Throughput Optimization for Massive MIMO Systems Powered by Wireless Energy Transfer

Gang Yang, Chin Keong Ho, Rui Zhang, and Yong Liang Guan

Abstract

This paper studies a wireless-energy-transfer (WET) enabled massive multiple-input-multiple-output (MIMO) system (MM) consisting of a hybrid data-and-energy access point (H-AP) and multiple single-antenna users. In the WET-MM system, the H-AP is equipped with a large number $M$ of antennas and functions like a conventional AP in receiving data from users, but additionally supplies wireless power to the users. We consider frame-based transmissions. Each frame is divided into three phases: the uplink (UL) channel estimation (CE) phase, the downlink (DL) WET phase, as well as the UL wireless information transmission (WIT) phase. Firstly, users use a fraction of the previously harvested energy to send pilots, while the H-AP estimates the UL channels and obtains the DL channels by exploiting channel reciprocity. Secondly, the H-AP broadcasts wireless energy to all users in the DL via energy beamforming. Thirdly, the users use the rest of the harvested energy to send their independent information to the H-AP simultaneously in the UL. To optimize the throughput and ensure rate fairness, we consider the problem of maximizing the minimum rate among all users. The variables are the time allocation for UL CE and DL WET, the energy allocation weights in the DL WET phase, as well as the fraction of energy used for CE. In the large-$M$ regime, we obtain the asymptotically optimal solutions and some interesting insights. We define a metric, namely, the massive MIMO degree-of-rate-gain (MM-DoRG), as the asymptotic UL rate normalized by $\log(M)$. We show that the proposed WET-MM system is optimal in terms of MM-DoRG, i.e., it achieves the same MM-DoRG as the case with ideal CE. Moreover, in the proposed WET-MM system, the users asymptotically achieve a common rate, ensuring the best possible rate fairness among users.

Index Terms

Massive MIMO, wireless energy transfer, energy beamforming, channel estimation, throughput, fairness, time allocation, energy allocation, asymptotic analysis, degree-of-rate-gain (DoRG)

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I. INTRODUCTION

Recently, far-field wireless energy transfer (WET) has emerged as a promising technology to address energy and lifetime bottlenecks for power-limited devices in wireless networks [1]. WET refers to using the radiative electromagnetic (EM) wave emitted from a power transmitter to deliver energy to a power receiver (see [1] and references therein). Since EM waves decay quickly over distances, to realize WET in practice, the EM energy needs to be concentrated into a narrow beam to achieve efficient transmission of power, also referred to as energy beamforming [2].

Simultaneous wireless information and power transfer (SWIPT) that was proposed in [3], has been extensively studied in literature, since it offers great convenience to mobile users with concurrent data and energy supplies. The authors in [4], [2] studied the performance limits of single-input-single-output (SISO) and multiple-input-multiple-output (MIMO) SWIPT systems, respectively, and characterized various achievable rate-energy (R-E) trade-offs by practical receiver designs. SWIPT has also been studied in fading channels [5], and orthogonal frequency division multiplexing (OFDM) systems [6]–[9], and other multiuser channel setups such as broadcast channels [10], [11], relay channels [12], [13], and interference channels [14], [15]. Moreover, [16] studied a hybrid network which overlays an uplink (UL) cellular network with randomly deployed power beacons that charge users wirelessly, while [17] studied a similar setup in cognitive radio networks with secondary users harvesting wireless power opportunistically from nearby primary users’ transmission.

Another emerging trend focuses on the study of using wireless power to support wireless communications, thus forming a wirelessly powered communication network (WPCN). In a WPCN, an access point (AP) with multiple antennas first broadcasts energy to multiple single-antenna users via downlink (DL) beamforming, then the users use the harvested energy to perform UL wireless information transmission (WIT) to the AP. The total time duration is divided into the DL WET and the UL WIT. Assuming perfect CSI, the single-user scenario was studied in [18], [19]. With finite-rate feedback, [18] optimized the time duration for DL WET to maximize a lower bound on the UL WIT rate. [19] maximized the energy efficiency of UL WIT, by jointly optimizing the time duration and transmit power for DL WET. Also assuming perfect CSI, the multiuser WPCN was studied in [20], [21]. In particular, [20] considered the scenario in which users use the harvested energy to send independent information to the AP through time division multiple access (TDMA). The sum throughput was maximized subject to user fairness by jointly optimizing the time allocation for DL WET and UL WIT. [21] maximized the minimum throughput among all users by a joint
design of the WET-WIT time allocation, the DL energy beamformer, and the UL transmit power allocation.

The knowledge of CSI is essential for both DL energy beamforming and UL information decoding in a WPCN. More accurate CSI contributes to higher efficiency of energy transfer and higher UL information rate. In practice, perfect CSI at the transmitter is not available due to various factors such as channel estimation error, feedback error, and time-varying channel. Typically, the power transmitter needs to perform channel estimation (CE) before transferring energy. Although a longer time duration for CE leads to more accurate CSI available at the transmitter, it reduces the WET and WIT duration, which may lead to less harvested energy and lower throughput. To optimize the throughput, there is thus a design freedom, that is the time spent for CE, WET and WIT.

For a point-to-point WET system, the effect of CE and feedback on energy beamforming was studied in our previous work [22], [23]. In particular, [22] investigated the dynamic allocation of time resource for CE and energy resource for WET. The optimal preamble length is obtained by solving a dynamic programming problem. The solution is a threshold-type policy that depends only on the channel estimate power, and hence allows a low-complexity WET system to be implemented in practice. [23] studied transmit energy beamforming by using one-bit feedback from the energy receiver, to comply with its practical hardware limitation. Based on the one-bit feedback information, the energy transmitter adjusts transmit beamforming and at the same time obtains improved estimates of the channel to the energy receiver.

Recently, massive MIMO has been proposed to enormously improve the transmission capacity of wireless communication networks by exploiting its large array gain at the base stations (BSs) [24]. Large antenna arrays can potentially reduce DL and UL transmit power through precoding and coherent combining, respectively [25]. Similarly, massive MIMO can also be used to effectively enhance the efficiency and distance of WET, which motivates our work.

In this paper, we consider a WET-enabled massive MIMO system (termed WET-MM), which consists of one hybrid data-and-energy access point (H-AP) with constant power supply and equipped with a large-scale antenna array, and a set of distributed single-antenna users that rely on the wireless power sent from the H-AP for UL transmission. We assume frame-based transmissions with the time-division-duplexing (TDD) protocol. Each frame is divided into three phases: the UL CE (i.e., channel estimation) phase, the DL WET (i.e., wireless energy transfer) phase, as well as the UL WIT (i.e., wireless information transmission) phase. Firstly, users use a fraction of the harvested
energy in the previous frames to send pilots, while the H-AP estimates the UL channels and obtains the DL CSI by exploiting channel reciprocity. Secondly, the H-AP broadcasts wireless energy to all users in the DL via energy beamforming with appropriately designed weights. Thirdly, the users use the rest of the harvested energy (after deducting that stored for the CE in the next frame) to send their independent information to the H-AP simultaneously in the UL. To optimize the throughput and ensure rate fairness, we consider the problem of maximizing the minimum rate among all users. The design variables are the time allocations for UL CE and DL WET (subject to a given total time for CE, WET and WIT of each frame), the energy allocation weights in the DL WET phase for different users, as well as the fraction of energy used for CE (versus WIT) at each user.

To the best of our knowledge, this paper is the first in the literature to consider the WET-MM system under imperfect CSI. For comparison, we consider a low-complexity system as a benchmark, namely the omnidirectional transmission massive MIMO (OT-MM) system, which uses isotropic beamformer for energy broadcasting. Besides being easy to implement, the OT without CSI may still perform sufficiently well for DL WET, because it may be difficult to obtain accurate CSI in massive MIMO systems. To investigate the optimality of the proposed WET-MM system under imperfect CSI, we also compare it to the ideal case with perfect CSI known at the H-AP. Here, we introduce a metric, namely, the massive MIMO degree-of-rate-gain (MM-DoRG), which is defined as the asymptotic scaling order of the UL rate with respect to \( \log(M) \), i.e.,

$$
\kappa \triangleq \lim_{M \to \infty} \frac{R}{\log M},
$$

where \( R \) is the data rate that depends on \( M \), and \( M \) is the number of transmit antennas at the H-AP. The MM-DoRG differs from the well-known degree-of-freedom (DoF) which instead considers the asymptotic scaling of \( R \) normalized to \( \log(\gamma) \) for large \( \gamma \), where \( \gamma \) is the signal-to-noise-ratio (SNR). In this paper, we use a lower bound on the achievable rate for analytical tractability, which is numerically shown to be tight. We consider the use of zero-forcing (ZF) and maximal-ratio-combining (MRC) detection in the UL WIT phase. We show that the proposed WET-MM system has the following advantages:

- In terms of MM-DoRG, the proposed WET-MM system is optimal, as it achieves the same MM-DoRG (\( \kappa_{\text{Ideal}} = 2 \)) as the ideal case with perfect CSI. Moreover, the proposed WET-MM system substantially outperforms the OT-MM system. With ZF detection, the WET-MM system achieves \( \kappa_{\text{WET-MM}} = 2 \), while the OT-MM system achieves \( \kappa_{\text{OT-MM}} = 1 \).
The proposed WET-MM system achieves the best possible rate fairness among users, while no fairness is guaranteed in the OT-MM system. For both ZF and MRC detection, all users in the proposed WET-MM system asymptotically achieve a common rate in the large-$M$ regime.

