Optimal Reverse-Pricing Mechanisms

Martin Spann 1 Robert Zeithammer 2 Gerald Häubl 3

December 18, 2008

Abstract

Reverse pricing is a market mechanism under which a consumer’s bid for a product or service leads to a sale if the bid exceeds a hidden acceptance threshold that the seller has set in advance. The specification of such a mechanism by the seller involves two key decisions: (1) the revenue model decision—that is, how to set his margin above cost (and thus the bid-acceptance threshold) and/or how to set a fee for the right to bid—and (2) whether to facilitate or hinder consumer learning about the bid-acceptance threshold. We analyze these interrelated decisions for a profit-maximizing firm selling to consumers with heterogeneous product valuations, derive the optimal revenue model, and characterize how the seller strategy should account for consumers learning about the bid-acceptance threshold. The optimal reverse-pricing mechanism is to charge a bidding fee upfront and then accept all bids above cost, rather than to charge a positive margin above cost (as is common practice). When consumers learn about the bid-acceptance threshold, the market becomes more efficient, bidding fees remain superior to margins in profitability, and both consumers and the seller can benefit. Specifically, we show that everyone benefits from consumers learning about the threshold when there are enough consumers, who are not interested in buying the product on the outside posted-price market.

Keywords: reverse pricing, name-your-own-price, analytical modeling, e-commerce.

1 Martin Spann: School of Business and Economics, University of Passau, D-94032 Passau, Germany, Phone: +49 (851) 509-2421, Fax: +49 (851) 509-2422, E-mail: spann@spann.de
2 Robert Zeithammer: The Anderson School of Management, UCLA, Los Angeles, USA, Phone: (310) 825-1862, E-mail: robert.zeithammer@anderson.ucla.edu
3 Gerald Häubl: School of Business, University of Alberta, Edmonton, AB, Canada T6G 2R6, Phone: (780) 492-6886, Fax: (780) 492-3325, E-mail: gerald.haebul@ualberta.ca

The authors thank Kursad Asdemir, Scott Fay, Yuanfang Lin, and Paul Messinger for their helpful comments on an earlier version of this paper.
1 Introduction

Reverse pricing (often also referred to as “name-your-own-price selling”) is an emerging market mechanism under which a consumer’s bid for a specific product leads to a sale if this bid exceeds the seller’s hidden acceptance threshold, which in turn depends on the seller’s cost (Spann and Tellis 2006). If a bid exceeds the seller’s threshold, the consumer receives the product and pays the bid amount. Some current examples of reverse-pricing sellers are Priceline (which specializes in airline tickets, hotel accommodations, and car rentals), some eBay sellers (with eBay’s “Best Offer” feature), and discount airlines such as Germanwings.

The essential property of most markets that allow consumers to name their own price is that the seller’s costs fluctuate too quickly and unpredictably for him to determine and post a selling price. Although such fluctuations can be due to variable input costs a producer faces, as in the case of discount airlines and fuel prices, we focus on the situation of an intermediary retailer who faces a highly variable wholesale price offered by upstream producers. For example, Priceline’s cost of an airline ticket for a given flight route varies greatly as a function current market conditions, thus making the determination and posting of the optimal posted prices for all eventualities and all flight routes difficult and prohibitively costly. In other words, the seller faces menu costs sufficiently high to make constantly changing and posting selling prices infeasible.

Menu costs were first discussed by Mankiw (1985) in the brick-and-mortar context, and Ghose and Gu (2007) show that they are non-trivial even in online retailing. Reverse pricing solves the pricing problem by soliciting consumer bids in real time and automatically evaluating them in light of the current wholesale price available once the bid is received.
Given the apparent importance of fluctuating costs in existing reverse-pricing markets, we cast the seller’s strategic decision as setting a bid-acceptance rule to maximize profits before the wholesale price becomes known. The problem is essentially static thanks to the intermediary assumptions: the producer’s price is a temporary exploding offer, and the prospective customers are those in the market for that particular product or service at that moment. For example, if Priceline does not sell a given seat on a flight to a customer interested in buying it right now, both the seat and the customer may disappear before another opportunity for a trade arises. We thus abstract away from dynamic yield-management issues that would arise if the seller were the producer of the good.

How should a seller design the reverse-pricing mechanism so as to maximize profit? The specification of a seller strategy involves two key decisions. First, where in the interaction with consumers should the seller generate revenue: by setting a positive margin above the wholesale price in specifying the bid-acceptance threshold, by charging consumers a bidding fee (akin to a subscription fee) for the right to submit a bid, or by some combination of both sources? Second, the seller must decide whether to help or hinder the bidders in their learning about the bid-acceptance threshold. Consumers may learn about the current threshold via repeated bidding—for example, using multiple identities as in Fay (2004)—or through communication with other bidders (Hinz and Spann 2008). Given that such learning is typically considered to be destructive to profits (see, e.g., Hinz and Spann (2008) and Priceline’s no re-bidding policy), understanding whether the seller is in fact better off preventing bidders from such learning is important. The answer to this design question is intertwined with the answer to the revenue-model question: knowing the threshold both erodes margins via lower bids of high-value consumers and potentially increases their willingness to pay a bidding fee by resolving uncertainty about surplus.
To characterize the optimal reverse-pricing mechanism, we analyze both seller decisions using a model of a small intermediary seller who faces a population of consumers and a premium posted-price competitor. Regarding the first decision, we find that under several different assumptions about consumers, bidding fees dominate margins in terms of expected profit. Bidding fees are a superior source of revenue because they allow more trades to occur via an implied lower bid-acceptance threshold, thereby making the market more efficient. Although similar to the intuition for the superiority of a two-part tariff to simple pricing, the argument for bidding fees is more subtle: unlike in the case of a simple two-part tariff, the reverse-pricing seller is not a monopolist and he does not capture the entire consumer surplus because of strategic bidding by the consumers. Despite these differences, we show that positive margins are not optimal, and the seller can always capture enough of the efficiency gains that arise from a switch to bidding fees. In addition to their profit advantage, bidding fees are also easier to implement than margins because they require less commitment from the seller: a margin seller needs to credibly reject potentially profitable trades in order to increase bids.

Regarding the second decision, the seller who makes revenue through bidding fees instead of margins sometimes prefers to voluntarily reveal the current bid-acceptance threshold to potential consumers (instead of trying to prevent them from discovering it). Intuitively, bidding fees “immunize” the seller against the low-profit consequences of consumers becoming informed about the threshold, and the additional information assures that only consumers with high-enough valuations enter and submit bids. Since high-valuation consumers bid as if their valuations were the premium outside posted price, this efficiency gain gradually vanishes as more high bidders enter the market. On the other hand, the additional information always allows the consumers to
capture a larger share of the gains from trade, so revealing the information is only profitable when the market does not have too many high-valuation consumers.

Our framework for specifying reverse-pricing mechanisms allows us to analyze market efficiency and distribution as a function of when the seller and consumers learn the wholesale price. When they learn this information in the beginning of the game, the proposed model is equivalent to posted pricing, so our results can be compared to this important benchmark. We find that the efficiency and seller profits move together, decreasing as the revelation of the wholesale price is delayed. In this sense, reverse pricing is second best to the standard posted pricing, and the menu-cost assumption about infeasibility of posted pricing is necessary to make reverse pricing optimal. On the other hand, consumers maximize their surplus when they can collect the information rent that arises from learning the wholesale price (or, equivalently, the bid-acceptance threshold) before they enter the market. Therefore, if technologically feasible, facilitating early consumer learning about the wholesale price/threshold may benefit both the seller and the consumers even when posted pricing is not possible. Of course, the menu costs that make posting selling prices infeasible also make posting all the thresholds difficult. Therefore, relying on communication among consumers may be necessary for their learning, and our managerial recommendation is that the seller may sometimes wish to enable instead of hinder such communication.

We organize the remainder of the paper as follows. Section 2 provides a brief review of the relevant literature. In section 3, we develop a model of consumer bidding behavior in a reverse-pricing market with an outside option, derive the optimal seller strategy for several key scenarios, and compare different specifications of the mechanism in terms of their implications for seller profit. Section 4 compares the efficiency and distributional properties of the reverse-pricing
market under these scenarios. We conclude in section 5 with a discussion of our findings, limitations of the present work, and directions for future research.

2 Relationship to Prior Research on Reverse Pricing

Interest in the rigorous examination of bidding behavior under reverse pricing has been growing. Most prior research in this area empirically analyzes bidding behavior and focuses on questions unrelated to the seller’s perspective. Some of this work examines behavioral aspects of bidding behavior: consumer preferences for different price-elicitation formats (Chernev 2003); the influence of emotions, evoked by the bidding process, on bidding behavior in reverse-pricing markets (Ding, Eliashberg, Huber, and Saini 2005); and the extent to which consumers behave rationally when making bidding decisions in such markets (Spann and Tellis 2006). Other empirical research attempts to infer certain consumer characteristics from observed bidding behavior. Hann and Terwiesch (2003) propose a model of bidding behavior under reverse pricing and use it to quantify consumer search cost from bid amounts. Spann, Skiera, and Schäfers (2004) develop and empirically test a model to simultaneously estimate consumers’ willingness to pay and the frictional costs based on their bidding behavior in a reverse-pricing setting. Hinz and Spann (2008) show that a consumer’s position in a social network can have a significant impact on bidding behavior as information regarding a seller’s bid-acceptance threshold diffuses through that network. However, none of the above articles characterize the optimal selling strategy in a reverse-pricing market.

Some recent articles are more closely relevant to the present work in that they examine some aspects of the seller strategy. Terwiesch, Savin, and Hann (2005) discuss the optimal bid-acceptance threshold and the influence of frictional costs on the threshold, based on the model
and results in Hann and Terwiesch (2003). Wang, Gal-Or, and Chatterjee (2005) investigate the case of a service provider that uses both a reverse-pricing and a posted-price channel. These authors show that offering a reverse-pricing channel alongside a posted-price channel can enhance service providers’ ability to target different market segments, thus providing a rationale for the existence of the reverse-pricing channel in addition to a posted-price channel. Amaldoss and Jain (2008) analyze joint bidding for multiple items at a reverse-pricing retailer and find that such joint bids can increase retailer profit.

