A Distributed Power Allocation Scheme for Sum-Rate Maximization on Cognitive GMACs

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Abstract—This paper considers a distributed power allocation scheme for sum-rate-maximization under cognitive Gaussian multiple access channels (GMACs), where primary users and secondary users may communicate under mutual interference with the Gaussian noise. Formulating the problem as a standard nonconvex quadratically constrained quadratic problem (QCQP) provides a simple distributed method to find a solution using iterative Jacobian method instead of using centralized schemes. A totally asynchronous distributed power allocation for sum-rate maximization on cognitive GMACs is suggested. Simulation results show that this distributed algorithm for power allocation converges to a fixed point and the solution achieves almost the same performance as the exhaustive search.

Index Terms—Distributed power allocation, cognitive radio, sum-rate maximization, non-convex QCQP, Gaussian multiple access channels.

I. INTRODUCTION

Investigation of both spatial and temporal spectrum utilization reveals the fact that not all the spectrum is in use all the time. Cognitive radio technology inspired by the observation turns out to be a promising technique for the efficient use of this unused spectrum, potentially allowing a large amount of spectrum to become available for future high bandwidth applications [1] [9]. Some works [2] - [5] make discussions on cognitive radio’s achievable rate from information theoretic point. In the seminal work [3], the achievable rate of a single cognitive radio user is provided under such constraints as (i) there is no interference to the primary user, and (ii) the primary encoder-decoder pair is oblivious to the presence of cognitive radio. In [4], they extend the result of [3] to the case with multiple cognitive radio users and characterize the cognitive radio’s achievable rate region for Gaussian multiple access channels (MACs). Maximization of the cognitive radio’s sum-rate on Gaussian MAC then raises the problem of the allocation of each cognitive user’s power ratio [5].

This paper considers a distributed power allocation problem to approximate the maximization of the sum-rate of cognitive radio on the Gaussian MACs. The maximization formulation is formulated as a standard nonconvex quadratically constrained quadratic problem (QCQP). In [4], they solve the same problem to result in a nonconvex quadratic optimization problem, whose solution is difficult to get. Thereby, they rather handle the nonconvexity through a heuristic scheme that uses Lagrangian multipliers, by which the solution can only be obtained through iterative numerical computations. When the complexity of the solution is an issue, the solution of a standard type of the nonconvex QCQP can be achieved more easily in polynomial time by applying semidefinite relaxation (SDR) [14]. Through the formulation by a standard nonconvex QCQP, reference [9] uses a conceptually simple SDR to solve the relaxation of the power allocation problem for the non-convex sum-rate maximization on cognitive GMACs, instead of using a heuristic algorithm in [4]. However, the power allocation schemes used in [4] and [9] are not practical because they are centralized methods which are not appropriate for a decentralized CR system. Distributed implementation of power allocation is important in CR systems for scalability reasons. We solve the same problem efficiently using iterative Jacobian method in a distributed way [15]. Performance of the solutions are made through Monte-Carlo simulations. We show that this distributed algorithm for power allocation converges to a fixed point and the solution achieves almost the same performance as the exhaustive search [4].

This paper extends the result of [10] to an asynchronous setting which better resembles the reality of cognitive networks and real environment simulations for cognitive radio and their convergence results. If the users change their powers at the same time, it is called synchronous update. In Section V, we consider a more general model, called totally asynchronous update model, which includes synchronous updates as special cases [15]. Under this model, the users may update at different rates, and relative delay between updates may also vary.
This means that some users may update their power, more frequently than others and they may use an outdated measurement of the interference caused from others [12] [13]. These features make the asynchronous power allocation appealing for all practical scenarios, cognitive radios, as it strongly relaxes the need for coordinating the users’ updating schedule.

