Shape Optimization of Single-Chamber Mufflers with Side Inlet/Outlet by Using Boundary Element Method, Mathematic Gradient Method and Genetic Algorithm

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Abstract

This paper is trying to build up the theoretical four-pole matrices for the single-chamber mufflers equipped with side inlet/outlet duct. In addition, by using the BEM simulation and the experimental works on silencers, the accuracy of above theoretical solution can be verified and reassured, accordingly. Moreover, applying the numerical optimizations of genetic algorithm (GA) and three kinds of mathematic gradient method – interior penalty function method (IPFM), exterior penalty function method (EPFM) and feasible direction method (FDM), the optimal design of silencers under constrained space can then be achieved.

In this paper, considering the flowing effect, the theoretical scalar function of muffler’s noise reduction is firstly deduced by using four-pole matrices method on the basis of one-dimensional plane wave theorem. Moreover, a numerical case of muffler’s shape optimization in dealing with the elimination of pure tone noise of 500 Hz under space constraints is introduced. Before optimizations being performed, the effective design variables can be selected by the computer-aided sensitivity analysis of design parameters in Matlab package. Besides, the starting points have been initialized first and then later taken into the Gradient methods for further optimization.

By using Gradient Methods and GA Method, several better design data are obtained. Consequently, the best design parameter with highest STL may be determined within them.

Key Words: Single-Chamber Muffler with Side Inlet/Outlet, Four-Pole Transfer Matrix, Mathematic Gradient Method, Boundary Element Method, Genetic Algorithm, Sound Transmission Loss, Optimal Design

1. Introduction

The study of expansion silencer used in diesel engine has already been started by Davis [1] in 1954. Thereafter, the influences of flowing effect and temperature gradient have been considered, and discussed by Prasad [2–4], Crocker [2] and Munjal [4–6] after 1970. Therefore, the researches on muffler design have been well addressed; however, the constrained problem has hardly been mentioned. Bernhard [7] has introduced the shape optimization of simple expansion mufflers by using design sensitivity matrices. The space volume of
the reactive silencer is still non-constrained, and the calculation of design sensitivity matrices is difficult especially for the mufflers, which are complicated. In the previous work [8], the four-pole matrix oriented from the one-dimensional plane theory was linked with graphic analysis to search for the optimal design parameters of a constrained single-chamber silencer; however, the accuracy is insufficient.

To recognize the model correctness of silencer performance in four-pole matrix method, the three-dimensional boundary element analysis is adopted in this study; in addition, to highly increase the acoustic performance, a new type of single-chamber silencer with side inlet/outlet has been introduced in this study.

Moreover, the maximal acoustic performance of silencer can be precisely and quickly achieved by using the advanced searching techniques of gradient method [9, 10] and genetic algorithm method [10–13]. Wherein the traditional gradient method has already been widely applied in engineering constrained problem for a long time; and are in good performance; besides, the genetic algorithm, a novel scheme oriented from Darwinian notion of natural selection and evolution has been prosperously adopted in many field during the nearest decade.

2. Nomenclature

This paper is constructed on the basis of the following notations:

- \( F(X) \): unmodified objective function
- \( g_i \): inequality constraints
- \( \text{gen}_\text{no} \): maximum no. of generation
- \( j \): imaginery unit
- \( J_m \): Bessel function of order \( m \)
- \( k \): wave number
- \( k_{r,m,n}^\pm \): wave number in \( r \)-direction
- \( L_i \): length at \( ith \) element (m)
- \( L_o \): total length of the muffler (m)
- \( M_i \): mean flow Mach number at \( ith \) element
- \( \text{OBJ}_i \): objective function
- \( p_c \): crossover ratio
- \( p_i \): acoustic pressure at \( ith \) node (Pa)
- \( p_m \): mutation ratio
- \( \text{popuSize} \): no. of population
- \( Q \): volume flow rate of venting gas (m\(^3\) s\(^{-1}\))
- \( r_p \): penalty parameter
- \( r_p' \): penalty parameter
- \( r, \theta, z \): axis of cylindrical coordinates
- \( S_i \): section area at \( ith \) element (m\(^2\))
- \( \text{STL}_i \): sound transmission loss (dB)
- \( t \): time (second)
- \( T_{ij} \): components of four-pole transfer matrix
- \( u_i \): acoustic particle velocity at \( ith \) node (m s\(^{-1}\))
- \( U_i \): acoustic volume velocity at \( ith \) node (m\(^3\) s\(^{-1}\))
- \( v_i \): acoustic mass velocity at \( ith \) node (kg s\(^{-1}\))
- \( Y_i \): characteristic impedance at \( i \)
- \( \rho_o \): air density (kg m\(^{-3}\))
- \( \Phi \): modified objective function

3. Theoretical Background

The acoustic field in silencer represented by nine points is shown in Figure 1. As indicated in Figure 1, the silencer includes five components; three of them are the straight ducts; and two of them are the side ducts. As derived in the Appendices A and B, individual four-pole transfer matrix with respect to straight ducts and side duct are described as [2–6].

\[
\begin{bmatrix}
    P_i \\
    \rho_o C_p U_i
\end{bmatrix} = e^{-j \beta M U_i (I - M^*)} \begin{bmatrix}
    b11^T & b21^T \\
    b22^T \\
\end{bmatrix} \begin{bmatrix}
    P_z \\
    \rho_o C_p U_2
\end{bmatrix}
\]

\( (1) \)
By the substitutions and rearrangement in Eqs. (1)–(5), the resultant system matrix of silencer can be deduced in the form

\[
\begin{bmatrix}
  p_2 \\
  \rho_c u_2
\end{bmatrix} = \begin{bmatrix}
  TS2
\end{bmatrix}
\begin{bmatrix}
  p_4 \\
  \rho_c u_4
\end{bmatrix}
\] (2)

\[
\begin{bmatrix}
  p_4 \\
  \rho_c u_4
\end{bmatrix} = e^{-\frac{i \omega L_3}{1 - M_3^2}} \begin{bmatrix}
  c11' \\
  c21'
\end{bmatrix}
\begin{bmatrix}
  \rho_c u_3 \\
  \rho_c u_4
\end{bmatrix}
\] (3)

\[
\begin{bmatrix}
  p_5 \\
  \rho_c u_5
\end{bmatrix} = \begin{bmatrix}
  TS4
\end{bmatrix}
\begin{bmatrix}
  p_7 \\
  \rho_c u_7
\end{bmatrix}
\] (4)

\[
\begin{bmatrix}
  p_7 \\
  \rho_c u_7
\end{bmatrix} = e^{-\frac{i \omega L_5}{1 - M_5^2}} \begin{bmatrix}
  c11' \\
  c21'
\end{bmatrix}
\begin{bmatrix}
  d11' \\
  d21'
\end{bmatrix}
\begin{bmatrix}
  \rho_c u_5 \\
  \rho_c u_7
\end{bmatrix}
\] (5)

By the substitutions and rearrangement in Eqs. (1)–(5), the resultant system matrix of silencer can be deduced in the form

\[
\begin{bmatrix}
  p_1 \\
  \rho_c u_1
\end{bmatrix} = e^{-\frac{i \omega L_1}{1 - M_1^2}} \begin{bmatrix}
  b11' \\
  b21'
\end{bmatrix}
\begin{bmatrix}
  TS2
\end{bmatrix}
\begin{bmatrix}
  c11' \\
  c21'
\end{bmatrix}
\begin{bmatrix}
  d11' \\
  d21'
\end{bmatrix}
\begin{bmatrix}
  \rho_c u_1 \\
  \rho_c u_2
\end{bmatrix}
\] (6)