The most related paper in the literature is Fay (2004), who also asks whether to limit consumers’ ability to learn the sellers’ bid-acceptance threshold through re-bidding. Contrary to the common belief that repeat bidding erodes profits by revealing information about the seller’s threshold, Fay shows that repeat bidding may leave profits unaffected as long as the seller anticipates rebidding and adjusts his threshold accordingly. Moreover, Fay (2004) also shows that the seller may sometimes wish to encourage rather than discourage repeat bidding. The key assumption of his model is that only two possible thresholds exist, arising from two possible inventory levels. This assumption both simplifies bidding and gives a lot of power to the seller through contingent pricing. In contrast, our model allows a continuum of thresholds in equilibrium, increasing realism. Our model is also complementary to Fay (2004) in that his seller is initially uncertain about inventory, whereas our seller is uncertain about marginal cost.

In contrast to prior work on optimal reverse-price selling, the present research examines not only whether to limit consumers’ ability to learn the sellers’ bid-acceptance threshold, but also the more fundamental and connected question of how to combine margins and bidding fees as a source of revenue. Instead of allowing bidders to gradually learn about the threshold through
repeated bidding, we analyze the limiting case in which bidders become fully informed about the threshold.

Our research relates to the auction and mechanism design literature in economics and marketing, which discusses extensively various aspects of optimal auction design, such as bidding fees and reserve prices (McAfee and McMillan 1987, Milgrom and Weber 1982, Samuelson 1985). In contrast to standard auctions, in which a seller can collect multiple bids, bidders in a reverse-pricing market do not compete with each other. However, much of the intuition from first-price sealed-bid auctions carries over to reverse pricing because a successful bidder pays his bid. We also demonstrate that the optimal bidding fee problem is a screening problem analogous but not identical to the problem Myerson (1981) analyzes of setting the optimal reserve price. We show in full generality that the optimal bidding fee always screens out more potential consumers than the optimal reserve price set by an auctioneer who can obtain the product for a wholesale price of zero.

In a nutshell, the contributions of this research relative to prior work on reverse pricing are as follows: We conceptualize the reverse-pricing seller as an intermediary uncertain about the wholesale price. The resulting model allows analysis and optimization of two key parameters of a reverse-pricing mechanism under the seller’s control: revenue model and information. In particular, we show that the optimal reverse-pricing mechanism charges a positive bidding fee and accepts all bids above the wholesale price rather than charging a margin above the wholesale price. Moreover, we account for the possibility of bidders discovering wholesale prices and show that both the seller and the consumers can sometimes benefit when consumer learning is facilitated, as long as the seller anticipates the learning and adjusts the fees accordingly.
3 Model of a Reverse-Pricing Market

3.1 Assumptions

We analyze the optimization problem of a small perishable-product intermediary seller who cannot set posted prices contingent on his highly variable wholesale costs. The seller is small in that his actions do not affect wholesale prices or the prevailing posted price in an outside market. We normalize the posted price in the outside market to be 1. The seller intermediates between producers and consumers of the product. The product is perishable in that neither the consumers nor the wholesale offers persist over time. Consequently, the seller can model his profit-maximization problem as a single-shot game between himself and the consumers who happen to be in the market at the moment. In the Priceline plane ticket setting, the producers are airlines, the product is a seat on a specific flight, and the product is perishable in the sense that each airline makes a short-lived exploding offer of a particular wholesale price for the seat to Priceline. Next, we discuss our assumptions about consumers and wholesale prices in more detail, starting with the former.

Consumers. A unit mass of risk-neutral consumers exists. The consumers have private valuations of the product drawn from some continuous distribution $H$ with support on $[0,M]$ for some $M \geq$ outside posted price = 1. We provide as many results as possible for a general $H$ but resort to $H=\text{Uniform}[0,M]$ with $M \geq 1$ and $H=\text{Exponential}(\beta)$ with $M=\infty$ when a general result is not tractable. Because $M \geq 1$, we have two distinct segments of consumers: those with a valuation less than 1 (“low consumers”) can only buy the product from the reverse-pricing seller, whereas others (“high consumers”) can also obtain a positive surplus by buying on the outside posted-price market. Since some consumers cannot afford the outside price, the outside market can be
considered a premium option. All consumers have unit demand in the sense that once they buy
the product from the reverse-pricing seller, they do not buy it again on the outside market.
Finally, consumers incur no frictional cost for bidding—an assumption we relax in the Appendix
to demonstrate the robustness of our results.

Wholesale Prices. The producers offer the product to the reverse-pricing seller at a
wholesale price $w$, which is between 0 and the outside posted price of 1. For analytical
tractability, we assume wholesale prices are distributed uniformly on $[0,1]$.

Timing of the Game. The difference between our model and a model of standard posted-
price retailing is that the wholesale price $w$ is not known when the seller sets its strategy, and
posting a selling price once $w$ becomes known is not practical. Instead, the seller must maximize
expected profits by specifying and announcing both his bidding fee and his bid-acceptance rule
before he knows $w$. The seller has the following two sources of revenue at his disposal:

1) The seller can charge each bidder a bidding fee $f$ regardless of whether an individual’s
   eventual bid is successful. This policy is similar to charging consumers for a referral service
   that matches them with producers, but it is not equivalent because the seller also keeps the
difference between the bid and the wholesale price when he accepts a bid.

2) The seller can commit to the minimum-margin strategy of only accepting bids that are at least
   a fixed margin $m \geq 0$ above the current wholesale price, that is, above $w+m$. Commitment is
   necessary because, *ex post* (i.e., once $w$ is realized), this strategy foregoes potentially
   profitable trades (when $w < bid < w+m$) in order to induce higher bids *ex ante*.

---

1 An equivalent assumption to the seller announcing the bid-acceptance rule would be that the consumers infer it
   from recent experience with the same seller and share this information through word-of-mouth (Hinz and Spann
   2008).

2 The $m \geq 0$ assumption implies that a seller cannot subsidize the product. We relax this assumption in the Appendix
   after demonstrating that the constraint binds, that is, that the optimal strategy given $m \geq 0$ involves $m=0$. 
Figure 1 illustrates the timing of the game. First, at Stage 1, the seller sets and announces his $(m,f)$ strategy. At Stage 2, consumers make their entry decision. Some low consumers do not enter because their expected surplus is negative. The consumers who do enter then submit their bids. Once the seller receives a bid (Stage 3), he queries the producers to determine the lowest current wholesale price and then applies the bid-acceptance rule committed to at Stage 1. The seller faces competition from the posted-price market because the bidders whose bids are unsuccessful, as well as those who chose not to submit a bid, have the opportunity to buy the product on the outside posted-price market at Stage 4.

The order of moves in Figure 1 implies that bidders are uncertain about $w$ when they submit their bids. Therefore, they must optimize against a distribution of bid-acceptance thresholds the seller’s strategy at Stage 3 implies, namely, Uniform $[m,1+m]$. This setup is analogous to the problem of a bidder in a first-price sealed-bid auction who faces one exogenous competitor who bids Uniform $[m,1+m]$. To balance the surplus from winning against the probability of winning, the optimal bidding strategy in such a situation is to “shade” the bid down from one’s valuation of the product. If the seller instead learns the wholesale price $w$ before Stage 1, the game is

---

Note that seller’s commitment to $m$ makes this a correct equilibrium belief about the bid-acceptance threshold.
reduced to a standard (posted) pricing problem in which the fee $f$ and the margin $m$ are interchangeable. We examine this important benchmark case first before solving the reverse-pricing model Figure 1 outlines.

### 3.2 Benchmark Model: Posted-Price Selling

When the wholesale price $w$ becomes public knowledge prior to Stage 1, a standard pricing problem results, with $w+m+f$ representing the “posted price.” Since consumers know $w$, they pay the fee $f$ and bid $w+m$ at Stage 2 when $w+m+f$ is less than their valuation of the product and less than the outside posted price (i.e., $w+m+f < \min(v,1)$). Since each entry thus generates revenue $f$ and automatically leads to a sale with margin $m$, $m$ and $f$ are interchangeable in this setting, and it is without loss of generality to let $m=0$. The seller’s profit is

$$\Pi(f \mid w) = f \Pr(w+f < v) = f \left[1-H(w+f)\right].$$

Maximizing $\Pi$ leads to the classic inverse-hazard condition of a zero-marginal-cost monopolist facing demand $D(f \mid w) = 1-H(w+f)$ (Tirole 1988):

$$f = -\frac{\frac{D(f \mid w)}{D'(f \mid w)}}{h(w+f)} = \frac{1-H(w+f)}{h(w+f)}.$$  

In the case of a uniform distribution of bidder valuations $H=\text{Uniform}[0,M]$, equation (2) becomes $f = \frac{(M-w)}{2}$, and a trade occurs if $w+f < 1 \Leftrightarrow w < 2-M$. When $w$ exceeds $2-M$, the posted-price seller abandons the low bidders and focuses on the high bidders by charging $f=1-w$. The latter obviously happens for all $w$ when $M>2$. When $M=1$, the seller is effectively a monopolist, and $M \in (1,2)$ results in the seller serving both consumer segments.

Posted-price selling is the optimal direct revelation mechanism for a seller who learns $w$ before Stage 1, because monopoly pricing coincides with the optimal reserve when a seller faces
only one bidder in an auction (Bulow and Roberts 1989). Therefore, even when consumers do not know $w$ before Stage 1, a seller who does know it can do no better than to reveal $w$ through posting a monopoly price. In other words, equation (2) characterizes optimal behavior even when bidders do not know $w$ before Stage 1.

This posted-price benchmark helps sharpen our research questions about reverse-pricing mechanisms. First, are bidding fees and margins still interchangeable from the seller’s perspective if $w$ is only revealed to the seller in Stage 3? Second, how will reverse pricing’s entry decision select consumers for trading (i.e., what will the conditional distribution of valuations be) when bidders need to decide whether to bid before knowing the threshold? And finally, how much worse off is the reverse-pricing seller as a result of his inability to use posted pricing contingent on wholesale prices?