The rest of this paper is structured as follows. Section II presents a model of the cognitive radio multiple access channel (MAC). Section III investigates the sum-rate maximization on GMACs. Section IV introduces a distributed power allocation for sum-rate maximization on cognitive GMACs. Section IV.A shows the convergence of the distributed power allocation and simple numerical results are presented in Section IV.B, to illustrate the effectiveness of our proposed algorithm. Section V provides a totally asynchronous distributed power allocation for sum-rate maximization on cognitive GMACs. Simulation results appear in Section VI. Section VII draws some conclusions.

II. MODELING OF COGNITIVE RADIO MULTIPLE ACCESS

Figure 1 describes the system model [4]. This model has a primary mobile station (MS) communicating with a primary base station (BS) and multiple secondary Ms, who desire access to a secondary BS using primary frequency bands without license. In Figure 1, $g_p$ is the path gain from the primary MS to the primary BS, $g_k$ ($k = 1, 2, ..., K$) are the path gains of the interference links from the respective secondary MSs to the primary BS, $h_k$ ($k = 1, 2, ..., N$) are the path gains from the respective secondary MSs to the secondary BS, and $h_p$ is the path gain of the interference link from the primary MS to the secondary BS. The average power of the primary MS is $p_p$, and the average power of the $k$-th secondary MS is assumed to be $p_k$ ($k = 1, 2, ..., N$).

In order to meet the CRs transmit requirements such as (i) no interference is created for the primary system, and (ii) the primary MS-BS link is oblivious to the presence of secondary MSs employing CRs, a scenario of only one secondary MS was studied in [3] by introducing a superposition coding method. The maximum rate of the CR is achieved by a mixed strategy of dirty paper coding and cooperation with the primary MS. The optimal strategy for superposition coding for secondary MS $k$ is as follows:

$$ X^n_k = \hat{X}_k^n + x_k \sqrt{\frac{p_k}{p_p}} X^n_p, \quad k = 1, 2, ..., K. $$ (1)

where the first term $\hat{X}_k^n$ denotes the codeword when the secondary MS $k$’s codeword $m_k$ is dirty-paper coded based on the primary MS’s codeword $m_p$, and the second term, the cooperation part, is the duplicate information of the primary MS, which is combined with the information from the primary MS. In addition, $x_k^2$ is the power ratio of the cooperation part. Figure 2 shows a simple superposition coding scheme for primary MS and secondary MSs using QPSK constellation. From the figure, it can be seen that the superposition coding signal of CR MS is the sum of the primary MS signal and CR MS signal.

With the above superposition coding strategy, the cognitive multiple access channel can be regarded as a GMACs with the common interference noncausally available at each transmitter. Based on Theorem 2 in [7], the capacity region of the later is equal to the capacity region of the standard GMACs without the common interference, which gives an achievable rate region of the cognitive multiple access channel:

$$ C_{max}(h, p, x) = \left\{ R : R(S) \leq \frac{1}{2} \log \left( 1 + \frac{\sum_{k=1}^{N} h_k p_k (1+x_k^2)}{N_x} \right) \right\} $$

(2)

where $h_k$ is the path gain from secondary MS $k$ to the secondary BS, $N_x$ is additive white Gaussian noise at the secondary BS, $h = (h_1, h_2, ..., h_N)^T$ and $p = (p_1, p_2, ..., p_N)^T$ are constants, and $x = (x_1, x_2, ..., x_N)^T$ is a variable vector. The data rate of the primary MS in the absence of any secondary MS is

$$ R_p = \frac{1}{2} \log(1 + \frac{g_p p_p}{N_p}) $$

(3)

where $g_p$ is the path gain from the primary MS to the primary BS, and $N_p$ is the additive white Gaussian noise at the primary BS. At the primary BS’s receiver, its data rate in the presence of secondary MSs using superposition coding can be expressed as follows:
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\[ R'_p = \frac{1}{2} \log \left( 1 + \frac{\left( \sqrt{g_pp_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k (1 - x_k^2) p_k} \right) \]  

where \( g_k \) is the path gain from secondary MS \( k \) to the primary BS. Eqn. (4) should be greater than the original rate (3) as if there were no interference from the secondary MSs, i.e., the following inequality must be satisfied:

\[ \log \left( 1 + \frac{g_pp_p}{N_p} \right) \leq \log \left( 1 + \frac{\left( \sqrt{g_pp_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k (1 - x_k^2) p_k} \right) \]

\[ \frac{g_pp_p}{N_p} \leq \frac{\left( \sqrt{g_pp_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k (1 - x_k^2) p_k} \]