Eq. (6) can subsequently be expressed as

\[
\begin{bmatrix}
  p_1 \\
  \rho_c u_1
\end{bmatrix} = \begin{bmatrix}
  TS1' \\
  TS2'
\end{bmatrix}
\begin{bmatrix}
  p_8 \\
  \rho_c u_8
\end{bmatrix}
\] (7)

Consequently, the sound transmission loss (STL) of silencer, which is derived on the basis of one-dimensional plane wave theory, is obtained as [6]

\[
\text{STL}(D_{d5}, L_{d1}, L_{d2}) = 20 \log \left( \frac{T11' + T12' + T21' + T22'}{2} \right) + 10 \log \left( \frac{S_b}{S_k} \right)
\] (8a)

where

\[
L_0 = L_2 + L_3 + L_{d4}; D_6 = L_1 + D_{d5} + L_5
\] (8b)

By using the formula of Eq. (8), the objective function used in following numerical optimization is established as

\[
\text{OBJ} = \text{STL}(Q, D_{d5}, L_{d1}, L_{d2}, L_1, L_5, D_1, D_2)
\] (9)

4. Model Check

Before performing the numerical optimal simulation on mufflers, an accuracy check of the mathematical model on the single-chamber muffler equipped with side inlet and outlet duct was performed and verified by using not only experimental data but also the SYSNOISE simulation, one kind of the commercialized boundary element method and used for the high degree wave’s analysis.

First of all, one set of design parameter shown in Figure 2 has been initialized and constructed. By using the impedance tube shown in Figure 3, which is used for the measurement of acoustical performance in silencer, the practical STL of this silencer with respect to spectrum has thus be acquired.

Secondly, the three-dimensional boundary element modeling in SYSNOISE has been constructed. The meshed profile of silencer is depicted in Figure 4. By acoustical simulation, the STL of silencer can also be obtained.

As a result, the STL curves of theoretical solution, experimental data as well as SYSNOISE’s simulated data are thus plotted and shown in Figure 5. As indicated in Figure 5, the accuracy comparisons within the
Figure 2. Detailed dimension of silencer used for experimental work.

Do = 0.4 (m); Lo = 0.8 (m); D1 = D2 = 0.0508m (2 inch); D45 = 0.2 (m);
L1 = L5 = 0.1 (m); L3 = 0.457 (m); L2 = L4 = 0.171 (m)

Figure 3. Acoustic impedance tube used for STL measurement in muffler.

Figure 4. Modeling and meshing for three-dimensional boundary element in SYSNOISE.

Figure 5. Profiles of STL curves in theory, sysnoise and experiment.
theoretical, the experiment data and BEM simulated data for the muffler model are in good agreement with each other.

Thus, the proposed fundamental mathematical model is valid under the theoretical cutoff frequency of $f_{c_1}$ which is given by

$$ f_{c_1} = \frac{1.84c}{\pi D} \left(1 - M_1^2\right)^{1/2} = 996 \text{ Hz} \tag{10} $$

where $D$ and $M_1$ refer to the maximum diameter and Mach number of muffler, respectively. Consequently, the developed model of single-chamber side muffler linked with numerical method is applied for the shape optimization in the following section.

5. Case Study

The noise control of a fan’s sound exhausted at the outlet is introduced as the numerical case. According to the sound pressure levels measured by octave band sound meter at 1 meter, the dominated (maximum) sound energy is recognized at frequency of 500 Hz. As shown in Figure 6, the available space, which is suitable for the elevated side inlet/outlet silencer is constrained as $0.4W \times 0.4H \times 0.8L$ (Lo = 0.8 M and Do = 0.4 M). The O.D. of exhaust pipe is confined to 0.0508 m. To efficiently depress the exhausted noise level, the sound energy emitted at the dominated frequency of 500 Hz is thus selected as the primary targeted frequency for noise elimination. Moreover, to obtain the maximal noise reduction, the shape optimization of silencer under space constraint is subsequently compulsory. In this study, both of the gradient methods (EPFM, IPFM, FDM) and genetic algorithm together with the graphic analysis will be applied in the following numerical assessments. The related design volume flow rate is confined to 0.8 (m$^3$/s).

