A Dual-Stage Control Design for High Track Per Inch Hard Disk Drives
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Abstract—This paper proposes an optimization based control scheme for a dual-stage hard disk drive servo system to maximize the tracking accuracy given the actuator bandwidth limitations without knowing the mechanical disturbance model. We proposed to add an outer loop, selected based on optimization of the Youla parameters, to the usual dual-stage controller for tracking accuracy improvement. The Youla parameters are bounded based on specifications of minimum bandwidth, phase margin, and gain margin, and maximum bandwidth limitations. Time domain and frequency domain simulations were performed to show the effectiveness of the design. The tracking accuracy with respect to various noise and disturbance levels was considered. The results suggested that for the noise and disturbance model considered, the mechanical disturbance must be reduced considerably to achieve higher TPI.

Index Terms—Dual-stage servo, hard disk drive, optimal control, servo system.

I. INTRODUCTION

As the recording density advances rapidly, the requirement for hard disk drive (HDD) servo systems becomes more demanding because improving the track density faster than improving the linear density is advantageous. To be competitive in the market, the high track per inch (TPI) access systems that have minimal cost hike, if not maximum reduction, are desirable. As usual, single stage voice coil motor (VCM) based servo loop bandwidth is increased whenever possible [2] whereas the more recent trend is to use dual-actuator systems [7]. The driving force is to increase the servo bandwidth for better disturbance rejection. At the same time, alternative substrates [6], improved spindle motor designs [4], and air flow designs [12] have reduced nonrepeatable runout signals, thereby facilitated the improvement of the track density even without changing the servo. Parallel to the servo and mechanical improvement, advances in signal processing techniques, head, media technology, chip on suspension have enabled faster and more accurate PES sampling, thus allowed and accelerated the use of higher servo bandwidth.

Due to the limitation of the actuator pivot nonlinearity, high frequency uncertainty, the effects of various external disturbances and noises, improved servo control designs via multirate control, higher order control, lower order control, and real time optimization techniques [22], [8], [3], [14] have been studied extensively over the years as a cost effective way of moving toward higher track density. Traditional way of control design is to follow design specifications such as a certain open loop 0 dB cross over frequency, error rejection transfer function peak, etc. To increase the servo loop bandwidth while avoiding the amplification of the unwanted noise and disturbances, detailed plant models, including noise and disturbance models, have been studied to facilitate the servo control and mechanical system (or access system) design [5], [13], [1]. Modern control theories allow the optimization of certain kinds of norms such as $H_2$ or $H_{\infty}$ norms, or even mixed $H_2/H_{\infty}$ norm with or without using the disturbance models. Either way, the designer has to justify how to translate their specifications to performance and robustness requirements. The conflict of robustness versus performance, the availability of and assumptions on plant, noise and disturbance models are always the major issues that limit the application of the control design methods.

To study the access system requirements for future TPI, this paper proposed a new control scheme for a dual-stage servo system to maximize the TPI given the actuator bandwidth limitations. The design is based on a normal dual-stage control system. We added an outer loop, selected based on optimization of the Youla parameters, to the secondary actuator control for tracking accuracy improvement. Time domain and frequency domain simulations were performed to show the effectiveness of the design. The projected TPI given various access systems limitations is also presented.

II. A DUAL-STAGE SERVO CONTROL DESIGN

The objective of the servo system during track following mode is to ensure the accurate positioning of the read/write head. To predict the TPI achievable for an HDD with different disturbance model, we considered a typical dual-stage servo system as shown in Fig. 1. This is a de-coupled two-loop design [19] which is a sort of two-level tracking. Fig. 2 shows the 6th order milli-actuator model we used for the control
One of the zeros is unstable. Thus the plant is nonminimum phase.

First we designed a notch which inverted the milli-actuator plant and maps the unstable pole to the left half plane when designing the controller [21], [15]. The unstable pole has a frequency of over 10 kHz, hence this mapping only changes the phase around that frequency, and has no effect on the magnitude at all. Since the milli-actuator modes appear in the VCM loop as well due to the coupling, it is reasonable for both loops to share a common notch filter. The sensitivity function of the scheme in Fig. 1 is decoupled, which is

\[
S = \frac{1}{(1 + NK_v G_w)(1 + NK_\mu G_\mu)}
\]

where \( G_v, G_\mu, K_v, K_\mu \) are the voice coil motor, the milli-actuator, and their respective controllers, \( N \) is the notch filter shared by the VCM and milli-actuator loops. By using such a scheme, from viewpoint of controller optimization, the VCM controller and the milli-actuator could be optimized separately.

