

# MULTIPATH EFFECTS ON ELLIPSOID HEIGHT POSITIONING USING KINEMATIC GPS TECHNIQUES

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**ABSTRACT:** This paper examines results of GPS testing conducted in 1998. The survey spanned four days, and collected over 10,000 repeat kinematic observations at two monuments spaced ~30 m apart. Occupations were made at the same sidereal time of day to demonstrate the repeatability of kinematic GPS positioning. These occupations were made at a 1-second epoch rate with observation periods lasting almost an hour. Two days of testing were made in this manner. The third day introduced a carbon substance (charcoal) at one setup to test that effect. On the last day of testing, a reflecting surface was placed at one setup with the intent of inducing carrier phase multipath. The paper documents the results of those tests as they relate to ellipsoid height repeatability.

## INTRODUCTION

The last decade has seen an increase in the use of GPS technology. Geodetic surveying first made use of static positioning methods as they were the most reliable. Kinematic GPS positioning techniques soon followed, and offered higher production rates when compared to static positioning techniques (Remondi 1985). Kinematic positioning became a routine surveying technique as methods of ambiguity resolution have become less cumbersome.

Investigations of multipath effects on carrier phase GPS positioning are not new. Georgiadou and Kleusberg (1988) developed the first useful mathematical models for carrier phase GPS multipath errors. Other investigators such as Elósegui et al. (1995) examined the effects of carrier phase GPS multipath at permanent GPS sites. Several investigators (Georgiadou and Kleusberg, 1988; Elósegui et al., 1995; Johnson et al., 1995; Jaldehag et al., 1996) have concluded that multipath primarily affects the vertical component of carrier phase GPS observations by a factor of two or three. Much of the multipath testing pre-dates advanced techniques such as on-the-fly (OTF) ambiguity resolution and wide-lane data processing. It is worthwhile to review the “classical” work on GPS carrier phase multipath in the light of today’s technological advances.

Currently, GPS is used increasingly with other measurement systems to provide positioning information. For example, measurement systems such as Light Detection and Ranging (LIDAR) rely upon carrier phase GPS to position the sensor. As the platform (aircraft) is moving, kinematic GPS techniques are used to position the sensor in three-dimensions.

LIDAR is a very popular way to develop mass points for orthophotography and compile digital elevation models (DEMs). As GPS provides the positioning information used in LIDAR, it is important to understand the contribution of kinematic techniques to the overall error budget. Often times, kinematic GPS surveys are used to provide comparisons with LIDAR-derived DEMs as a quality control check. Understanding GPS positioning errors is essential to any airborne GPS application such as LIDAR. Any vertical GPS error will directly influence the accuracy of any LIDAR product.

Using classical survey methods, vertical control surveys reference measurements to the geoid, thereby giving an orthometric height ( $H$ ). Measurements using GPS give an ellipsoid height ( $h$ ). GPS heights are combined with geoid heights ( $N$ ) to obtain orthometric heights (elevations). GEOID99 is the latest geoid height model published by NGS. Ellipsoid height differences are combined with geoid height differences to obtain orthometric height differences through the relationship

$$\Delta H \equiv \Delta h - \Delta N \quad (1)$$

where

$\Delta H$  = the difference in orthometric height (differential leveling);  
 $\Delta h$  = the difference in ellipsoid height (GPS observations); and  
 $\Delta N$  = the difference in geoid height (geoid height model).

It should be noted that ellipsoid heights were used for this article. Since most engineering and mapping projects are referenced to an orthometric height surface, GPS users requiring orthometric heights need to perform geoid modeling. Procedures for geoid modeling are outside the scope of this document; however improper geoid modeling techniques could increase vertical errors associated with kinematic GPS positioning. Users seeking a more detailed treatment of GPS-derived orthometric heights should refer to Zilkoski (1990) and Zilkoski et al. (1997).

Multipath is defined as a reflected GPS satellite signal that arrives at a receiver by more than one path. If the antennae are kept at the same locations, the effects of multipath repeat every sidereal day. This is due to the fact that multipath is dependent upon the combined geometries of the receiver, satellites, and reflectors (Georgiadou and Kleusberg, 1988; Hoffman-Wellenhof et al., 1997, p. 126; Leick, 1995, p. 311). Differential GPS positioning can be grouped into two broad categories: code positioning and carrier phase positioning. Differential code positioning gives meter-level precision. Differential carrier phase positioning gives centimeter-level precision. For the purposes of this discussion, kinematic GPS positioning is defined as the ability to position at the centimeter-level while an antenna is moving.

