An energy based peridynamic state-based failure criterion

Christian Willberg

christian.willberg@dlr.de

Martin Rädel

martin.raedel@dlr.de

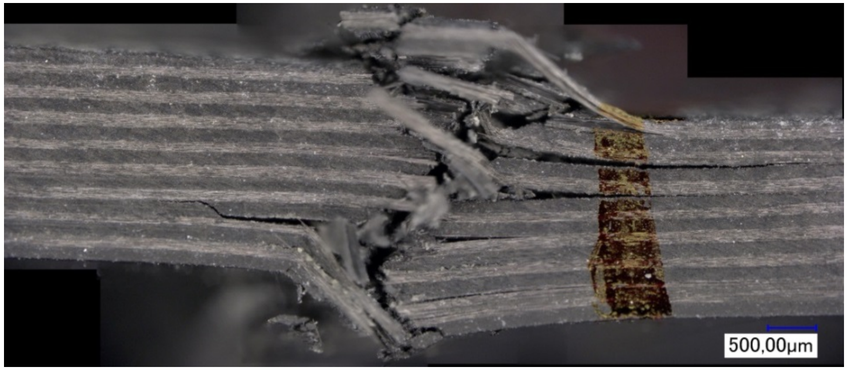
S03.05 Damage and fracture mechanics
GAMM 2018 21/03/2018

Outline

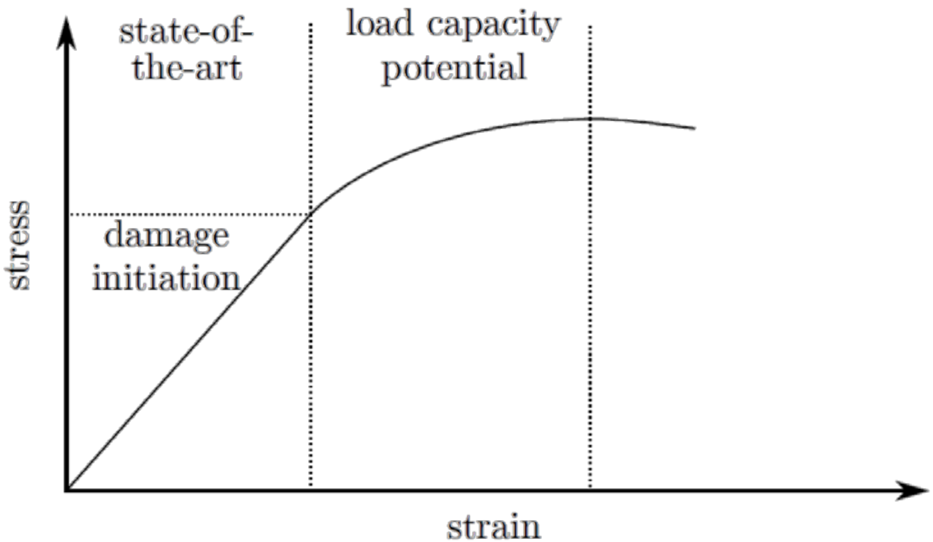
1. Motivation
2. Peridynamics
3. Damage model
4. Convergence
5. Comparison – Critical Stretch
6. Example
7. Conclusion

Motivation

- Challenges:
  - Exploitation of fiber reinforced plastics (FRP) lightweight potential limited
  - Missing reliability of failure predictions



- Goals:
  - Increase understanding of failure mechanisms
  - Reduce number of experiments
  - Derive improved failure criteria for design process of structures



Motivation - Continuum mechanics vs. Peridynamic approach

1. The medium is continuous (a continuous mass density field exists)
2. Internal forces are contact forces (material points interact only if they are separated by zero distance)
3. The deformation is twice continuously differentiable (this assumption is relaxed
4. The conservation laws of mechanics apply (conservation of mass, linear momentum, and angular momentum)[1]

$$\mathrm{div}(\boldsymbol{\sigma}) + \mathbf{b} = \rho\ddot{\mathbf{u}}$$

[1]Bobaru, F.; Foster, J. T.; Geubelle, P. H. & Silling, S. A. Handbook of peridynamic Modeling *CRC Press,* **2016**

Motivation - Continuum mechanics vs. Peridynamic approach

1. ~~The medium is continuous (a continuous mass density field exists)~~
2. ~~Internal forces are contact forces (material points interact only if they are separated by zero distance)~~
3. ~~The deformation is twice continuously differentiable (this assumption is relaxed~~
4. The conservation laws of mechanics apply (conservation of mass, linear momentum, and angular momentum)[1]

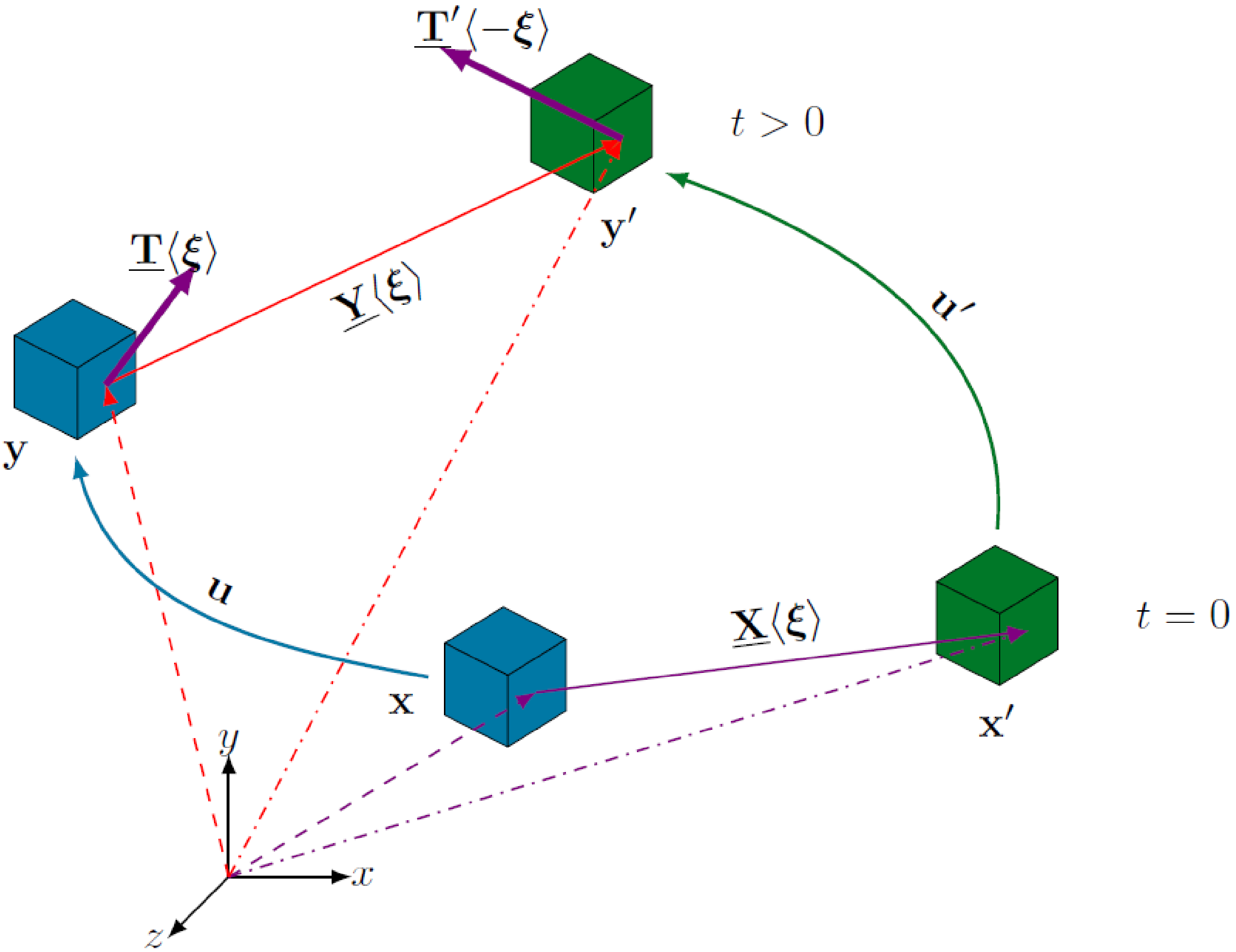
$$\mathrm{div}(\boldsymbol{\sigma}) + \mathbf{b} = \rho\ddot{\mathbf{u}}$$

$$\int_H \left(\underline{\mathbf{T}}(\mathbf{x}, t)\langle \mathbf{x}' - \mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{x}', \mathrm{t})\langle \mathbf{x} - \mathbf{x}'\rangle\right) dV + \mathbf{b} = \rho\ddot{\mathbf{u}}$$