For ZF detection, to achieve a desired common rate, the maximal distance that can be supported by the proposed WET-MM system scales as $O \left( M^{\frac{1}{u}} \right)$, where $u$ is the path-loss exponent. In contrast, in the OT-MM system, the maximal distance scales as $O \left( M^{\frac{1}{2u}} \right)$. In other words, to support the same distance, the proposed WET-MM system requires less antennas, roughly a square root of the number of antennas that is required by the OT-MM system.

The proposed WET-MM system is of low complexity, since only the conventional beamforming and detection is required. When the simplest MRC detection is used, the proposed WET-MM system still significantly outperforms the OT-MM system, due to the exploitation of (imperfect) channel estimation to increase the harvested energy in the DL WET.

The rest of this paper is organized as follows: Section II presents the system model. The problem formulation is then given in Section III. We derive the achievable rate in Section IV. The asymptotic analysis is given in Section V, followed by the asymptotically optimal solutions given in Section VI. Numerical results are given in Section VII. Finally, Section VIII concludes the paper.

Notation: Scalars are denoted by letters (or Greek letters), vectors by boldface lower-case letters, and matrices by boldface upper-case letters. $I$ and $0$ denote an identity matrix and an all-zero vector, respectively, with appropriate dimensions. For a matrix $A$ of arbitrary size, $A^*$, $A^T$, $A^H$ denote the conjugate, the transpose and the conjugate transpose of $A$, respectively. For a diagonal matrix $D$ of order $K$, $D^{\frac{1}{2}}$ denotes the diagonal matrix whose $k$-th diagonal entry is the square root of the $k$-th diagonal entry of $D$. $E(\cdot)$ denotes the statistical expectation. The distribution of a circularly symmetric complex Gaussian (CSCG) random vector with mean $\mu$ and covariance matrix $\Sigma$ is denoted by $\mathcal{CN}(\mu, \Sigma)$. “$\sim$” stands for “distributed as”. $\|a\|_2$ denotes the Euclidean norm of a complex vector $a$. $O(\cdot)$ denotes the big-O order. “$\longrightarrow$” denotes the convergence as $M \to \infty$.

II. SYSTEM MODEL

We consider a wirelessly powered communication network (WPCN) consisting of a hybrid data-and-energy access point (H-AP) with $M$ antennas, and $K$ single-antenna users. Each user uses the harvested energy to power its UL information transmission. We assume that the H-AP and all users are perfectly synchronized and operate with a TDD protocol. We consider frame-based transmissions over flat-fading channels on a single frequency band.
As shown in Fig. 1, the length of one frame is fixed, given by $T$ seconds. Each frame consists of three phases. In the first CE phase of time period $\tau T$ ($0 < \tau < 1$) seconds, the users send orthogonal training pilots, and the H-AP estimates the UL channels and obtains the DL CSI by exploiting channel reciprocity. Then in the second WET phase of time period $\alpha T$ ($0 < \alpha < 1$) seconds, the H-AP delivers energy via beamforming, and the users harvest energy from the received RF signals. In the final WIT phase of the remaining time period $(1 - \tau - \alpha)T$ seconds, all users transmit information to the H-AP simultaneously in a space-division-multiplexing-access (SDMA) manner. For convenience, we normalize $T = 1$ in the rest of this paper without loss of generality.

![Frame Structure](image)

Fig. 1: Frame Structure

Let $G \triangleq [g_1, g_2, \cdots, g_K]^T$ be the UL $M \times K$ channel matrix between the H-AP and the $K$ users, i.e., $g_{mk} \triangleq [G]_{mk}$ is the channel coefficient between the $m$-th antenna of the H-AP and the $k$-th user. We model the channel matrix $G$ as

$$G = HB^{1/2},$$

where $H$ is the $M \times K$ matrix of independent Rayleigh fading coefficients between the H-AP and $K$ users, i.e., $[H]_{mk} = h_{mk} \sim \mathcal{CN}(0,1)$, and $B$ is a $K \times K$ diagonal matrix, with $[B]_{kk} = \beta_k$ denoting the (long-term) path loss of the channel between the H-AP and user $k$ that is assumed to be constant over frames and taken to be known a priori at both the H-AP and user $k$.

A. Uplink Channel Estimation Phase

In the CE phase, all users simultaneously transmit mutually orthogonal pilot sequences of length $L$ ($L \geq K$) symbols, which allows the H-AP to estimate the channels. User $k$ transmits a pilot sequence with power $q_k$. Define $D = \text{diag}\{q_1, q_2, \cdots, q_K\}$. The pilot sequences used by $K$ users
can be represented by an \( L \times K \) matrix \( \Phi D^{\frac{1}{2}} \), where \( \Phi \) is of size \( L \times K \) and satisfies \( \Phi^H \Phi = I_K \) to preserve orthogonality of the pilots \[25\]. The received signal at the H-AP is thus given by

\[
Y_p = G \left( \Phi D^{\frac{1}{2}} \right)^T + N,
\]

where \( N \) is an \( M \times L \) matrix with independent and identically distributed (i.i.d.) elements each distributed as \( \mathcal{CN}(0, \sigma^2) \). Given \( Y_p \), the minimum mean-square-error (MMSE) estimate of \( G \), denoted by \( \hat{G} = [\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K]^T \), is given by \[26\]

\[
\hat{G} = Y_p \left( \Phi^* BD \Phi^T + \sigma^2 I_L \right)^{-1} \Phi^* D^{\frac{1}{2}} B = Y_p \Phi^* (BD + \sigma^2 I_K)^{-1} D^{\frac{1}{2}} B. \tag{4}
\]

Denote the estimation error by \( E = \hat{G} - G \). From the property of MMSE estimation, \( E \) is independent of \( \hat{G} \). From (4), the elements of the \( k \)-th column of the estimation error matrix \( E \) are random variables with zero mean and variance

\[
\sigma^2_{e,k} = \sigma_{e,k}^2(L, q_k) = \frac{\beta_k}{1 + \beta_k L q_k / \sigma^2}, \tag{5}
\]

**B. Downlink Energy Transfer Phase**

In the DL WET phase, the \( M \times 1 \) transmitted signal is given by \( \sqrt{p_{dl}} w(\hat{G}) \), where \( p_{dl} \) is the transmit power for DL WET, and the beamformer \( w(\hat{G}) \) is to be designed depending on the channel estimate \( \hat{G} \), subject to \( \| w(\hat{G}) \|_2 = 1 \). Assuming channel reciprocity, the received equivalent baseband signal in one symbol period at user \( k \) is written as

\[
z_k = \sqrt{p_{dl}} g_k^H w(\hat{G}) + n_{0,k}, \tag{6}
\]

where the receiver noise is \( n_{0,k} \sim \mathcal{CN}(0, \sigma_0^2) \). For analytical tractability, we take the beamformer in (6) as a linear combination of the normalized channel estimates \( \hat{g}_k \)'s, i.e.,

\[
w^*(\hat{G}) = \sum_{k=1}^K \sqrt{\xi_k} \frac{\hat{g}_k}{\| \hat{g}_k \|_2}, \tag{7}
\]

where the weights \( \xi_k \)'s are subject to the condition \( \sum_{k=1}^K \xi_k = 1 \). The choice of the beamformer in (7) is motivated by the observation that this beamformer is asymptotically optimal, because the channel vectors \( g_k \)'s for all users are orthogonal when \( M \) tends to infinity \[24\]. The weights \( \xi_k \)'s represent the energy allocation for DL WET among users, which will be optimized later in Section \[VI\].
We assume the energy due to the ambient noise in (6) cannot be harvested. We also assume the energy harvested by user $k$ in one symbol period equals the energy of the equivalent base band signal in (6) by the law of energy conservation; our results remain valid if a fixed energy loss, or a fixed fraction energy loss, is incurred. Hence, the expected harvested energy by user $k$ is given by

$$Q_k(L, \alpha, \xi_k) = \alpha \mathbb{E} \left( p_{dl} \left| g_k^H w \left( \hat{G} \right) \right|^2 \right). \tag{8}$$

We assume that users have infinite battery storage for storing the harvested energy. Note that the energy used for pilot transmission is drawn from the harvested energy in previous frames. At steady state, we assume a fraction $\rho$ ($0 < \rho < 1$) of the expected harvested energy $Q_k(L, \alpha, \xi_k)$ is used by users to send the pilots. The amount of pilot energy affects the accuracy of CSI and thus the efficiency of DL WET. Under this energy-splitting scheme, the harvested energy for user $k$, denoted as $E_k(\alpha, \rho, \xi_k)$, will be derived in Section IV.

C. UL Data Transmission Phase

In the UL WIT phase, the received baseband signal vector at the H-AP is given by

$$y = Gx + n, \tag{9}$$

where $x = [x_1, x_2, \ldots, x_K]^T$ with $x_k = \sqrt{p_k}s_k$, where $p_k$ is the transmit power of user $k$, and $s_k \sim \mathcal{CN}(0, 1)$ is its information-carrying signal that is independent of signals of other users. Here, $n \sim \mathcal{CN}(0_M, \sigma^2 I_M)$ is the noise vector with the same noise variance as in the UL CE phase.