When selecting the controller parameters, although some state space based control design can minimize the standard deviation of the position error signal (PES), or \( \sigma_{pes} \), directly without selecting the weighting functions, these methods require the knowledge of accurate noise and disturbance models [16]. In general, higher order models can represent the disturbance better but the resultant control scheme will be of higher order; The numerical stability is very often poor when solving higher order Riccati equations to design such controllers. Using lower order disturbance models makes it easier to find an optimal solution; however, the model reduction process may defeat the purpose of doing so. In addition, the discretization of either the controller or the plant model given the limited PES sampling frequency is an approximation process which degrades the optimality of the controllers obtained [23].

The block diagram of the proposed dual-stage control system is shown in Fig. 3. In the figure, the dashed lines represent that the signal is digital, while the continuous lines represent analog signals. As shown in Fig. 3, the actuators to be controlled were analog while the controllers were digital ones with limited sampling frequency. The analog plant with sampled PES for feedback was modeled as

\[
\dot{x}(t) = \hat{A}x(t) + \hat{B}_1 u_1(t) + \hat{B}_2 u(t),
\]

\[
y(t) = Cx(t) + u_2(t),
\]

\[
y_m(k) = y(k) + v(k)
\]

where \( \hat{A} \) means perturbed variables, \( A \in \mathbb{R}^{n \times n}, B_1, C^T \in \mathbb{R}^{n \times r} \) and \( B_2 \in \mathbb{R}^{n \times 2} \) represent the nominal model. \( x(t) \in \mathbb{R}^n \) is state, \( y(t), y_m(k) \) are controlled and measured outputs, \( u(t) \in \mathbb{R}^1 \) is the control signal. \( u_1(t), u_2(t), v(k) \in \mathbb{R}^1 \) are process and measurement noise respectively.

Ideally, both the VCM controller \( K_v \) and secondary-stage controller \( K_\mu \) need to be tuned to achieve good performance. Previous research has shown that the overall performance of the dual-stage servo is robust against the VCM. Therefore we assumed that the VCM controller does not need to be modified once it was optimized; The performance improvement of the dual-stage servo system by tuning the VCM controller is negligible. Thus a higher track density is achieved just by milli-actuator controller optimization, after successful two-loop decoupling in Fig. 1.

In our previous work [15], the controller optimization was performed by tuning the proportional integral, or PI, controller for the milli-actuator once the notch filter was designed. In this paper, the parameters for \( K_v, K_\mu \) can be obtained using existing methods such as PQ method [17], LQG/LTR design, or even a simple lead-lag compensator. Some merits of Youla parameter were exploited and the coefficients of finite impulse response (FIR) Youla parameter were tuned to achieve a better performance. For ease of analysis, we took out the integrator in \( K_\mu \) in Fig. 3. Since \( K_v \) and \( K_\mu \) are normally designed based on models and design specifications and the model may deviate from the real situation when implemented, in the next section we will discuss how to pick the Youla parameter \( Q(z) \) that will further improve the servo performance.
III. CONTROL OPTIMIZATION

From [20], we know that the set of all internally stabilizing discrete time controllers can be parameterized in terms of a free parameter \( Q \in RH_{\infty} \) as:

\[
K'_\mu = F(t(K_\mu, Q), \quad (3)
\]

where

\[
K_\mu : \begin{bmatrix}
\Phi_{\mu} - \Gamma_{\mu}F - LC_{\mu} & L & -\Gamma_{\mu} \\
-F & 0 & -I \\
-C_{\mu} & I & 0
\end{bmatrix}
\]

is the original controller for the secondary-stage actuator. Such a controller with update period \( T_\mu \) has the following realization

\[
\dot{x}(k + 1) = (\Phi_{\mu} - \Gamma_{\mu}F - LC_{\mu})\dot{x}(k) \\
+ L(y_m(k) - y_r) - \Gamma_{\mu}u_q(k),
\]

\[
u_q(k) = -F\dot{x}(k) - u_q(k),
\]

\[
c_q(k) = -C_{\mu}\dot{x} + (y_m(k) - y_r),
\]

\[
u_q(z) = Q(x)\nu_q(z),
\]

\[
(4)
\]

where \((\Phi_{\mu}, \Gamma_{\mu}, C_{\mu})\) is the step invariant discrete-time nominal model of milli-actuator compensated with notch filter and augmented with an integrator. \( \dot{x}(k) \in \mathbb{R}^{n_\mu} \) is controller state and \( y_r \in \mathbb{R}^1 \) is the reference. \( F \) and \( L \) are selected such that \( \Phi_{\mu} - \Gamma_{\mu}F \) and \( \Phi_{\mu} - LC_{\mu} \) are stable. \( c_q(k) \) and \( u_q(k) \) are the input and output of Youla parameter \( Q(z) \) respectively.