6. Gradient Method

Before optimization being performed, the three-dimensional sensitivity of silencer’s acoustic performance to the design variables is analyzed first and shown as Figures 7~12. As the Figures indicated that both of $L_3$ and $D_{45}$ are the main design parameters. In addition, they also reveal that the smaller $D_1$ and $D_2$ will result in the higher acoustic performance in STL; therefore, the minimum values of $D_1$ and $D_2$ are determined by 0.0508 m simultaneously. Consequently, the OBJ function in Eq. (8) is then simplified as

$$ OBJ(X_1, X_2) = STL(L_3, D_{45}) \tag{11} $$

By using three kinds of gradient techniques, including IPFM, EPFM and FDM, the shape optimization on
silencer is then performed. The mathematic algorithms of optimal searching with respect to these techniques are depicted in Figures 13~15 individually; In addition, the mathematic forms of these techniques are described in Appendix C. Consequently, the optimal results with respect to three algorithms are shown in Tables 1~3 respectively.

7. Genetic Algorithm [10–13]

The concept of Genetic Algorithms, first formalized by Holland [10] and extended to functional optimization by D. Jong [11] later involves the use of optimization search strategies patterned after Darwinian notion of na-
For the optimization (minimum) of the objective function \((OBJ)\), the design parameters are \((X_1, X_2, \ldots X_k)\). As the bit length of \(bit_n\), which in terms of the chromosome, was chosen first, the interval of the design parameter \((x_k)\) with \([Lb, Ub]_k\) was thereafter mapped to the band of binary value. The mapping system between the variable interval of \([Lb, Ub]_k\) and the \(k^{th}\) binary chromosome of

\[
\begin{align*}
\text{Start} \\
\text{Given: } X^0, r_p, \gamma \\
\text{Minimize } \phi(X, r_p) \text{ as an unconstrained function} \\
\text{Converged?} \\
\text{Yes} \quad \text{Exit} \\
\text{No} \\
r_p \leftarrow \gamma r_p
\end{align*}
\]

**Figure 13.** Algorithm of exterior penalty function method.

\[
\begin{align*}
\text{Start} \\
\text{Given: } X^0, r_p, \gamma, s_p, \gamma^* \\
\text{Minimize } \phi(X, r_p, r_p^*) \\
\text{as an unconstrained function} \\
\text{Converged?} \\
\text{Yes} \quad \text{Exit} \\
\text{No} \\
r_p \leftarrow \gamma r_p \\
\gamma^* \leftarrow \gamma^* + \gamma
\end{align*}
\]

**Figure 14.** Algorithm of interior penalty function method.

\[
\begin{align*}
\text{Start} \\
\text{Choose: } X^0, s, \theta_0 \\
q \leftarrow 0 \\
X \leftarrow X^0 \\
q \leftarrow q + 1 \\
F \leftarrow F(X) \\
S \leftarrow S_{\text{feasible}} \\
\text{Solve one-dimensional} \\
\text{Search problem} \\
X^{*+1} \leftarrow X^* + \alpha S^* \\
\text{Converged?} \\
\text{Yes} \quad \text{Exit} \\
\text{No}
\end{align*}
\]