The object of this paper is to investigate the effects of multipath on carrier phase GPS positioning, specifically as they relate to GPS-derived ellipsoid height repeatability. The paper is organized as follows. The first section introduces the topic. Mathematical models for multipath are reviewed in the second section. The third section examines test results of continuous kinematic GPS. The multipath testing included in this paper occurred over a period of four days; over 10,000 repeat ellipsoid height comparisons were made at the same sidereal time of day over the four days. Discussion of the test results and their context with previous work is given in the next section. The last section summarizes the findings of this article, and offers a perspective on future work.

## MATHEMATICAL MODELS FOR CARRIER PHASE GPS MULTIPATH

This section introduces the basic mathematical models for carrier phase GPS multipath. The models used in this section are those developed previously by Georgiadou and Kleusberg (1988) to describe multipath effects. They are reviewed in this section for the benefit of readers unfamiliar with the basic formulae. Those seeking a more thorough explanation of these models should refer to Georgiadou and Kleusberg (1988). One assumption for these models is that the multipath source is both a single reflector and a planar surface. In practice, this assumption is an exception rather than a rule. That being said, the object of this discussion is to illustrate multipath effects on 1) amplitude and 2) duration (frequency).

Multipath amplitude is defined as the ranging error caused by the reflected carrier phase GPS signal. Maximum multipath amplitude is defined as

$$\Psi_{max} = \pm \arcsin \phi \quad (2)$$

where

$\Psi_{max}$  = maximum carrier phase multipath error;  
 $\phi$  = phase shift.

The maximum multipath error occurs when  $\phi = 1$  or when there is a  $90^\circ$  phase shift. Converting this phase shift to range equals  $\lambda/4$  or, with  $\lambda = 19.0$  ( $L_1$  wavelength), a maximum change in range of approximately 4.8 cm.

The phase shift ( $\phi$ ) represents the angle of incidence for the GPS signal reflecting off the planer surface. There are two basic carrier phase frequencies,  $L_1$  (1575.42 Mhz) and  $L_2$  (1227.60 Mhz). These frequencies can also be expressed as a wavelength, where the  $L_1$  wavelength equals 19.0 cm and the  $L_2$  wavelength equals 24.4 cm. Advanced processing techniques may use a linear combination of both frequencies, giving a signal wavelength larger or smaller than the  $L_1$  or  $L_2$  carrier phase.

Dual-frequency, full-wavelength GPS receivers currently available use a technique known as wide-laning to more quickly resolve the integer bias. This technique is especially useful for applications such as kinematic and fast-static positioning. Wide-laning is achieved through the linear combination of the  $L_1$  (19.0 cm wavelength) and  $L_2$  (24.4 cm wavelength) through signal squaring, giving an effective 86.2 cm wavelength (Hoffman-Wellenhof et al., 1997, p. 217). This wide-lane solution is then used in a step-wise fashion to estimate the whole number of cycles for the  $L_1$  carrier phase. This situation becomes more complicated as the GPS baseline length increases due to the possibility of unmodeled ionospheric effects (Hoffman-Wellenhof et al., 1997, p. 219). Even if all systematic errors related to the atmosphere have been resolved while wide-laning, one still needs to contend with multipath. Using Equation 2, this gives a maximum multipath error of 21.5 cm using the 86.2 cm wide-lane wavelength.

Multipath error frequency is expressed as

$$f_{\psi} = (2 h / \lambda) \cos \theta \, d\theta / dt \quad (3)$$

where

$f_{\psi}$  = multipath error frequency;  
 $\theta$  = satellite elevation above the reflector plane;  
 $\lambda$  = is the carrier wavelength (in mm);  
 $h$  = height of phase center (in m);  
 $d\theta / dt$  = rate of change of the elevation angle (in mrad/sec.).