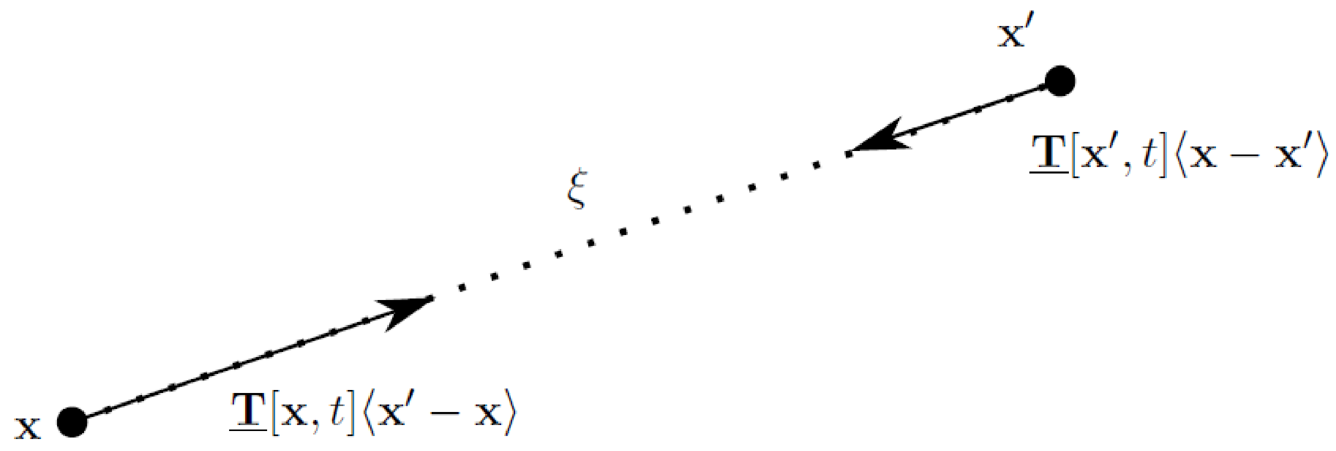
$$\lim_{H\to 0}\int_H \left(\underline{\mathbf{T}}\,(\mathbf{x}, t)\langle \mathbf{x}' - \mathbf{x}\rangle - \underline{\mathbf{T}}(\mathbf{x}', \mathrm{t})\langle \mathbf{x} - \mathbf{x}'\rangle\right) dV = \mathrm{div}(\boldsymbol{\sigma})$$

[1]Bobaru, F.; Foster, J. T.; Geubelle, P. H. & Silling, S. A. Handbook of peridynamic Modeling *CRC Press,* **2016**

Peridynamics



Peridynamics – ordinary state based



$$= \int_{\mathcal{H}} \left(\underline{\mathbf{T}}\left[\mathbf{x}, t\right] \left\langle \mathbf{x}' - \mathbf{x} \right\rangle - \underline{\mathbf{T}}\left[\mathbf{x}', t\right] \left\langle \mathbf{x} - \mathbf{x}' \right\rangle \right) \mathrm{d}V + \mathbf{b}\left(\mathbf{x}, t\right)$$

Peridynamics – ordinary state based

$$W_{CM} = \frac{1}{2} K \left[\epsilon_{kk}\right]^2 \delta_{ij} + 2G \left[\epsilon_{ij}^d\right]^2 \overset{!}{=} W_{PD}$$

$$\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle = \mathbf{F}\boldsymbol{\xi} = \mathbf{F}\langle\mathbf{x}' - \mathbf{x}\rangle \quad \forall\boldsymbol{\xi} \in \mathcal{H}$$

- For small deformations and isotropic material

$$\underline{x} = |\underline{\mathbf{X}}\langle\boldsymbol{\xi}\rangle| \qquad \underline{y} = |\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle| \qquad \underline{e}\langle\boldsymbol{\xi}\rangle = \underline{y} - \underline{x}$$

$$\underline{e}\langle\boldsymbol{\xi}\rangle = |\mathbf{F}\boldsymbol{\xi}| - |\boldsymbol{\xi}| = \epsilon_{ij}\xi_i \frac{\xi_j}{|\boldsymbol{\xi}|}$$

$$\underline{e}^d\langle\boldsymbol{\xi}\rangle = \epsilon_{ij}^d \xi_i \frac{\xi_j}{|\boldsymbol{\xi}|} \qquad \underline{e}^i\langle\boldsymbol{\xi}\rangle = \epsilon_{ii}\xi_i \frac{\xi_i}{|\boldsymbol{\xi}|}$$

$$W_{PD} = \frac{A}{2} \int_{\mathcal{H}} \underline{\omega}\langle\boldsymbol{\xi}\rangle \left[\epsilon_{ij}^d \xi_i \frac{\xi_j}{|\boldsymbol{\xi}|}\right]^2 \mathrm{d}V_{\boldsymbol{\xi}} + \frac{B}{2} \int_{\mathcal{H}} \underline{\omega}\langle\boldsymbol{\xi}\rangle \left[\epsilon_{ii} \xi_i \frac{\xi_i}{|\boldsymbol{\xi}|}\right]^2 \mathrm{d}V_{\boldsymbol{\xi}}$$

DLR

Peridynamics – ordinary state based

$$A = \frac{3K}{m_V} \quad \text{and} \quad B = \frac{15G}{m_V}$$

$$m_V = \int_{\mathcal{H}(\mathbf{x})} \underline{\omega}\langle\boldsymbol{\xi}\rangle \underline{x x}\, \mathrm{d}V_{\boldsymbol{\xi}} \qquad \theta = \frac{3}{m_V} \int_{\mathcal{H}(\mathbf{x})} \underline{\omega}\langle\boldsymbol{\xi}\rangle \underline{x e}\langle\boldsymbol{\xi}\rangle\, \mathrm{d}V_{\boldsymbol{\xi}}$$

$$\underline{t}\langle\boldsymbol{\xi}, t\rangle = \frac{\underline{\omega}\langle\boldsymbol{\xi}\rangle}{m_V}\left[3K\theta\underline{x} + 15G\underline{e}^{d}\right]$$

$$\underline{\mathbf{T}} = \underline{t}\frac{\underline{\mathbf{Y}}}{|\underline{\mathbf{Y}}|}$$

Damage model

- Could be included via the influence function
- For programming reasons the history dependend scalar value representing the damage function is split from the the influence function

$$\chi(\boldsymbol{\xi}, t) = \begin{cases} 1 & \text{no failure} \\ 0 & \text{failure} \end{cases}$$

- Critical stretch model

$$s_C = \sqrt{\frac{G_{0C}}{\left[3G + \left(\frac{3}{4}\right)^4 \left(K - \frac{5G}{3}\right)\right]\delta}}$$

- Critical energy model by Foster et al.

$$W_C = \frac{4G_{0C}}{\pi\delta^4}$$

DLR

Damage model

$$W_{\text{bond}} = 0.25\chi(\underline{e}\langle\boldsymbol{\xi}\rangle, t)\left\{\underline{t}\left[\mathbf{x}, t\right] - \underline{t}\left[\mathbf{x}', t\right]\right\}\underline{e} < W_C$$