**Figure 15.** Algorithm of feasible direction method.

<table>
<thead>
<tr>
<th>Item</th>
<th>Starting point</th>
<th>Optimal point in IPFM</th>
<th>Optimal STL-dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L3 (m)</td>
<td>D45 (m)</td>
<td>L3 (m)</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.15</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>0.15</td>
<td>0.20001</td>
</tr>
<tr>
<td>6</td>
<td>0.31</td>
<td>0.2</td>
<td>0.31</td>
</tr>
<tr>
<td>7</td>
<td>0.42</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.42</td>
<td>0.15</td>
<td>0.4568</td>
</tr>
<tr>
<td>9</td>
<td>0.42</td>
<td>0.2</td>
<td>0.42</td>
</tr>
<tr>
<td>10</td>
<td>0.53</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>0.53</td>
<td>0.15</td>
<td>0.4568</td>
</tr>
<tr>
<td>12</td>
<td>0.53</td>
<td>0.2</td>
<td>0.53</td>
</tr>
<tr>
<td>13</td>
<td>0.64</td>
<td>0.1</td>
<td>0.64</td>
</tr>
<tr>
<td>14</td>
<td>0.64</td>
<td>0.15</td>
<td>0.64</td>
</tr>
<tr>
<td>15</td>
<td>0.64</td>
<td>0.2</td>
<td>0.64</td>
</tr>
</tbody>
</table>
was then built. The encoding from x to $B2D$ can be performed as

$$B2D_i = \text{integer}\left\{ \frac{x_i - Lb_i}{Ub_i - Lb_i} (2^{m_i} - 1) \right\}$$  \hspace{1cm} (12)$$

The initial population was built up by randomization. The parameter set was encoded to form a string which represented the chromosome. By evaluation of the objective function ($OBJ$), the whole set chromosome of $[B2D_1, B2D_2, \ldots, B2D_k]$ that changed from binary form to decimal form was then assigned a fitness by decoding the transformation system individually

$$\text{fitness} = OBJ(X_1, X_2, \ldots, X_k)$$  \hspace{1cm} (13a)$$

where

\begin{table} 
\centering
\begin{tabular}{|c|c|c||c|c||c|}
\hline
Item & \multicolumn{2}{c||}{Starting point} & \multicolumn{2}{c||}{Optimal point in EPFM} & \multicolumn{1}{c|}{Optimal STL-dB} \\
\hline
& L3 (m) & D45 (m) & L3 (m) & D45 (m) & \\
\hline
1 & 0.2 & 0.1 & - & - & Not available \\
2 & 0.2 & 0.15 & - & - & Not available \\
3 & 0.2 & 0.2 & - & - & Not available \\
4 & 0.31 & 0.1 & - & - & Not available \\
5 & 0.31 & 0.15 & - & - & Not available \\
6 & 0.31 & 0.2 & - & - & Not available \\
7 & 0.42 & 0.1 & - & - & Not available \\
8 & 0.42 & 0.15 & - & - & Not available \\
9 & 0.42 & 0.2 & - & - & Not available \\
10 & 0.53 & 0.1 & - & - & Not available \\
11 & 0.53 & 0.15 & - & - & Not available \\
12 & 0.53 & 0.2 & - & - & Not available \\
13 & 0.64 & 0.1 & - & - & Not available \\
14 & 0.64 & 0.15 & - & - & Not available \\
15 & \textbf{0.64} & \textbf{0.2} & \textbf{0.456} & \textbf{0.272} & \textbf{144.5} \\
\hline
\end{tabular}
\caption{Starting point and optimal result in EPFM}
\end{table}