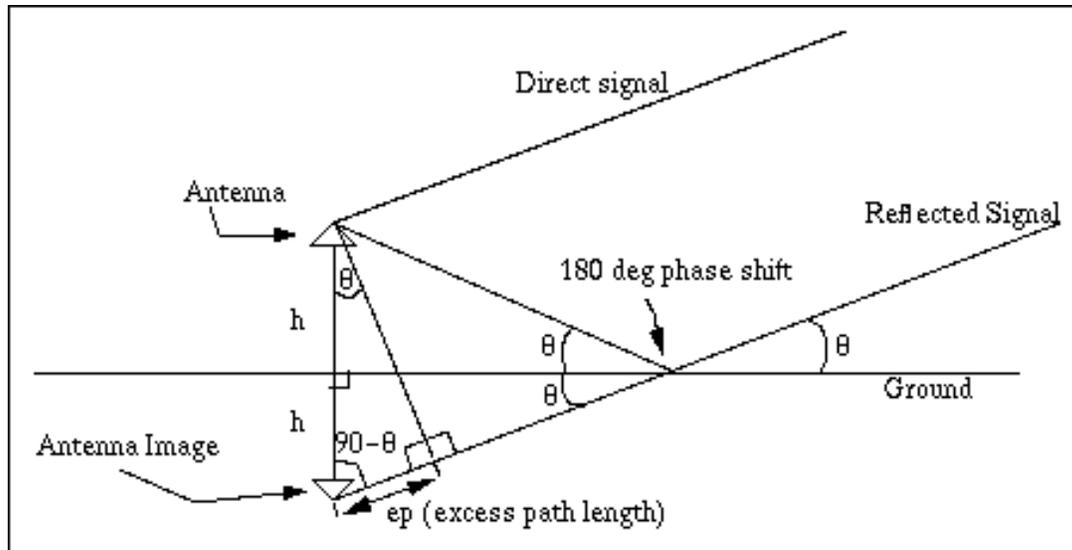
For average values of  $\theta$  and  $d\theta / dt$ ,

$$\theta = 45^\circ \text{ and } d\theta / dt \approx 0.07 \text{ mrad/sec.} \quad (4)$$

Therefore, a reflecting plane 30 cm below the antenna phase center using the 19.0 cm  $L_1$  wavelength and assuming the average values in Equation 3 gives a multipath period of 107 minutes. Using the same basic assumptions, but with a reflecting plane located 10 m below the antenna phase center gives a multipath period of only 3.2 minutes.

Analyzing Equation 4, one can see that multipath error frequency is influenced by four factors. First, the frequency is proportional to the distance of the reflector to the antenna phase center. Multipath will contaminate the GPS signal for a longer period of time the closer a reflector is to an antenna. Next, the multipath frequency is inversely proportional to carrier wavelength. For instance, carrier phase multipath using the ~86 cm wide-lane wavelength will show a lower frequency than the ~19 cm  $L_1$  carrier phase wavelength. Third, multipath error frequency is proportional to the cosine of elevation ( $\theta$ ) of the satellite above the reflector. Multipath will last longer as a satellite nears the horizon. Last, multipath error

frequency is proportional to the rate of change of the satellite elevation above the reflector. Multipath will last for a longer period when the rate of change of the satellite elevation is lower. Figure 1 shows the geometric relationships in multipath at a typical GPS station per Johnson (1995).



**Figure 1.** Geometric relationships in multipath (Johnson 1995).

## MULTIPATH TESTING

*Test Methodology.* Field tests of multipath were performed in June 23 through 26, 1998 in La Puente, California. The location of the test was the parking lot for a California Department of Transportation (CALTRANS) survey field office. Previously, the parking lot had a test network established using static GPS methods. These static results are used as the initial baseline for the four days worth of kinematic positioning. Forty- to fifty-minutes worth of continuous kinematic GPS data were collected from the same two stations over the four days. Occupations were timed to coincide in sidereal time with the ~4 minute daily change in satellite position. Six or more satellites were visible at all times during the four days of testing. The positional dilution of precision (PDOP) was below three at all times during the four days of testing. GPS data was collected above a 15-degree elevation mask. All observations are unadjusted. GPS data was reduced using GPSurvey version 2.30 in a continuous kinematic mode using precise orbits computed by the National Geodetic Survey (NGS). Integer ambiguity resolution was estimated through OTF techniques (Remondi 1991). GPS results over the four days achieved fixed integer solutions.