Taking into account of the energy consumption for UL pilots, the energy left for UL transmission is $(1 - \rho)E_k(\alpha, \rho, \xi_k)$. Thus, the transmit power is written as

$$p_k = p_k(\tau, \alpha, \rho, \xi_k) = \frac{(1 - \rho)E_k(\alpha, \rho, \xi_k)}{1 - \tau - \alpha}. \tag{10}$$

In the UL WIT phase, all users simultaneously transmit to the H-AP. The H-AP adopts a linear detector $A = [a_1, a_2, \ldots, a_K]$ to detect the information for all users. Specifically, a ZF or MRC detector which performs well for large $M$ [24], is written as

$$A = \begin{cases} \hat{G} \left( \hat{G}^H \hat{G} \right)^{-1}, & \text{for ZF} \\ \hat{G}, & \text{for MRC}. \end{cases} \tag{11}$$
From (9), the signal after using the detector $A$ is obtain as $r = A^H G x + A^H n$. In particular, the detected signal associated with user $k$, denoted by $r_k$, is written as

$$ r_k = \sqrt{p_k} a_k^H \hat{g}_k s_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} a_k^H \hat{g}_i s_i - \sum_{i=1}^K \sqrt{p_i} a_k^H e_i s_i + a_k^H n, $$

where $a_i$, $\hat{g}_i$, and $e_i$ are the $i$-th column of $A$, $\hat{G}$ and $E$, respectively. The last three terms in (12) are treated as interference and noise, and assumed to be Gaussian distributed to maximize entropy. Hence, the achievable rate of the UL transmission from user $k$ is given by

$$ R_k = R_k(\tau, \alpha, \rho, \xi_k) \triangleq (1 - \tau - \alpha) \log \left(1 + \frac{1}{\gamma_k}\right), $$

where the signal-to-interference-plus-noise-ratio (SINR) is

$$ \gamma_k = \frac{p_k |a_k^H \hat{g}_k|^2}{\sum_{i=1, i \neq k}^K p_i |a_k^H \hat{g}_i|^2 + |a_k^H a_k|^2 \sum_{i=1}^K p_i \sigma_i^2 + |a_k^H a_k|^2 \sigma^2}. $$

The exact expression for the rate $R_k$ is not analytically tractable. Instead, we focus on obtaining an analytically tractable lower bound on the rate achieved by using the harvested energy. From the convexity of the function $\log \left(1 + \frac{1}{x}\right)$ and Jensen’s inequality [27], the rate $R_k$ is lower-bounded as

$$ R_k \geq \tilde{R}_k = \tilde{R}_k(\tau, \alpha, \rho, \xi_k) \triangleq (1 - \tau - \alpha) \log \left(1 + \frac{1}{\gamma_k}\right) \frac{1}{\left(1 + \frac{1}{\gamma_k}\right)^{-1}}. $$

We shall observe numerically in Section VII that the derived lower bound is tight for massive MIMO systems. In the sequel, we use $\tilde{R}_k$ as the achievable rate.

D. Benchmark and Ideal Case

For performance comparison, we consider the OT-MM benchmark, for which the frame structure is obtained by exchanging the order of the UL CE and DL WET phases in Fig. 1. The H-AP first uses the isotropic beamformer $w = [1 \ 1 \ \cdots \ 1]^T / \sqrt{M}$ for energy broadcasting, followed by UL CE and UL WIT. A fraction $\rho$ of the harvested energy is used for CE, with the rest of the energy used for WIT. Hence, channel estimation is employed only for data detection, but not for WET.
Also, we shall compare the proposed WET-MM system with the ideal case, in which the H-AP is assumed to use perfect CSI for all users to perform DL WET and UL information decoding. The CE phase in the frame structure in Fig. 1 is thus removed, i.e., \( \tau = 0, \rho = 0, \sigma^2_{e,k} = 0, \forall k \). Clearly, the proposed WET-MM system with estimated CSI cannot perform better than the ideal case.

### III. Problem Formulation

In this section, we give the problem formulation. To optimize the throughput and achieve user fairness, we maximize the minimum rate for all users, by optimizing over the CE time \( \tau \), the WET time \( \alpha \), the energy allocation weights \( \xi_k \)'s, as well as the energy-splitting fraction \( \rho \). Denote the vector \( \xi = [\xi_1, \xi_2, \cdots, \xi_K]^T \). We have the following problem formulation

\[
\begin{align*}
\text{(P}_1\text{)} \quad & \max_{\tau, \alpha, \rho, \xi} \min_{1 \leq k \leq K} \tilde{R}_k(\tau, \alpha, \rho, \xi_k) \\
\text{s. t.} \quad & \sum_{k=1}^{K} \xi_k = 1 \\
& 0 \leq \tau + \alpha \leq 1 \\
& \tau \geq 0, \alpha \geq 0, \xi_k \geq 0, \forall k = 1, \cdots, K \\
& 0 \leq \rho \leq 1,
\end{align*}
\]

where (16b) is due to the power constraint at the H-AP for energy beamforming, and (16c) is due to the frame duration constraint. The achievable rate \( \tilde{R}_k(\tau, \alpha, \rho, \xi_k) \) is given in (15).

For fixed \( M \), solving Problem \( \text{(P}_1\text{)} \) is challenging, because of the nonlinear relationship between the information rate and the variables \( \tau, \alpha, \rho \) and \( \xi \). In our system model, the variables are coupled in the UL CE, DL WET and UL WIT phases as follows:

- The CE time \( \tau \) and the fraction of the harvested energy used for pilots \( \rho \) affect the CSI accuracy.
- The CSI accuracy affects the amount of harvested energy in the DL WET phase, the signal power and the interference power for data detection in the UL WIT phase.
- Besides the CSI accuracy, the amount of harvested energy also depends on the energy allocation weights \( \xi \) and the WET time \( \alpha \).
- Moreover, the interference from other users depends on the energy harvested by them, the time allocation \( \tau, \alpha \), as well as the energy-splitting fraction \( \rho \).

Nevertheless, it is possible to obtain interesting asymptotical solutions and insights in the large-\( M \)
regime. In the sequel, we obtain analytical expression to the rate in Section IV followed by the asymptotic analysis in Section V and then obtain the asymptotically optimal solutions in Section VI.

IV. Analysis on Achievable Rate

Before deriving the achievable rate in Section IV-B, we first obtain the harvested energy.

A. Harvested Energy

Using the beamformer in (7), the harvested energy is derived in the following lemma.

Lemma 1. With the beamformer in (7), the total harvested energy by user $k$ is given by

$$Q_k(L, \alpha, \xi_k) = \alpha p d l \xi_k \beta_k M \left[ 1 - \frac{(M-1)\sigma^2}{M(\beta_k L q_k + \sigma^2)} \right] + \alpha p d l \beta(1 - \xi_k).$$  \hspace{1cm} (17)

Proof: See Appendix A.

The first term in (17) is the harvested energy from the beam directed toward user $k$, which is dominant when $M$ is large, while the second term represents the energy harvested from beams directed toward other users but still harvested by user $k$. We observe that $Q_k(L, \alpha, \xi_k)$ is an increasing function of the (discrete) pilot length $L$. For the case of single user with perfect CSI, i.e., $\xi = 1$ and equivalently $\sigma^2 = 0$, we obtain the harvested energy $Q(\alpha) = \alpha p d l \beta M$, as expected.

The term $L q_k$ in (17) represents the energy used by user $k$ for pilot transmission. By assumption, a fraction $\rho$ of the harvested energy $Q_k(L, \alpha, \xi_k)$ is used for pilots. Taking into account this energy-splitting scheme, we further obtain the harvested energy and the variance of channel estimation, in the following lemma.

Lemma 2. With the beamformer in (7) and the energy-splitting fraction $\rho$, the total harvested energy by user $k$ is given by

$$E_k(\alpha, \rho, \xi_k) = \frac{\alpha p d l \beta_k (\xi_k(M-1) + 1) - \frac{\sigma^2}{\beta_k \rho} + \sqrt{\left( \alpha p d l \beta_k (\xi_k(M-1) + 1) - \frac{\sigma^2}{\beta_k \rho} \right)^2 + \frac{4 \alpha p d l \sigma^2}{\rho}}}{2}.$$

Moreover, the variance of the channel estimation error of user $k$ is rewritten as

$$\sigma_{e,k}^2 = \sigma_{e,k}^2(\alpha, \rho) = \frac{\beta_k \sigma^2}{\beta_k \rho E_k(\alpha, \rho, \xi_k) + \sigma^2}.$$

(19)
Proof: By replacing the term \(Lq_k\) in (17) by \(\rho Q_k(L, \alpha, \xi_k)\), we obtain the equation with \(Q_k(L, \alpha, \xi_k)\) in both sides, which can be written as a quadratic equation in \(Q_k(L, \alpha, \xi_k)\). Solving for \(Q_k(L, \alpha, \xi_k)\) and discarding the negative solution, we obtain the harvested energy for user \(k\) in (18). From (5), the variance of channel estimation error is obtained as in (19).

B. Achievable Rate for WET-MM System with Estimated CSI

For ZF detection, from (11), we have

\[
A = \hat{G}(\hat{G}^H \hat{G})^{-1} \text{ and } a_i^H \hat{g}_j = \delta_{ij}, \text{ where } \delta \text{ is the Dirac delta function.}
\]

The corresponding achievable rate is obtained in the following lemma.