\begin{table} 
\centering
\begin{tabular}{|c|c|c||c|c||c|}
\hline
Item & \multicolumn{2}{c||}{Starting point} & \multicolumn{2}{c||}{Optimal point in FDM} & \multicolumn{1}{c|}{Optimal STL-dB} \\
\hline
& L3 (m) & D45 (m) & L3 (m) & D45 (m) & \\
\hline
1 & 0.2 & 0.1 & 0.2 & 0.1 & 9.5 \\
2 & 0.2 & 0.15 & 0.2 & 0.15 & 14.6 \\
3 & 0.2 & 0.2 & 0.2 & 0.2 & 19.1 \\
4 & 0.31 & 0.1 & 0.2 & 0.147 & 14.38 \\
5 & 0.31 & 0.15 & 0.21 & 0.2 & 18.9 \\
6 & 0.31 & 0.2 & 0.31 & 0.2 & 11.03 \\
7 & \textbf{0.42} & \textbf{0.1} & \textbf{0.4568} & \textbf{0.1922} & \textbf{219.8} \\
8 & 0.42 & 0.15 & 0.4568 & 0.1888 & 218.9 \\
9 & 0.42 & 0.2 & 0.42 & 0.2 & 11.6 \\
10 & 0.53 & 0.1 & 0.4568 & 0.1903 & 213 \\
11 & 0.53 & 0.15 & 0.4568 & 0.1879 & 218.7 \\
12 & 0.53 & 0.2 & 0.53 & 0.2 & 19.1 \\
13 & 0.64 & 0.1 & 0.468 & 0.2 & 21.4 \\
14 & 0.64 & 0.15 & 0.5436 & 0.2 & 18.8 \\
15 & 0.64 & 0.2 & 0.64 & 0.2 & 11.5 \\
\hline
\end{tabular}
\caption{Starting point and optimal result in FDM}
\end{table}
\[ X_k = B2D_k \times (U_b - L_b) / (2^{\text{bit}_n} - 1) + L_b \]  

(13b)

By using the probabilistic computation weighted by the relative fitness, pairs of chromosome were selected as parents. The weighted roulette wheel selection was then applied. For the \( n \) set of parent in the mating pool, the weighted roulette wheel for the \( kk \)th individual was represented as

\[
\frac{\text{fitness}_{kk}}{\sum_{i=1}^{n} \text{fitness}_i} \times 100\%
\]

(14)

During a GA optimization, one set of trial solutions was chosen and “evolved” toward an optimal solution.

During the GA optimization, one pair of offspring was generated first from the selected parent by crossover with a probability of \( pc \). Genetically, mutation occurred with a probability of \( pm \) of which the new and unexpected point was brought into the GA optimizer’s search domain. To prevent the best gene from the disappearing and improve the accuracy of optimization during reproduction, the elitism scheme of keeping the best gene (one pair) in the parent generations was thus presented and developed. The process was terminated when number of generations exceeded a pre-selected value of \( \text{gen}_\text{no} \).

The operations in GA method are pictured in Figure 16.

According to Eq. (9), the objective function (OBJ) is defined as

\[ \text{OBJ}(X_1, X_2, X_3, X_4) = STL(L_3, D_4, D_1, D_2) \]  

(15)

Wherein \( L_3, D_4, D_1 \) and \( D_2 \) are selected as the design parameters during GA optimization.

By adopting the same setting of GA operators in our previous works, the related values are preset as [10]

\[ \text{bit}_n = 40; \text{popuSize} = 60; \text{gen}_\text{no} = 500 \]  

(16)

The constrained conditions for four variables are

\[ (L_b, U_b)_1 = (0.1, 0.2); (L_b, U_b)_2 = (0.1, 1.0); (L_b, U_b)_3 = (0.0508, 0.2); (L_b, U_b)_4 = (0.0508, 0.2) \]  

(17)

To reach a better solution during GA optimization, four cases in varying the GA operators of crossover, mutation as well as elitism are carried out and described as below:

Case 1: crossover (\( pc = 0.8 \)) + mutation (\( pm = 0.05 \)) +

---

**Figure 16.** GA operation during optimization.
elitism (Elt = 1)

Case 2: crossover (pc = 0.8) + mutation (pm = 0.05)
Case 3: mutation (pm = 0.05) + elitism (Elt = 1)
Case 4: crossover (pc = 0.8) + elitism (Elt = 1)

The simulated results with respect to four cases are illustrated in Figure 17 and Table 4. As indicated in Figure 17, case 1, in which all the GA operators are adopted concurrently, keeps the better value of STL at 500 Hz obviously.