*Equipment.* The survey used a pair of Trimble 4000 Ssi receivers with Compact  $L_1/L_2$  groundplane antennae. The Trimble 4000 Ssi is a dual-frequency, full-wavelength GPS receiver. Receivers were set to collect data on a 1-second epoch rate. The survey used 2-m fixed-height tripods at both setups for the four days. The antennae were orientated to a common direction for the four days of testing.

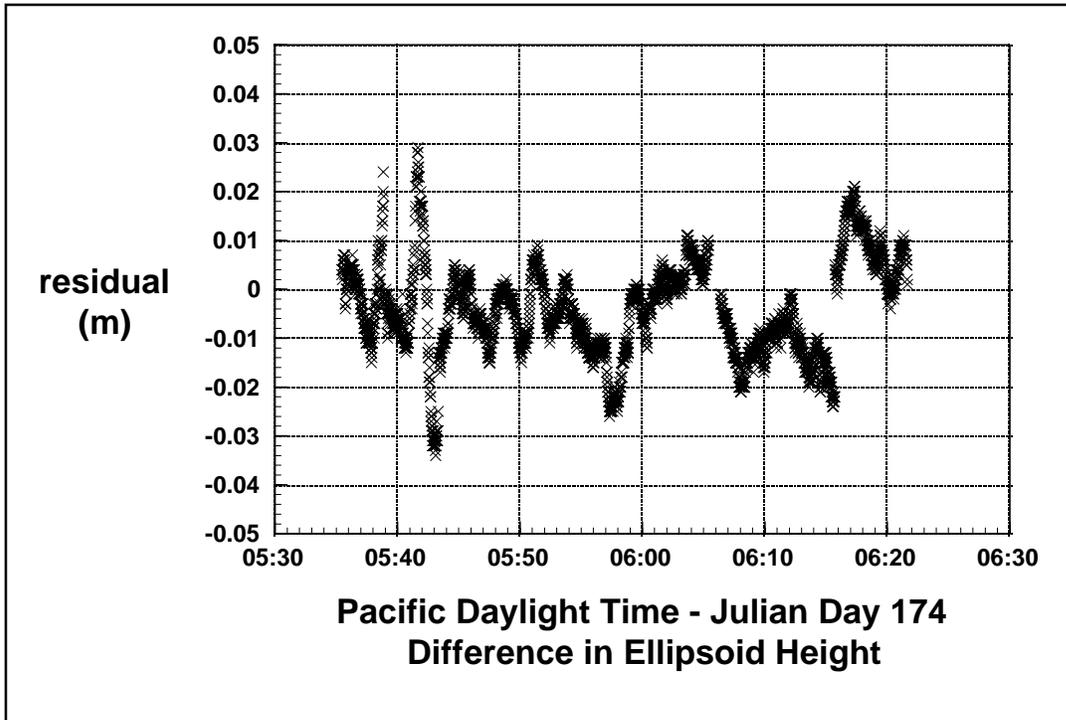


Figure 2. Ellipsoid height repeatability, Julian day 174.