### 3.3 Model of Reverse Pricing

In this section, we analyze the main proposed model, in which the seller learns the wholesale price only after consumers have submitted their bids (i.e., at Stage 3 of Figure 1). We solve the model by backward induction. Stage 4 behavior of the consumers is obvious, and Stage 3 behavior of the seller is pre-determined by the commitment assumption. We extend the model of Spann and Tellis (2006) to account for the outside option, and characterize consumers’ bidding behavior at Stage 2 as follows:

**Proposition 1**: Bidders with valuation $v \geq m$ bid $b(v \mid m) = \begin{cases} \frac{m+v}{2} & \text{for } v < 1 \\ \frac{m+1}{2} & \text{for } v \geq 1 \end{cases}$ at Stage 2.
The proofs of Proposition 1 and all subsequent propositions are in the Appendix. It is sufficient to characterize bidding for valuations that exceed the margin \( m \) because bidders with valuations below \( m \) cannot obtain a positive surplus in the market.

Intuitively, Proposition 1 follows because bidders maximize their expected surplus by trading off a higher probability of acceptance associated with a higher bid against obtaining a greater surplus (i.e., the difference between their valuation \( v \) and their bid \( b \)) if their bid is accepted. The probability of acceptance in turn follows from the Uniform \([m, 1+m]\) distribution of the bid-acceptance threshold. A bidder shades her bid in that \( b(v|m) < v \) for the same reason bidders in a first-price sealed-bid auction do: winning means paying one’s bid, and shading is necessary to leave room for surplus.\(^4\) The Stage 4 option to buy at a posted price implies a ceiling on bids. Because of the outside option, all high bidders with \( v > 1 \) mimic the behavior of a bidder with \( v = 1 \). As a consequence, the shape of \( H \) above 1 does not affect the reverse-pricing seller’s profit. Also, the expected surplus \( S(v,m) \) of consumers with \( v > 1 \) is qualitatively different from that of consumers with \( v < 1 \):

\[
S(v,m) = \begin{cases} 
\frac{(v-m)^2}{4} - f & \text{for } m < v \leq 1 \\
1 - \frac{(1-m)^2}{4} - f & \text{for } v > 1 
\end{cases}
\]

For a derivation of equation (3), see the proof of Proposition 1. The functional form of the surplus is instructive with respect to bidders’ entry decisions. First, low-valuation bidders face quadratic surplus because higher valuations and/or lower margins increase both the probability of bid acceptance and the surplus upon acceptance. These two components multiply to produce the convex function from which the bidding fee is simply subtracted as a direct transfer to the seller.

Another insight concerns the high bidders: they receive the outside-option surplus $v - 1$ plus a “gambling bonus” equivalent to the surplus of the highest low consumer (with $v \leq 1$ and hence uninterested in the posted-price market). Thus, when $f > 0$, satisfying the low consumers is critical for the reverse-pricing seller to be viable:

**Proposition 2:** High $(v > 1)$ bidders enter the reverse-pricing market if at least some low $(v \leq 1)$ bidders enter. No low bidders enter when $v \equiv m + 2\sqrt{f} > 1$. When $v \leq 1$, low bidders with valuations in $[v, 1]$ enter.

Proposition 2 describes the entry behavior for any continuous distribution of valuations $H$. Given the outside posted-price competitor, the reverse-pricing seller must please the low consumers to make any profit at all. The influence on entry of the two revenue sources—the bidding fee $f$ and the margin $m$—is quite similar. Both reduce entry and both increase the revenue the entrants provide. However, they are clearly not interchangeable as in the benchmark case of section 3.2. In particular, the fee enters $v$ as a square root because the surplus function in equation (3) is quadratic in $m$ but linear in $f$. To solve the model, we now analyze how the seller chooses between these two sources of revenue at the first stage of the game.

**Optimal Seller Strategy.** At Stage 1, the reverse-pricing seller maximizes his expected profit $\Pi(m, f)$ by selecting the optimal combination of bidding fee $f$ and margin $m$. The seller must first consider consumers’ entry behavior, which does not depend on the realization of the wholesale price and delivers a revenue $f$ when a bidder enters. Then the seller must average over all possible bidders who enter, considering all possible wholesale prices for each to see whether a sale would occur for every combination of bid and wholesale price. As a result, as long as
$m + 2\sqrt{f} \leq 1$ (i.e., at least some low bidders enter), the seller obtains the following total expected profit:

$$\Pi(m, f) = \left[1 - H\left(m + 2\sqrt{f}\right)\right]f + \int_{m+2\sqrt{f}}^1 \pi(m|v) dH(v) + \left[1 - H(1)\right]\pi(m|1)$$

(4)

where $\pi(m|v) \equiv \int_0^{v-m/2} \left(\frac{v+m}{2} - w\right) dw = \left(\frac{v-m}{2}\right)\left(\frac{v+3m}{4}\right)$.

See Appendix A3 for a detailed derivation of the profit function. The profit $\Pi(m, f)$ consists of three intuitive parts. The first term is the revenue from bidding fees, the second is the expected revenue from low entrants, and the third is the expected revenue from high entrants.

The second term of the profit function in equation (4) requires further dissection and explanation. It considers all possible valuations $v$ of consumers who will enter, and for each such $v$, it computes the seller’s ex-ante expected profit from a particular entrant with that valuation. This profit, denoted $\pi(m|v)$, arises from the seller integrating over all wholesale prices that are accepted ($w$ s.t. $w < b(v|m) - m$), while collecting the contribution $b(v|m) - w$ for each such $w$. Note that increasing the margin $m$ only increases $\pi(m|v)$ when $m$ is small relative to $v$:

$$d\pi/dm = (v - 3m)/4,$$ because a greater $m$ not only leads to higher bids, but it also tightens the bid-acceptance criterion. See Figure 2 for an illustration of these two effects. Because the margin acts on both the bid-acceptance threshold and the bidding function, a unit increase in $m$ results in at most a quarter-unit increase in profits: bidders only let half of the margin “pass through” as they increase their bids by only a half unit, and only wholesale prices below $v/2 < 1/2$ result in an accepted bid. In mathematical terms, $d\pi/dm |_{m=0} = v/4 \leq \sqrt{4}$. This bound will be critical for the seller’s ultimate decision as to whether to use a fee or a margin as a revenue source.
Figure 2: The Effect of Margin $m$ on Bid Amounts and Seller Profit $\pi(m \mid v)$

Note to Figure 2: The thick line shows the baseline ($m=0$) profit from a bidder with valuation $v$ as a function of the wholesale price $w$. The area under the line is the total baseline profit averaged over $w$ low enough to accept the bid. A positive $m$ raises the profit function to the dashed line but simultaneously lowers the acceptance criteria. The area marked $+$ is added to total baseline profit, and the area marked $-$ is subtracted.

To maximize profit $\Pi(m, f)$, the seller first finds the optimal bidding fee for every possible level of margin $m$. This contingent optimal fee satisfies the following first-order condition that equates the marginal cost of raising $f$ slightly with its marginal benefit:

$$FOC_f : \left( m + \frac{3\sqrt{f}}{2} \right) h(m + 2\sqrt{f}) = 1 - H\left( m + 2\sqrt{f} \right).$$

Increasing $f$ slightly has the marginal benefit of all bidders above $v$ paying a higher bidding fee (RHS of equation (5)). The marginal cost of increasing $f$ slightly (LHS of equation (5)) is the threshold bidder with valuation $v$ not entering anymore, which results in a loss of both the
marginal bidding fee that bidder would pay \( f \cdot (d\pi/df) = \sqrt{f} \) and the loss of the marginal profit from that consumer not bidding \( \pi(m | \varphi) \cdot (d\pi/df) = m + \sqrt{f}/2 \). Both of these components of the marginal cost are weighted by the density \( h(\varphi) \) of the marginal bidder occurring in the first place. The first-order condition implies the optimal bidding fee schedule contingent on \( m \), including the special case of a seller using \( m=0 \), perhaps because he cannot commit to rejecting some potentially profitable offers at Stage 3:

**Proposition 3:** When \( m - \frac{1-H(m)}{h(m)} < 0 \) and \( m + \frac{3x}{4} - \frac{1-H(m+x)}{h(m+x)} \) is an increasing function of \( x \), the optimal bidding fee contingent on the margin \( m \) is positive. When \( m=0 \), the optimal bidding fee satisfies: \( \frac{3}{4} \varphi = \frac{1-H(\varphi)}{h(\varphi)} \) when \( \varphi = 2\sqrt{f} < 1 \).

The first condition of Proposition 3 ensures that the margin \( m \) alone is not already screening out too many low bidders, and the second condition is a single-crossing condition analogous to a monotone hazard-rate condition on \( H \).

The \( m=0 \) case of Proposition 3 reveals that the problem of the zero-margin reverse-pricing seller is a screening problem: setting the optimal fee is equivalent to setting the screening level \( \varphi \), below which consumers stay out of the market. This screening characterization allows us to rank the optimal bidding fee relative to fees (screening levels) of two other sellers: a pure referral service and an optimal auctioneer who can obtain the product for \( w=0 \):

**Proposition 4:** The zero-margin reverse-pricing seller charges a lower bidding fee than a zero-marginal-cost referral service that matches consumers up with producers. In contrast, the zero-margin reverse-pricing seller charges a higher bidding fee than Myerson's (1981) optimal auctioneer, who can obtain the product for a wholesale price of zero.
A remarkable aspect of Proposition 4 is that it holds for every valuation distribution $H$. This generality is possible because all three sellers considered are solving similar screening problems. The referral service raises its fee above that of the reverse-pricing seller because it is only concerned with losing the marginal fee and not the profit contribution of the marginal bidder. In contrast, the optimal auctioneer with $w=0$ uses the screening equation (2) characterizes, and effectively charges a monopoly posted price for the product.

The key finding of the present work is that setting the margin $m$ to zero and using the optimal contingent bidding fee in line with Proposition 3 is often the globally optimal policy of the seller. We first demonstrate this result for a uniform distribution of valuations and then argue why it is likely to hold for many other distributions as well. Let $H(x) = x/M$ on $[0,1]$ and arbitrary on $[1,M]$. In other words, assume $1/M$ low bidders are distributed uniformly on $[0,1]$ and an additional mass $(M-1)/M$ of arbitrarily distributed high bidders are above 1. For this uniform distribution of low bidders, the optimal fee contingent on $m$ based on Proposition 3 becomes

\[
  \sqrt{f^*(m)} = \frac{2M - 4m}{7} \quad \text{whenever } 2m < M \leq \frac{7 + m}{4},
\]

where $2m < M$ ensures $m$ does not screen too much, and the upper bound on $M$ ensures participation because it must be that $v = (4M - m)/7 \leq 1$. The linear relationship between $\sqrt{f}$ and $m$ allows us to limit our search for the optimal selling strategy $(f^*, m^*)$ to a straight line $(2M - 4m)/7$ in the $(\sqrt{f}, m)$ space. This search does not turn out an internal solution, thereby resulting in the following proposition:

**Proposition 5:** When $H(x) = x/M$ on $[0,1]$, the optimal selling strategy uses zero margin $m$ and positive fee $\sqrt{f^*} = \min\left(\frac{2M}{7}, \frac{1}{2}\right)$. 