We formulate a sum-rate maximization problem under the constraint (5). We are interested in the maximum sum-rate of the CR network is expressed as follows:

\[ R_s = \frac{1}{2} \log \left( 1 + \frac{\sum_{k=1}^{N} h_k p_k (1 - x_k^2)}{N_s} \right) \]

III. SUM-RATE MAXIMIZATION ON COGNITIVE GMACS

This section formulates a sum-rate maximization problem within the achievable rate region, whose solution will be the power allocation to each secondary MS that maximizes the sum-rate. The formulation is made as a nonconvex quadratic problem for cognitive GMACs [9], where the mixed strategy of dirty-paper precoding and cooperation with the primary MS is adopted. The solution then determines the allocation of each secondary MSs power ratio for the cooperation part, \( x_k^2 \) in \( p_k \).

maximize \( \sum_{k=1}^{N} h_k p_k (1 - x_k^2) \)  
subject to \( \frac{g_pp_p}{N_p} \leq \frac{\left( \sqrt{g_pp_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k p_k (1 - x_k^2)} \) \[ 0 \leq x_k \leq 1 \text{ for all } k \]

where \( N_p \) is the additive white Gaussian noise at the primary BS, \( g = (g_1, g_2, ..., g_N)^T \), \( h = (h_1, h_2, ..., h_N)^T \) and \( p = (p_1, p_2, ..., p_N)^T \) are constants, and \( x = (x_1, x_2, ..., x_N)^T \) is a variable vector.

Reference [4] does not observe that (7) could be formulated as a standard type of quadratically constrained quadratic problem (QCQP). Instead [4] provides a heuristic and iterative search that requires high complexity. First, this new letter changes (7) into a standard QCQP minimization problem. (See the details in the top of the next page.)

minimize \( x^T H x + z \)  
subject to \( x^T G x + 2b^T x + c \leq 0 \) \[ 0 \leq x \leq 1, \]

where, \( H = \text{diag}(h_k p_k) \) is an \( N \times N \) diagonal matrix, and \( z = -\sum_{k=1}^{N} h_k p_k \). The problem (8) is a QCQP, but unfortunately \( G \) is not positive semi-definite, which makes the problem nonconvex. In general, a nonconvex QCQP is hard to solve. However, using semi-definite relaxation (SDR), the solution can be obtained with much lower complexity. The problem (8) could be formulated as following.

minimize \( \text{Tr}(H X) + z \)  
subject to \( \text{Tr}(G X) + 2b^T x + c \leq 0 \) \[ \begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \succeq 0, \] \[ 0 \leq x \leq 1, \]

which is an SDP problem. This is called the SDP relaxation of the standard nonconvex QCQP. Since it is an SDP problem, it is easy to solve. In addition, we provide a Lagrangian relaxation of sum-rate maximization on cognitive GMAC in Appendix I.

IV. DISTRIBUTED POWER ALLOCATION FOR SUM-RATE MAXIMIZATION ON COGNITIVE GMACS

The solutions of [4] and SDP are centralized methods. They are not practical for the implementation in CR systems which are decentralized. Distributed implementation of power allocation is important in CR systems for scalability reasons. Therefore, in this section, we provide a distributed algorithm for cognitive GMACs by using Iterative Jacobian method [15]. We suggest a distributed power ratio allocation for CR systems.

