# Subcarrier Allocation with Minimum Data Rate Constraint in OFDMA Wireless Channels

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## Abstract

*In this paper, we present a novel and efficient Subcarrire Allocation algorithm with Minimum Data RAte (SAMDRA) constraint for multiuers orthogonal frequency division multiple access (OFDMA) wireless networks. The proposed algorithm attempts to exploit both the time diversity and frequency diversity in the wireless channel. A concept of service expectation is used to indicate how users hope to be served and have their quality of service (QoS) guaranteed. The subcarrier allocation problem is decomposed into two stages. The first stage is to decide which user to be served based on the service expectation and channel quality of users. In the second stage, a subcarrier is chosen for the current user in order to maximize the average data rate. Numerical results demonstrate a significant improvement in the average date rate under the constraint of minimum data rate guarantee for users.*

## 1. Introduction

Recently, broadband wireless access networks (BWA), such as IEEE 802.16(d, e) and IEEE 802.11(a, b, g) have been rapidly developed and are being put into operation. Unlike wireline channels, broadband wireless channels in BWA networks are prone to frequency selective fading. The orthogonal frequency division multiple access (OFDMA) scheme is considered as a basic technique for BWA because of its high immunity to inter-symbol interference and frequency selective fading. As a method of sharing a wireless channel, OFDMA also bring new challenges to the resource allocation problem which takes both the QoS requirement and channel efficiency or average data rate into account.

OFDMA is based on the orthogonal frequency division multiplexing (OFDM) technique where the total available bandwidth is divided into several narrow subcarriers. This allows users to spread their information on the subcarriers under the control of the subcarrier allocation algorithm so as to improve the total average data rate and at the same time guarantee QoS requirements. OFDMA matches well to the multiuser and frequency selective fading scenario where different subcarriers have different channel qualities for different users. The main goal of subcarrier allocation in OFDMA wireless channels is, on one hand, to improve the average data rate by exploiting both the time diversity in the time domain and frequency diversity in the frequency domain of the wireless channel. On the other hand, the subcarrier allocation algorithm should try to guarantee the prescribed QoS requirements of users.

The problem of subcarier allocation in OFDMA systems has been recently studied quite extensively. In reference [2], subcarriers are allocated to users based on the current channel condition and current buffer state as well as the measured ratio of arrival rate to throughput for each user. In reference [3], a delay utility function based subcarrier allocation scheme is presented for real time services, where the main goal of the proposed scheme is to maximize the aggregate utility. However, these algorithms do not take any QoS requirements into account, thus it has no guarantee of the minimum service quality of users. In reference [4], Wong etc present a new method to solve the subcarrier allocation problem in OFDMA system by proportionally distributing the subcarriers among users based on their QoS requirements. Wong's algorithm is more conservative in the sense of channel efficiency improvement although it can guarantee a proportional fairness among users. Other works on this topic are [5]~[7], which try to combine the subcarrier allocation with the power allocation problem.

In this paper, we focus on subcarrier allocation in wireless OFDMA channel. A novel and efficient subcarrire allocation algorithm (SAMDRA) is proposed. Numerical results show that the algorithm in this paper can maximize the overall average data rage under the constraint of minimum data rates guarantee of users by exploiting both the time diversity and frequency diversity in the wireless channel.

The rest of the paper is organized as follows. In section II, the OFDMA wireless system model is introduced.

In section III, we formulate the problem of subcarrier allocation with minimum data rate constraint and propose our solution. Some implementation related issues are also discussed in this section. Simulation results are given in section IV. Section V is the conclusions.

## 2. System Model

In this paper, we consider the downlink OFDMA cell system, where there is only one base station (BS) and multiple users. The wireless channel between the base station and users consists of K subcarriers, which are shared by M users under the control of the subcarrier allocation algorithm in BS. The users measure the current channel qualities based on the received signals and then send this information back to BS in a predefined feedback channel. The scheduler in BS then allocates the subcarriers to users based on the channel quality information and QoS requirements of users. Based on the result of subcarrier allocation, bits of users data are then modulated into the subsymbols and passed to the subcarriers allocated to them. All the subsymbols belonging to different users are then combined into a single symbol in the IFFT module.