As both the fee and the margin screen out some low-valuation bidders, they might appear equivalent at first glance. However, the margin also changes the entrants’ bidding strategy. Because bidders shade their bids half way down from their valuation toward \( m \) (Proposition 1), the margin increases the bids of the entrants less than it increases the acceptance threshold \( w + m \). Therefore, increasing the margin slightly reduces the amount of trading the entrants achieve, thus diminishing market efficiency (i.e., the size of the pie). This reduction in efficiency is large enough to hurt seller profit as well. Consequently, generating revenue through a bidding fee, which does not distort bidding, is always better than generating revenue through a margin, which does.

**Illustrative Example.** For concreteness, suppose the seller is Priceline and the product is a plane ticket from New York to London, for which \( v = 1 \) corresponds to $1000—the lowest outside posted price competitors such as Expedia and Travelocity offer. Assume \( M = \frac{2}{3} \), indicating that two thirds of the prospective Priceline customers value the ticket below the lowest posted price and one third value it above that amount. According to Proposition 5, the optimal bidding fee is \( f^*(0) = \frac{6}{70} \approx $183 \), which screens at level \( v(0) = \frac{2}{3} \approx $857 \). Thus, most low consumers do not enter. Those low consumers who do enter bid \( \min(v/2, \frac{2}{3}) \), resulting in bids between $429 and $500. The expected social welfare \( W \) realized through Priceline is the difference \( v - w \) when there is a trade, that is, when valuation exceeds \( v \) and the bid exceeds \( w \). Assuming the high bidders are also Uniform on \([1, \frac{2}{3}]\), the welfare is \( W(0, \frac{6}{70}) \approx $198 \), with an overall probability of trading of about 21 percent. The seller’s profit is \( \Pi(0, \frac{6}{70}) \approx $130 \). (See Appendix A8 for the calculations of welfare and profit for this example.)
Now consider the opposite of \( f^\ast(0) \) along the locus of potential solutions defined in equation (6); that is, the margin \( m = \frac{\gamma}{4} \approx \$750 \), for which \( \sqrt{f^\ast(m)} = 0 \). The entry-screening level \( \gamma \) is now lower than under \( f^\ast(0) \): it is equal to \( m \). Moreover, the entrants now bid more, namely, \( \min((v + m)/2,(1 + m)/2) = \min(v/2 + \gamma/4, \gamma) \), resulting in bids between \$750 and \$875. This result all sounds good for both welfare and seller profit, but bids are now accepted only when they exceed \( w + m \), that is, when \( w < v/2 - m/2 \approx v/2 - \$375 \). The resulting welfare is \( W(\gamma/4, 0) \approx \$59 \). A comparison with \( W(0, \gamma_0) \) immediately reveals the reason so little welfare is realized with the margin: although slightly more low bidders enter, many more of their bids and the high bidders’ bids are rejected, resulting in only a 5.2 percent probability of a trade occurring. Among the high bidders, for example, instead of wholesale prices below \$500 resulting in a trade as in \( W(0, \gamma_0) \), only wholesale prices below \$125 now lead to a trade. Even if the seller appropriated all these gains as profit, he would be worse off than with \( \Pi(0, \gamma_0) \).

Therefore, the seller must be better off with the optimal fee and no margin. It can be shown that the seller actually makes \( \Pi(\gamma/4, 0) = 97/2304 \approx \$42 \)—about two thirds less than with the bidding fee \( f^\ast(0) \) and no margin.

Of course, \( m = \gamma/4 \) is not the optimal margin to charge when bidding fees are restricted to zero by assumption as opposed to allowed to be optimal contingent on the margin. It can be shown (see Appendix B1) that the optimal margin to charge without bidding fees is actually always less than \( \gamma/4 \), and the \( H=\text{Uniform}[0, \gamma/4] \) assumption implies the optimal margin \( m = (9 - \sqrt{41})/10 \approx \$260 \). The social welfare with this (rather low) margin is quite high at \( (14\sqrt{41} - 31)/300 \approx \$195 \), but the seller only makes about half of this welfare in profit \( \Pi(9 - \sqrt{41}/10, 0) = (61 + 41\sqrt{41})/3600 \approx \$89 \). Therefore, although society is almost as well off with \( (m^\ast, 0) \) as with \( (0, f^\ast(0)) \), the seller is still worse off than with \( (0, f^\ast(0)) \) because the
consumers receive a much larger portion of the gains from trade as their surplus. This concludes the example.

The uniform assumption does not drive the key qualitative result that the seller prefers fees over margins. To gain insight into the proof of Proposition 5 and see why the result generalizes to other valuation distributions $H$, consider the profit function $\Pi$ along the locus of contingent solutions equation (5) characterizes, parameterized by $m$:

$$
(7) \quad \Pi(m \mid FOC_f) = \int_{v(m)}^{1} \left[ f^*(m \mid M) + \pi(m \mid v) \right] dH(v) + (1 - H(1)) \left[ f^*(m \mid M) + \pi(m \mid 1) \right],
$$

where $f^*(m \mid M)$ is the optimal contingent fee satisfying equation (5), the integral is profit from low bidders, and the remainder is profit from high bidders (who all mimic the bidder with valuation 1). Equation (7) adds the per-entrant profit $f+\pi$ across all entrants, weighted by their likelihood in the distribution $H$. Starting with $m=0$, consider reshuffling the source of revenue from $f$ to $\pi$ by reducing $f$ by one dollar while increasing $m$ to stay on the $FOC_f$ locus. Since reducing one source leads to increasing the other, the effect on $v(m)$ of the reshuffling from $f$ to $m$ is second-order. The first-order impact of the reshuffling is on the profit $f+\pi$ obtained from each entrant. Recall that, because of bid shading, the seller cannot get more than 25 cents on the dollar from an increase in margin (i.e., $d\pi/dm < \frac{1}{4}$) regardless of $H$. Therefore, a one-dollar decrease in $f$ (i.e., a unit reduction of per-entrant revenue) along the locus would have to be accompanied by an increase in $m$ by at least four dollars (i.e., $dm/d(-f) > 4$) in order for $\pi(m \mid v)$ to compensate for the drop in $f(m \mid M)$. Such an increase is impossible for $H=$Uniform because $dm/d(-f)\big|_{m=0} = 49M/16 < 4$ (see equation (6)). The proof of Proposition 5 makes this argument rigorous for the Uniform $H$, but an analogous argument will work for other distributions as well. We provide another tractable example in the next paragraph.
Consider the exponential distribution \( H(x) = 1 - e^{-\beta/b} \), which is convenient because of its constant inverse hazard rate that is equal to its mean \((1 - H(x))/h(x) = \beta = E(x)\). Proposition 3 shows that the optimal fee is \( \sqrt{f^*(m)} = 2(\beta - m)/3 \) when \( 4\beta - 3 < m < \beta \). Thus, we need \( \beta < \frac{3}{4} \) for enough low bidders such that \( \nu < 1 \) with \( m = 0 \). Since the locus of contingent fees is again a straight line in the \((\sqrt{f}, m)\) space, showing that we can extend the qualitative predictions of Proposition 5 to the exponential case:

**Proposition 6:** When \( H(x) = 1 - e^{-\beta/b} \) on \([0,1]\), the optimal selling strategy uses zero margin \( m \) and positive fee \( \sqrt{f^*} = \min \left( \frac{2\beta}{3}, \frac{1}{2} \right) \).

Propositions 5 and 6 illustrate that the seller’s search for the optimal mix of fees and margins often ends up in a corner solution, with the bidding fee as the sole source of revenue. Although we do not have a completely general result, we argue that bidding fees dominate margins often because they increase the efficiency of the market. This intuition is analogous to the completely general intuition behind two-part tariffs outperforming simple pricing in posted-price markets.

At least four questions arise from the seller’s preference for fees over margins: First, given that the increased efficiency of the market can leave the seller better off, would he benefit even more if he revealed the wholesale price (or, equivalently, the implied bid-acceptance threshold) to consumers? In other words, should the seller try to disguise the thresholds or post them for everyone to see? Second, given that the no-subsidy constraint binds, would the seller actually want to subsidize the margin and increase the bidding fee above \( f^*(0) \)? Third, how does the prescription for bidding fees change when bidders experience additional frictional costs associated with participating in the reverse-pricing channel? And finally, how would a second-best seller unable to use bidding fees set its margin? When would such a seller be considerably
worse off than one who can use a bidding fee? We address the first question in the following section (because the answer is surprising) and the remaining ones in the Appendix (because they are straightforward extensions and/or robustness checks of our model).

3.4 Model with Informed Consumers

As discussed in the introduction, bidders may learn the bid-acceptance threshold early, for example, through communication with other bidders. Knowing the threshold is equivalent to knowing the seller’s wholesale price because margin $m$ is assumed to be common knowledge. To analyze consumer bidding behavior in that case, as well as the reverse-pricing seller’s best response in anticipation of it, we assume the limiting case of consumers knowing the upcoming wholesale price with certainty prior to making their entry decision. That is, consumers learn $w$ at the beginning of Stage 2 in Figure 1—a situation between the benchmark case of Section 3.2 and the main model of Section 3.3. Is consumers’ advance knowledge of the wholesale price bad news for the reverse-pricing seller? And how should the seller change his strategy at Stage 1 when he anticipates consumers learning $w$?

