_i(j,k)$$

where $(Y, Z(j,k))$, $0 \le j \le m$, $0 \le k \le l$, are vectors of finite Fourier series coefficients of moments of the data defined by

$$Z(j,k) = \int_0^T [u(t)]^j [y(t)]^k \mathbf{f}(t)dt \tag{10}$$

$$Y = Z(0,1).$$

These vector valued functionals of moments formed from the data can be calculated using an FFT algorithm as discussed in [8].

After introducing some notation, a rearrangment of (9) leads to the following linear equation in the coefficient parameters:

$$-CY = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} H(m,l;i,j,k)\theta(i,j,k) \tag{11}$$

where the $(2L+1-\bar{n})\times 3$ matrix $H(m,l;i,j,k)$ and 3×1 vector $\theta(i,j,k)$ are defined by[6]

$$H(m,l;i,j,k) = CD^{i-1}\left[\frac{1}{m}D^{\delta_{k1}} Z(0,k)\,,\ \frac{1}{l}Z(j,0)\,,\ Z(j,k)\right] \tag{12}$$

and

$$\theta(i,j,k) = \begin{bmatrix}\alpha_i(k)\\ \beta_i(j)\\ \gamma_i(j,k)\end{bmatrix}$$

respectively.

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[5] Notice that $\Phi(t)$ is also a modulating function of order n for any $n \le \bar{n}$.

[6] The reason for including the reciprocals $(1/m, 1/l)$ in the first two columns of (12) is to account for the expansion of the first two double sums in (9) to a triple sum so that (11) and (9) are equivalent.

Except for the re-parametrization and notational differences, (11) is equivalent to the regression equation derived in [8] and, as such, can be used as the starting point for formulating a least squares estimate of the coefficient parameters under the proviso that the structural parameters (n,m,l) are known. However, the emphasis here is on obtaining values (n_0,m_0,l_0) which are correct insofar as the given input-output date is concerned. For this purpose, it will be assumed that (11) contains a sufficient number of algebraic equations to permit a one-shot solution, pending some nondegeneracy conditions on the data, regardless of the actual values (n_0,m_0,l_0) as long as (2) is satisfied. Since the number of unknown coefficient parameters is possibly as large as $\bar{n}(\bar{l}+\bar{m}+\bar{l}\bar{m})$ and since the number of rows in the matrix C is $(2L+1-\bar{n})$, it follows that L must be chosen in the general nonzero input situation to satisfy the inequality:

$$2L+1-\bar{n} \;\geq\; \bar{n}(\bar{l}+\bar{m}+\bar{l}\bar{m}) \quad \text{or} \quad 2L \;\geq\; \bar{n}(\bar{l}+\bar{m}+\bar{l}\bar{m}+1)-1. \tag{13}$$

If on the other hand the input is zero on $[0,T]$, i.e., $u(t)=0,\ 0\leq t\leq T$, and if the time interval $[0,T]$ has been preceded by an interval during which all the system modes have been excited, then (3) reduces to

$$0 \;=\; y(t)+\sum_{i=1}^{n}\sum_{k=1}^{l}\alpha_i(k)p^{\,i-1+\delta_{k1}}[y(t)]^k$$

and (11) can be replaced by the simpler expression

$$-CY \;=\; \sum_{i=1}^{n}\sum_{k=1}^{l}H_0(i,k)\theta_0(i,k) \tag{14}$$

where the $(2L+1-\bar{n})$ dimensional column vector $H_0(i,k)$ and the scalar $\theta_0(i,k)$ are defined by

$$H_0(i,k) \;=\; CD^{\,i-1+\delta_{k1}}Z(0,k) \quad \text{and} \quad \theta_0(i,k) \;=\; \alpha_i(k). \tag{15}$$

In this special situation there are at most $\bar{n}\bar{l}$ unknown coefficient parameters that can be determined given the data $[0,y(t)]$ on $[0,T]$ so that the condition on L can be replaced by the less stringent inequality

$$2L \;\geq\; \bar{n}(\bar{l}+1)-1. \tag{16}$$

In summary for this section, the problem of structure determination for the class of polynomial differential systems modeled by (3) can be stated as finding the smallest triple indexed "discrete time" integers (n_0,m_0,l_0) and "control vector" triple indexed sequence $\theta(i,j,k)$ such that (11) is satisfied given the discrete indexed "system matrix" $H(m,l;i,j,k)$ and the output vector CY. This problem will have a solution only if L has been chosen to satisfy (13), or less restrictively (16) if the zero input prevails on $[0,T]$, and only if the vector CY is reachable for some (n,m,l) satisfying (2), a condition which reflects back on the input-output data $[u(t),y(t)]$ on $[0,T]$ as will be seen below.