Each user has its own queue to buffer the randomly incoming packets and the size of the queue is assumed to be infinite so that no packet will be lost because of queue overflow. We assume that there are enough packets to be scheduled for all the admitted users so there is no need for the scheduler to check the queue state in case of bandwidth waste for empty queues. One subcarrier can only be allocated to a single user during a single time slot, while one user can occupy multiple subcarriers based on the result of subcarrier allocation. Let $k \in (1, 2, 3, \; ..... \; , K)$ denote the subcarrier index, and $m \in (1, 2, 3, ....., M)$ denote the user index and $n$ denote distant time index. The duration of each time slot in OFDM is $T_s$ . We use $(A_1[n], A_2[n], ..., A_K[n])$ to denote the result of subcarrier allocation at time $T_s * n$ , where $A_k[n]$ indicates that subcarrier $k$ is allocated to user $A_k[n]$ during time $T_s * n$ and $A_k[n] \in (1, 2, 3, ....., M)$ . Therefore, the problem of subcarrier allocation in OFDMA is essentially to choose a proper user for all the subcarriers belonging to a time slot.

## 3. SAMDRA Subcarrier Allocation Scheme

To formulate the subcarrier allocation problem in a time varying OFDMA wireless channel we make some simplifying assumptions as follows:

1) The system operates in a time slot base which means that the scheduler does the subcarrier allocation time slot by time slot. In this case, the scheduler only decides how to distribute the subcarriers in the current time slot among admitted users. This assumption is a little different from what happens in real systems. For example in IEEE 802.16 the system works on the frame basis. One frame consists of multiple time slots based on the duration of the frame and that of single time slot. In this case, the scheduler has to decide how to allocate all the subcarriers in current frame to different users, where the problem becomes more complex.
2) The channel quality of a subcarrier is stable during a single time slot and channel qualities of different subcarriers for the same user are iid processes and are time-varying.
3) The scheduler in the base station knows exactly the channel quality information of the subcarriers at the beginning of each time slot. In real word, channel quality information is collected by the signal receiver and is periodically sent to the base station in a predefined channel.
4) Power is equally distributed over all subcarriers so that the maximum feasible data transmission rate is only decided by the current channel quality in SNR and required BER of users.

### 3.1. Problem Formulation

We use similar notations and terminologies as that in reference [2] and [3] to formulate the problem. The total bandwidth of the wireless channel is B Hz and if we use $\triangle f$ Hz to stand for the bandwidth of each subcarrier, we have $\triangle f$ = B/K. We use $C_{i,k}[n]$ to denote the data transmission rate of user i on subcarrier k during the n[th] time slot. $C_{i,k}[n]$ is expressed as follows:

$$C_{i,\,k}[n] = \Delta f * \log_2(1 + \beta * \rho_{i,k}[n]) \qquad (1)$$

where $\rho_{i,k}[n]$ is the signal to noise rate (SNR) of user i on subcarrier k at time $T_s * n$ and $\beta = -1.5 / \ln(5 * BER)$ [2]. We use $S_i[n]$ to denote the set of subcarriers indices assigned to user i during time slot n. We have

$$S_i(n) = \{k : A_k[n] = i\} \qquad (2)$$

and

$$S_i[n] \cap S_j[n] = \varnothing, \;\; \forall \mathrm{i} \neq \forall \mathrm{j}, \qquad (3)$$