As in the benchmark case of Section 3.2, knowing $w$ makes consumers enter and bid $\min(w + m, 1)$ when $\min(v, 1) - m - f - w > 0$. Also as in the benchmark case, fees and margins are completely interchangeable, and we can set $m=0$ without loss of generality. Note that bidding fees are much easier to implement because they do require less commitment than margins. Since $m=0$, the seller collects all of his revenue through the fee, resulting in the following profit function:

$$\Pi(f) = f \Pr(entry \mid f) = f \int_{f}^{1} (v - f) dH(v) + f (1 - f) [1 - H(1)].$$
where the first term in $\Pr(entry \mid f)$ averages the probability that $w < v - f$ over the valuations of the low bidders. The second term adds the probability that $w < 1 - f$ times the probability of a high bidder occurring. The following proposition captures the optimal fee:

**Proposition 7:** When potential consumers learn the wholesale price before their entry decision, the optimal bidding fee is a solution to $2f = E[\min(1, v) \mid v > f]$.

Proposition 7 prescribes charging half of the expected net valuation (minimum of $v$ and 1) of bidders who have a positive probability of trading. As this net valuation cannot exceed 1, $f$ approaches $\frac{1}{2}$ as the probability of high bidder rises for all $H$. In the case of $H = \text{Uniform}[0, M]$, the optimal fee is $\left(2M - \sqrt{4M^2 - 6M + 3}\right)/3$, which becomes $1 - \sqrt{3}/3 \approx \$423$ in the plane-ticket example of Section 3.3. How this optimal fee compares to that of Proposition 5 is not clear a priori, but the plane-ticket example suggests the fee is much higher when the potential consumers learn the wholesale price $w$ in advance. This finding makes intuitive sense because informed consumers do not have to gamble at entry time and shade their bids afterward. Instead, they are simply consumers at a posted price of $w$.

What is the impact of this information time shift on seller profits and social welfare? Should the seller help or hinder the consumers in their learning of the bid-acceptance threshold? To answer these key managerial questions, the next section compares the efficiency and distributional properties of all three variants of the model: the benchmark posted-price model, the reverse-pricing model with bids preceding the wholesale realization, and the model with informed consumers.
4 Efficiency and Distribution Properties of Reverse Pricing

We now analyze the three different scenarios of Sections 3.2, 3.3, and 3.4 in terms of market efficiency, consumer surplus, and seller profit. We label the regimes according to when consumers first learn the wholesale price $w$ (see Figure 1):

1) before Stage 1 (posted-price selling of Section 3.2),
2) before Stage 2 (informed consumers of Section 3.4), and
3) before Stage 3 (reverse-pricing model of Section 3.3).

We assume $H=\text{Uniform}[0, M]$ to make the computation of social welfare well defined, that is, to fix the distribution of high valuations above the outside posted price. We start by focusing on the monopoly situation ($M=1$) in which consumers can only buy the product from the reverse-pricing seller, and then we discuss which of the qualitative conclusions are robust to $M>1$ (where the reverse-pricing seller competes for the high-valuation consumers with the outside posted-price market) and which are not.

4.1 Efficiency: Expected Gains from Trade in Monopoly ($M=1$)

Social welfare reaches its maximum value $\bar{W}$ when all the potential gains from trade are realized, that is, when trading occurs whenever a bidder’s valuation $v$ exceeds the wholesale price $w$ (which happens half the time due to $w$ and $v$ being distributed iid):

\begin{equation}
\bar{W} = \Pr(v > w) E(v - w \mid v > w) = \int_0^1 \int_{v-w}^1 (v-w) dv dw = \frac{1}{6}.
\end{equation}

Since some bidders (those with $0 < v < w$) never enter the market under each wholesale-revelation regime, the positive bidding fees in each regime immediately imply that the market is not efficient. This inefficiency is merely the dead-weight loss of monopoly in the reverse-pricing
setting. Denote $W_T$ the social welfare realized when $w$ is first revealed at the beginning of Stage $T$. The first two regimes (i.e., $w$ becomes known before Stage 1 or Stage 2) only involve the inefficiency due to reduced entry because all of the entrants’ gains from trade are realized. Since entry occurs in those regimes when $v > w + f$, the realized welfares are simply

$$W_T = \Pr(v > w + f_T)E(v - w|v > w + f_T),$$

where the regime 1 fee (a.k.a. the “posted price”) depends on $w$ according to $f_1 = (1 - w)/2$, whereas the regime 2 fee (the fee that anticipates informed consumers) does not depend on $w$ ($f_2 = 0$). Therefore,

$$W_1 = \Pr \left( v > w + \frac{1 - w}{2} \right) E \left( v - w | v > w + \frac{1 - w}{2} \right) = \int_0^{1 - w/2} \int_0^{1 - w/2} (v - w) dw dv = \frac{1}{8}$$

(10)

$$W_2 = \Pr \left( v > w + \frac{1}{3} \right) E \left( v - w | v > w + \frac{1}{3} \right) = \int_0^{1 - 1/3} \int_0^{1 - 1/3} (v - w) dw dv = \frac{10}{81}$$

Revealing the wholesale price at Stage 1 is slightly more efficient because the seller can adjust the bidding fee accordingly, which allows more trading to occur. (The ex ante trading probabilities are $\frac{1}{4}$ in regime 1 and $\frac{2}{9}$ in regime 2.) See Figure 3 for an illustration of when trading occurs under the three different regimes. Note that the increase in trading probability under the first regime is much greater than the increase in realized social welfare because more high-$w$ trades occur that involve a low $(v - w)$ contribution to welfare.

The third regime (i.e., reverse-pricing proper) involves an additional inefficiency because entrants bid less than their valuation, skipping some profitable trades in order to attain a positive surplus. Since we already developed this intuition in the example of Section 3.3, we simply note that the optimal fee with $M=1$ is $\frac{1}{\sqrt{2}}$, resulting in the screening level $\frac{1}{\sqrt{2}}$, and welfare:

$$W_3 = \int_{\frac{4}{7}}^{\frac{\sqrt{2}}{2}} (v - w) dv dw = \frac{279}{2744} \approx 0.101 < \frac{10}{81}.$$
Figure 3 illustrates the dramatically reduced trading probability under this regime. Whereas the trading probability drops by almost a third from the second regime, the social welfare is reduced by only 22 percent because, although the relatively smaller gains of bidders with lower valuations are not realized, the large gains close to the upper left corner of Figure 3 still occur.

Figure 3: Trading as a Function of when Wholesale Price First Becomes Known

The equilibrium incidence of trading when \( w \) is revealed at:

- Stage 1
- Stage 2
- Stage 3

Note: The area above the dashed 45-degree line represents the potential gains from trade, and the three differently shaded areas represent the trading probabilities under the three regimes. The isoquants of the conditional gains from trade are parallel with the 45-degree line, and the gains increase toward the upper left corner.

4.2 Distribution: Seller Profit and Consumer Surplus for a Monopoly \((M=1)\)

The previous section illustrates that the efficiency of the market increases as the wholesale price is revealed earlier and earlier, with the largest drop occurring as the revelation occurs at Stage 3 because of bid-shading: \( W_1 \approx 1.01W_2 > W_2 \approx 1.22W_3 > W_3 \). This section shows that the expected
seller profit is ordered in the same way as the social welfare (see Table 1 for the profit calculations). Therefore, a monopolist seller facing only low bidders \((M=1)\) prefers the wholesale price to be publicly revealed as early as possible: \(\Pi_1 \approx 1.12\Pi_2 > \Pi_2 \approx 1.08\Pi_3 > \Pi_3\).

Table 1: Seller Profit and Consumer Surplus

<table>
<thead>
<tr>
<th>Regime</th>
<th>Consumer Surplus</th>
<th>Seller Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(S_1 = W_1 - \Pi_1 = \frac{1}{24})</td>
<td>(\Pi_1 = E_w \left[ \max \left( f(1-w-f) \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>(W_1 \approx 0.041)</td>
<td>(= \frac{1}{4} E_w \left[ (1-w)^2 \right] = \frac{1}{12} \approx 0.083)</td>
</tr>
<tr>
<td>2</td>
<td>(S_2 = W_2 - \Pi_2 = \frac{4}{81})</td>
<td>(\Pi_2 = \max \left( \int f \Pr(w &lt; v - f) dv \right) )</td>
</tr>
<tr>
<td></td>
<td>(W_2 \approx 0.049)</td>
<td>(= \max \left[ \frac{1}{2} f(1-f)^2 \right] = \frac{2}{27} \approx 0.074)</td>
</tr>
<tr>
<td>3</td>
<td>(S_3 = W_3 - \Pi_3 = \frac{45}{1372})</td>
<td>(\Pi_3 = \max \left( \int \left( \frac{v}{2} \right)^2 + \frac{1}{2} \left( \frac{v}{2} \right)^2 \right) )</td>
</tr>
<tr>
<td></td>
<td>(W_3 \approx 0.033 &lt; S_2)</td>
<td>(= \int \left( \frac{v}{2} \right)^2 + \frac{1}{2} \left( \frac{v}{2} \right)^2 ) dv</td>
</tr>
</tbody>
</table>

The intuition for the profits increasing as information appears earlier is as follows: The profit increase from \(\Pi_2\) to \(\Pi_1\) is simply the principle of optimality. Because the consumers face the same information in both cases, the “informed” seller who knows \(w\) at Stage 1 can always replicate the \(\Pi_2\) profits of the “uninformed” seller by charging the same fee \(f_2 = \frac{1}{2}\) for every \(w\). Since the informed seller actually charges different prices for different \(w\), the inequality is strict: \(\Pi_1 > \Pi_2\).

The \(\Pi_2 > \Pi_3\) inequality shows that letting consumers know about the wholesale price before they enter but after the seller sets the bidding fee is beneficial to the seller. On the face of it, this result is surprising because the better-informed consumers capture a larger chunk of the
pie: the information about $w$ allows the informed entrants to capture the entire surplus $(v-w)$, whereas uninformed entrants have to share the pie with the seller via bidding. This intuition about the share of the pie is obviously correct. Consider what would happen if the seller did not anticipate the informed consumers and simply used the bidding fee that is optimal for the uninformed consumers, namely, $f_3 = (\frac{\lambda}{2})^2$. See Figure 3 to appreciate how low this bidding fee is relative to the optimal fee of $f_2$. Given this lower fee, the seller would receive only 

$$\frac{1}{2} f_3 \left(1 - f_3\right)^2 \approx 0.034 < \Pi_3$$

because the informed entrants would all bid $w$. The market would be efficient, and the consumers would receive a lion’s share of the gains from trade. However, this intuition about the share of gains is incomplete because it ignores the strategic response of the seller to the informed consumers in the form of dramatically increasing the bidding fee: raising the fee to about $\frac{\lambda}{4}$ matches the $\Pi_3$ profit, and raising it further to the optimal $f_2 = \frac{\lambda}{2}$ exceeds it. Even at this high level, the second regime is much more efficient $W_2 \approx 1.22W_3$, so, unsurprisingly, the seller can be better off.