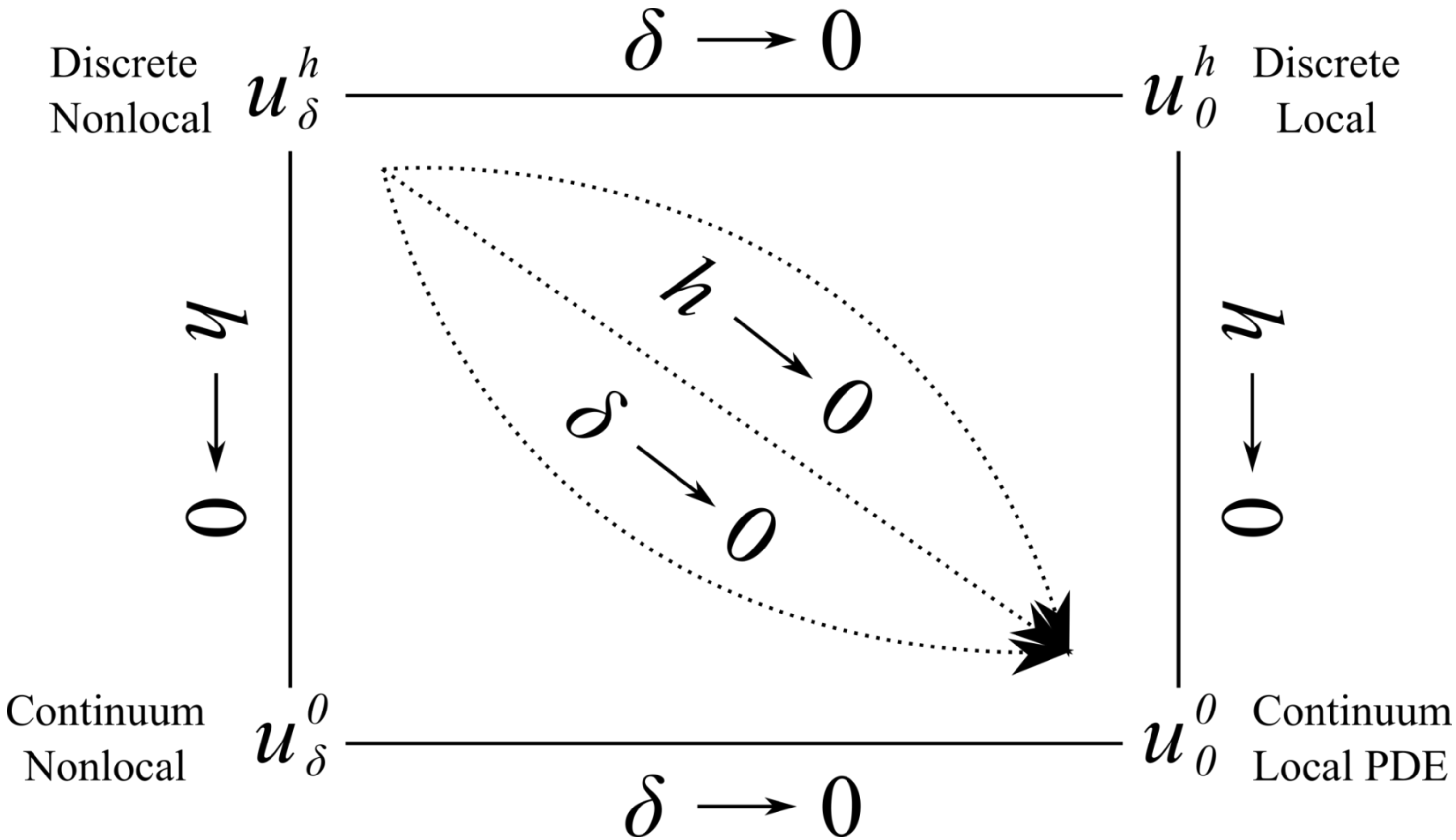
$$\underline{t}\left[\mathbf{x}, t\right] = \chi(\underline{e}\langle\boldsymbol{\xi}\rangle, t)\left(\frac{3K\left[\mathbf{x}, t\right]\theta\left[\mathbf{x}, t\right]}{m_V\left[\mathbf{x}, t\right]}\underline{\omega x} + \frac{15G\left[\mathbf{x}, t\right]}{m_V\left[\mathbf{x}, t\right]}\underline{\omega e}^d\left[\mathbf{x}, t\right]\right)$$

$$\underline{t}\left[\mathbf{x}', t\right] = \chi(\underline{e}\langle\boldsymbol{\xi}\rangle, t)\left(\frac{3K\left[\mathbf{x}', t\right]\theta\left[\mathbf{x}', t\right]}{m_V\left[\mathbf{x}', t\right]}\underline{\omega x} + \frac{15G\left[\mathbf{x}', t\right]}{m_V\left[\mathbf{x}', t\right]}\underline{\omega e}^d\left[\mathbf{x}', t\right]\right)$$

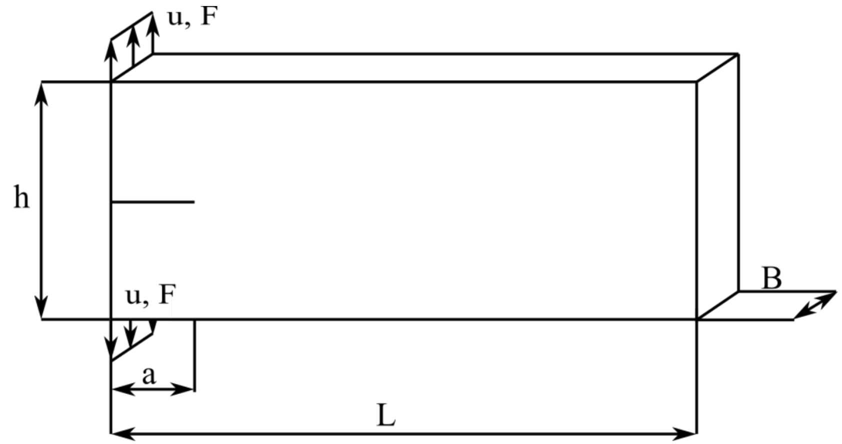
$$\theta\left[\mathbf{x}, t\right] = \frac{3}{m_V\left[\mathbf{x}, t\right]}\int_{\mathcal{H}(\mathbf{x})}\underline{\omega x e}\,\mathrm{d}V_{\boldsymbol{\xi}} \qquad \underline{e}^d\left[\mathbf{x}, t\right] = \underline{e} - \frac{\theta\left[\mathbf{x}, t\right]\underline{x}}{3}$$

$$\theta\left[\mathbf{x}', t\right] = \frac{3}{m_V\left[\mathbf{x}', t\right]}\int_{\mathcal{H}(\mathbf{x}')}\underline{\omega x e}\,\mathrm{d}V_{\boldsymbol{\xi}} \qquad \underline{e}^d\left[\mathbf{x}', t\right] = \underline{e} - \frac{\theta\left[\mathbf{x}', t\right]\underline{x}}{3}$$