**Lemma 3.** With MMSE channel estimate \(\hat{G}\), ZF receiver and \(M \geq K + 1\), an achievable UL rate of user \(k\) is given by

\[
\tilde{R}^{ZF}_k = (1 - \tau - \alpha) \log \left( 1 + \frac{(M - K)\beta_k^2 \rho E_k(\alpha, \rho, \xi_k)}{\sigma^2 \left( \beta_k \rho + \frac{\sigma^2}{E_k(\alpha, \rho, \xi_k)} \right) \left( 1 - \tau - \alpha \right) + \sum_{i=1}^K \beta_i E_i(\alpha, \rho, \xi_i) + \sigma^2 \right),
\]

(20)

where the harvested energy \(E_k(\alpha, \rho, \xi_k)\) is given by (18).

**Proof:** See Appendix [B].

For MRC detection, from (11), we have \(A = \hat{G}\). The achievable rate is obtained in Lemma [4]

**Lemma 4.** With MMSE channel estimate \(\hat{G}\), MRC detection and \(M \geq 2\), an achievable UL rate of user \(k\) is given by

\[
\tilde{R}^{MRC}_k = (1 - \tau - \alpha) \log \left( 1 + \frac{(M - 1)\beta_k^2 \rho E_k(\alpha, \rho, \xi_k)}{\beta_k \rho + \frac{\sigma^2}{E_k(\alpha, \rho, \xi_k)} \left( 1 - \tau - \alpha \right) + \sum_{i=1}^K \beta_i E_i(\alpha, \rho, \xi_i) + \beta_k \sigma^2 \right),
\]

(21)

where the harvested energy \(E_k(\alpha, \rho, \xi_k)\) is given by (18).

**Proof:** See Appendix [C].

For ZF detection, the interference in (20) is due to only the channel estimation error, while for MRC detection, the interference component in (21) comes from both the channel estimation error and the multi-user interference (MUI). Comparatively, MRC detection has lower complexity of computation, since all antennas independently apply a matched filter \(\hat{g}_k\) to maximize the signal power for user \(k\).
C. Achievable Rate for Benchmark and Ideal Case

1) OT-MM System: It can be shown that the harvested energy by user $k$ is $E_{k,\text{OT-MM}}(\alpha) = \alpha p_d \beta_k$. By replacing $E_{k}(\alpha, \xi_k, \rho)$ in (20) by $E_{k}(\alpha)$, the achievable rate for ZF is given by

$$
\tilde{R}_{k,\text{OT-MM}}^\text{ZF}(\tau, \alpha, \rho) = (1 - \tau - \alpha) \log \left( 1 + \frac{(M - K) \alpha p_d \beta_k^3 \rho}{\sigma^2 \beta_k \rho + \sigma^2 \alpha p_d \beta_k} \right) \cdot (22)
$$

Similarly, from (21), the achievable rate for MRC is thus given by

$$
\tilde{R}_{k,\text{OT-MM}}^\text{MRC}(\tau, \alpha, \rho) = (1 - \tau - \alpha) \log \left( 1 + \frac{(M - 1) \alpha p_d \beta_k^3 \rho}{\beta_k \rho + \sigma^2 \alpha p_d \beta_k} \left( \sum_{i=1}^{K} \alpha p_d \beta_i^2 \right) + \beta_k \sigma^2 \right) \cdot (23)
$$

2) Ideal Case: With perfect CSI, i.e., $\sigma_{e,k}^2 = 0$, the ideal case is the special case for the analysis in Sections IV-A and IV-B. With the beamformer in (7), the harvested energy by user $k$ is given by

$$
E_{k}(\alpha, \xi_k) = \alpha p_d W_k \beta_k M + \alpha p_d \beta_k (1 - \xi_k). \quad (24)
$$

Moreover, the achievable rate is given by

$$
\tilde{R}_{k,\text{Ideal}}(\alpha, \xi_k) = \begin{cases} 
(1 - \alpha) \log \left( 1 + \frac{E_{k}(\alpha, \xi_k)(M - K) \beta_k}{(1 - \alpha) \sigma^2} \right), & \text{for ZF} \\
(1 - \alpha) \log \left( 1 + \frac{E_{k}(\alpha, \xi_k)(M - 1) \beta_k}{\sum_{i=1, i \neq k}^{K} E_{i}(\alpha, \xi_i) \beta_i + (1 - \alpha) \sigma^2} \right), & \text{for MRC}. 
\end{cases} \quad (25)
$$

V. ASYMPTOTIC ANALYSIS

In this section, we analyze the UL achievable rate in the large-$M$ regime.

A. Asymptotic Analysis for WET-MM System with Estimated CSI

To obtain analytical insights, we consider the massive MIMO regime where the number of transmit antennas at H-AP, $M$, is sufficiently large. Specifically, we assume

$$
M \gg \max_{1 \leq k \leq K} \frac{\sigma^2}{\alpha p_d \beta_k^3 \xi_k}. \quad (26)
$$
Then, from (18), the asymptotic harvested energy at user $k$ is given by

$$E_k(\alpha, \xi_k, \rho) \rightarrow E_k^{\text{asym}}(\alpha, \xi_k) \triangleq \alpha p_d \beta_k \xi_k M. \quad (27)$$

**Remark 1 (Discussion on the asymptotically harvested energy).** The asymptotically harvested energy $E_k^{\text{asym}}$ is achieved when $M$ is sufficiently large such that the $M$-dependent term in (18) is dominant over other terms. We note that the $E_k^{\text{asym}}$ is independent of $\rho$, as the $\rho$-dependent terms in (18) are negligible.

For the condition (26), we assume that $\alpha, \rho$ and $\xi_k$ are arbitrarily fixed and independent of $M$. However, we noted that when $\alpha = O(M^{-2\nu})$ for $0 < \nu < \frac{1}{2}$, the condition (26) still holds, and $E_k^{\text{asym}}(\alpha, \xi_k) = O(M^{1-2\nu})$ increases as $M$ increases. This observation will be used in Section VI.

Also, we observe that the asymptotically harvested energy in (27) approaches the harvested energy in (24) for the ideal case with perfect CSI. That is, with the proposed scheme of energy splitting between UL CE and UL WIT, the energy beamforming with estimated CSI asymptotically achieves the DL-WET performance limit that is achieved by the ideal case.

Moreover, the asymptotic rate is given in Theorem 1 for ZF, and in Theorem 2 for MRC.

**Theorem 1.** For fixed $\tau$, $\alpha$, $\rho$ and $K$, when $M$ is sufficiently large such that (26) is satisfied, the asymptotically achievable rate of user $k$ for ZF detection is given by

$$\tilde{R}_k^{ZF} \rightarrow (1 - \tau - \alpha) \log \left( 1 + \frac{M(M - K)\alpha p_d \beta_k^2 \xi_k \rho}{\sigma^2 \left[ K + \frac{(1 - \tau - \alpha) \rho}{1 - \rho} \right]} \right). \quad (28)$$

**Proof:** Taking $M \rightarrow \infty$ and assuming (26) holds, from (20) in Lemma 3 and (27), the asymptotically achievable rate is derived in (28). \hfill \blacksquare

**Theorem 2.** For fixed $\tau$, $\alpha$, $\rho$ and $K$, when $M$ is sufficiently large such that

$$M \gg \max_{1 \leq k \leq K} \max \left\{ \frac{\sigma^2}{\alpha p_d \rho \beta_k^2 \xi_k}, \frac{\sigma^2 (1 - \tau - \alpha)}{\alpha p_d \beta_k \xi_k (1 - \rho)} \right\}, \quad (29)$$

the asymptotically achievable rate of user $k$ for MRC detection is given by

$$\tilde{R}_k^{MRC} \rightarrow (1 - \tau - \alpha) \log \left( 1 + \frac{(M - 1) \beta_k^2 \xi_k}{\sum_{i=1, i \neq k}^{K} \beta_i^2 \xi_i} \right). \quad (30)$$

\footnote{Later we shall consider $\alpha, \rho, \xi_k$ as variables to be optimized.}
Proof: Taking $M \to \infty$, and assuming $M \gg \max_{1 \leq k \leq K} \frac{\sigma^2(1-\tau-\alpha)}{\alpha_p \xi k_1(1-\rho)}$ and (26) holds, from (27), the noise power in (21) in Lemma 5 is negligible compared to the MUI power, and the asymptotically achievable rate is further derived in (30).

For MRC detection, the asymptotic rate is independent of the energy-splitting fraction $\rho$. This is because both the signal power and the MUI power in (21) are proportional to $\rho$. The effect of $\rho$ is thus cancelled. Thus, $\rho$ can be arbitrarily chosen in $(0, 1)$. Also, we note that the condition in (29) can be easily satisfied as long as $\rho$ does not approach 1.

For the proposed WET-MM system, the asymptotic SINR in (28) for ZF is of order $O(M^2)$, since the MUI is cancelled. For MRC, however, the asymptotic SINR in (30) is of order $O(M)$, due to the MUI. The factor $(M-1)$ in the asymptotical SINR is due to the maximum ratio combining. The MM-DoDR of the proposed WET-MM system is thus given immediately by the following theorem.