We have already established that the trading mechanisms with informed consumers (regimes 1 and 2) are much more efficient than keeping consumers uninformed. When do the consumers capture most of these gains? Table 1 implies that consumers benefit most when the seller is uninformed because they get a better deal when the wholesale price happens to be low, that is, when consumers’ surplus is large. Note in Figure 3 that the $T=2$ regime conducts more trades for low $w$. Unsurprisingly, the efficiency gain from informed consumers (regimes 3 vs. 2) benefits the consumers. Interestingly, the intuition about the share of pie also reverses: letting consumers know about the wholesale price before they enter but after sellers set the bidding fee gives consumers a larger share of a larger pie.
In summary, shifting revenue generation from margins to bidding fees motivates the seller to make the wholesale price as transparent as possible before consumers enter. Not forcing consumers to gamble at both entry time and bidding time allows many more trades to occur because consumers enter more often and do not skip potentially profitable trades in order to get a good deal when they win. As the seller and consumers share the resulting efficiency gains more equally, everyone benefits in both absolute and relative terms. The seller could benefit further by somehow learning the wholesale price even earlier (i.e., before setting the bidding fee), but he would not share these additional efficiency gains, thereby leaving the consumers worse off. More importantly, the situation would be equivalent to a continuum of monopolistic posted-price markets for the product, with different posted prices for every different wholesale price.

Thus, we have established that the reverse-pricing channel with a constant bidding fee and publicly observable wholesale prices is less profitable for the seller than charging different posted prices for the different wholesale prices. The profit difference \((\Pi_1 - \Pi_2)\) is the cost of not customizing prices all the time—the price of simplicity. Much of the profit decrease (5/6 to be precise) accrues to the consumers as surplus, and only a small portion (1/6) is lost as additional inefficiency. Therefore, the reverse-pricing channel can be nearly as efficient as a posted-price channel, is simpler for the seller to optimize, and results in increased consumer surplus. We can draw on Fay (2008) and also argue that relative to using posted prices, reverse pricing is less likely to lead to a reaction of other sellers in the posted-price market.

### 4.3 Efficiency and Distribution with High Bidders \((M>1)\)

Sections 4.1 and 4.2 show that, in a monopoly, \(\Pi_1 > \Pi_2 > \Pi_3\), indicating that the seller prefers to reveal the wholesale price to the consumers as early as possible. The efficiency of the market
sorts the same way. However, the consumers prefer knowing the wholesale price before entry, but the seller does not know the wholesale price before setting his fees: \( S_1 < S_2 > S_3 \).

Including some high bidders with a small \( M > 1 \) would not affect these orderings because the models are continuous in \( M \) at 1: the optimal fees, and hence welfare and profit, change continuously as \( M \) increases, starting with the monopoly \( M = 1 \). In general, the fees \( f_T \) increase to capture the higher profits from the high bidders instead of the lower profits from the marginal low bidders. What would happen if \( M \) were substantially greater than 1?

Suppose \( M \) grows without bounds. In the limit, the reverse-pricing seller only serves the high bidders by charging \( f_2 = \sqrt{f_1} = \frac{1}{2} \) and making sure they are willing to participate in reverse pricing instead of simply buying in the posted-price market. Since they all enter and bid \( \frac{1}{2} \), wholesale prices below \( \frac{1}{2} \) result in a transaction. The welfare calculation is simple for the seller in regime 3 because \( \sqrt{f_1} = \frac{1}{2} \) is constant once \( M \) reaches 7/4:

\[
W_3 \left( M \mid M > \frac{7}{4} \right) = \frac{1}{M} \int_0^{\frac{1}{2}} (v - w) dw dv = \frac{2M^2 - M - 1}{8M}.
\]

The welfare the seller who faces informed consumers in regime 2 achieves can be expressed in the limit as \( M \) grows large and \( f_2 \) approaches \( \frac{1}{2} \). In that limit, the seller only serves the high consumers who mimic the \( v = 1 \) consumer and buy when \( w + f_2 < 1 \Leftrightarrow w < \frac{1}{2} \). This trading rule is the same as in regime 3, so \( W_2 \rightarrow W_3 \left( M \mid M > \frac{7}{4} \right) \), and the efficiency gains of releasing information gradually evaporate. However, for finite \( M \), the \( W_2 > W_3 \) relationship is obviously preserved even with high bidders. An analogous argument can show \( W_1 \rightarrow (M - 1)/2 > W_2 \), and the full ordering of welfares generalizes from the monopoly case.
In terms of seller profit, the principle of the optimality intuition for $\Pi_1 > \Pi_2$ is fully general, so this inequality will continue to hold for all $M$. In contrast, $\Pi_2 > \Pi_3$ continues to hold only for low $M$ because as $M$ grows large in the limit, the seller in regime 2 gets $f_2(1 - f_2) \rightarrow \gamma_4$ from the high bidders, whereas the seller in regime 3 gets the bidding fee $f_3 = \gamma_4$ plus the profit $\pi(0|1) = \gamma_6$ from bids exceeding $w$. Therefore, $\Pi_2 \rightarrow f_2(1 - f_2) = \gamma_4 < \gamma_4 + \gamma_6 = f_3 + \pi(0|1) \leftrightarrow \Pi_3$. It can be shown numerically that the two profit functions cross at $M \approx 1.43$. The benchmark posted-price seller makes at least as much as he does abandoning the low bidders and charging $f_1 = 1 - w$, which alone would result in a superior profit of $\Pi_1 = ((M - 1)/M)E(1 - w) = (M - 1)/2M > \max(\Pi_2, \Pi_3)$. Thus, we have shown the final proposition, which ties together all the efficiency and distribution results:

**Proposition 8**: Suppose $H =$Uniform $[0,M]$. For all $M \geq 1$, revealing the wholesale price to consumers earlier in the game is more efficient, and the posted-price benchmark seller that results from revealing the wholesale price at the beginning of the game attains a higher profit than any other type of seller. For a reverse-pricing seller who does not know the wholesale price at the time he sets his fee structure, revealing the wholesale price to consumers when $1 \leq M < 1.43$ is more profitable, and vice versa for higher $M$.

The intuition for the switch in the profitability of informing consumers is as follows: with few high bidders, $\Pi_2 > \Pi_3$ because the efficiency gain from informing consumers outweighs the increased information rent of the consumers (as discussed in the previous section). As the proportion of high consumers increases, the efficiency advantage gradually disappears (see above), but the information rent remains in the form of the informed consumers’ unwillingness to bid above the wholesale price.
5 Discussion

This paper analyzes the design of an optimal reverse-pricing mechanism from the perspective of a profit-maximizing seller. Specification of such a mechanism involves two key decisions: (1) the revenue model, namely, whether to set a margin above cost (and thus set the bid-acceptance threshold) and/or charge a fee for the right to bid, and (2) whether to facilitate or hinder consumer learning about the bid-acceptance threshold. Bidders may learn about the acceptance threshold through repeated bidding or communication with other bidders.

Two key implications for reverse-pricing sellers result from our analyses. First, the optimal revenue model is to charge a bidding fee and then accept all bids above the wholesale price instead of charging a margin above the wholesale price. A bidding fee is preferable because it allows more trades to occur and because it does not require commitment to reject potentially profitable trades. In contrast, a margin increases the bid-acceptance threshold (to cost + margin), thus reducing the amount of trading achieved (i.e., reducing efficiency). There is nothing special about zero margin: if subsidies on the wholesale price are possible, it is optimal for the seller to increase the bidding fee and subsidize high wholesale prices via a negative margin (see Appendix B2).

The second major implication of our analyses is that if consumers discover the bid-acceptance threshold (e.g., via word of mouth) before they make their entry and bidding decisions, the market becomes more efficient, and both the consumers and the seller can benefit. Here, charging bidding fees instead of margins motivates the seller to actually make the acceptance threshold as transparent as possible before bidders enter. Not forcing bidders to gamble at both entry time and bidding time allows many more trades to occur. Moreover, since the seller and the bidders share the resulting efficiency gains more equally, everyone benefits in
both absolute and relative terms. Therefore, consumers’ discovery of the bid-acceptance threshold can be a win-win situation, and sellers may even actively promote consumers’ learning of these thresholds. However, the result of the seller benefiting from such an early revelation of the acceptance threshold depends on the distribution of valuations across consumers in the market. As the proportion of high-valuation consumers increases, the seller may be better off not revealing the bid-acceptance threshold early because the information rent from these consumers dominates the efficiency gains.

The results of our analyses also provide interesting insights into the phenomenon of (“live”) shopping communities, in which retailers may charge consumers a fee for the opportunity to enter and buy short-term promotional offers. Furthermore, retailers may even charge consumers a fee for the discovery of the selling price. For instance, the German online shopping site www.rabattschlacht.de, at which prices fluctuate continuously, charges consumers 0.90€ to learn the current price of a product.

Bidding fees can be viewed as analogous to subscriptions, which are quite common in discount retailing (BJ’s, Sam’s Club, etc.). When interpreting the bidding fee as a subscription, the key simplifying assumption is that the consumer’s surplus from bidding at Stage 2 is an accurate characterization of the consumer surplus that arises from her aggregate purchases over the lifetime of the subscription, with these purchases reflecting successful reverse-pricing bids rather than posted-price transactions. By showing that bidding fees are optimal in such a situation, we have provided a model of a subscription-based reverse-pricing discounter.

---

5 For example, sellers may wish to support online communities such as www.biddingfortravel.com or www.betterbidding.com.
We derive our results under the simplifying assumption of no frictional costs for ease of exposition. This is not a limitation because, as we show in Appendix B3, our basic qualitative results would not change if we relaxed this assumption. In contrast, the assumption that the outside market is not a strategic player is necessary for our results, and what would happen if more active competition were allowed is unclear. The analysis of the dynamics of competition between reverse-pricing and posted-price sellers (see Fay 2008) is an important area for future research.