(2) and (3) indicate that each subcarrier can only be assigned to one user during a single time slot while one user can occupy multiple subcarriers during a single time slot. The allocated data rate of user i during time slot n with the subcarrier assignment $S_i[n]$ is denoted by $r_i[n]$, which is expressed as follows:

$$r_i[n] = \sum_{k \in S_i[n]} C_{i,k}[n] \qquad (4)$$

According to the definition of QoS requirements for best effort users in IEEE 802.16, we apply $R_i^{\min}$ in bits per second as the prescribed minimum data rate, which means that user i should receive at least $R_i^{\min}$ bps in order to maintain an acceptable level of service quality. So if we use $E(r_i[n])$ as the average service received by user i at time $T_s * n$, then the scheduler must try to maintain the following condition:

$$E(r_i[n]) \geq R_i^{\min} \quad (5)$$

The main idea of the subcarrier allocation is to improve the average data rate as much as possible while proportionally distributing the subcariers among users based on their OoS requirements in terms of minimum data rate $R_i^{\min}$. The resulting problem can be formulated as an optimization problem under constraints as follows:

$$\max E\left( \sum_{k=1}^{K} C_{A_k[n],k}[n] \right) \quad (6)$$

subject to

$$\text{CS1: } S_i[n] \cap S_j[n] = \varnothing, \ \forall \text{i} \neq \forall \text{j} \quad (7)$$

$$\text{CS2: } S_1 \cup S_2 \cup .....S_M = \textit{set of subcarriers} \quad (8)$$

$$\text{CS3: } E(r_i[n]) \geq R_i^{\min} \quad \text{i = 1, 2, ….., M} \quad (9)$$

where $S_i(n) = \{k : A_k[n] = i\}$. CS1 comes from the OFDMA restriction that each subcarrier in a single time slot can only be allocated to one user. CS2 indicates that all the subcarriers are allocated to users during each time slot. CS3 comes from the long-term minimum data rate requirements as defined in IEEE 802.16(d, e). In other words, the objective of this problem is to maximize the average data rate during each subcarrier allocation cycle subject to the physical constraints (CS1, CS2) and to the constraint that the QoS requirements (CS3) of users in the media access control (MAC) layer are satisfied.

Obviously, we can solve the optimization problem above by listing all the combinations of users and subcarriers. Because there are M users and K subcarriers, the overall number of possible subcarrier assignments is $M^K$. This approach would be unrealistic because the complexity increase exponentially with the user number and normally the number of subcarriers is also very large. Next we propose a suboptimal solution which dramatically decreases the complexity of the algorithm above while still providing a near optimal average data rate and minimum data rate guarantee for users.

### 3.2. Proposed Suboptimal Solution

We try to apply the idea in reference [8] designed for CDMA-HDR system to the multicarrier OFDMA system. In [8], the scheduler serves the user with the highest ratio of $DRC_i(t)/R_i(t)$ at each decision time. $DRC_i(t)$ is the current data rate of user i, which is an indication of current channel quality. Higher $DRC_i(t)$ indicates better channel quality and vice versa. $R_i(t)$ is the average data rate received by user i by time t. In other words, the algorithm prefers to serve users with good channel quality and relatively bad service quality. The algorithm can improve the channel efficiency and distribute the time slots among users proportionally by exploiting the time diversity of the dynamic wireless channel.

In this paper, we apply the above idea into a multiple subcarriers OFDMA system where the scheduler tries to exploit not only the time varying diversity but also frequency (multiple subcarriers) diversity in the wireless OFDMA channel. Instead of $R_i(t)$, we apply a *service expectation* to indicate the degree users are expecting to get served to achieve their QoS guarantees. Our proposed solution includes two stages during each scheduling decision process. Stage one is to select a best user to serve based on the service expectation and the current channel qualities of users. The scheduler prefers to serve users with relatively better channel quality and high service expectation. In the second stage, the scheduler chooses for the user the best subcarrier from the available subcarriers so as to improve the average data rate for the overall channel efficiency. One allocation cycle ends when all the subcarreirs in the current time slot are allocated to users.