**Theorem 3.** For a WET-MM system, the maximal asymptotic MM-DoDR of any user $k$ is given by

$$\kappa_{WET-MM} = \begin{cases} 2, & \text{for ZF} \\ 1, & \text{for MRC} \end{cases}$$

(31)

**Proof:** Clearly, from Theorem 1 and Theorem 2 we have

$$\kappa_{WET-MM} = \lim_{M \to \infty} \frac{\tilde{R}_k}{\log M} = \begin{cases} 2(1-\tau-\alpha), & \text{for ZF} \\ 1-\tau-\alpha, & \text{for MRC} \end{cases}$$

For ZF, let $\tau \to 0$, $\alpha = O(M^{-2\nu}) \to 0$, where $0 < \nu < \frac{1}{2}$. For MRC, let $\tau \to 0$, and $\alpha = O(M^{-\varphi}) \to 0$, where $0 < \varphi < 1$. The maximal asymptotic MM-DoRG is thus obtained in (31).

**B. Asymptotic Analysis for Benchmark and Ideal Case**

The MM-DoRG for the OT-MM benchmark and the ideal case is given in the following theorem.

**Theorem 4.** The maximal asymptotic MM-DoDR for the OT-MM benchmark is given by

$$\kappa_{OT-MM} = 1, \quad \text{for ZF and MRC.}$$

(32)

The maximal asymptotic MM-DoDR for the ideal case of the WET-MM system is given by

$$\kappa_{Ideal} = \begin{cases} 2, & \text{for ZF} \\ 1, & \text{for MRC} \end{cases}$$

(33)
Proof: For the OT-MM benchmark, the asymptotic achievable rate is still given in (22) for ZF, and in (23) for MRC. Note that the asymptotic SINR is of order \( O(M) \). By definition, the maximal asymptotic MM-DoDR for the OT-MM benchmark is obtained in (32).

For the ideal case of the WET-MM system, from (24), we obtain that the harvested energy 
\[ E_k(\alpha, \xi_k) \longrightarrow \alpha p_0 \xi_k \beta_k M. \]
Then, the achievable rate for ZF is given from (25) by
\[
\tilde{R}_k^{ZF}(\alpha, \xi_k) \longrightarrow (1 - \alpha) \log \left( 1 + \frac{\alpha p_0 \beta_k^2 \xi_k M (M - K)}{\sigma^2(1 - \alpha)} \right). \tag{34}
\]
The achievable rate for MRC is given from (25) by
\[
\tilde{R}_k^{MRC}(\alpha, \xi) \longrightarrow (1 - \alpha) \log \left( 1 + \frac{(M - 1) \beta_k^2 \xi_k}{\sum_{i=1, i\neq k}^{K} \beta_i^2 \xi_i} \right). \tag{35}
\]
The asymptotic SINR of ZF in (34) is of order \( O(M^2) \), while the asymptotic SINR for MRC in (35) is of order \( O(M) \). By definition, the maximal asymptotic MM-DoDR is thus obtained in (33). ■

Remark 2 (Optimality in terms of MM-DoRG). Clearly, the rate and thus the MM-DoRG of the ideal case with perfect CSI is the performance limit for any practical system with estimated CSI. For finite \( M \), the performance of both energy transfer and information decoding for the proposed WET-MM system is degraded due to imperfect CSI. In the large-\( M \) regime, however, the system is optimal in terms of DoRG, as it achieves the same MM-DoRG as the ideal case. This is explained intuitively as follows: as in Remark 1, the proposed WET-MM system with estimated CSI asymptotically achieves the same DL-WET performance as the ideal case. Also, with fixed UL transmit power, the decoding SINR in (20) and (21) for the proposed WET-MM system is of order \( O(M) \), which is the same as the SINR in (25) for the ideal case.

Remark 3 (Advantage over the OT-MM benchmark). For ZF detection, the DoRG achieved by the proposed scheme is double of that in the OT-MM system, and also double of that of the conventional UL massive MIMO system without DL WET [25]. This performance gain comes from the fact that the UL transmit power is proportional to \( M \) (due to the beamforming gain in the WET phase).

Moreover, we shall see in Remark 4 in Section VI that the proposed WET-MM system asymptotically achieves the best possible rate fairness among users (i.e., a common rate), while no fairness is guaranteed in the OT-MM system.
VI. ASYMPTOTICALLY OPTIMAL SOLUTION

Following the asymptotic analysis in Section V, in this section, we derive the asymptotically optimal solution to the minimum rate maximization Problem (P1) in Section VI-A for the proposed WET-MM system, and in Section VI-B for the ideal case and the OT-MM benchmark.

A. Asymptotically Optimal Solution for WET-MM Systems

1) Asymptotically Optimal Energy Allocation Weights: Due to the distance-dependent signal attenuation, the users that are far from the H-AP harvest less energy, and moreover, they have to perform UL transmission using higher power to overcome the propagation. This is referred as the “double near-far” problem [20], which intuitively leads to unfairness among users, with significantly lower rates achieved by far users. Problem (P1) thus minimizes this unfairness. In the large-M regime, the optimal \( \xi_k \) that maximizes the minimum rate is given in the following lemma.

**Lemma 5.** When \( M \) satisfies the condition in (26), for both ZF and MRC detection, the minimum achievable rate is maximized when the energy allocation weight is chosen as

\[
\xi_k^* = \frac{1}{\sum_{i=1}^{K} \frac{1}{\beta_i^2}}.
\]

(36)

**Proof:** See Appendix D.

That is, the optimal \( \xi_k^* \) is inversely proportional to the square of the long-term path loss of user \( k \), which compensates the long-term path loss in both the DL WET phase and the UL WIT phase.

2) Asymptotically Optimal Energy-Splitting: Here, we assume \( \tau \) and \( \alpha \) to be arbitrary. We first obtain the asymptotically optimal \( \rho^* \) for Problem (P1) in the following lemma.

**Lemma 6.** For ZF detection, when \( M \) satisfies (26), the asymptotically optimal \( \rho_{ZF}^* \) is given by

\[
\rho_{ZF}^*(\tau, \alpha) = \frac{\sqrt{K}}{\sqrt{K + \sqrt{1 - \tau - \alpha}}}. \tag{37}
\]

For MRC detection, when \( M \) satisfies (29), the asymptotically optimal energy-splitting fraction \( \rho_{MRC}^* \) that maximizes the achievable rate is arbitrary in \((0, 1)\).

**Proof:** For ZF detection, given \( \tau, \alpha \), the asymptotic SINR in (28) is proportional to the function

\[
f(\rho) = \frac{\rho}{K + \frac{(1-\tau-\alpha)\rho}{1-\rho}}.
\]
It can be shown that \( f(\rho) \) is a quasiconcave function of \( \rho \). The optimal \( \rho^* \) that maximizes \( f(\rho) \) is obtained by solving \( f'(\rho) = 0 \) where \( f'(\rho) \) is the derivative of \( f(\rho) \). The solution is given by (37).

For MRC detection, the asymptotic rate in (30) is independent of \( \rho \). Hence, the energy-splitting coefficient \( \rho \) can be arbitrary chosen in \((0, 1)\).

For ZF detection, we observe that the asymptotically optimal \( \rho^*_{\text{ZF}} \) increases as the number of users \( K \) increases. It implies that a higher fraction of harvested energy should be used for CE for larger \( K \). This is because the asymptotic rate is more interference-limited for large \( K \), and hence more accurate CSI is required to decrease the interference that comes from channel estimation error.

3) Asymptotically Optimal Time Allocation: The asymptotically optimal \( \tau^* \) and \( \alpha^* \) is given in the following Lemma 7 for ZF detection, and in Lemma 8 for MRC detection.

**Lemma 7.** For ZF detection, when \( M \) is sufficiently large to satisfy (26), the asymptotically optimal time allocation for CE and WET is given by

\[
\tau^*_{\text{ZF}} \rightarrow 0, \quad \alpha^*_{\text{ZF}} = O(M^{-2\nu}) \rightarrow 0, \quad \text{where } \nu > 0, \text{ and } \nu \rightarrow 0. \tag{38}
\]

**Proof:** Suppose the condition (26) is satisfied. We define the asymptotic SINR in (28) as a function \( g(\tau) \) of \( \tau \). Then the derivative of \( g(\tau) \) is derived as

\[
g'(\tau) = \frac{(1 - \tau - \alpha)\rho}{(1 - \rho)K + (1 - \tau - \alpha)\rho} - \log \left( \frac{M(M - K)\alpha p_d \beta_k^2 \xi_k \rho}{\sigma^2 K + \frac{(1 - \tau - \alpha)\rho}{1 - \rho}} \right). \tag{39}
\]

Clearly, the above derivative is negative, due to the assumption of large \( M \). Hence, we choose \( \tau^*_{\text{ZF}} \rightarrow 0 \) to maximize \( g(\tau) \).

Suppose \( \alpha = O(M^{-2\nu}) \), for \( 0 < \nu < \frac{1}{2} \); otherwise, the asymptotically harvested energy in (27) decreases as \( M \) increases, which is suboptimal. Define \( \epsilon(M) \) such that \( \epsilon(M) \rightarrow 0 \) for sufficiently large \( M \). From (28), the asymptotic rate is rewritten as

\[
\tilde{R}^\text{ZF}_k \rightarrow (1 - \tau - \epsilon(M)) \log \left( 1 + \frac{M^{2(1-\nu)} p_d \beta_k^2 \xi_k \rho}{\sigma^2 K + \frac{(1 - \tau - \epsilon(M))\rho}{1 - \rho}} \right)
\rightarrow (1 - \tau) \log \left( 1 + \frac{M^{2(1-\nu)} p_d \beta_k^2 \xi_k \rho}{\sigma^2 K + \frac{(1 - \tau)\rho}{1 - \rho}} \right). \tag{39}
\]

We choose \( \nu \rightarrow 0 \) to maximize the asymptotic rate. Hence, \( \alpha^*_{\text{ZF}} = O(M^{-2\nu}) \), where \( \nu \rightarrow 0 \).
Lemma 8. For MRC detection, when $M$ is sufficiently large to satisfy (29), the asymptotically optimal time allocation for CE and WET is given by

$$
\tau^*_\text{MRC} \longrightarrow 0, \quad \alpha^*_\text{MRC} = O\left(M^{-\varphi}\right) \longrightarrow 0,
$$

(40)

where $0 < \varphi < 1$ is arbitrary.