Our analyses are based on the (common) assumption of rational, utility-maximizing consumers. Follow-up work is needed to empirically validate our analytical results. Psychological factors might cause actual bidding behavior in a reverse-pricing context to depart from the normative model we use to characterize it. For instance, examining consumer reactions to bidding fees would be an interesting line of research. We suspect that consumers might (irrationally) be more willing to pay a margin than to pay a bidding fee for entry. For example, consumers may use an overriding “rule” to never pay for anything other than the product itself (e.g., information or service), a situation akin to the one Amir and Ariely (2007) describe. A deep understanding of the behavioral aspects of bidding in reverse-pricing markets will require much empirical work, but the analytical findings we report here provide a normative benchmark against which to develop and test behavioral departures and extensions.

The analyses we have presented in this article have important implications for sellers who wish to apply a reverse-pricing mechanism. Our findings provide guidance as to how such firms should determine the optimal reverse-pricing mechanism in terms of how to set a bidding fee or margin, whether to facilitate or hinder consumer learning about the bid-acceptance threshold, and how best to adjust the revenue model when such consumer learning is anticipated.
Appendix A: Proofs of Propositions

A1. Proof of Proposition 1

A bidder with valuation \( v \) who has decided to enter and bid an amount \( x \) has the expected surplus

\[
\Pr(x \text{ accepted}|m)(v-x) + \mathbf{1}(v > 1)\Pr(x \text{ denied}|m)(v-1) = (x-m)(v-x) + \mathbf{1}(v > 1)(1+m-x)(v-1).
\]

Maximizing the expected surplus over \( x \) yields the result

\[
(A1) \quad b(v|m) = \arg\max_x (x-m)(v-x) + \mathbf{1}(v > 1)(1+m-x)(v-1) = \begin{cases} \frac{m+v}{2} & \text{for } v < 1 \\ \frac{m+1}{2} & \text{for } v \geq 1 \end{cases} \quad QED \text{ Prop 1.}
\]

The ex-ante overall expected surplus \( S(v,m) \) from entering and bidding \( b(v|m) \) is (see equation (3)):

\[
(A2) \quad (b(v|m)-m)(v-b(v|m)) + \mathbf{1}(v > 1)(1+m-b(v|m))(v-1) - f.
\]

A2. Proof of Proposition 2

To see the first part, note that when the gambling bonus of the highest low bidder is negative, the high bidders will just skip the reverse-pricing option and buy in the posted-price channel to get \( v-1 \) for sure. Since for every \( v > 1 \), \( S(v,m|v > 1) > S(v,m|v < 1) \), entry by low bidders implies entry by high bidders. To derive the entry threshold of the low bidders, note that a rational bidder with \( v < 1 \) decides to bid when \( (v-m)^2/4-f > 0 \). A quadratic equation yields the valuation entry threshold \( v \): \( v^2 - 2mv + (m^2 - 4f) = 0 \) & \( v > m \Rightarrow v = m + 2\sqrt{f} \). Thus, we have a useful constraint: at least some low bidders participate \( \Leftrightarrow m + 2\sqrt{f} < 1 \). Q.E.D.

A3. Detailed Derivation of the Expected Seller Profit

The expected seller profit of equation (4) results from solving the following expected profit expression up to \( H \):

\[
\Pi(m,f) = \Pr(y < v)f + E_{w,v} \left[ \mathbf{1}(b > w+m)(b-w) \right] =
\]

\[
(A3) \quad = \left[ 1 - H(v) \right] f + \int_0^1 \mathbf{1} \left( \frac{v-m}{2} > w \right) \left( \frac{v+m}{2} - w \right) dwdH(v) + \left[ 1 - H(1) \right] \int_0^{1-m} \left( \frac{1+m}{2} - w \right) dw.
\]
A4. Proof of Proposition 4

In terms of the LHS of equation (5), the referral service only loses the marginal bidding fee on the margin, so his optimal screening is characterized by

\[
\sqrt{\bar{f}} = \frac{1 - H \left( \frac{2 \sqrt{\bar{f}}}{f} \right)}{h \left( \frac{2 \sqrt{\bar{f}}}{f} \right)} \quad \text{i.e.} \quad \frac{1}{2} \bar{V} = \frac{1 - H \left( \bar{V} \right)}{h \left( \bar{V} \right)} .
\]

The pure intermediary thus charges a higher fee and screens out more bidders than the reverse-pricing seller. The optimal auctioneer, who can somehow get the same product for zero cost, uses Myerson’s (1981) screening level (called “reserve price” in the auction literature) that satisfies

\[
R = \left( 1 - H \left( R \right) \right) / h \left( R \right) . \quad Q.E.D.
\]

A5. Proof of Proposition 5

Consider the profit function restricted to the locus of points that satisfy the \( FOC_f \) of equation (5). For clarity, let \( K = \sqrt{f} \), so the locus is a straight line \( K = (2M - 4m)/7 \), and the screening level is \( \bar{v} = m + 2K = (4M - m)/7 \). To reduce the number of ratios, rescale the profit by multiplying it through by 8M:

\[
\hat{\Pi} (m) \equiv 8M \Pi \left( m, \left( \frac{2M - 4m}{7} \right)^2 \right) =
\]

\[
= \frac{8}{7^3} (3M + m)(2M - 4m)^2 + \int_{\frac{4M - m}{7}}^1 (v - m)(v + 3m) \, dv + (M - 1)(1 - m)(1 + 3m) .
\]

The derivative of \( \hat{\Pi} \) is negative along the entire locus because it is proportional to the quadratic

\[
7^2 d\hat{\Pi} / dm = -11m^2 - 38Mm - 64M^2 + 98M - 49, \quad \text{which is negative for all } m \text{ on the locus and all } M \text{ weakly greater than one}. \quad \text{Since profit is thus decreasing in } m \text{ along the locus of possible solutions, the globally optimal solution is captured by the } FOC_f \text{ given } m = 0 . \quad Q.E.D.
A6. Proof of Proposition 6

The screening level is \( v(m) = m + 2\sqrt{f^*(m)} = (4\beta - m)/3 \), so profit \( \Pi(m \mid FOC_f) \) in equation (7) can be differentiated in \( m \) and confirmed decreasing for every \( \beta \):

\[
(A6) \quad \frac{d\Pi(m \mid FOC_f)}{dm} < 0 \iff \frac{4}{3\beta} - \frac{2m}{3\beta} e^{\frac{3m}{\beta}} < 0
\]

A7. Proof of Proposition 7

The first-order conditions of the profit function in equation (8) are

\[
(A7) \quad 0 = \Pr(entry \mid f) + f \frac{d\Pr(entry \mid f)}{df} \iff 2f = \frac{1 - H(1) + \int_v^f v dH(v)}{1 - H(f)}.
\]

On the RHS is the expected “net valuation” (minimum of \( v \) and 1) of bidders who have a positive probability of trading: \( E(\min(1,v) \mid \min(1,v) > f) \). The FOC says to charge half of this expected net valuation. \( Q.E.D. \)

A8. Welfare and Profit Calculation of Example in Section 3.3

\[
(A8) \quad W(0, 9/49) = \frac{2}{3} \int_0^{1/7} (v-w) dw dv + \frac{2}{3} \int_0^{3/7} (v-w) dw dv = \frac{271}{1372} \approx $198
\]

\[
(A9) \quad \Pi(0, 9/49) = \left[ 1 - \frac{2}{3} \int_0^{6/7} \frac{9}{49} + \frac{2}{3} \int_0^{1/8} v^2 dv + \frac{11}{3} \right] = \frac{461}{3528} \approx $130
\]

\[
(A10) \quad W(3/4, 0) = \frac{2}{3} \int_0^{3/4} \int_0^{(\frac{v-3}{2})} (v-w) dw dv + \frac{2}{3} \int_0^{3/4} (v-w) dw dv = \frac{15}{256} \approx $59
\]
Appendix B: Extensions: No Fees, Subsidies, and Frictional Cost

B1. No Bidding Fees Allowed

Although not globally optimal, considering the optimal margin for situations that do not allow fees is interesting. To this end, we return to the case of general $H$. The first-order condition in $m$ is more complicated because $m$ affects not just entry but also the bidding strategy of all entrants:

$$\text{(A11) } FOC_m : 0 = \frac{\partial \Pi}{\partial m} = \left[ \int_{m + 2 \sqrt{f}}^\infty \left( \frac{v - 3m}{4} \right) dH(v) + (1 - H(1)) \left( \frac{1 - 3m}{4} \right) \right] - h(m + 2 \sqrt{f}) \sqrt{f} \left( m + \frac{3 \sqrt{f}}{2} \right).$$

The first term delineated by brackets shows the benefit of raising $m$ slightly: all bidders above $v$ bid more, and the net effect on profits is positive when $d\pi/dm\bigg|_{v=0} > 0$, that is, when $\sqrt{f} > m$. The marginal cost of increasing $m$ slightly is analogous to the above case of $f$ (equation (5)): an increase in $m$ screens out the marginal bidder with valuation $v$, and both his marginal bidding fee $f$ and his bid $m\sqrt{f} + f/2$ are lost. The second term combines these costs because they are both weighted by the density $h(v)$ of the marginal bidder occurring.

Starting at no margin, increasing $m$ slightly results in a much smaller downside than upside: not only is no fee lost, but the profit $\pi(m \mid v)$ the marginally non-entering bidder generates is zero. Therefore, the optimal margin must be positive because

$$\text{(A12) } \left. \frac{\partial \Pi}{\partial m} \right|_{m=0, f=0} = \left( \frac{1}{4} \right) \left[ 1 - H(1) + \int_0^1 v dH(v) \right] > 0.$$

The restricted $FOC_m$ shows that the balance of losing profit from low bidders and gaining profit from the high bidders, where $3m - v$ is clearly positive for low-enough entrants with $v$ above but near $v = m$, determines the optimal margin:

$$\text{(A13) } FOC_m \text{ given } f = 0 : (1 - H(1)) (1 - 3m) = \int_m^1 (3m - v) dH(v).$$

In the uniform case, the optimal margin without a bidding fee is
Interestingly, committing to a positive margin is always optimal, even when no high bidders enter the market: raising everyone’s bids is worth the loss of a small fraction of the lowest bidders, who do not generate positive profits anyway. In the uniform case, the optimal margin to commit to is $\frac{2}{3}$. L’Hopital’s rule shows that the limit of $m^*$ as $M$ grows to infinity is $\frac{13}{5}$.