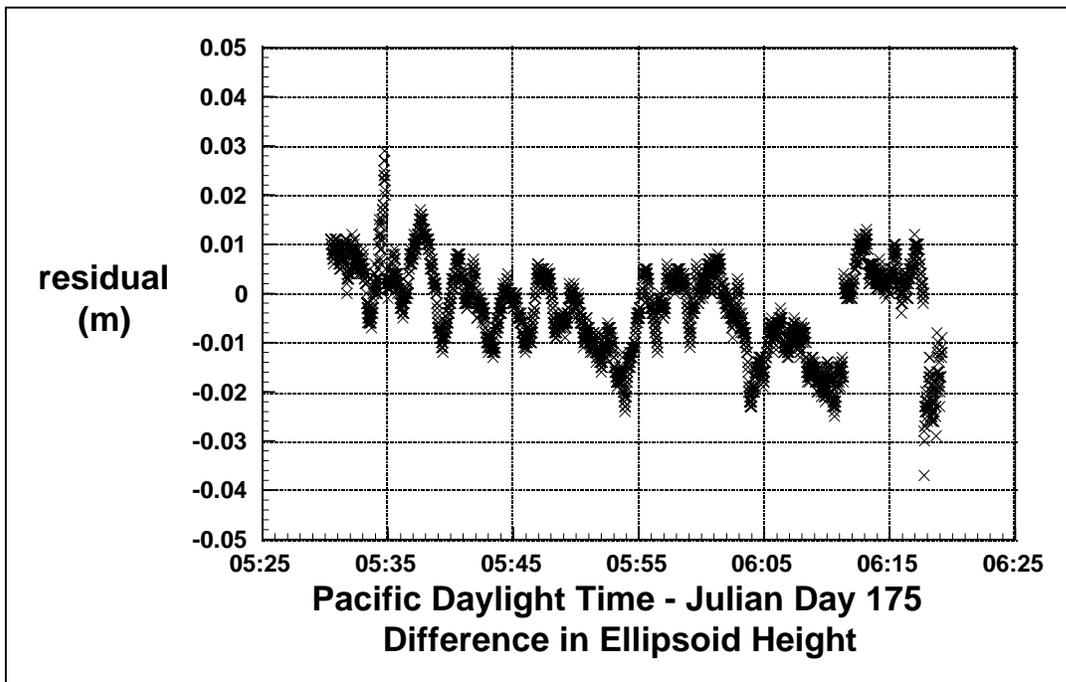


Figure 3. Ellipsoid height repeatability, Julian day 175.

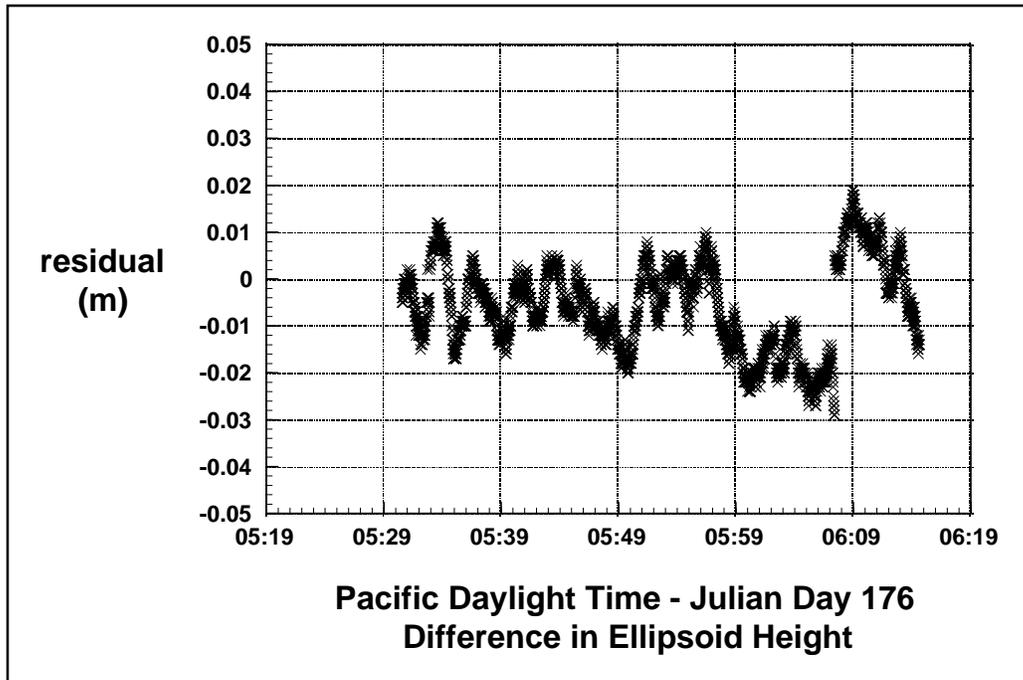


Figure 4. Ellipsoid height repeatability, Julian day 176.