Lagrangian of (8) is defined as,

\[ L(x, \lambda) = x^T (H + \lambda G) x + 2\lambda b^T x + \lambda c + z \]

\[ 0 \leq x \leq 1. \]

The solution of \( L(x, \lambda) \) exists in the following condition,

\[ \frac{\partial L}{\partial x} = (H + \lambda G) x + 2\lambda b = 0. \]

The problem is the same as solving linear system equation. In order to solve this problem in a distributed fashion, we apply Jacobian algorithm [15].

\[ (H + \lambda G) x + \lambda b = 0 \]
\[-(g_0p_0 + N_p) \sum_{k=1}^{N} g_k p_k x_k^2 - N_p \left( \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right) \sum_{j \neq k}^{N} \sqrt{g_j p_j x_j} \]

\[-2N_p \sqrt{g_0 p_0} \sum_{k=1}^{N} \sqrt{g_k p_k x_k} + g_0 p_0 \sum_{k=1}^{N} g_k p_k \leq 0 \]

\[x^T G x + 2b^T x + c \leq 0 \]

\[G = \begin{bmatrix}
-(g_0p_0 + N_p)g_1 p_1 & -N_p \sqrt{g_1 p_1 g_2 p_2} & \cdots & -N_p \sqrt{g_1 p_1 g_N p_N} \\
-N_p \sqrt{g_2 p_2 g_1 p_1} & -(g_0p_0 + N_p)g_2 p_2 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-N_p \sqrt{g_N p_N g_1 p_1} & \cdots & \cdots & -(g_0p_0 + N_p)g_N p_N
\end{bmatrix} \]

\[b = \begin{bmatrix}
-N_p \sqrt{g_0 p_0 g_1 p_1} & \cdots & \cdots & -N_p \sqrt{g_0 p_0 g_N p_N}
\end{bmatrix}^T \]

\[c = g_0 p_0 \sum_{k=1}^{N} g_k p_k \]

\[(I + \lambda H^{-1} G)x + \lambda H^{-1} b = 0 \quad (13)\]

\[x = -\lambda H^{-1} G x - \lambda H^{-1} b \quad (14)\]

Now, for the purpose of solving (12) in a distributed fashion, consider the following iterative power allocation procedure:

\[x^{(n+1)} = -\lambda H^{-1} G x^{(n)} - \lambda H^{-1} b \quad (15)\]

or equivalently for each MS \(k\)

\[x^{(n+1)}_k = \frac{1}{\alpha_k} \left\{ \left( g_0 p_0 g_k p_k x^{(n)}_k \right) + N_p \sqrt{g_k p_k x^{(n)}_k} \right\} \]

where \(\alpha_k = \sum_{j=1}^{N} \sqrt{g_j p_j x_j}^{(n)}\). Taking into the power ratio constraint, let us define the mapping,

\[T(x^{(n+1)}) = \left\{ \min \left\{ 1, -\lambda H^{-1} G x^{(n)} - \lambda H^{-1} b \right\} \right\}^+, \quad (17)\]

where \([z]^+ = \max\{z, 0\}\). We summarize this simple distributed algorithm in the top of the next page.

### A. Convergence

In this section, we show that the optimal power allocation ratio will be converged. In the general iterative method, let \(\alpha_1, \alpha_2, \ldots\) be the eigenvalue of the iteration matrix \(-\lambda H^{-1} G\) and define \(\rho(-\lambda H^{-1} G) = \max_{\alpha_k} |\alpha_k|\). The constant \(\rho(-\lambda H^{-1} G)\) is called the spectral radius of \(-\lambda H^{-1} G\). The following result guarantees that the optimal power allocation ratio will be found.

**Proposition 1:** For any initial \(x^{(0)}\), the mapping \(T\) converges with geometric rate to a power ratio vector for each MS \(k\), if such power allocation is feasible.