We define $u_i[n]$ as the *service expectation* that user i has during time slot n to meet the user's QoS guarantee. We use $R_i[n]$ as the *current received data rate* which is the serve rate at the end of the current time slot n. $u_i[n]$ is a function of $R_i[n]$ and $R_i^{\min}$.

$$u_i[n] = f(R_i[n] - R_i^{\min}) \quad (10)$$

$f(R_i[n] - R_i^{\min})$ is a concave decreasing function of $R_i[n] - R_i^{\min}$ which indicates that users with high $R_i[n]$ have low *service expectations* and are less likely to be served since their service qualities are in relatively good states. According to the definition of constraint (9), we believe that a user is well served when their received data rate is larger than $R_i^{\min}$, so there is no need to care for QoS of those users whose current data rates are larger than $R_i^{\min}$. Taking the above concerns into account, we define $u_i[n]$ as follows:

$$u_i[n] = \begin{cases} 1; & \text{if } \mathrm{R_i[n]} \geq \mathrm{R_i^{min}} \\ \lambda_i * \exp\left(-\frac{R_i[n] - R_i^{\min}}{R_i^{\min}}\right); & \textit{otherwise} \end{cases} \quad (11)$$

From the definition of $u_i[n]$ in (11) we can see that the scheduler divides all the users into two kinds: users with $R_i[n] \geq R_i^{\min}$ and users with $R_i[n] < R_i^{\min}$. Users whose current received data rate $R_i[n]$ is less than $R_i^{\min}$ have higher service expectations. On the other hand, users' service expectations are fixed to 1 when their current received data rates are larger than $R_i^{\min}$.

Based on the above discussion, our proposed SAMDRA subcarrier allocation algorithm works as follows during each scheduling cycle:

Step 1. Predict the *current data rates* of users based on the past subcarrier allocation results and then calculate users' *service expectations* by

$$u_i[n] = f(R_i[n] - R_i^{\min}).$$

Step 2. Select a user i from all the users as the *current user* to serve. The *current user* is the one who is the most urgent to be served based on their current service expectation and current channel quality.

Step 3. Select an *optimal subcarrier* k from the available subcarrier set for the *current user* i, so that the channel efficiency is locally optimal.

Step 4. Allocate the *optimal subcarrier* k to the *current user* i.

Step 5. Update the current user's *current data rates* and users' *service expectations*.

Step 6. Repeat Step 2 to Step 4 until all the subcarriers are allocated to users.

In step 1, the *current data rate* is predicted based on the past data rate $R_i[n-1]$ and the subcarrier allocation in the current time slot. We use $\hat{R}_i[n]$ as the predicted *current data rate* and we have

$$\hat{R}_i[n] = (1-\rho) * R_i[n-1] + \rho * \Delta r_i[n], \qquad (12)$$

where $R_i[n-1]$ is the data rate of user i at the beginning of time slot $T_s * n$, $\Delta r_i[n]$ is the data rate increase during time $T_s * n$, $\hat{R}_i[n]$ is the service performance at the end of time slot $T_s * n$, and $\rho$ is a small system parameter which will be analyzed in the simulation section. Because at the beginning of the allocation, no subcarrier is allocated to any users, i.e., $\Delta r_i[n] = 0$, we have:

$$\hat{R}_i[n] = (1-\rho) * R_i[n-1] \qquad (13)$$

The most important mechanism in this algorithm is to decide which user to serve at each scheduling decision point or the serving sequence of users from the start of allocation to the end of allocation. In Step 2, we take both QoS and channel quality into account when choosing a user to serve. A user with the better *current channel quality* and high *service expectation* is selected as the *current user* at each subcarrier allocation decision time. $\hat{R}_i[n]$ is updated every time a subcarrier is allocated to a user. User's current channel quality is that of the best subcarrier channel quality among the unallocated subcarriers. The idea for choosing the best unallocated subcarrier channel as users' current channel is as follows. We only allocate one subcarrier to the current user at each allocation decision time, so there is no need to be concerned with other subcarrirers with bad channel qualities. Based on the above, user i is served according to:

$$i = \arg\max_i (u_i[n] * \max_{k \in D_l} C_{i,k}[n]) \qquad (14)$$

In step 3, the unallocated subcarrier with the best channel quality is allocated to the current user. Obviously this can improve the average data rate. We select subcarrier k for the current user at each allocation decision time where

$$k = \arg\max_{k \in D_l} C_{i,k}[n] \qquad (15)$$

The *current data rate* of the *current user* i $\hat{R}_i[n]$ is updated after the subcarreir k with data rate $C_{i,k}[n]$ is allocated to the current user i as follows:

$$\hat{R}_i[n] = \hat{R}_i[n] + \rho * C_{i,k}[n] \qquad (16)$$

Next, we present and analyze some properties of the SAMDRA subcarrier allocation by simulations.

**Pseudocode for SAMDRA Algorithm**

1. Initialize $D_l$ = all the subcarriers set.
2. Estimate users' current data rates by
   $\hat{R}_i[n] = (1-\rho) * R_i[n-1]$.
3. Calculate users' service expectation $\bar{u}_i[n]$ by
   $u_i[n] = f(\hat{R}_i[n], R_i^{\min})$.
4. Do while ( $D_l \neq \varnothing$ ) {
   4.1. Choose the *current user* i by
   $i = \arg\max_i (u_i[n] * \max_{k \in D_l} C_{i,k}[n])$.
   4.2. Select subcarrier k from the available subcarriers k by
   $k = \arg\max_{k \in D_l} C_{i,k}[n]$.
   4.3. Allocate subcarrier k to user i, i.e.,
   $A_k[n] = i$.
   4.4. Delete subcarrier k from $D_l$.
   4.5. Update $\hat{R}_i[n]$ by $\hat{R}_i[n] = \hat{R}_i[n] + \rho * C_{i,k}[n]$.
   4.6. Update $u_i[n]$ by $u_i[n] = f(\hat{R}_i[n] - R_i^{\min})$.
   }

## 4. Numerical Results

We run the simulation based on the implementation of IEEE 802.16d, where there is one base station and the number of users is increased based on the requirement of simulations. We assume there are 32 subcarriers (subchannels) in the wireless channel between the base station and users. A nine-sate Markov chain is used to emulate the frequency selective multipath channel according to reference [9]. We try to emulate different channel qualities by modifying the transmission matrix.

### 4.1. Channel Efficiency Improvement

Figure 1 shows the overall channel efficiency improvement when the number of users increases. We compare the channel efficiency improvement of SAMDRA against the Best Channel First (BCF) scheme and Wong's algorithm in reference [4]. BCF chooses the best channel quality user on each subcarrier, so this is the optimal solution in the sense of maximizing channel efficiency. From figure 1, we can see that the overall channel efficiency of SAMDRA increases as a function of the number of users and the channel efficiency of SAMDRA is almost equal to that of BCF especially when the number of admitted users is less than 30. This comes from the fact that frequency diversity increases when more and more users enter the network and the subcarrier allocation algorithm can choose relatively the best channel quality user for each subcarrier during each time slot. On the contrary, Wong's algorithm has a lower channel efficiency compared with that of SAMDRA and BCF for Wong's algorithm restricts the maximum number of sucarriers that can be allocated to a user during each allocation cycle in order to guarantee the weighted proportional fairness among users. This idea is more conservative in the sense of channel efficiency improvement.

When the number of users is larger than 30, the channel efficiency of SAMDRA is decreased as the number of users increases. This is because the system now is almost overloaded and the subcarrier allocation algorithm must take the QoS requirements in terms of data rate into account. Figure 1 also shows the effect of system parameter $\rho$, which decides the tradeoff between data rate guarantee and channel efficiency. As indicated in figure 1, higher $\rho$ results in higher channel efficiency and lower $\rho$ leads to lower channel efficiency.