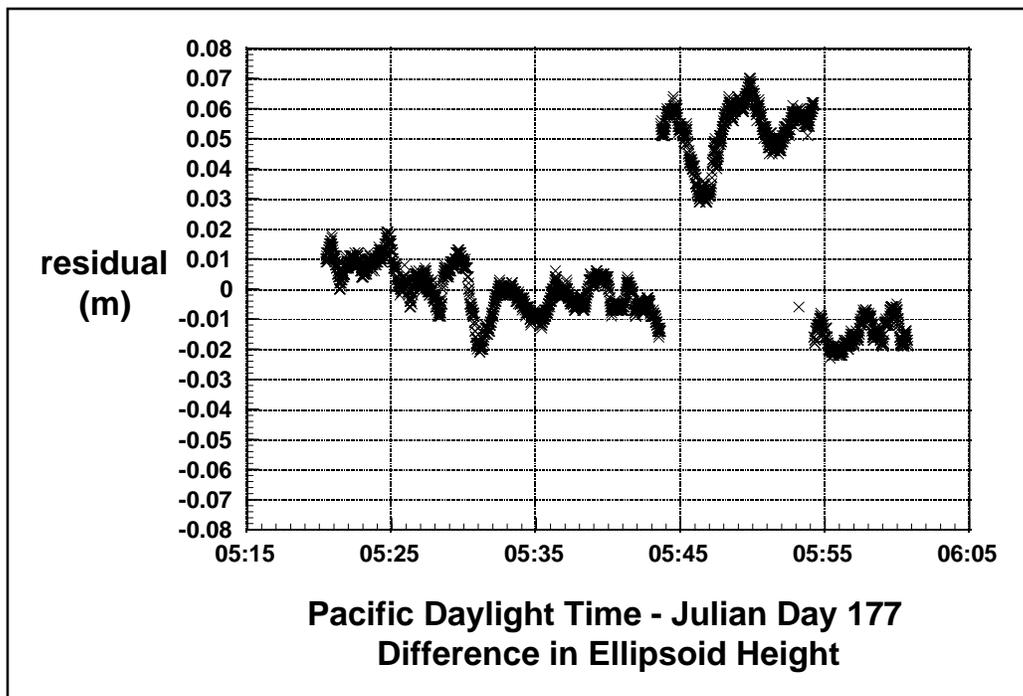


Figure 5. Ellipsoid height repeatability, Julian day 177.

*GPS testing – Julian days 174 and 175.* Test results from the two days are shown in Figures 2 and 3. On Julian day 174, the results show a range of +3- to -3.5-cm. Please note that the range of these differences occurs in a two-minute time span starting at 5:41am local time. As expected, the differences tend to fall near the mean (zero), but vertical drift is definitely evident. Julian day 175 shows a similar result with differences in the same range. The data from Julian day 175 contains slightly more data than from Julian day 174. The same feature is noted from day 174 (starting at 5:37am), a little less than 4 minutes earlier, but without the magnitude of that feature. What is notable about the two data sets is the similarity of the patterns. Although the patterns appear to be similar, they are not of the same magnitude from day-to-day when looking at the peaks. The similarity between these two datasets implies a systematic effect correlated with sidereal time. On Julian day 174, a distinct break is noted at ~6:15 am, corresponding to a change in the satellite constellation with the addition of a new satellite (SV 15).

*GPS testing – Julian day 176.* The goal of the last two days of the project was to isolate a single satellite (SV 11) and induce multipath at one of the antenna locations. SV 11 was selected because it had a clear line of sight from only one of the GPS setups. Three marks were made on the asphalt parking lot using a lumber crayon at the setup with the clear line of sight to SV 11. These marks were made on an approximate 145-degree azimuth from true north, so as to correspond with the azimuth from the GPS antenna to SV 11. The middle mark was made about 2.1 m away from the setup. The distance of this middle mark was selected to correspond with the height of the L<sub>1</sub> phase center above the ground (2.062 m), and is the intended point of incidence for the last two days of testing. This also corresponds to the maximum multipath as noted in the 90-degree phase shift shown in Equation 2.

Julian day 176 introduced a bed of charcoal at the point of incidence. Figure 4 shows the results of the testing from Julian day 176. Comparing the results from Julian day 176 gives essentially the same pattern as shown in Julian days 174 and 175. The differences range from +2- to -3-cm for the comparisons on Julian day 176. As in the two previous days, the patterns between the three days appear to match well, however the magnitude of those differences appear to be different. The introduction of charcoal at the point of incidence had no apparent influence on the GPS positions as noted in Figure 4.