**Proof:** Consider the weighted maximum norm \(\|x\|_W = \|Wx\|_\infty\) and consistent weighted matrix norm \(\|-\lambda H^{-1} G\|_W = \|W \cdot (-\lambda H^{-1} G \cdot W)\|_\infty\) where \(W\) is a nonsingular matrix. It can be shown that \(-\lambda H^{-1} G\) is nonnegative and therefore has a positive real eigenvalue equal to spectral radius to \(\rho(-\lambda H^{-1} G)\). By (18), Th 3.7, a necessary condition for a feasible solution (17) to exist that \(\rho(-\lambda H^{-1} G) < 1\). Let \(e = (e_i)\) be the Perron-Frobenius eigenvector of \(-\lambda H^{-1} G\), that is \(-\lambda H^{-1} G e = \rho(-\lambda H^{-1} G) e\) and choose \(W = \text{diag}\{1/e_i\}\), then

\[\|T(x^{(n)}) - x^*\|_W \leq \|-\lambda H^{-1} G x^{(n)} - \lambda H^{-1} G b - x^*\|_\infty \]

\[= \|\|\lambda H^{-1} G x^{(n)} - \lambda H^{-1} G b\|_\infty \|

\[\leq \|\lambda H^{-1} G\|_W \cdot \|x^{(n)} - x^*\|_W \]

Therefore,

\[\|x^{(n)} - x^*\|_\infty \leq \rho(-\lambda H^{-1} G)^n \cdot \|x^{(0)} - x^*\|_W \quad (18)\]

so if \(\rho(-\lambda H^{-1} G) < 1\) and the fixed point is within the feasible power range, \(T\) is a pseudo-contraction mapping with respect to the weighted maximum norm and converges to a unique fixed point given by \(x^* = T(x^*)\) which is the solution to \((I + \lambda H^{-1} G)x + \lambda H^{-1} b = 0\). Q.E.D.

It is also possible to prove convergence of \(T\) by verifying that it is a standard interference function [11], which in addition guarantees its asynchronous convergence. It should be also pointed out that the algorithm will converge to a fixed point.
Initialization
A primary MS at the cell edge broadcasts its $g_p p_p$ and $N_p$.
A primary BS initializes $\lambda^{(0)}$, and $x_k^{(0)}$ and broadcasts $\lambda^{(0)}$, $x_k^{(0)}$, and $\sum_{j=1}^{N} \sqrt{g_{j} p_{j} x_{j}^{(0)}}$.

Secondary MS Power Allocation Algorithm:
Each MS updates power ratio until it converges.
At time $(t)$
\[
x_k^{(n+1)} = \min \left\{ 1, \frac{\lambda}{x_k} \left( (g_p p_p g_k p_k) x_k^{(n)} + N_p \sqrt{g_k p_k \Lambda^{(n)}} + N_p \sqrt{g_{j} p_{j} x_{j}^{(n)}} \right) \right\}^{+}
\]
The primary BS broadcasts $\Lambda^{(n)} = \sum_{j=1}^{N} \sqrt{g_{j} p_{j} x_{j}^{(n)}}$.
When $x_k^{(n)}$ converges to $x_k^{(t)}$, MS transmit to primary BS with $x_k^{(t)}$.

Primary BS Algorithm:
Compute SINR with received power $x_k^{(t)}$ from all secondary MSs,
\[
\text{If } \frac{g_p p_p}{N_p} > \left( \sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k^{(t)}} \right)^2
\]
\[N_p + \sum_{k=1}^{N} g_k p_k (1 - (x_k^{(t)})^2)\]
Update $\lambda^{(t+1)} = \lambda^{(t+1)} + \Delta$ and broadcast $\lambda^{(t+1)}$ to each MS
Else
Stop
End

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<td>P. Cheng’s method [4]</td>
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<td>Distributed power allocation</td>
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V. ASYNCHRONOUS DISTRIBUTED POWER ALLOCATION FOR SUM-RATE MAXIMIZATION ON COGNITIVE GMACS

The synchronous model of the last section assumes that updates at the secondary MSs and the secondary BSs are synchronized to occur at time $n = \{1, 2, \ldots\}$. In this section, we will extend the model to an asynchronous setting which better resembles the reality of cognitive networks. In such network, the secondary MSs may be located at different distance from the BS. Network state may be proved by different secondary MSs at different rates, e.g., the power control in a cognitive network are sent at different rates by different secondary MS and secondary BS. These complications make our distributed computation system consisting of secondary MSs and secondary BSs asynchronous. In such a system, some secondary MSs may communicate faster and execute more iterations that others, some processors may communicate more frequently than others and the communication delay may be substantial.