Convergence



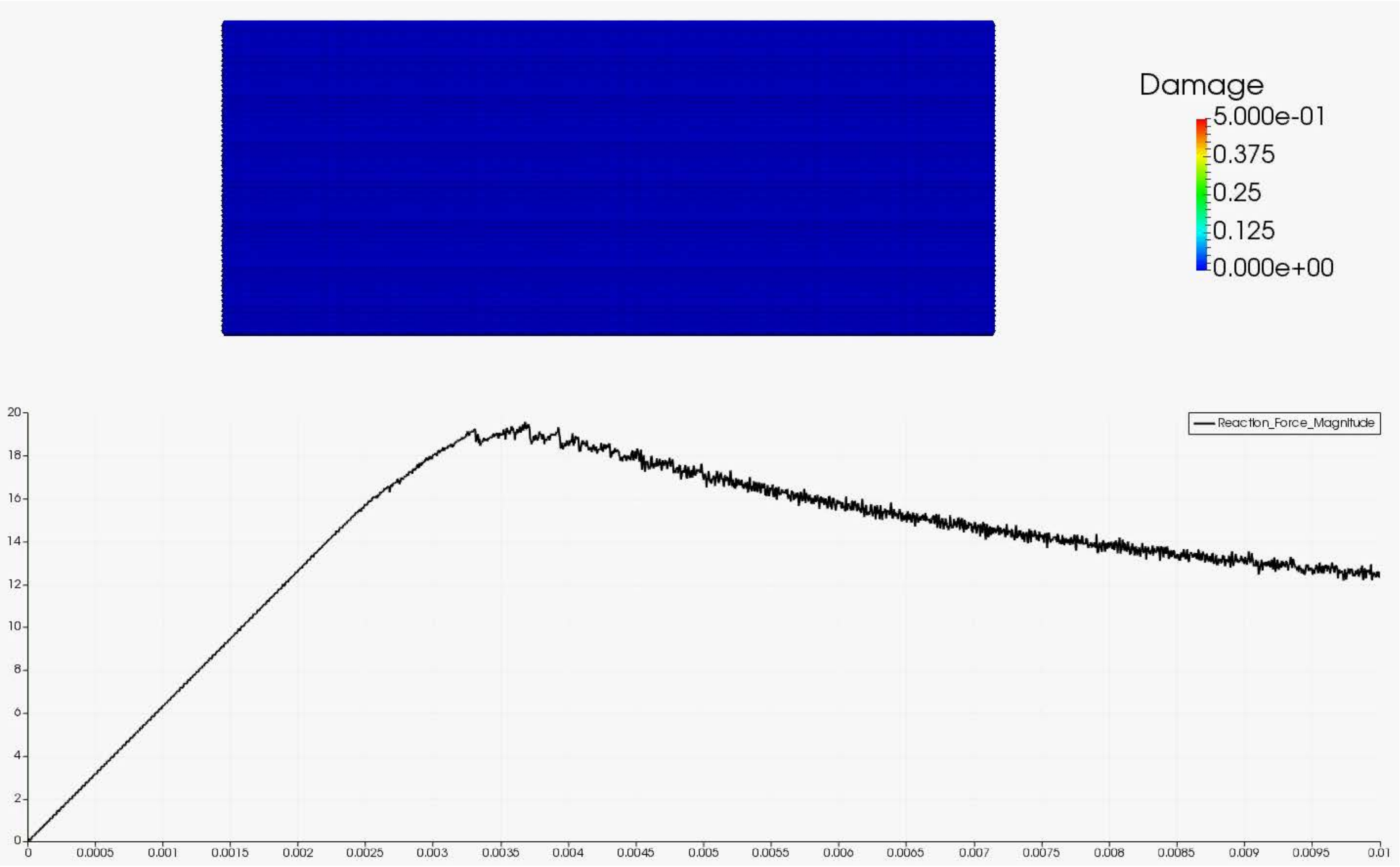
Convergence



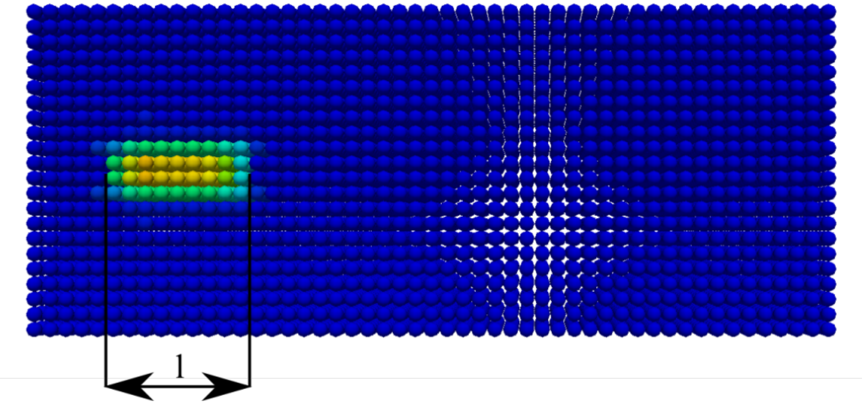
Geometry	a	h	L	B
	0.005m	0.02m	0.05m	0.006m
Material	**Bulk Modulus**	**Shear Modulus**	**Density**	**G_0**
	1.75E+09 Nm^{-2}	8.08E+08 Nm^{-2}	2000 kgm^{-3}	12 Nm^{-1}

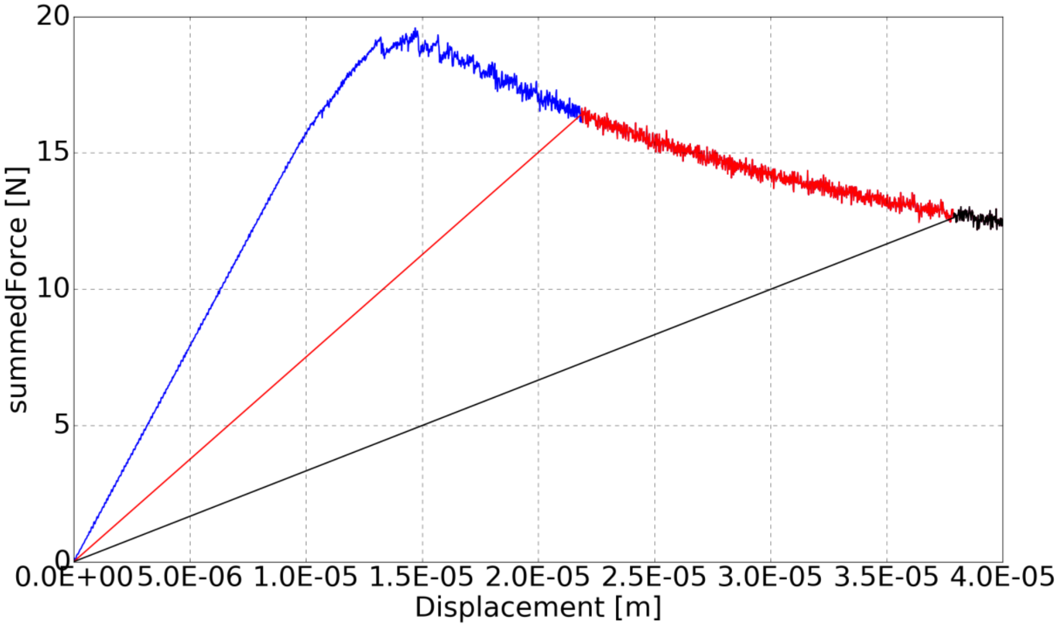
Mesh	2.01dx	3.01dx	4.01dx	5.01dx
0.0005	0.001005	0.001505	0.002005	0.002505
0.00033	0.000663	0.000993	0.001323	0.001653
0.00025	0.000503	0.000753	0.001003	0.001253
0.000125	0.000251		0.000501	