Proof: From (30), the asymptotically optimal $\tau$ and $\alpha$ should be chosen as small as possible. To satisfy (29), we choose $\alpha$ to tend to zero at the rate of order of $O\left(M^{-\varphi}\right)$, for $0 < \varphi < 1$.

For ZF detection, the optimal $\alpha^*_\text{ZF}$ in Lemma 7 approaches zero fairly slowly with $M$, as will be verified numerically in Section VII. For MRC detection, however, the optimal $\alpha^*_\text{MRC}$ in Lemma 8 approaches zero faster with $M$, when $\varphi$ is chosen to approach one.

4) Asymptotically Maximal Minimum Rate: The asymptotically maximal minimum achievable rate for Problem ($P_1$) is thus given in the following theorem.

Theorem 5. The asymptotically maximal minimum rate for the proposed WET-MM system is

$$
\tilde{R} \longrightarrow \begin{cases} 
\log \left(1 + \frac{M^2 \rho_{\text{MRC}}}{\sigma^2 (\sqrt{K} + 1)^2 \sum_{i=1}^{K} \beta_i^2}\right), & \text{for ZF} \\
\log \left(1 + \frac{M-1}{K-1}\right), & \text{for MRC.}
\end{cases}
$$

(41)

Proof: For MRC, from Lemma 8 and Theorem 2, the asymptotically maximal rate for user $k$ is obtained in (41). For ZF, from Lemma 6 and Lemma 7, the asymptotically optimal $\rho^*_\text{ZF} = \frac{\sqrt{K}}{\sqrt{K} + 1}$. From Lemma 7 and Theorem 1, the asymptotically maximal rate for user $k$ is obtained in (41). The fact that each user achieves the same asymptotically maximal rate completes the proof.

Remark 4 (Fairness comparison). We see that the proposed WET-MM system asymptotically achieves the best possible rate fairness among users (i.e., a common rate), while no fairness is guaranteed in the OT-MM system. For the OT-MM system, from (22) and (23), the rate achieved by users located far away may be extremely low, since no energy allocation is employed.

5) Asymptotic Scaling Law of Supporting Distance: Last, we derive the scaling law of the distance required to support a given achievable rate by the WET-MM system. Let $d_k$ be the distance between user $k$ and the H-AP. For convenience, we rewrite $d_k = d_0 r_k$, where $r_k$ represents the relative distance of user $k$ normalized to a reference distance $d_0$. Without loss of generality, we use the
long-term fading model to model the path loss for user $k$ as

$$\beta_k = \beta_0 (d_0 r_k)^{-u}, \quad (42)$$

where $\beta_0$ is the loss for a distance of one meter, and $u > 1$ is the path-loss exponent.

For ZF detection, we have the following scaling law for the supporting distance.

**Theorem 6.** When the reference distance $d_0 \ll O \left( M^{\frac{1}{2u}} \right)$, for a ZF detector, to achieve a rate $R_0$ for all users, the supported reference distance $d_0$ is scaled according to the following law

$$d_0 \longrightarrow O \left( M^{\frac{1}{2u}} \right). \quad (43)$$

**Proof:** For ZF detection, substituting (42) and the optimal $\beta_k^*$ into the condition in (26), from Lemma 6 and Lemma 7 we obtain the required assumption for the analysis to hold as

$$d_0 \ll \left( \frac{\alpha_{ZF}^* \rho_{ZF}^2 \beta_0^2 M}{\sigma^2 \sum_{k=1}^{K} r_k^{2u}} \right)^{\frac{1}{2u}} = O \left( M^{\frac{1}{2u}} \right). \quad (44)$$

From (41) and (42), given a desired rate $R_0$ for all users, the supported reference distance is

$$d_0 = \left( \frac{M(M-K)\alpha_{ZF}^* \rho_{ZF}^2 \beta_0^2}{\sigma^2(\sqrt{K}+1)^2 2R_0 \sum_{k=1}^{K} r_k^{2u}} \right)^{\frac{1}{2u}}. \quad (45)$$

Taking $M \rightarrow \infty$, we obtain the scaling law in (43).

Theorem 6 implies that the supporting reference distance $d_0$ scales as $O(M^{\frac{1}{2u}})$ with respect to $M$, when $d_0$ is smaller than some critical distance that is of order $O(M^{\frac{1}{2u}})$. We take an example to see this the implication. We set the path loss exponent $u = 3$. In the asymptotic regime, $M$ should be increased by $8$ times, if a reference distance $2d_0$ is desired. By performing similar analysis, we obtained that for the ideal case, $M$ should also be increased by $8$ times, while for the OT-MM system, $M$ should instead be increased by $64$ times. Clearly, the number of transmit antennas required by the proposed WET-MM system is a square root of that required by the OT-MM system.

Note that if $d_0$ is larger than the condition distance, the large-$M$ analysis becomes untractable, as the asymptotical harvested energy given by (18) is too complicated to do further analysis.

**Remark 5 (Supporting distance for MRC detection).** When the reference distance $d_0$ is smaller than some critical distance that is of order $O(M^{\frac{1}{2u}})$, a fixed asymptotic rate in (41) is achieved for MRC.
detection. There is no similar scaling law of \( d_0 \) with respect to \( M \). This is because the asymptotic rate is limited by the MUI.

**Remark 6 (Supporting number of users).** From (41), it can be shown that for a desired rate \( R_0 \) for all users, the supporting number of users \( K \) has the following scaling law

\[
K \longrightarrow O(M), \quad \text{for ZF and MRC.} \tag{46}
\]

This result is slightly surprising, since so far ZF outperforms MRC in the large-\( M \) regime. The reason can be contributed to the fact that in the previous analysis \( K \) is taken to be constant. For ZF detection, the condition \( M \gg K \) is always satisfied such that there is enough degree of freedom for cancelling the MUI. In (46), however, \( K \) is a variable, and hence ZF loses its advantage compared to MRC. In this case, more advanced detection like maximum likelihood (ML) detection, is required to obtain better scaling law. However, the complexity of ML detection is prohibitively high.

**B. Asymptotically Optimal Solutions for Benchmark and Ideal Case**

1) **OT-MM Benchmark:** For both ZF and MRC detection, the achievable rates in (22) and (23) are functions of \( \tau, \alpha \) and \( \rho \). Hence, the asymptotically optimal \( \tau_{\text{OT}}, \alpha_{\text{OT}}^*, \rho_{\text{OT}}^* \) that maximizes the achievable rate can be obtained by numerical search.

2) **Ideal Case:** Similar to Section VI-A1, it can be shown that the asymptotically optimal energy allocation weights \( \xi^* \) is given by (36). Thus, from (34) and (35), the achievable rate is given by

\[
\tilde{R}_{k,\text{Ideal}}(\alpha) \longrightarrow \begin{cases} 
(1-\alpha) \log \left( 1 + \frac{\alpha p_{dl} M (M-K)}{\sigma^2 (1-\alpha) \sum_{i=1}^{K} \frac{1}{\beta_i^2}} \right), & \text{for ZF} \\
(1-\alpha) \log \left( 1 + \frac{M}{K-1} \right), & \text{for MRC.} 
\end{cases} \quad \tag{47}
\]

For ZF detection, similar to Lemma 7 it can be shown from (47), the optimal \( \alpha_{\text{Ideal}}^* = O(M^{-2\nu}) \rightarrow 0 \), where \( \nu > 0 \), and \( \nu \rightarrow 0 \). For MRC, similar to Lemma 8 from (47), we have that the optimal \( \alpha_{\text{Ideal}}^* = O(M^{-\nu}) \rightarrow 0 \), where \( 0 < \nu < 1 \) is arbitrary.

**VII. Numerical Results**

In this section, we present numerical results to validate our results. Due to space limitation, we give only the numerical results for ZF detection which performs the same or better than MRC detection from our analysis. We set \( K = 2 \) and \( p_{dl} = 1 \) Watt. The frame length is fixed as 5 \( \mu s \),
which is normalized in the simulations. The carrier frequency is 5 GHz, and the bandwidth is 100 KHz. We set the power spectrum density of noise as $-120$ dBm/Hz, which implies the noise power $\sigma^2 = -70$ dBm. We use the long-term fading model $\beta_i = 10^{-3}d_i^{-3}$, where the distance $d_1 = 2$ m and $d_2 = 4$ m. We use 1000 channel realizations in the Monte Carlo simulation. The step size for $\tau$, $\alpha$ and $\rho$ is chosen as 0.00025, 0.0005 and 0.0005, respectively.

First, we fix $M = 200$. By numerical search, the optimal time for CE and for WET is $\tau^{\star} = 0.00625$, $\alpha^{\star} = 0.1240$, respectively. From the analytic result in (37), the asymptotically optimal $\rho^{\star} = 0.6023$. From (41), the maximal minimum rate is thus 9.5812 bps/Hz. These results will be verified in Figs. 2 and 3.