We now present the totally asynchronous version of distributed power allocation for sum-rate maximization on cognitive GMACs and prove its convergence. Our asynchronous model and the convergence proof follow the approach of [11] [12] [13] [15].

Let $x_i^{(n)}$ be the power allocation of MS $i$ at time $n$.

B. A simple numerical example

This section provides a simple numerical example of the convergence of power allocation ratio. The example assumes that the path gain vector of the interference link from secondary MS to primary BS is $g = [10 9 8 7 6 5 2 1 3]$ and that the path gain vector from secondary MS to secondary BS is $h = [20 18 16 14 12 10 14 4 2 6]$ and $g_p$ is 1. The average power of primary user is $p_p = 1$. The average power of each secondary user is $p_k = 10$, while the additive white Gaussian noise of primary and secondary BS are $N_p = 1$. Figure 3 shows the fast convergence of the iterative algorithm of each secondary MS’s power allocation. Table I shows that the distributed implementation achieved almost the same performance as P. Cheng’s method [4] and SDR [9].

![Fig. 3. Convergence of Power ratio](image_url)
so that the power allocation vector at time $n$ is $x^{(n)} = [x_1^{(n)}, x_2^{(n)}, ..., x_N^{(n)}]^T$. We assume that MS $i$ may not have access to the most recent values of the components of $x^{(n)}$. This occurs when user $i$ has outdated information about the received power at certain BS. At time $n$, suppose that when MS $i$ adjusts his power at time $n$, he only has delayed information about other users at time $n$. At time $n$, let $\tau(n)_i$ denote the most recent time for which $x_i$ is known to user $j$. Note that $0 \leq \tau(n)_i \leq n$. If MS $i$ adjusts its power allocation at time $n$, that adjustment is performed using power vector, 

$$x^{(\tau(n)_i)} = [x_1^{(\tau(n)_i)}, x_2^{(\tau(n)_i)}, ..., x_N^{(\tau(n)_i)}]^T.$$  

(19)

We assume a set of times $O = \{0, 1, 2, ...\}$ at which one or more components $x_i^{(n)}$ of $x^{(n)}$ are updated. Let $O^i$ be the set of times at which $x_i^{(n)}$ is updated. At times $n \notin O^i$, $x_i^{(n)}$ is left unchanged. Given the Set $O^1, ..., O^N$, the totally asynchronous power allocation algorithm is defined as,

$$x_i^{(n+1)} = \begin{cases} 
T_i(x^{(\tau(n))}) & n \in O^i \\
\text{otherwise} & \end{cases}$$  

(20)

We assume the sets $O^i$ are infinite and given any time $n_0$, there exists $n_1$ such that $\tau_i^{(n)} \geq n_0$ for $n \geq n_1$. Convergence of the totally asynchronous standard power allocation algorithm will be proven by the asynchronous convergence theorem from [15] as stated below in theorem 1. We note that $x$ and $f(x)$ in the statement of theorem 1 represent the power allocation vector $x$ and iteration function $T(x)$ in the context of this work.

Starting from an initial power allocation vector $x$, $n$ iterations of the standard power allocation algorithm produces the power allocation vector $T^n(x)$.

**Lemma 1** [15]: If $x$ is a feasible power allocation vector, then $T^n(x)$ is a monotone decreasing sequence of feasible power allocation vector that converges to a unique fixed point $x^*$.

**Proof**: Let $x(0) = x$ and $x(n) = T^n(x)$. Feasibility of $x$ implies $x(0) \geq x(1)$. Suppose $x(n-1) \geq x(n)$. Monotonicity implies $T(x(n-1)) \geq T(x(n))$. That is, $x(n) \geq T(x(n)) = x(n+1)$. Hence, $x(n)$ is a decreasing sequence of feasible power allocation vectors. Since the sequence $x(n)$ is bounded below by zero, this implies the sequence must converge to a unique fixed point $x^*$. Q.E.D.