Convergence



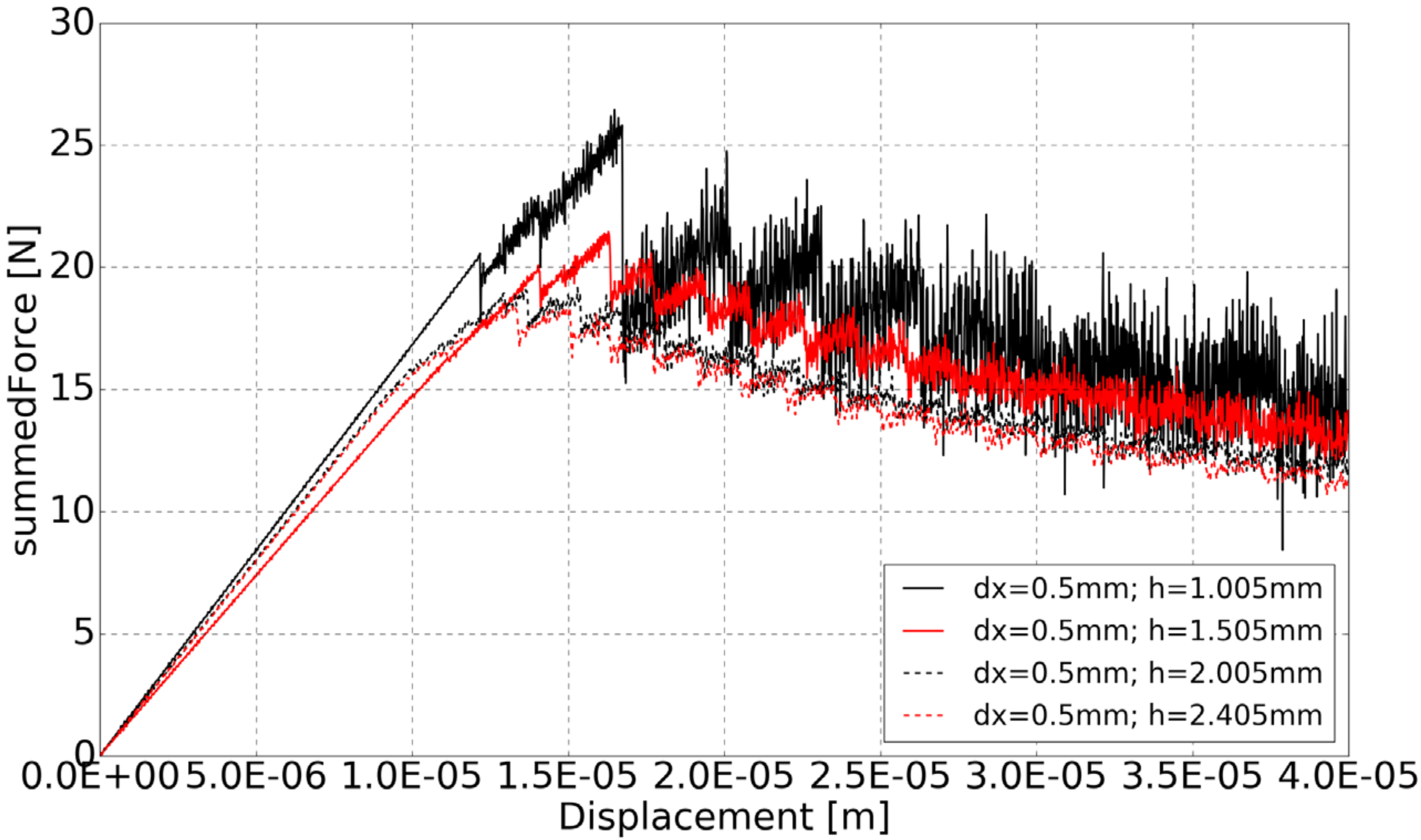
Convergence



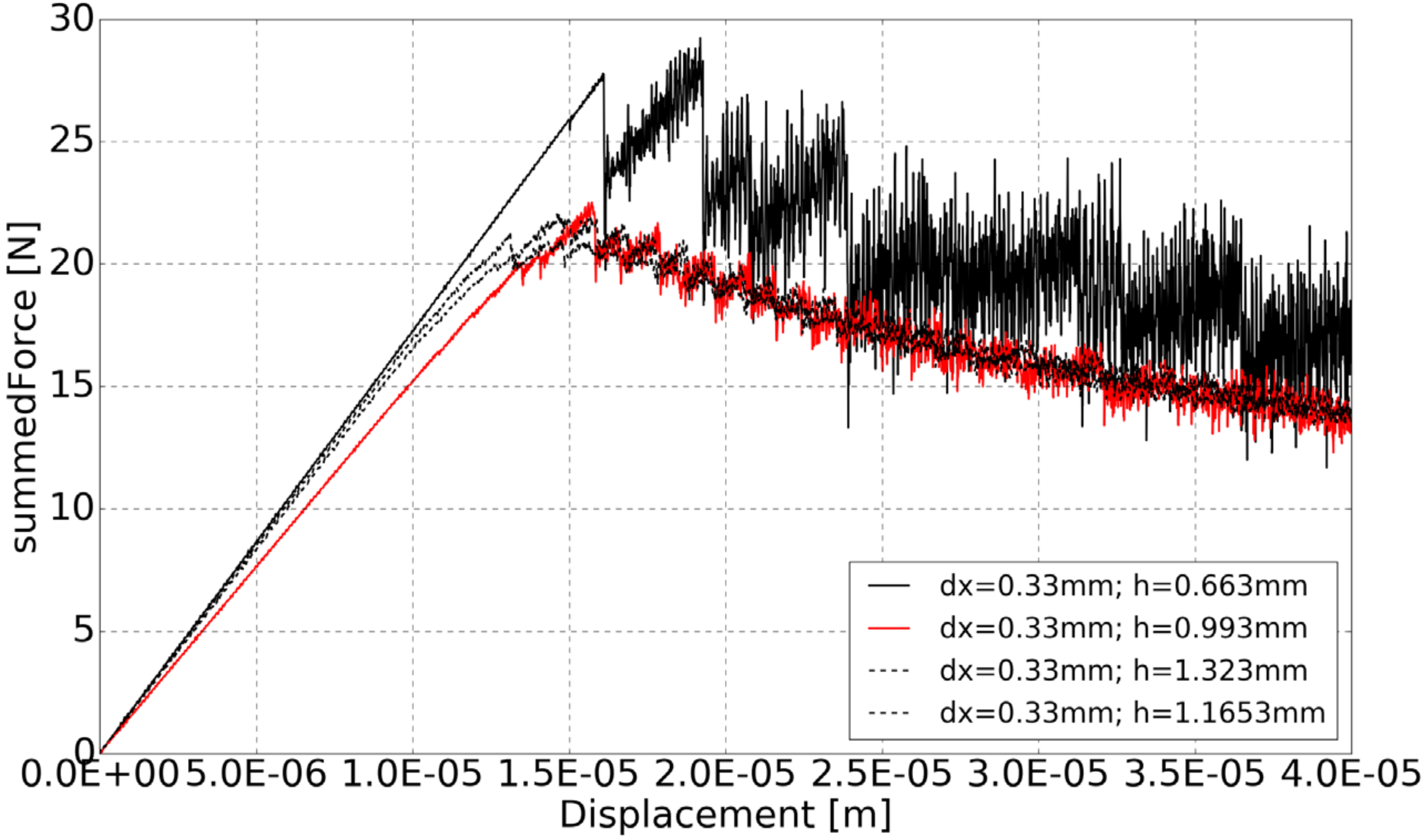


	Line 1	Line 2
δ	G_0	G_0
[m]	[N/m]	[N/m]
$2.015 \cdot 10^{-3}$	12.8	11.4
$3.015 \cdot 10^{-3}$	13.1	12.9
$4.015 \cdot 10^{-3}$	11.1	11.3
$5.015 \cdot 10^{-3}$	11.2	11.9