### 4.2. Minimum Data Rate Guarantee

Figure 2 shows the data rate of users as a function of system load. We increase the system load by increasing the number of users. In this scenario, we allocate user 1 user 2 and user 3 with minimum data rate of 30kbps (480 packet per second) 20kbps (320 packet per second) and 10kbps (160 packet per second) respectively. The other users have the same minimum data rate of 30kbps and all the users have the same channel quality defined by the channel state transmission matrix earlier in subsection A. As indicated in figure 2, when the number of users increases, the date rates of user 1 user 2 and 3 decrease to the minimum data rate prescribed.

In figure 3, we assign users 1~3 with different channel qualities and continue the process above. We find that for a user with bad channel its data rate will be reduced to its minimum data rate constraint than for users with good channel qualities.

Figure 4 shows the users' data rates as a function of time t. At this time, the system is near to overloaded. We set user 1 with bad channel quality and user 2 and user 3 have relatively better channel qualities. The three users have the same minimum data rate requirements of 20kbps. We can see that user 1 now can only get the minimum data rate of 20kbps, while user 2 and user 3 are better served than user 1. This indicates that the subcarrier allocation algorithm is more likely to allocate the subcarriers to users with better channel qualities in order to improve the overall channel efficiency.

## 5. Conclusions

In this paper, we have considered the subcarrier allocation problem in OFDMA wireless channels and we proposed a new subcarrier allocation algorithm, SAMDRA. SAMDRA exploits the time and frequency domain diversity in OFDMA wireless channel. It's shown through simulations that the proposed SAMDRA scheme can perform better than other previous reported schemes in terms of achieving overall channel efficiency while meeting the minimum data rate constraint.

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# 7. Appendix

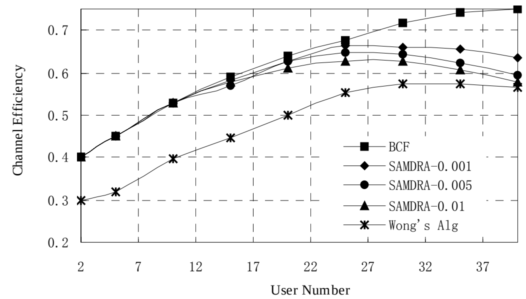


Figure 1. Channel Efficiency vs System Load

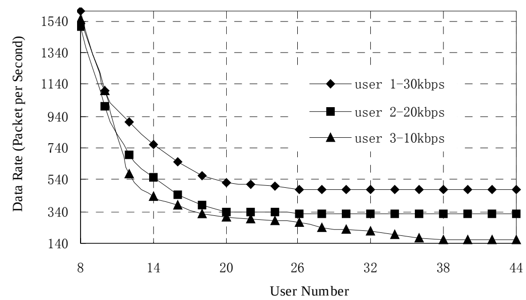


Figure 2. Different Level Data Rate Guarantee

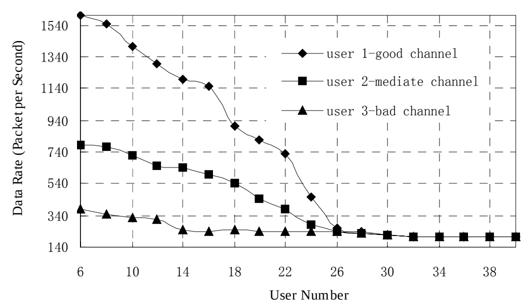


Figure 3. Data Rate Guarantee for Different Channel Quality users

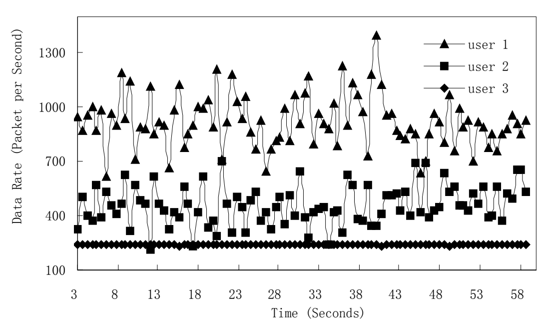


Figure 4. Data Rate for Different Channel Quality users As Function of Time