**Lemma 2** [15]: If $T(x)$ is feasible, then starting from $z$, the all zero vector, the power allocation algorithm produces a monotone increasing sequence of power vector $T^n(z)$ that converges to the fixed point $x^*$.

**Proof**: Let $z(n) = T^n(z)$. We observe that $z(0) < x^*$ and that $z(1) = T(z) \geq z$. Suppose $z \leq z(1) \leq ... \leq z(n) \leq x^*$, monotonicity implies

$$x^* = T(x^*) \geq T(z(n)) \geq T(z(n-1)) = z(n-1).$$  

(21)

That is, $x^* \geq z(n+1) \geq z(n)$. Hence the sequence of $z(n)$ is nondecreasing and bounded above by $x^*$. Q.E.D.

**Theorem 1** [11]: (Asynchronous Convergence Theorem) If there is a sequence of nonempty set $\{X(n)\}$ with $X(n+1) \in X(n)$ for all $n$ satisfying the following conditions:

1. Synchronous Convergence Condition: For all $n$ and $x \in X(n)$, $f(x) \in X(n+1)$. If $\{y^n\}$ is a sequence such that $\{y^n\} \in X(n)$ for all $n$, then every limit point of $\{y^n\}$ is a fixed point of $f$.

2. Box condition: for every $n$, there exists $X_n(n) \in X$, such that $X(n) = X_n(n) \times X_2(n) \times ... X_N(n)$ and the initial solution estimate $x(0)$ belongs to the set $X(0)$, then every limit point of $f$.

A proof of this theorem can be found in [15].

**Theorem 2** [11]: If $T(x)$ is feasible, then from any initial power allocation vector $x$, the asynchronous distributed power allocation algorithm converges to $x^*$.

**Proof**: Let $z$ denote the all zero vector. Feasibility implies the existence of the fixed point $x^*$. Given an initial power allocation vector $x$, we can choose $\alpha \geq 1$ such that $\alpha x^* \geq x$.

We define

$$X(n) = \{x: T^n(z) \leq x \leq T^n(\alpha x^*)\}$$  

(22)

For all $n$, the set $X(n)$ satisfies the box condition. Lemmas 1 and 2 imply $X(n+1) \subset X(n)$ for all $n$ and $\lim_{n \to \infty} T^n(z) = \lim_{n \to \infty} T^n(\alpha x^*) = x^*$. Hence any sequence $\{x(n)\}$ such that $x(n) \in X(n)$ for all $n$ must converge $x^*$. Since the initial power allocation vector $x$ satisfies $x \in X(0)$, the asynchronous convergence theorem implies to the fixed point $x^*$. Q.E.D.

**VI. SIMULATION RESULTS**

We consider the system model in IS-95, DS-CDMA uplink as primary and cognitive radio system sharing a common frequency. We consider the cell structure as Figure 4 and 5, where a primary hexagonal cell includes a small cognitive radio cell. MS’s maximum power is assumed to be 10 mW and bandwidth 1.2288 MHz. The COST 231 Hata urban propagation model is used for the link gain between BS and MSs:

$$\begin{cases} 
31.5 + 3.5 \log(d), & \text{if } d > 0.035 \text{ km} \\
31.5 + 3.5 \log(0.035), & \text{if } d < 0.035 \text{ km} 
\end{cases}$$  

(23)

Lognormal shadowing with mean 0 dB and standard deviation 8 dB is assumed. Thermal noise power is -130 dBm. Other parameters to be mentioned are summarized in Table II. Regarding simulation equipments, we use a PC with Intel(R) Core(TM) 2 Duo E8400 @ 3.00GHz CPU and 4 GB of RAM. Figure 6 and 7 show the fast convergence of the iterative algorithm of each secondary MS’s power allocation with users 5 and 10, respectively. Table III shows the average objective values of SDR, P. Cheng’s method and distributed power