For fixed $\rho = 0.6085$, Fig. 2 is the contour plot of the rate against the UL CE time $\tau$ and the DL WET time $\alpha$, for user 1 and 2. Specifically, the maximal minimum rate is achieved as 9.5492 bps/Hz, at $\tau^{\star} = 0.00625$, $\alpha^{\star} = 0.1240$. Furthermore, the achieved rate is 9.9436 bps/Hz and 9.5492 bps/Hz for user 1 and user 2, respectively, which is almost the same. The asymptotically maximal minimum rate is 9.5812 bps/Hz, which approximates the actually maximal minimum rate 9.5492 bps/Hz closely. Hence, the asymptotic analysis coincides with the numerical results. For fixed time allocation $\tau^{\star} = 0.00625$, $\alpha^{\star} = 0.1240$, the data rate is plotted in Fig. 3 against the energy-splitting fraction $\rho$. We see that the rate is a quasi-concave function of $\rho$, and the unique optimal energy-splitting fraction $\rho^{\star} \approx 0.6$ is the star-marker point in Fig. 3. This coincides with the analytic result.
Next, we compare the optimal resource allocation solutions for different $M$. The numerical results are shown in Table I. We use the normal notation for the analytically asymptotic results, and use the notations with the bar head for the numerical results. We observe that as $M$ tends to infinity, the optimal CE time $\tau^*$ tends to 0 quickly, while the optimal WET time $\alpha^*$ tends to 0 at a slower rate. This observation coincides with the results in Lemma 7. From Lemma 6 and Lemma 7, we obtain that the asymptotically optimal energy-splitting ratio $\rho^*_{\text{asym}} = 0.5858$, which is approached by the numerically obtained optimal $\rho^*$ in Table I.

TABLE I: Optimal resource allocation versus $M$

<table>
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<th>$M$</th>
<th>$\tau^*$</th>
<th>$\alpha^*$</th>
<th>$\rho^*$</th>
<th>$\rho^*_{\text{asym}}$</th>
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<td>0.6051</td>
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<td>7.8056</td>
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<td>13.7247</td>
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</table>
Fig. 4 compares the achievable rates to that of the OT-MM benchmark and the ideal case, where the x-axis is in the logarithm scale. We see that the proposed WET-MM system achieves higher rates than the OT-MM system. We also observe that the proposed WET-MM system achieves more than 70% of the performance limit of minimum rate achieved by the ideal case with perfect CSI, even though there are only 50 antennas at the H-AP. The observations numerically shows the efficiency of the proposed system.

Recall that the MM-DoRG $\kappa$ is defined in (1) as the asymptotic slope of rate $R$ with respect to $\log M$. We then observe from Fig. 4 that the DoRG of the proposed WET-MM system achieves the same DoRG as the ideal case, i.e., $\kappa_{\text{WET-MM}} = \kappa_{\text{Ideal}} = 2$, which is double of the DoRG of the OT-MM benchmark. Hence, the proposed WET-MM system is optimal in terms of DoRG.

Also, we observe from Fig. 4 that to achieve a max-min rate at 4 bps/Hz for the distance $d = 4$ m, the OT-MM system requires about 800 antennas at the H-AP, while the proposed WET-MM requires only 25 antennas, being roughly a square root of that required by the OT-MM system.

Finally, Fig. 5 compares the user fairness. Using the derived energy allocation weights for DL WET in (36), users in the proposed WET-MM system asymptotically achieves the same rate. In contrast, for the OT-MM system, the far user has a much lower rate than the near user, even in the massive MIMO regime. Hence, better user fairness is achieved in the proposed system.

VIII. CONCLUSION

This paper studies a WET-enabled massive MIMO system with estimated CSI. The minimum rate among users is maximized, by jointly optimizing the allocation of time resource and energy resource. With ZF detection, the proposed WET-MM system achieves the maximally achievable MM-DoRG with perfect CSI, which is double of the MM-DoRG of the OT-MM system. Moreover, for a desired data rate, the WET-MM system supports the same AP-user distance as the OT-MM system with less antennas, roughly a square root of that required for the OT-MM system. Also, the WET-MM system achieves the best possible fairness among users, since all users asymptotically achieve a common rate. The WET-MM is a promising system to provide high data rate and overcome the energy bottleneck for wireless devices. We believe that there are other interesting issues to be addressed in future, such as the effect of antenna correlation, the effect of imperfect channel reciprocity resulting from calibration error, other fading channel model with line-of-sight, etc.
Comparison of Max−Min Rate

Comparison of Rate Fairness

Fig. 4: Comparison of achievable rates versus $M$.

Fig. 5: Comparison of rate fairness versus $M$. 

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Appendix A

Proof of Lemma 2

Substituting (7) into (5), the harvested energy in the DL WET phase at user \( k \) is rewritten as

\[
Q_k(L, \alpha) = \alpha \mathbb{E}_{\hat{G} \mid G} \left[ \mathbb{E}_{G \mid \hat{G}} \left( \frac{p_{dl} \xi_k \| g_k^H \hat{g}_k \|^2}{\| g_k \|^2} + \frac{p_{dl} \sqrt{\xi_k} (g_k^H \hat{g}_k)^H}{\| g_k \|^2} \sum_{i \neq k} \frac{\sqrt{\xi_i} g_i^H \hat{g}_i}{\| g_i \|^2} \right) + \frac{p_{dl} \sqrt{\xi_k} g_k^H \hat{g}_k}{\| g_k \|^2} \sum_{i \neq k} \frac{\sqrt{\xi_i} (g_i^H \hat{g}_i)^H}{\| g_i \|^2} + \sum_{i \neq k} \sum_{j \neq k} \frac{p_{dl} \sqrt{\xi_k} \xi_j (g_k^H \hat{g}_k)^H (g_j^H \hat{g}_j)^H}{\| g_i \|^2 \| g_j \|^2} \right) \right].
\]

(48)

In the sequel, we investigate the four terms in (48). Recall \( \hat{g}_k = g_k + e_k \). From the property of MMSE estimation, \( \hat{g}_k \) is independent of \( e_k \). Conditioned on \( \hat{g}_k \), \( g_k \) is distributed as \( \mathcal{CN}(\hat{g}_k, \sigma_{e,k}^2 I_M) \).

The conditional correlation matrix of \( g_k \) is thus

\[
\mathbb{E}_{g_k \mid \hat{g}_k} (g_k^H g_k) = \sigma_{e,k}^2 I_M + \hat{g}_k \hat{g}_k^H.
\]

(49)

Conditioned on the channel estimate \( \hat{G} \), the harvested energy in the first term is rewritten as

\[
\mathbb{E}_{G \mid \hat{G}} \left( \frac{p_{dl} \xi_k \| g_k^H \hat{g}_k \|^2}{\| g_k \|^2} \right) = \frac{p_{dl} \xi_k E_{\hat{G} \mid g_k} \left( g_k^H g_k \right) \| g_k \|^2}{\| g_k \|^2} = p_{dl} \xi_k \left( \sigma_{e,k}^2 + \hat{g}_k \hat{g}_k^H \right).
\]

(50)

From the fact that \( \hat{g}_k \sim \mathcal{CN}(0_M, (\beta_k - \sigma_{e,k}^2) I_M) \), we have that \( \mathbb{E}_{\hat{g}_k} \left( \hat{g}_k^H \hat{g}_k \right) = M (\beta_k - \sigma_{e,k}^2) \), where \( \sigma_{e,k}^2 \) is given by (5). Hence, the first term in (48) is obtained as

\[
\alpha \mathbb{E}_{\hat{G} \mid G} \left[ \mathbb{E}_{G \mid \hat{G}} \left( \frac{p_{dl} \xi_k \| g_k^H \hat{g}_k \|^2}{\| g_k \|^2} \right) \right] = \alpha p_{dl} \xi_k \beta_k \left[ M - \frac{(M-1)\sigma^2}{\beta_k L q_k + \sigma^2} \right].
\]

(51)

Define \( \tilde{g}_i \equiv \frac{\hat{g}_i}{\| g_i \|^2} \), and \( \tilde{g}_k \equiv \| g_k \|^2 \hat{g}_k \). From (49), the second term in (48) is rewritten as

\[
\alpha \mathbb{E}_{\hat{G} \mid G} \left[ \mathbb{E}_{G \mid \hat{G}} \left( \frac{p_{dl} \sqrt{\xi_k} (g_k^H \hat{g}_k)^H}{\| g_k \|^2} \sum_{i \neq k} \frac{\sqrt{\xi_i} g_i^H \hat{g}_i}{\| g_i \|^2} \right) \right] = \alpha \mathbb{E}_{\hat{G}} \left[ p_{dl} \sqrt{\xi_k} g_k^H \hat{g}_k \sum_{i \neq k} \frac{\sqrt{\xi_i} \| g_i \|^2 \hat{g}_i}{\| g_i \|^2 \| g_k \|^2} \right] = \alpha \left[ p_{dl} \sqrt{\xi_k} \sigma_{e,k}^2 \sum_{i \neq k} \sqrt{\xi_i} \| g_i \|^2 \| g_k \| \| \hat{g}_i \| \| g_i \|^2 \hat{g}_i \right] + \alpha \left[ p_{dl} \sqrt{\xi_k} \sum_{i \neq k} \sqrt{\xi_i} \| g_i \|^2 \| g_k \|^2 \| g_i \| \| g_k \| \| \hat{g}_i \| \| g_k \| \| g_i \|^2 \hat{g}_i \right] \mathrm{I} = 0.
\]