*GPS testing – Julian day 177.* Julian day 177 introduced a metal road sign at the point of incidence. The goal of this day's test is that the placement of the sign will induce multipath in the GPS signal. Results from Julian day 177 are shown in Figure 5. What is most notable in the results from this day is the 5- to 6-cm change in ellipsoid height as seen beginning at 5:44. This apparent bias in the ellipsoid height lasts for approximately 10 minutes (600 seconds). The remainder of the observations during Julian day 177 shows the same repeat pattern at the same sidereal time as the three previous days. With the apparent bias, the differences range from +7- to -2-cm for the comparisons on Julian day 177. Removing the apparently biased data gives a range of +2- to -2-cm for the comparisons. Excluding the biased data from Julian day 177 shows similar patterns over the four days of testing, but they appear to differ somewhat in magnitude.

The satellite azimuth and elevation table for Julian day 177 at this location gives an approximate rate of change for SV 11 during this test of  $d\theta/dt \approx 0.10$  and an average of 45-degrees above the horizon during the observations taken on Julian day 177. The height of the phase center above the asphalt parking lot was 2.062 m using the fixed height tripod. Assuming a 19.0 cm L<sub>1</sub> carrier phase wavelength and using the values associated with test made on Julian day 177, applying Equation 3 gives a multipath period of 10.86 minutes (652 seconds).

## DISCUSSION

Results of this test confirm the earlier findings of Georgiadou and Kleusberg (1988) regarding carrier phase GPS multipath effects. Comparisons during the four days show a definite correlation with sidereal time; these effects have been noted by other investigators (Georgiadou and Kleusberg, 1988; Elósegui et al., 1995; Johnson et al., 1995; Jaldehag et al., 1996). Before multipath can be positively identified as the error source in the testing, several things must first be demonstrated.

Height of the phase center is one component in multipath frequency as noted in Equation 3. The survey used 2-m fixed-height tripods on all four days of testing, so instrument height variations are not an error source in this study. The survey used identical antennae during the four days of testing. Recent tests by Mader (1998) indicate that large errors (up to 10 cm) can contaminate vertical GPS solutions when phase center modeling has been either ignored or mis-modeled. This is due to the fact that the phase center is not a physical point, and differs as a GPS satellite changes in azimuth and elevation. Different models of antennae require different phase center modeling to eliminate differential antennae bias. Phase center modeling is not an error source for the four days of testing because identical antennae were used. Finally, antenna phase center variations can occur with similar antenna when they have not been orientated to a common direction. Antenna phase center variation is not an error source because both antennae were orientated to a common direction.

Antenna phase center variations and multipath are the two error sources that will repeat after every sidereal day (Georgiadou and Kleusberg 1988). Antenna phase center variations can be ruled out as an error source, leaving multipath as the only remaining candidate to explain the apparent systematic differences correlated with sidereal time.

Airborne GPS applications such as LIDAR rely on kinematic GPS observations to accurately position the sensor. This presents something of a dilemma to airborne GPS users as they rely upon a single epoch to position the sensor. Multipath can present additional challenges when performing airborne GPS (Leick, 1995, p. 312). Ground- and aircraft-based GPS antennae have different characteristics. Multipath can be reduced in ground applications by using low-gain GPS antennae. While in the air, the receiver must maintain lock while undergoing dynamics in situations such as banking, and therefore require a high-gain GPS antenna. Airborne receivers may experience significant multipath by reflections off the airframe. As a result, airborne GPS applications tend to be more prone to multipath error than terrestrial GPS applications, especially with low-elevation satellites.

Analysis of test results indicates that multipath effects can be averaged over time. This may be adequate for static and fast-static positioning, however multipath can create serious issues using kinematic techniques. This fact was first noted by Georgiadou and Kleusberg (1988):

Probably, the effect of multipath on relative positioning results can be reduced through averaging if the observation session is sufficiently longer than the longest multipath error period. But this certainly would increase the cost and time consumption of GPS observations beyond the acceptable level for routine surveying applications. *The effect of carrier signal multipath may be especially dangerous in kinematic relative GPS positioning since error reduction through averaging is not possible anymore* (emphasis added).

Current guidelines for precise GPS ellipsoid height positioning require minimum occupations of 30-minutes each for 2-cm ( $2\sigma$ ) ellipsoid heights (Zilkoski et al. 1998). GPS ellipsoid height positioning at a 5-cm tolerance ( $2\sigma$ ) does not have the same minimum time restrictions. Was the requirement of a 30-minute minimum session length a conscious attempt by NGS to reduce multipath? This is unknown, however the authors of these guidelines evidently saw the need for sufficient averaging to achieve the 2-cm ( $2\sigma$ ) ellipsoid height standard.