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation/Assumption</th>
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<tbody>
<tr>
<td>Cellular layout</td>
<td>Hexagonal grid</td>
</tr>
<tr>
<td>Primary Cell radius</td>
<td>1000 m</td>
</tr>
<tr>
<td>Cognitive Cell radius</td>
<td>500 m</td>
</tr>
<tr>
<td>Chip rate</td>
<td>1.2288 Mcps</td>
</tr>
<tr>
<td>Time slot length</td>
<td>0.625 msec</td>
</tr>
</tbody>
</table>
allocation scheme, where channel generations are 10000 times. In simulation, there are a primary user (red *) and 10 secondary users (blue +) who are randomly generated. Simulation results show that a distributed power allocation has lower performance than SDR but it has almost the same performance of P. Cheng’s method [4].

VII. CONCLUSION

This paper formulated the distributed power allocation problem to approximate the maximization of the sum-rate of cognitive radio on the Gaussian MAC, obtaining a solution by applying an iterative Jacobian method instead of using centralized methods. Distributed implementation of power allocation is important in CR systems for scalability reasons.

<table>
<thead>
<tr>
<th>TABLE III SIMULATION RESULTS (10 SECONDARY USERS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semidefinite relaxation (SDR)</td>
</tr>
<tr>
<td>P. Cheng’s method [4]</td>
</tr>
<tr>
<td>Distributed power allocation</td>
</tr>
</tbody>
</table>

Through the distributed formulation of a sum-rate maximization on Gaussian MAC by Jacobian method, one can thus have insight on the solution of power- and rate-optimization problems of cognitive radio in future research by a very efficient computational solution.

APPENDIX I: LAGRANGIAN RELAXATION OF SUM-RATE MAXIMIZATION ON COGNITIVE GMACs

Lagrangian duality provides lower bounds on the optimal value of the problem and simplifies the computation of the lower bound (or optimal in some cases) on the optimal value of the nonconvex QCQP. Lagrangian of (8) is,

\[ L(x, \lambda, \mu) = x^T(H + \lambda G)x + 2(\lambda b + \frac{\mu}{2})^T x + \lambda c - \mu^T 1 + z, \]

and the dual function is,

\[ g(\lambda, \mu) = \inf_{x \geq 0} L(x, \lambda, \mu) \]

\[ = \begin{cases} 
  y - w^T(H + \lambda G)^{-1} w & H + \lambda G \succeq 0, \\
  -\infty & \text{otherwise,}
\end{cases} \]

w ∈ \( \mathcal{R}(H + \lambda G) \)

where \( y = c - b^T H^{-1} \).
where \( y = \lambda c - \mu^T 1 + z \) and \( w = \lambda b + \frac{\mu}{2} \).

Then, the use of Schur complement allows the dual problem to be expressed as an SDP:

\[
\begin{align*}
\text{maximize} & \quad \phi + \lambda c - \mu^T 1 + z \\
\text{subject to} & \quad \lambda \geq 0, \quad \mu \geq 0 \\
& \quad H + \lambda G \geq \left( \frac{\lambda b + \frac{\mu}{2}}{2} \right)^T / 2 - \phi \\
& \quad \phi, \lambda \in \mathcal{R} \quad \text{and} \quad \mu \in \mathcal{R}^N.
\end{align*}
\]

This result is also known as S-procedure in control theory. The main insight is that while the original problem is possibly nonconvex and numerically hard to solve, its dual can be expressed as an SDP and is easy to solve. Regarding the complexity, it is known that SDR has a complexity order of \( O(N^3) \) where \( N \) is the number of variables in problems [16]. The number of variables for sum-rate maximization problem on Gaussian cognitive MAC problem addressed in this letter is \( N \) which denotes the number of users. While as in the previous approach [4], the problem is solved using a heuristic search algorithm which requires considerations of values (0 or not) for all \( x_k \), which leads the complexity order to \( O(2^N) \), an NP hard problem.

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