The objective of the track following servo control was to maintain a minimum tracking error under the actuator parameter variations and different noise and disturbance (such as windage and disk flutter) conditions. Real time optimization and adaptive control based methods [14] used measured PES and thus the optimal control may not necessarily optimal for the true PES; However, our previous result in [10] show that when the bandwidth of the measurement noise was more than two times wider than the closed-loop servo bandwidth, the difference of optimality could be less than 2%. Next, we will minimize the measured PES by tweaking on the Youla parameters assuming the true PES was also minimized. In this case, the objective function was expressed as [11]

\[
\sigma^2_{\text{PES}} = ||\tilde{T}_{yaw}||^2
\]

\[
= ||[T_{11} + T_{12}(I - T_2 Q)^{-1}QT_2]\tilde{W}_{aw}||^2
\]

\[
= ||[T_{11} + T_{12}(I + O(Q)QT_2)]\tilde{W}_{aw}||^2
\]

\[
= q^T Hv + \delta^T q + c + O(q),
\]

\[
\approx q^T Hv + \delta^T q + c,
\]

\[
(5)
\]

where \( \tilde{W}_{aw} \) are the models of disturbances and noise process. \( T_{11}, T_{12}, T_{21} \) and \( T_{22} \) are systems independent of \( Q \). Furthermore,

\[
T_{22} : \begin{bmatrix}
\Phi_{\mu} - \Gamma_{\mu}F - LC_{\mu} & LC_{\mu} & -\Gamma_{\mu} \\
-G_{\mu}F & F & -\Gamma_{\mu} \\
-C_{\mu} & C_{\mu} & 0
\end{bmatrix}
\]

Obviously, \( T_{22} = 0 \) and thus \( O(q) = 0 \) if the milli-actuator parameters are not perturbed. At that time, the deviation square of PES \( \sigma^2_{\text{PES}} \) is just a 2nd-order function of \( q \) without any approximation. \( q = [q_0 \cdots q_{N-1}]^T \) is the impulse response of Youla parameter. \( H, H \in \mathbb{R}^{N \times 1}, \ b \in \mathbb{R}^N, O(Q) \) is the sum of a series of system \( Q, O(q) \) is a lager-than-2nd-order function of \( Q \)’s impulse response \( q \). Equation (5) shows that \( \sigma^2_{\text{PES}} \) could be approximated as a quadratic function of FIR Youla parameters under some conditions that let the \( H_2 \) norm of \( O(Q) \) less than a small positive real number. Such conditions usually require that the real plant of milli-actuator is not perturbed too much. The corresponding optimal solution for FIR Youla parameter shown in quadratic function (5) is

\[
q_{opt} = -\frac{1}{2}H^{-1}b.
\]

The parameters of quadratic function \( \Theta \) consisting of \( H, b \) and \( \epsilon \) defined in (5) can be obtained by

\[
\Theta = (X^T X)^{-1}X^T Y
\]

\[
(7)
\]

where regressor \( X = [x_{(1)}^T \cdots x_{(p)}^T]^T, x_{(i)} = [\delta^T q_k \cdots \delta^T q_k] (j, k = 0, \cdots, N - 1; i = 1, \cdots, p) \) with \( p \) the number of Youla parameters sets tested, and \( Y = [\sigma^2_{\text{PES}}^1 \cdots \sigma^2_{\text{PES}}^p]^T \) are determined by the PES corresponding to each set of parameter. Generally, for an FIR Youla parameter with \( N \) coefficients, we need to change the \( Q \) at least \((N + 1)(N + 2)/2 \) times and collect the corresponding PES to determine \( \Theta \).