Use of a groundplane had only limited results on multipath mitigation. This is also consistent with the results of other investigators (Elósegui et al., 1995; Johnson et al., 1995; Jaldehag et al., 1996) who have examined multipath effects at permanent GPS sites.

What is most intriguing about these results is how multipath magnitude varies over the four days of testing. This same effect was shown on an earlier study on real-time kinematic repeatability conducted by Satalich and Ricketson (1998). This type of effect was also evident in tests performed by NGS used to establish the ellipsoid height guidelines, (Zilkoski, personal communication, 1998) hence the requirement of those guidelines to observe a repeat occupation on a different day and at a different time of day. Further research in this area is clearly justified.

Inducing multipath at one of the setups was also successful. From the practical point of view, understanding the characteristics of multipath effects on kinematic GPS techniques is extremely useful. Results from tests such as these are useful to the practicing land surveyor who wants to position points within a specified tolerance. Understanding multipath characteristics is an essential part of minimizing multipath effects.

In a calculated manner, a metal sign was introduced into the experiment with the specific purpose of inducing multipath in that day's GPS observations. A metal sign was chosen because it is reported to be a cause of carrier phase GPS multipath. It was the only new element introduced into the testing from the three previous days worth of work. As noted in the results, the multipath amplitude and frequency match remarkably well with models developed previously by Georgiadou and Kleusberg (1988), giving mathematical support to the argument of carrier phase multipath occurring on Julian day 177.

Charcoal was placed at the point of incidence on Julian day 176 because of its properties as a radar-absorbing material. Charcoal is used at some Continuous Operating Reference Stations (CORS) to reduce multipath (Kenneth Hudnut, personal communication, 1998). Use of charcoal at the point of incidence was used as control for the work performed on Julian day 177. Other physical methods of multipath reduction include crumpled aluminum foil located under CORS sites to diffuse multipath occurring from the supporting concrete pillar. Effects of multipath diffusion are more thoroughly explained by van Nee (1995, p. 84-85).

Current research on multipath reduction focuses on the use of microwave absorbers at permanent GPS sites (Elósegui et al., 1995). In this study, it was shown that using choke ring GPS antenna without the microwave absorber causes multipath errors of several centimeters in vertical GPS results. Other researchers are examining the properties of surface reflectivity for GPS applications (Kavak et al. 1996). Our current understanding of multipath errors indicates snow, water, and metallic surfaces are multipath inducers. Carbon products such as charcoal are multipath inhibitors. Additional work related to surface reflectivity is another area of research related to multipath mitigation.

## CONCLUSION

*Findings.* This paper examines multipath effects on GPS-derived ellipsoid heights during a four-day period. Over 10,000 repeat GPS occupations were performed during the four days of testing. Results of the tests are consistent with other work in the field of GPS carrier phase multipath. Multipath was deliberately induced on the fourth day of testing to show the effects of carrier phase multipath on kinematic GPS observations. Kinematic GPS observations over the four days validate the multipath models developed originally by Georgiadou and Kleusberg (1988).

Results of these tests indicate that multipath can be a significant source of error in kinematic GPS positioning. Understanding these effects on GPS repeatability is important so that the system can be used within the desired tolerance for the survey at hand.

*Recommendations.* Investigations should continue in multipath as it relates to GPS-derived ellipsoid heights. These efforts should concentrate on either eliminating or minimizing the effects of GPS multipath. Suggested areas of research include multipath reflectivity, multipath absorbers, and improved groundplane antennae. Research should also continue in improving algorithms for detecting and reducing multipath effects during baseline reduction.

Short of a breakthrough in one of these areas, GPS users have relatively few options. One can attempt to not observe near areas of high multipath, but surveyors seldom have that luxury especially in today's competitive environment. Multipath effects can be disregarded, however one is then forced into the situation of having to live with the consequences of those results. Finally, surveyors have the option of averaging the multipath over a longer period of time. This is impossible in the case of airborne GPS positioning. What is evident is that no easy answers exist to solve GPS multipath errors.

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