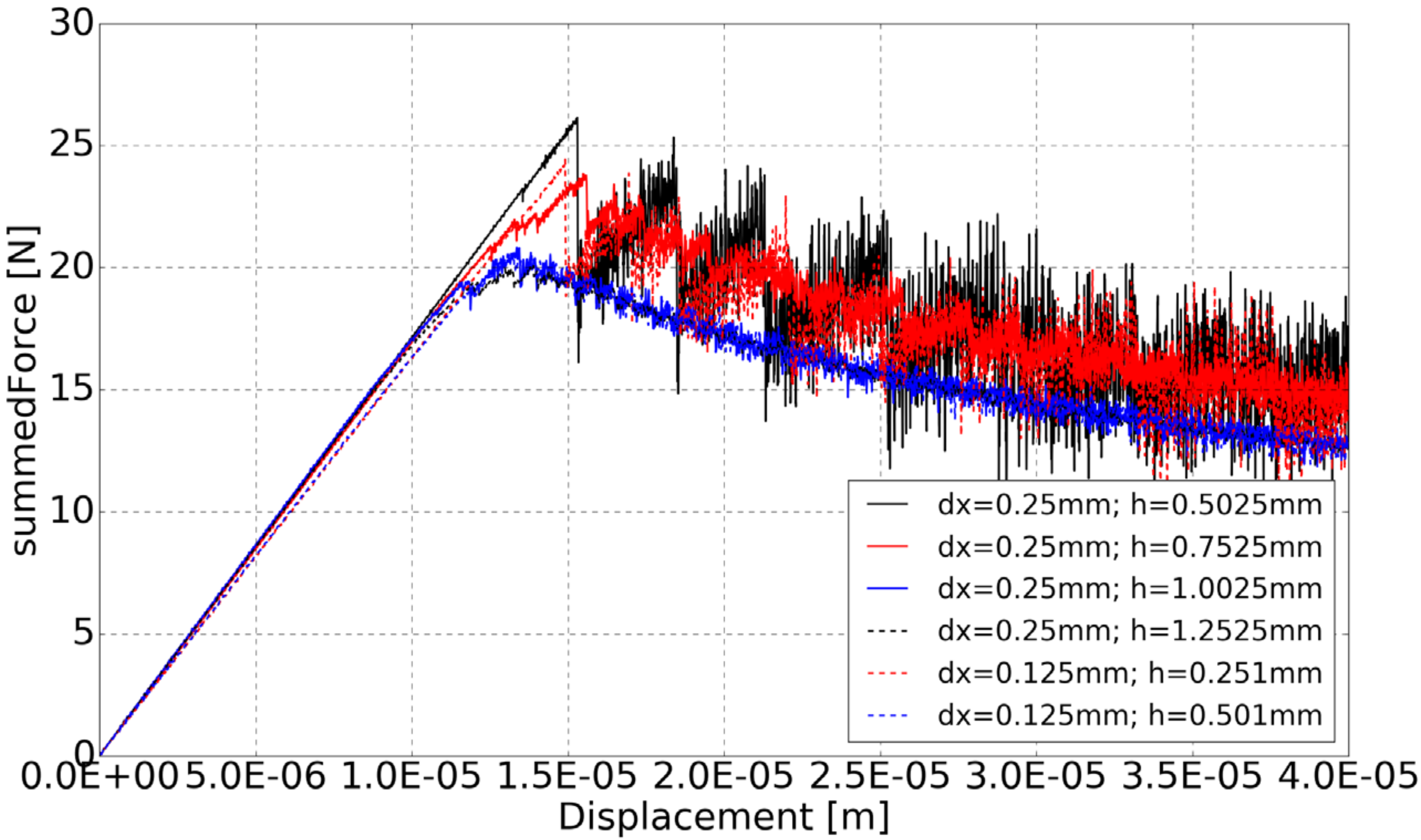
Convergence - Results



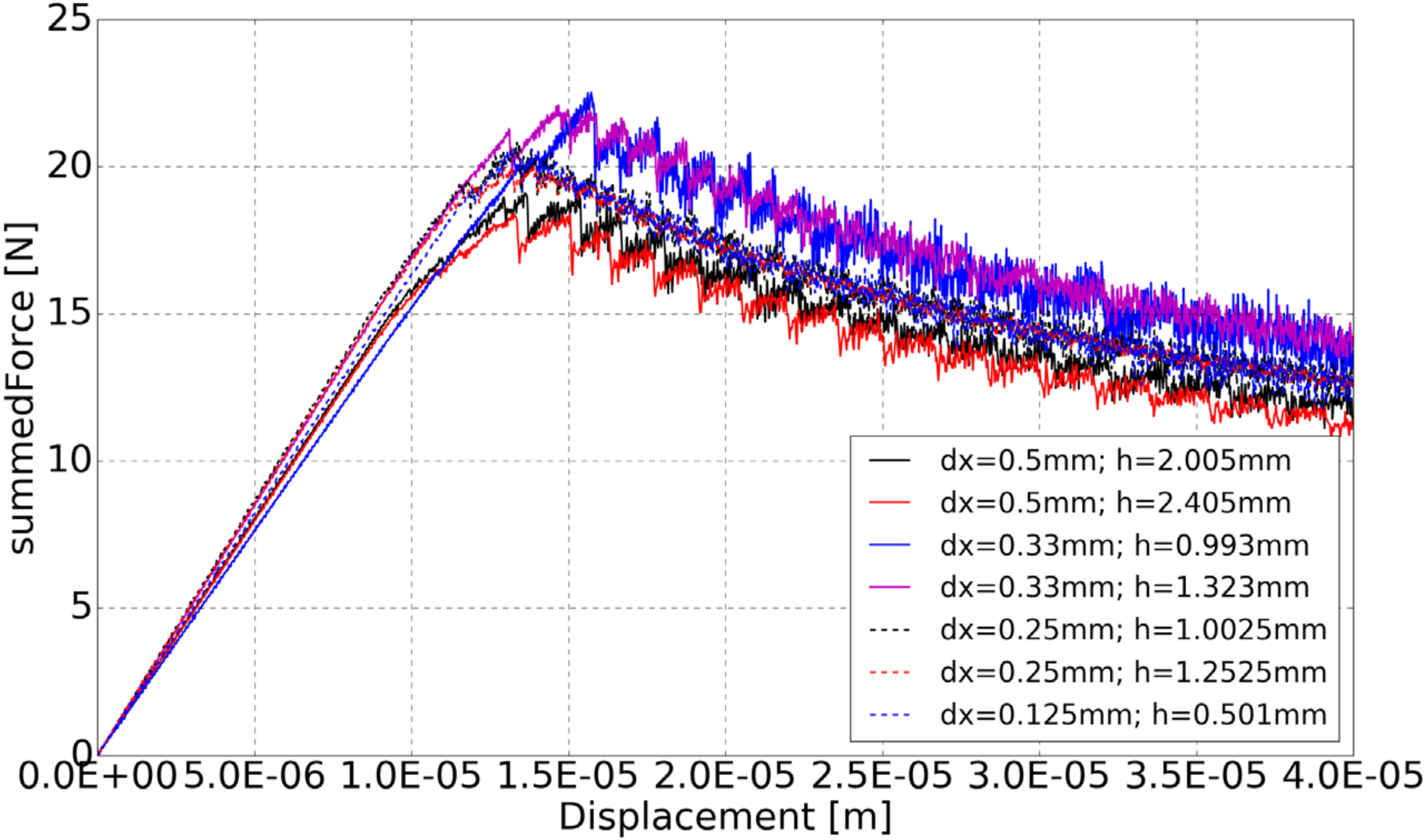
Convergence - Results



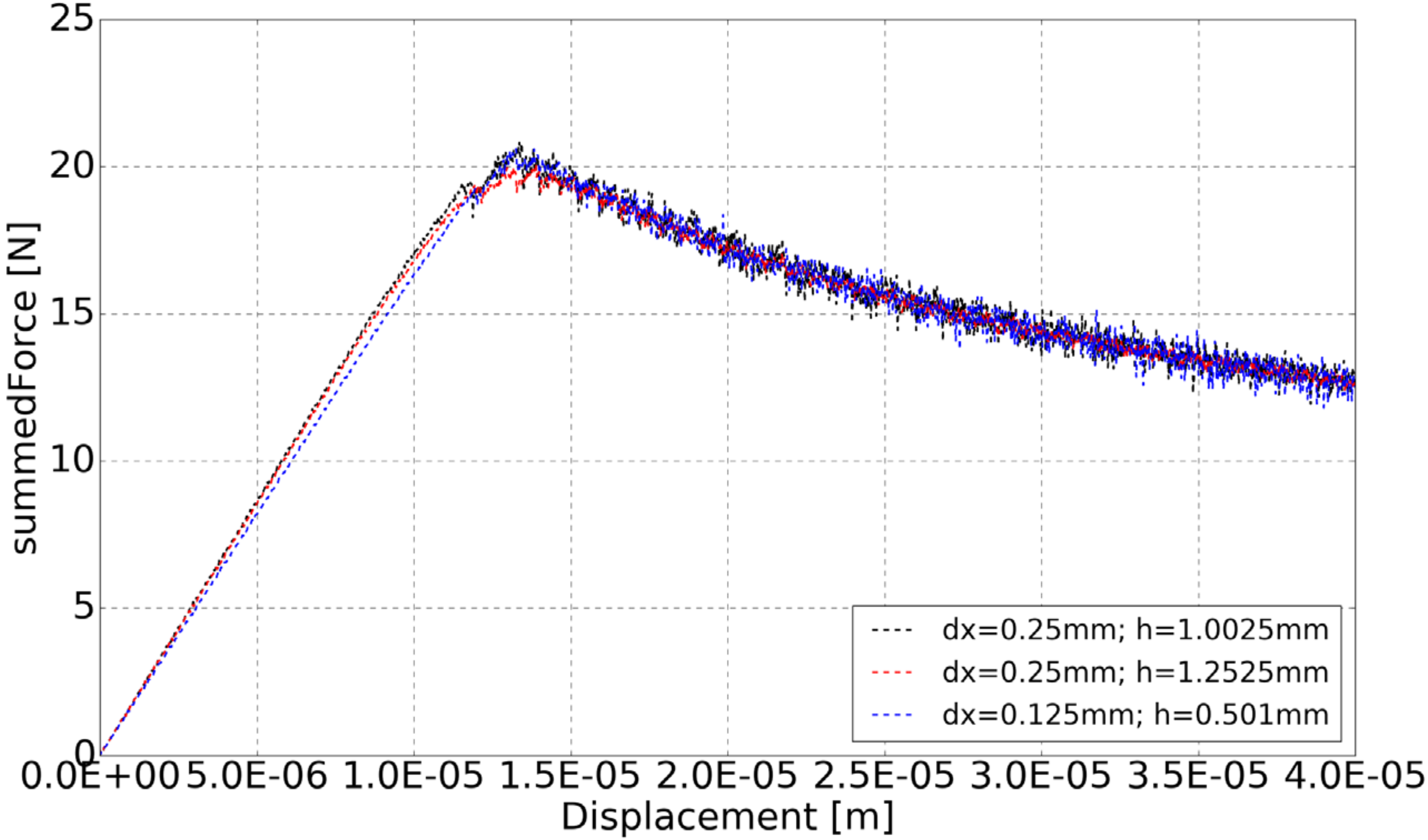
Convergence - Results



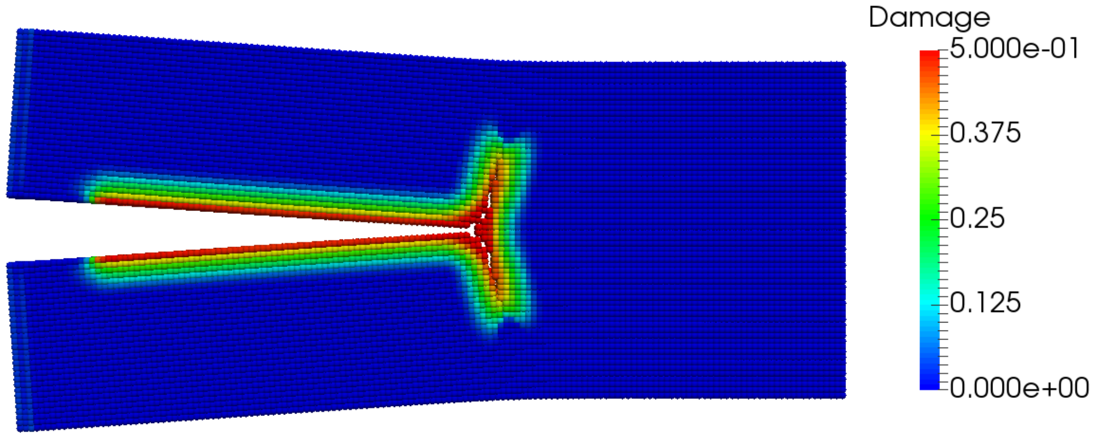
Convergence - Results



Convergence - Results

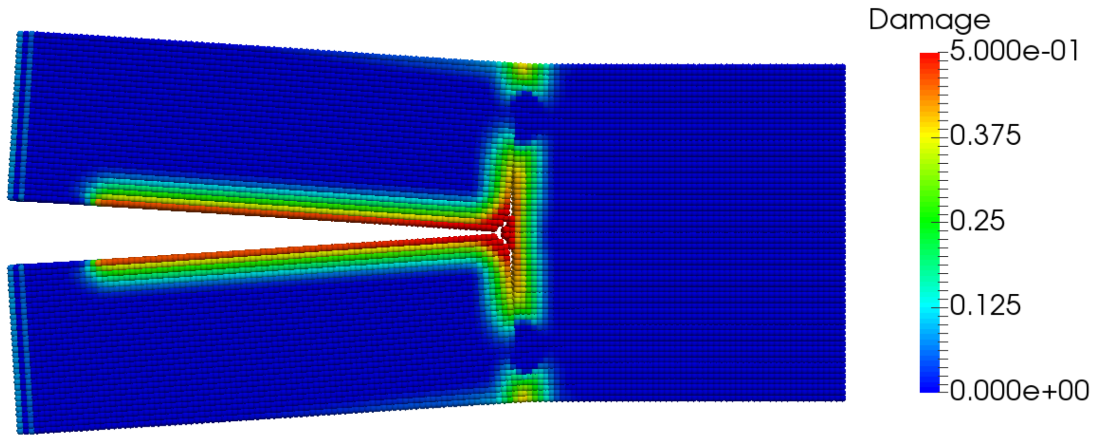


Comparison – Critical Stretch



Critical Stretch

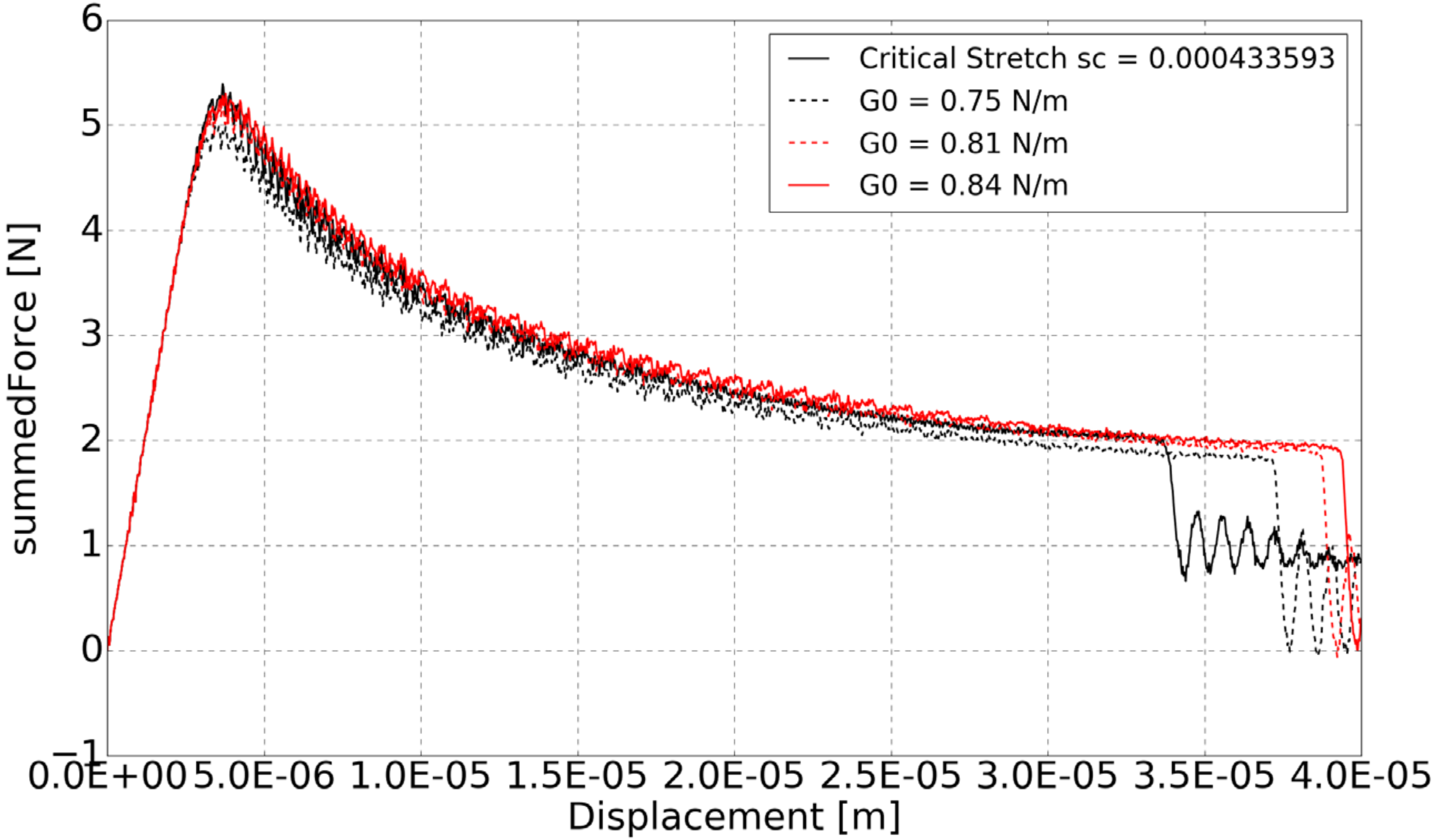
$s_c = 0.000433593$
$K = 1.75E09\ N/m^2$
$G = 8.08E8\ N/m^2$
$\delta = 0.002505\ m$
$\underline{G_0 = 12\ N/m}$



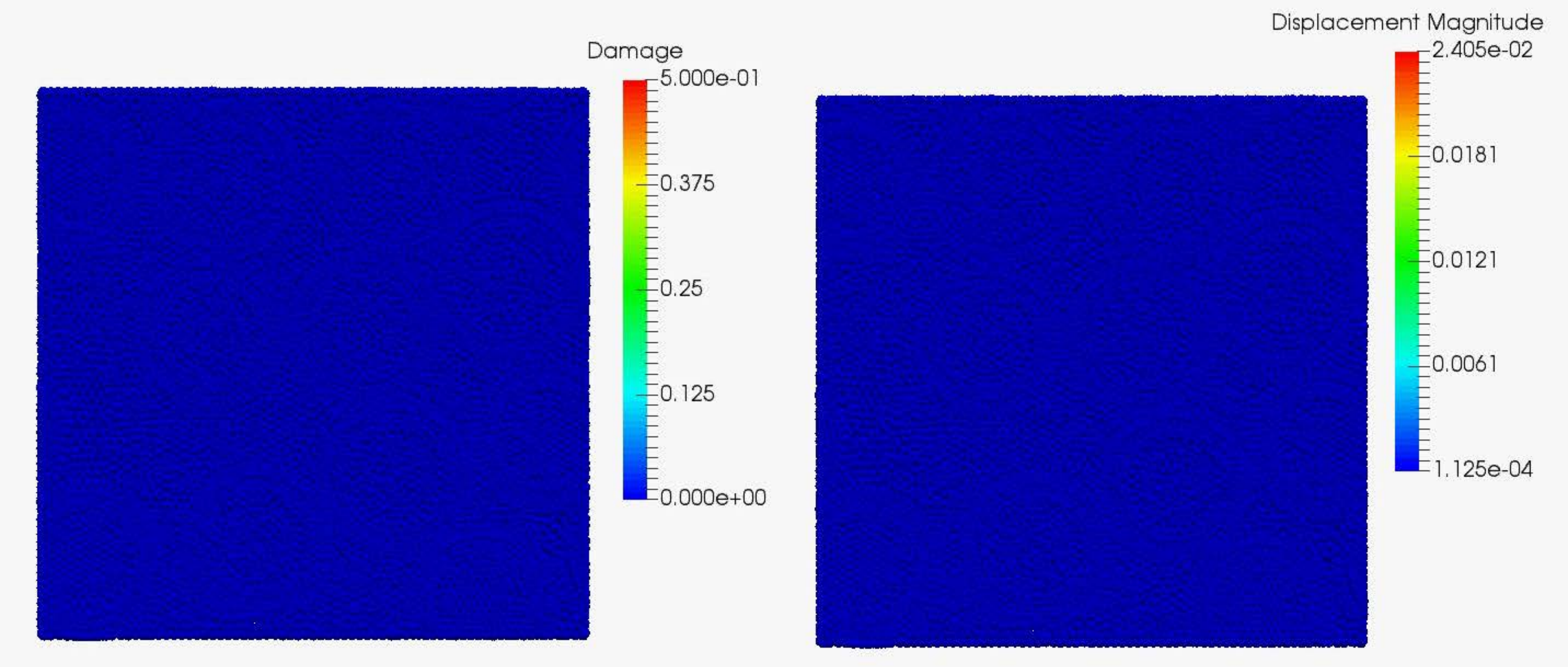
Critical Energy

$G_0 = 0.75\text{-}0.84\ N/m$

Comparison – Critical Stretch



Example – RVE


Damage
5.000e-01
0.375
0.25
0.125
0.000e+00
Displacement Magnitude
2.405e-02
0.0181
0.0121
0.0061
1.125e-04

Conclusion

- The energy criterion from Foster et al. has been implemented and tested due to its convergence