Alternatively, \( \Theta \) can be obtained via the following recursive least square algorithm (RLSA):

\[
\hat{\Theta}(i) = \hat{\Theta}(i - 1) + K(i)x^T(i)\epsilon(i)
\]

\[
K(i) = K(i - 1) - \frac{K(i - 1)x^T(i)K(i - 1)}{1 + x^T(i)K(i - 1)x(i)}
\]

\[
\epsilon(i) = y(i) - x(i)\hat{\Theta}(i - 1).
\]

(8)

With the added control \( Q_{opt} \) which is optimized, the disturbance and noise rejection was better compromised because the initial design was mainly based on frequency domain specifications whereas the Youla parameter was selected based on maximization of the TPI given servo bandwidth limitations.

Youla parameterization can guarantee the stability of the system. The designer only needs to find the parameter for the chosen design specifications. However, the stability of the system is not guaranteed when plant model has uncertainty. In the case of HDD actuator control, [11] proposed to use the \( H_{\infty} \) norm of the sampled data system, which includes the actuator model and its uncertainty, sampler (analog to digital convertor) and holder (digital to analog convertor) as a criterion during parameter fine tuning. In this paper, we used the usual way of bandwidth limitation and guaranteed phase and gain margins to bound the parameter optimization process. The detail will be shown in the next section.

IV. AN APPLICATION EXAMPLE

Following the above optimization procedure, dual-stage servo loops were designed for a 3.5 inch disk model spun at 5400 rpm. Using the approaches discussed in [5], we can model the corresponding mechanical disturbances including windage effects, disk flutter, and measurement noise, including electronic noise and media noise. Using these data, we can
test our control design algorithm via simulation when the dual-stage actuator prototype drive is not available.

First, we obtained a suitable feedback gain $F$ and estimator gain $L$ of the observer based state feedback controller (4). In our case, the $K_u$ and $K_f$ were lead lag compensators and LQG compensators respectively. The corresponding closed-loop system Bode plots are shown as dashed line in Fig. 5.

Second, we decided the bounds of $q_0$ and $q_1$ based on phase margin and gain margin requirements. For simplicity, we called the open-loop 0 dB crossover frequency the servo bandwidth, or BW for short. Fig. 4 shows the stable regions for $q_0$ and $q_1$ if a minimum 600 Hz BW, 6 dB of gain margin, 30 degree of phase margin (PM), are required, and a maximum of 2000 Hz predefined BW for robustness reasons. Based on Fig. 4, we set the following bounds for $q_0$ and $q_1$ by some linear constraints:

$$\alpha_i q_0 + \beta_i q_1 \leq \gamma_i \quad (i = 1, 2 \ldots) \quad (9)$$

For the limitations we described previously, we have $\alpha_1 = -0.8916$, $\alpha_2 = -1$, $\alpha_3 = 1.5644$, $\alpha_4 = 0.5079$, $\beta_1 = -1$, $\beta_2 = 0$, $\beta_3 = 1$, $\beta_4 = 1$, $\gamma_1 = -0.526 \times 10^3$, $\gamma_2 = -0.9659 \times 10^4$, $\gamma_3 = 2.2221 \times 10^4$, $\gamma_4 = 0.443 \times 10^4$. Apparently, if the secondary stage actuator has a higher resonant frequency, the allowable BW can be higher, and thus the robust stable region for $q_0$ and $q_1$ is also wider. For example, if the secondary stage actuator supports 3000 Hz BW, we may let $\alpha_3 = 0.6914$, $\alpha_4 = 1.4786$, $\beta_3 = 1$, $\beta_4 = -1$, $\gamma_3 = 0.6630 \times 10^4$, $\gamma_4 = 0.0062 \times 10^4$.

Next we used the controller $U_f(K, Q)$ to control the actuator, obtain the PES, and calculate the $\sigma_{PES}$. After that, we changed the parameters of $Q$ 30 times, and obtained 30 sets of PES and the corresponding $\sigma_{PES}$. Then we used (7) to calculate $\Theta$, and update $q_{k+1} = q_k - \alpha(2Hq_k + b)$, where $\alpha$ was the pre-set step size. The updated $q$ at each step was checked against with (9) until we reached the optimal point or arrived at the boundary of (9). For the 1st order Youla parameter in our case, the optimal solution could be obtained using (6) and (7) after a minimum of 6 iterations of changing the parameters of $Q$.

In our example system, the transfer functions before and after the optimization is shown in Fig. 5. The corresponding open loop transfer function is shown in Fig. 6.

Fig. 7 shows the step response of the system. The VCM covers most of the DC movement, while the secondary stage actuator acted during the transient period. The settling time is about 0.4 ms and the rise time is shorter than 0.2 ms.