3. MINIMUM NORM-TIME OPTIMAL SOLUTION

As formulated above, structure determination is a simplified version of a discrete system optimal control problem considered by Sarachik and Kranc [9], although the discrete time is triple indexed. Following the analysis used in [9], consider the linear functional derived from (11):

$$-\lambda' CY \;=\; \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\lambda' H(m,l;i,j,k)\theta(i,j,k) \tag{17}$$

where λ is an arbitrary $(2L+1-\bar{n})$ dimensional column vector. The right hand side of (17) will be viewed as an inner product for fixed values of (n,m,l) by defining the 3×1 vector function

$K(i,j,k)$ as

$$K(i,j,k) = H'(m,l;i,j,k)\lambda. \tag{18}$$

Using (18) and the inner product notation $<\ ,\ >$, (17) is represented by

$$-\lambda' CY = <K,\theta>. \tag{19}$$

Taking absolute values of both sides of (19) and applying the Schwarz inequality to the right hand side:

$$|\lambda' CY| \leq \| K \| \| \theta \| \tag{20}$$

where the norm on θ is defined by

$$\| \theta \| = \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} \theta'(i,j,k)\theta(i,j,k) \right]^{\frac{1}{2}} \tag{21}$$

and the norm on K is similarly defined.

From the inequality (20) it can be deduced that if the problem has a solution for an arbitrary vector CY, it must be the case that

$$\| \theta \| \geq \| K^* \|^{-1}$$

where $K^*(i,j,k) = H'(m,l;i,j,k)\lambda^*$ and λ^* is that value of λ which solves the minimization problem

$$\text{Min} \| K \| = \left[\lambda' \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} H'(m,l;i,j,k) H(m,l;i,j,k) \lambda \right]^{\frac{1}{2}}$$

subject to

$$-\lambda' CY = 1.$$

The latter problem is equivalent to minimizing the quadratic form

$$\| K \|^2 = \lambda' W(n,m,l)\lambda + \mu(1+\lambda' CY) \tag{22}$$

where μ is a scalar Lagrange multiplier and $W(n,m,l)$ is the Gram matrix defined by[7]

$$W(n,m,l) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} H(m,l;i,j,k) H'(m,l;i,j,k). \tag{23}$$

Assuming the inverse of $W(n,m,l)$ exists for some (n,m,l), the minimizing value of λ subject to the indicated constraint is found to be

$$\lambda^* = -\frac{W^{-1}(n,m,l)CY}{Y'C'W^{-1}(n,m,l)CY}.$$

If a solution to the above minimization problem exists for some (n_0,m_0,l_0) satisfying (2), a solution exists for all (n,m,l) satifying the following

$$n_0 \leq n \leq \bar{n}, \quad m_0 \leq m \leq \bar{m}, \quad l_0 \leq l \leq \bar{l}. \tag{24}$$

This is due to the monotonicity property of the Gram matrix (23) in that using matrix inequalities in the sense of symmetric positive semidefiniteness:

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[7] Note that $W(n,m,l)$ can be interpreted as an output controllability Grammian for the problem at hand.

$$W(n_0,m_0,l_0) \leq W(n,m,l)$$

holds for all (n,m,l) satisfying (24). This coupled with the fact that the norm (21) is a monotonically increasing function of (n,m,l) allows for the conclusion that structure determination for the model (3) given the data at hand corresponds to the smallest triple of integers which satisfy the determinant inequality:

$$\det W(n_0-1,m_0-1,l_0-1) \leq \epsilon_0 \tag{25}$$

together with one or more of the following inequalities:

$$\det W(n_0,m_0,l_0) > \epsilon_0 \quad \text{or} \quad \det W(n_0-1,m_0,l_0) > \epsilon_0$$

$$\det W(n_0-1,m_0-1,l_0) > \epsilon_0 \quad \text{or} \quad \det W(n_0-1,m_0,l_0-1) > \epsilon_0$$

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$$\det W(n_0,m_0,l_0-1) > \epsilon_0$$

where ϵ_0 is a positive parameter selected by the user to discriminate between the theoretical zero value and small threshold effects of noise, roundoff errors, and the like.