- The criterion is able to represent the energy release rate
- 2dx meshes of any discretization lead to overestimation of the crack initiation load
- 4-5dx shows the best results + converge; <2% difference in results
- Difference between the standard method (critical stretch) and critical energy has been shown
- Use case has been shown for complex fiber matrix model

All presented models (end of March) and source code can be found here

Rädel, R. & Willberg, C. PeriDoX Repository
https://github.com/PeriDoX/PeriDoX

References

Silling, S. A.; Epton, M.; Weckner, O.; Xu, J. & Askari, E. Peridynamic States and Constitutive Modeling in *Journal of Elasticity,* **2007***, 88*, 151-184

Bobaru, F.; Foster, J. T.; Geubelle, P. H. & Silling, S. A. Handbook of peridynamic Modeling *CRC Press,* **2016**

Foster, J. T.; Silling, S. A. & Chen, W.
An Energy based Failure Criterion for use with Peridynamic States in *International Journal for Multiscale Computational Engineering,* **2011***, 9*, 675-688

Rädel, M.; Bednarek, A.-J. & Willberg, C. Influence of probabilistic material distribution in peridynamics to the crack initiation *6th ECCOMAS Thematic Conference on the Mechanical Response of Composites: COMPOSITES 2017,* **2017**

Willberg, C.; Rädel, M. & Wiedemann, L. “A mode-dependent energy-based damage model for peridynamics and its implementation” in *Journal of Mechanics of Materials and Structures,* **2018**, in review

Thank you!

Dr.-Ing. Christian Willberg

DLR
Institute of Composite Structures
and Adaptive Systems

Lilienthalplatz 7
38108 Braunschweig
Germany

Phone: +49 531 295 - 2232
Email: christian.willberg@dlr.de

All presented models (end of March) and source code can be found here

Rädel, R. & Willberg, C. PeriDoX Repository
https://github.com/PeriDoX/PeriDoX