Fig. 8 shows the histogram of the PES before and after the optimization. From this figure, we see that the tracking accuracy can be improved considerably depending on how well the initial controller was selected.
V. TPI PREDICTION USING THE PROPOSED CONTROL SCHEME

Now that we have built the tool to design controller to optimize the tracking accuracy, next we will study the sensitivity of the tracking accuracy with respect to different levels of the noise and disturbance. The results gave us some suggestions on how to improve the access system design for higher TPI. Compared with the results in [9] which were obtained via manually tuning the secondary stage controller parameters, the projected TPI's are 10–30% higher because of the use of the proposed optimization scheme.

A. Case 1: Mechanical Disturbance, PES Noise and Servo Bandwidth

Assuming that all the mechanical disturbance including windage, disk flutter, etc., change proportionally, we can obtain the PES $3\sigma$ versus disturbance and PES noise of different amplitude when the controller is optimized for each case, with and without all RRO taken out. To see the effectiveness of TPI improvement via higher maximum allowable servo bandwidth, the sensitivity of the PES $3\sigma$ versus the servo bandwidth from around 600 Hz up to 3 kHz with all the RRO's taken out is shown in Fig. 9. From this figure, it seems for the disturbance model considered, the highest TPI achievable was only around 40 000 regardless of the servo bandwidth improvement to above 3 kHz; A 100 kTPI was achievable when the mechanical disturbance level was reduced by 80%; If the measurement noise level was also reduced by 80%, the achievable TPI could be higher than 100 000 with a 1 kHz servo bandwidth.

B. Case 2: Tracking Performance versus Torque Disturbance, Disk Flutter and Servo Bandwidth

The result in the previous section showed that the mechanical disturbance affects the TPI more than the PES noise, thus we would like to see which factor, roughly classified as torque disturbance and disk flutter, is more effective to improve the tracking accuracy.

The PES $3\sigma$ as a function of servo bandwidth given different torque disturbance and disk flutter is shown in Fig. 10 for the case all RRO were taken out, and in Fig. 11 for full RRO present. Obviously, taking out all the RRO components is a very effective way to increase the TPI; Also it seems that reducing the disk flutter is more effective than reducing the torque disturbances, for the same percentage of amplitude reduction.

C. Discussion

Roughly speaking, air flow induced disk vibration/flutter is proportional to $\rho R^2 \omega^2 / \delta E d$, where $R$ is the disk outer radius $\rho$ gas density, and $\omega$ is the angular speed, $t$ is the disk thickness, $E$ is the disk substrate Young’s modulus, $\delta$ is the disk substrate damping. From the above simulation results, it seems that reducing the mechanical disturbance level (such as by using spindle motors and servo writing technologies that generate lower repeatable and nonrepeatable runouts, stiffer and thicker substrate with better damping, or even smaller disk and appropriate rotating speed) are effective ways to increase the TPI; For larger form factor drives with higher rotating speeds, improved airflow (by changing the actuator shape and base plate design, or even smaller sliders) and high bandwidth,
dual-stage if necessary, actuators are also important. Since the mechanical disturbance is a function of the spindle speed, higher rpm drives need higher servo bandwidth.

From the sensitivity analysis results shown above, it appears that when TPI goes higher and higher, the PES sensing noise should be reduced as well. This suggests that we need to improve the read back signal quality via head, media, and signal processing innovations, and use more servo bursts with better efficiencies to go to beyond 100 kTPI.

It should be noted here that the accuracy of the prediction is dominated by the accuracy of the noise and disturbance models and their scalings, the selection of initial controller and the order of the Youla parameter. Furthermore, the effect of external disturbances which might be a limiting factor in higher TPI drives was not considered.

VI. CONCLUSION

This paper presented a dual-stage servo system with an add-on controller that maximizes the tracking accuracy given the actuator bandwidth limitations without knowing the mechanical disturbance model. We proposed to add an outer loop, selected based on optimization of the Youla parameters, to the usual dual-stage controller for tracking accuracy improvement. Time domain and frequency domain simulations were performed to show that the proposed method can include frequency domain design specifications by bounding the Youla parameter when improving the tracking accuracy. Although predicting the future TPI is difficult due to the interaction of mechanical components and shift of the mechanical disturbance peaks, this paper studied the case where various noise and disturbance amplitudes were reduced, and the new control design was employed. The results show that to go to higher TPI, we should improve the HDD mechanical system as a whole instead of just pushing the bandwidth alone.

REFERENCES