Nothing is claimed about the existence or uniqueness of such a triple (n_0,m_0,l_0) satisfying (25). About all that can be claimed at this point is that if a minimizing triple can be found satisfying (25), then that triple (n_0,m_0,l_0) solves the structure determination problem ***relative to the data at hand***. More analysis is needed to find conditions on the input $u(t)$ which would guarantee the existence of a solution. The problem is complicated by the fact that the modulating functions act as a filter on the data and that the various polynomials arising from the differential operators in (3) may have some common factors, i.e., they may not be coprime. Nonetheless, (25) is a relatively easy test to perform in consideration of the complexity of structure determination problems, and any minimizing solution to (25) is a likely candidate for the structure parameters pending additional tests involving a variety of inputs, initial conditions, etc.

Assuming the existence and determination of a minimizing triple (n_0,m_0,l_0) satisfying (25), the system coefficient parameters for (3) can be obtained using the least squares procedure of [8], or if sufficient confidence exists for the one-shot data at hand, the conditions for equality in the Schwarz inequality (20) yield the parameters directly as

$$\theta^*(i,j,k) = -H'(m_0,l_0;i,j,k)W^{-1}(n_0,m_0,l_0)CY \tag{26}$$

$$1 \leq i \leq n_0, \quad 1 \leq j \leq m_0, \quad 1 \leq k \leq l_0.$$

In the special case of the zero input on $[0,T]$, $W(n,m,l)$ is to be replaced by

$$W_0(n,l) = \sum_{i=1}^{n}\sum_{k=1}^{l} H_0(i,k)H_0'(i,k) \tag{27}$$

where $H_0(i,k)$ is defined by (15). Assuming existence and determination of a minimizing integer pair (n_0,l_0) satisfying

$$\det W_0(n_0-1,l_0-1) \leq \epsilon_0 \tag{28}$$

together with one or more of the following inequalities:

$$\det W_0(n_0,l_0) > \epsilon_0 \quad \text{or} \quad \det W_0(n_0,l_0-1) > \epsilon_0$$

$$\det W_0(n_0-1,l_0) > \epsilon_0,$$

the corresponding relation to (26) in this special situation is

$$\theta_0^*(i,k) = \nu_i^*(k) = -H_0^l(i,k)W_0^{-1}(n_0,l_0)CY \tag{29}$$

$$1 \le i \le n_0, \quad 1 \le k \le l_0.$$

4. CHOICE OF (L,T) AND NOISE CONSIDERATIONS

Suppose the system bandwidth ω_b is approximately known. Although the value of ω_b is likely to be input dependent for a nonlinear system, it seems reasonable to presume that such a value may be available from practical considerations. Clearly the highest frequency $L\omega_0$ in the modulating functions should be chosen comparable to the system bandwidth else higher frequence noise will be present in the vectors $Z(j,k)$, i.e.

$$L\omega_0 = L2\pi/T \approx \omega_b. \tag{30}$$

Assuming the inequality (13) is approximately satisfied with equality, it then follows that a guideline for choosing T is

$$T \approx (\bar{n}(\bar{l}+\bar{m}+\bar{l}\bar{m}+1)-1)\pi/\omega_b. \tag{31}$$

In the special case of a zero input on $[0,T]$, the analogous relation for a guideline in choosing $T=T_0$ is

$$T_0 \approx (\bar{n}(\bar{l}+1)-1)\pi/\omega_b. \tag{32}$$

5. CONCLUSIONS

The determinant inequalities derived in (25) and (28) indicate ways of testing for the structural parameters of the model (3), given upper bounds for the indexes and assuming the system belongs to this class of nonlinear models. Although relatively simple in appearance, this simplicity undoubtedly belies a variety of the difficulties that might be encountered in a practical situation. Not only will the presence of significant amounts of noise invalidate the conclusions, but there are potential difficulties resulting from the projections on the data due to the modulating functions and the possibility of common factors in the polynomials corresponding to the differential operators in (3). The noise problem can be mitigated to some extent by a careful choice in modulating function frequencies as explained earlier. In fact, there is no need to pick the first L commensurable sinusoids; rather, the basis set $\{\sin l\omega_0 t, \cos l\omega_0 t\}$, $0 \le t \le T$, $\omega_0 = 2\pi/T$, $l \in (l_1, \ldots l_L)$ will suffice, where $(l_1, \ldots l_L)$ is any suitably chosen integer set. The possibility of modes in the data which are orthogonal to one or more of the modulating functions is practically unimportant if the number of such modulating functions is sufficiently large, i.e., L is chosen sufficiently large, to encompass all the essential information in the data. However, such a choice may then require a very long time interval $[0,T]$ or high frequency noise problems may be encountered, etc. Further analysis will be needed to resolve these questions.

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