### B2. Subsidies Allowed

Suppose the seller can credibly promise the consumers he will subsidize the wholesale price. For simplicity, we assume $H=\text{Uniform } [0,1]$. Let $s>0$ be the subsidy in that the seller will accept $b > w-s$. This assumption means nobody has an incentive to bid above $1-s$. However, the outside option means nobody wants to bid more than $(1-s)/2$ anyway, so this assumption does not bind the computations. The bidder surplus function becomes exactly like $b(v|m)$ with $m<0$:

\[
(A15) \quad b(v|s) = \arg\max_x \text{Pr}(x+s > w)(v-x) + 1(v > 1)\text{Pr}(x+s < w)(v-1) = \begin{cases} \frac{v-s}{2} & \text{for } v < 1 \\ \frac{1-s}{2} & \text{for } v \geq 1 \end{cases}.
\]

Because the form is the same as in the previously solved case, the optimized surplus functions are the same:

\[
(A16) \quad S(v,s) = \begin{cases} \frac{(v+s)^2}{4} - f & \text{for } m < v \leq 1 \\ v-1 + \frac{(1+s)^2}{4} - f & \text{for } v > 1 \end{cases}.
\]

Therefore, the participation decisions are also the same: the entry threshold is $v = \min \{0, -s + 2\sqrt{f}\}$. As long as $2\sqrt{f} > s$, the profit computations therefore carry exactly as before, as does the locus of potential solutions $\sqrt{f} = 2(1+2s)/7$. It is immediately clear that $2\sqrt{f} > s$ on the locus—some screening of low consumers is desirable even with subsidies. Since everything carries through, the profit function along the locus carries through as well:
In the feasible range, where \( v < 1 \), the cubic profit equation has a unique maximum at \( s = 5/11 \). The corresponding fee is \( \sqrt{f} = 6/11 \), so the screening level is \( v = 7/11 \). We thus have a solution to the unconstrained problem: When wholesale-price subsidies are possible, the optimal selling strategy sets the bidding fee to \( (\%_1)^2 \) and subsidizes each wholesale price by \( \%_1 \). Only bidders with \( v \in [\%_1,1] \) enter. The result is striking in that the subsidy of 0.45 far exceeds the bidding fee of 0.3. This seemingly good deal makes sense because of the randomness of the wholesale prices: not all entrants get the subsidy. As the high \( v \) shows, only very high bidders actually find the deal worthwhile.

B3. Frictional Costs

Suppose bidders incur frictional costs related to their entry and bidding in the reverse-pricing channel, perhaps because of the opacity of the product or the time costs associated with participating (Ratchford 1982). It is reasonable to assume these costs are proportional to valuation: higher bidders care more about the exact details of the product, and their time is worth more. Suppose an entrant loses a fraction \( c < 1 \) of her valuation whether or not she gets the product. When she wins, she still enjoys the full value of the product. Therefore, the bidding after entry remains unaffected. However, the surpluses guiding the entry-decisions change:

\[
(A18) \quad S(v, m) = \begin{cases} 
\frac{(v-m)^2}{4} - f - cv & \text{for } m < v \leq 1 \\
(v-1) + \frac{(1-m)^2}{4} - f - cv & \text{for } v > 1
\end{cases}
\]

Two participation thresholds appear: the low bidders participate when

\[
(v-m)^2 - 4f - 4cv > 0 \Leftrightarrow v > v = 2c + m + 2\sqrt{c^2 + f + cm},
\]

where we need to constrain \( m^2 < 4f \) in order to guarantee the smaller root of the participation condition is negative. This lower bound
on participating low bidders is analogous to the \( \nu \) in the basic model. A high participation threshold arises because the high bidders are less and less keen on the reverse-pricing channel as their valuations increase. Their gambling bonus is only positive when
\[
1 < v < \bar{v} \equiv \left( (1 - m)^2 - 4f \right) / 4c \quad \text{where} \quad (1 - m)^2 > 4(c + f) \quad \text{ensures} \quad \bar{v} > 1, \quad \text{and the constraint binds only when} \quad \bar{v} \quad \text{is inside the support of} \quad H. \quad \text{Note that increased screening at the bottom via either} \quad f \quad \text{or} \quad m \quad \text{now also implies increased screening at the top. Will bidding fees still be optimal?}
\]

As a consequence of the second participation threshold, the shape of \( H \) above 1 is no longer irrelevant, so let \( H \) be Uniform \([0, M]\) for simplicity. The seller’s profit function becomes
\[
(A19) \quad MP(m, f) = (\bar{v} - v) f + \int_v^\infty \frac{\min(v, 1) - m}{2} \frac{\min(v, 1) + 3m}{4} dv.
\]

The FOC in \( f \) is
\[
(A20) \quad \frac{\partial \Pi}{\partial f} = (\bar{v} - v) - \frac{\partial v}{\partial f} \left[ f + \frac{(v - m)}{2} \left( \frac{v + 3m}{4} \right) \right] + \frac{\partial \bar{v}}{\partial f} \left[ f + \frac{(1 - m)}{2} \left( \frac{1 + 3m}{4} \right) \right],
\]

where the last term is new and the first term missing the \((1 - \bar{v})\) consumers. Therefore, the optimal bidding fee should be smaller than without the frictional cost.

The form of the \( FOC_f \) is algebraically involved. To simplify, we focus on the case \( m=0 \). The changes in threshold become \( \partial v / \partial f = 1 / \sqrt{c^2 + f} \) and \( \partial \bar{v} / \partial f = -1 / c \). Therefore, a unit increase in \( f \) spurs away more high bidders than low bidders: \( |\partial v / \partial f| < |\partial \bar{v} / \partial f| \). Re-parameterize the problem from the \((c, f)\) space to \((c, K)\) space, where \( K = \sqrt{c^2 + f} \). In this space, the first-order condition that determines \( f \) is
\[
0 = \partial \Pi / \partial K = K - 4(4K - c) (c + K)^2.
\]

A solution of optimal \( K \) as a function of \( c \) is possible but messy because we are dealing with a cubic equation. Nevertheless, we can derive several properties of optimal bidding fees from the first-order condition without explicitly solving it: First, the slope of profit is positive at \( f=0 \) as long as \( c \) is small enough: \( f=0 \) means that \( K = c \), so
\[
\frac{\partial \Pi}{\partial f} \bigg|_{f=0} > 0 \Leftrightarrow \frac{\partial \Pi}{\partial K} \bigg|_{K=c} > 0 \Leftrightarrow 48c^2 < 1. \quad \text{Therefore, the optimal fee is positive as long as the frictional cost} \quad c \quad \text{is small enough in the sense} \quad c < \sqrt{1/48} \approx 0.14. \quad \text{Second, we can investigate how}
the seller should respond to an exogenous increase in the frictional costs. It may seem that the
seller should always compensate by reducing the fee, yet some screening is always optimal.
Explicitly obtaining the comparative static of \( f \) in \( c \) is difficult, but implicit differentiation of the
\( FOC_k \) yields how optimally selected \( K \) varies with \( c \):

\[
(A21) \quad \frac{dK^*}{dc} = \frac{4(c + K)(7K - 3c)}{1-8(c + K)(c + 6K)} > \frac{32c^2}{1 - 112c^2} > 1 \Leftrightarrow c > \frac{1}{12},
\]

where the first inequality follows from \( K > c \). Since \( \frac{\partial K}{\partial c} < 1, \frac{dK^*}{dc} > 1 \) means \( f^* \) must be
increasing in \( c \). Given that \( v = 2c + 2K \), this finding is striking: for a unit increase in \( c \) (none of
which results directly in revenue), the seller who does not change his bidding fee automatically
loses between 2 and 3 units of low customers. When \( f^* \) is increasing in \( c \), the seller finds
screening away even a few more customers to be profitable.

The appeal of frictional costs proportional to valuation is the second entry threshold at the
top, above which consumers simply buy at the posted price market. However, as \( c \) gets extremely
small, this second threshold vanishes (for a fixed \( M \)). To analyze very small costs, considering
only the bottom threshold is therefore necessary. Keep the \( m=0 \) assumption, and without loss of
generality set \( M=1 \). Then the first-order condition simplifies to

\[
(A22) \quad 0 = \frac{\partial \Pi}{\partial f} = 1 - v - \frac{\partial v}{\partial f} \left[ f + \frac{v^2}{8} \right] = 1 - 2c - 2K = \frac{3K^2 - c^2 + 2cK}{2K}.
\]

At \( K=c \), \( \frac{\partial \Pi}{\partial f} = 1 - 6c > 0 \Leftrightarrow c < \frac{1}{6} \). Therefore, a positive fee is once again optimal as
long as \( c \) is small enough. A simple quadratic equation characterizes the optimal \( K \):

\[
K^* = \left(1 - 3c + \sqrt{1 - 6c + 16c^2}\right)/7. \quad K^* \text{ has several notable features: First, setting } c=0 \text{ produces the}
\]
now familiar \( K = \frac{1}{2} \) result from the model without frictional costs. Second, \( dK^*/dc < 0 \), so a rise
in cost always leads to an aggressive drop in fee—so aggressive that the drop in fee overwhelms
the rise in \( c \) so as to make \( c^2 + f \) go down. In other words,
Equation (A23) leads to a useful rule of thumb: when the frictional costs rise by a unit, dropping the bidding fee at least two frictional costs is optimal. For example, suppose the products are plane tickets from New York to London, \( v = 1 \) corresponds to $1000, and Priceline bidders experience a participation cost equal to 1 percent of their valuation of the ticket (\( c = 0.01 \)). Then Priceline should charge a bidding fee of about $76. Only bidders with valuations greater than \( v = $574 \) will enter. The impact of the bidding fee on the amount of entry is enormous; \( v \) would be only $40 with no fee. Now suppose the cost suddenly rises to 2 percent. \( \frac{df^*}{dc} < -2c \) means the fee has to fall at least $1000 \( \times 2c \cdot \Delta c = $2 \). In fact, the new optimal fee is about $72. Figure A1 plots the optimal bidding fee as a function of frictional costs, demonstrating the nearly linear relationship that gives rise to the rule of thumb.

Figure A1: Optimal Bidding Fee as a Function of Percentage Frictional Costs (M=$1000)
References