(52)

where (a) is from the fact that \( \hat{g}_k \) and \( g_k \) are independent zero-mean random vectors, for any \( i \neq k \).
The third term in (48), which is the conjugate of the second term in (48), is similarly obtained as

\[
\alpha \mathbb{E}_G \left[ \mathbb{E}_G | \hat{G} \left( \frac{p_{dl} \sqrt{\xi_i \hat{g}_{ik}^H \hat{g}_{ik} \sum_i \sqrt{\xi_i} (\hat{g}_{ik}^H \hat{g}_{ik})^H}{\| \hat{g}_i \|^2} \right) \right] = 0. \tag{53}
\]

The fourth term in (48) is rewritten as

\[
\begin{align*}
\alpha \mathbb{E}_G \left[ \mathbb{E}_G | \hat{G} \left( \sum_{i \neq k} \sum_{j \neq k} \frac{p_{dl} \sqrt{\xi_i \xi_j} (\hat{g}_{ik}^H \hat{g}_{ik})^H}{\| \hat{g}_i \|^2} \quad \| \hat{g}_j \|^2 \right) \right] \\
= (a) \alpha p_{dl} \mathbb{E}_G \left[ \sum_{i \neq k} \xi_i \hat{g}_{i}^H (\sigma_{e,k}^2 I_M + \hat{g}_{ik}^H \hat{g}_{ik}) \hat{g}_{i} \right] + \sum_{i \neq k} \sum_{j \neq i, k} \sqrt{\xi_i \xi_j} \hat{g}_{i}^H (\sigma_{e,k}^2 I_M + \hat{g}_{ik}^H \hat{g}_{ik}) \hat{g}_{j} \\
= (b) \alpha p_{dl} \beta_k \sum_{i \neq k} \xi_i, \tag{54}
\end{align*}
\]

where (a) is from (49), and (b) is from the fact that \( \mathbb{E}_G (\hat{g}_{ik}^H \hat{g}_{ik}) = (\beta_k - \sigma_{e,k}^2) I_M \), as well as the fact that \( \hat{g}_i \) and \( \hat{g}_j \) are independent zero-mean random vectors, for any \( i \neq j \).

Substituting (51), (52), (53) and (54) into (48), we obtain the harvested energy as in (17).

APPENDIX B

PROOF OF LEMMA 3

With ZF, we have \( A = \hat{G} \left( \hat{G}^H \hat{G} \right)^{-1} \). The achievable rate of user \( k \) can thus be rewritten as

\[
\tilde{R}_k^{ZF} = (1 - \tau - \alpha) \log \left( 1 + \left( \mathbb{E} \left( \left( \sigma^2 + \sum_{i=1}^K p_i \sigma_{e,i}^2 \right) \left( \frac{\left( \hat{G}^H \hat{G} \right)^{-1}}{p_k} \right)_{kk} \right) \right)^{-1} \right). \tag{55}
\]

Note that \( \hat{G}^H \hat{G} \) is an \( K \times K \) central complex Wishart matrix with \( M(M > K) \) degrees of freedom. From [28], we have that for \( M \geq K + 1 \)

\[
\mathbb{E} \left( \left( \hat{G}^H \hat{G} \right)^{-1} \right)_{kk} = \frac{1}{(M - K)(\beta_k - \sigma_{e,k}^2)}. \tag{56}
\]

Substituting (56) into (55), we obtain that

\[
\tilde{R}_k^{ZF} = (1 - \tau - \alpha) \log \left( 1 + \frac{p_k (\tau, \alpha, \rho) (M - K)(\beta_k - \sigma_{e,k}^2) (\alpha, \rho)}{\sigma^2 + \sum_{i=1}^K p_i (\tau, \alpha, \rho) \sigma_{e,i}^2 (\alpha, \rho)} \right). \tag{57}
\]

The result in (20) is obtained, by substituting the error variance in (19) into (57).
APPENDIX C

PROOF OF LEMMA 4

With MRC, $a_k = \hat{g}_k$. Define $\tilde{g}_i = \frac{\hat{g}_i^H \hat{g}_i}{\|\hat{g}_i\|}$. The achievable rate of user $k$ is written as

$$ \tilde{R}_k^{MRC} = (1 - \tau - \alpha) \log \left( 1 + \left( \mathbb{E} \left\{ \frac{\sum_{i=1,i \neq k}^K p_i|\tilde{g}_i|^2 + \sum_{i=1}^K p_i \sigma_{e,i}^2 + \sigma^2}{p_k \|\hat{g}_k\|^2} \right\} \right)^{-1} \right). $$ (58)

It can be easily shown that $\tilde{g}_i \sim \mathcal{CN}(0, \beta_i - \sigma_{e,i}^2)$. It is further noted that the conditional probability density function (pdf) $f(\tilde{g}_i|\hat{g}_k) = f(\tilde{g}_i)$, where $f(\hat{g}_i)$ is the marginal pdf of $\tilde{g}_i$. Then we have

$$ \mathbb{E} \left\{ \frac{\sum_{i=1,i \neq k}^K p_i|\tilde{g}_i|^2 + \sum_{i=1}^K p_i \sigma_{e,i}^2 + \sigma^2}{p_k \|\hat{g}_k\|^2} \right\} = \left( \sum_{i=1,i \neq k}^K p_i \beta_i + p_k \sigma_{e,k}^2 + \sigma^2 \right) \mathbb{E} \left\{ \frac{1}{p_k \|\hat{g}_k\|^2} \right\}. $$ (59)

It can be shown that the random variable $Z_k = \frac{2}{\beta_k - \sigma_{e,k}^2} \hat{g}_k^H \hat{g}_k$ follows central chi-square distribution with $2M$ degrees of freedom. Then, we have $\mathbb{E} \left\{ \frac{1}{Z_k} \right\} = \frac{1}{2(M-1)}$. Hence, it holds that

$$ \mathbb{E} \left\{ \frac{1}{p_k \|\hat{g}_k\|^2} \right\} = \frac{2}{p_k (\beta_k - \sigma_{e,k}^2)} \mathbb{E} \left\{ \frac{1}{Z_k} \right\} = \frac{1}{p_k (M-1)(\beta_k - \sigma_{e,k}^2)}. $$ (60)

Substituting (59) and (60) into (58), we obtain that

$$ \tilde{R}_k^{MRC} = (1 - \tau - \alpha) \log \left( 1 + \frac{p_k(\tau, \alpha, \rho)(M-1)(\beta_k - \sigma_{e,k}^2(\alpha, \rho))}{\sum_{i=1,i \neq k}^K p_i(\tau, \alpha, \rho) \beta_i + p_k(\tau, \alpha, \rho) \sigma_{e,k}^2(\alpha, \rho) + \sigma^2} \right). $$ (61)

The result in (21) is obtained, by substituting the error variance in (19) into (61).

APPENDIX D

PROOF FOR LEMMA 5

We rewrite the asymptotic achievable rate in (28) for ZF detection as follows,

$$ \tilde{R}_k^{ZF} \longrightarrow (1 - \tau - \alpha) \log \left( 1 + C_k(\tau, \alpha, \rho) \xi_k \right), $$ (62)

where $C_k(\tau, \alpha, \rho)$ is the multiplicative coefficient of $\xi_k$ in the logarithm of (28).
We consider the case in which the first \((K - 1)\) users have the same rate \(\tilde{R}_{ZF}\) and the \(K\)-th user achieves a higher rate, i.e., \(\tilde{R}_{ZF}^1 = \tilde{R}_{ZF}^2 = \cdots = \tilde{R}_{ZF}^{K-1} = \tilde{R}_{ZF} < \tilde{R}_{ZF}^K\). Clearly, one can always increase the minimum rate \(\tilde{R}_{ZF}\) by increasing \(\xi_1, \xi_2, \cdots, \xi_{K-1}\) and decreasing \(\xi_K\), subject to the constraint \(\sum_{i=1}^{K} \xi_i = 1\). Then the minimum rate among users is maximized when \(\tilde{R}_{ZF} = \tilde{R}_{ZF}^K\).

The same argument can be extended to other cases where less than \((K - 1)\) users have the same UL rate. That is, the minimum rate is maximized when all users achieves the same rate; otherwise, one can always increase the minimum rate by adjusting \(\xi_k\)’s. Let \(\tilde{R}_{ZF} = (1 - \tau - \alpha) \log(1 + \gamma)\), where the common SINR \(\gamma = C_k(\tau, \alpha, \rho)\xi_k\). From the constraint \(\sum_{i=1}^{K} \xi_i = 1\), we obtain that

\[
\xi_k^* = \frac{\gamma}{C_k(\tau, \alpha, \rho)} = \frac{1}{C_k(\tau, \alpha, \rho) \sum_{i=1}^{K} \frac{1}{C_i(\tau, \alpha, \rho)}} = \frac{1}{\beta_k^2 \sum_{i=1}^{K} \frac{1}{\beta_i^2}}.
\]

Hence, the asymptotically optimal \(\xi_k^*\) only depends on the long-term path loss \(\beta_i\)’s of all users.

For MRC detection, by using the Karush-Kuhn-Tucker conditions [27], it can be shown that the minimum rate is maximized when all users achieves the same rate. The energy allocation weight \(\xi_k^*\) is thus given by (36).

**References**


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