8. Results and Discussion

By using the numerical results optimized by three kinds of gradient methods and one kind of GA method and plotting their STL curves together, the results are clearly illustrated in Figure 18 respectively. As indicated in Figure 18, their profiles are similar for each other; besides, the maximal STLs with respect to four kinds of different design-parameter sets are precisely occurred at 500 Hz concurrently. Consequently, there are multiple solutions for the shape optimization of muffler under space constraint.

9. Conclusion

The interest in maximal sound transmission loss (STL) of a single-chamber side muffler under space is arising. On the basis of one-dimensional plane wave theory, the relevant four-pole matrices of silencer and system matrix are deduced, accordingly.

By using both of the experimental data and the three-dimensional BEM simulated data, the accuracy of theoretical model with four-pole matrix method is then be verified and reassured with the comparisons of the simulated results for each other.

One case in eliminating the fan’s sound energy at 500 Hz is exemplified. Before optimization being performed, the sensitivity of silencer’s acoustic performance with respect to each variable has been analyzed;

![Figure 17](image1.png)

**Figure 17.** STL with respect to frequency for four cases in GA optimization.

![Figure 18](image2.png)

**Figure 18.** STL with respect to frequency for four kinds of optimal searching techniques (IPFM, EPFM, FDM, GA).

<table>
<thead>
<tr>
<th>CASE</th>
<th>Control parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pc</td>
<td>pm</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.05</td>
</tr>
</tbody>
</table>
and therefore, the appropriate parameters can be selected.

Using three kinds of gradient searching techniques, including EPFM, IPFM and FDM, the optimal solutions can be acquired individually. Moreover, the GA optimization has been prosecuted by varying the GA operators of crossover, mutation and elitism. Consequently, as the results revealed that the maximal STL are precisely located at the targeted frequency of 500 Hz.

This study definitely offers a simple advance to not only comprehensively arrange the best shape design in muffler but also satisfactorily compromise the effects for the constraint problem which is occasionally occurred in basement or other building.

### Acknowledgements

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### Appendix A

#### Transfer Matrix of Straight Duct

For a three dimensional wave with moving medium, the resultant wave governing equation is [2–5]

\[
\frac{\partial^2}{c^2} - c^2 \frac{\nabla^2}{\nabla^2} \cdot p = 0
\]  

(A1)

where the Laplacian \(\nabla^2\) with respect to the cylindrical system is

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}
\]  

(A2)

By using separation of variables method in Eq. (A1) and (A2), it yields

\[
p(r, \theta, z, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} J_m(k_{r,m,n}r) e^{i\omega t} \left( \frac{k_{r,m,n}}{k_0 - Mk_{r,m,n}^2} C_{1,m,n} e^{-\beta_{r,m,n}z} + \frac{k_{r,m,n}^2}{k_0 + Mk_{r,m,n}^2} C_{2,m,n} e^{\beta_{r,m,n}z} \right)
\]  

(A4)

\[
k_{r,m,n}^2 = \frac{M_k r_k + \left( k_0^2 - (1 - M_k^2)k_{r,m,n}^2 \right)^{1/2}}{1 - M_k^2}
\]  

(A5)

For the fundamental mode of \((m = 0, n = 0)\), only a plane wave would propagate if the frequencies of \(f\) are smaller than cut-off frequency of \(f_c\).

Where

\[
f_c = \frac{1.84c}{\pi D} (1 - M_k^2)^{1/2}
\]  

(A6)

For one dimensional wave propagating in a symmetric straight duct, the acoustic pressure and particle velocity are reduced as

\[
p(z,t) = \left( C_1 e^{-k z (1+M_k)} + C_2 e^{k z (1-M_k)} \right) e^{i\omega t}
\]  

(A7)

\[
u(z,t) = \left( \frac{C_1}{\rho_o c_o} e^{-k z (1+M_k)} - \frac{C_2}{\rho_o c_o} e^{k z (1-M_k)} \right) e^{i\omega t}
\]  

(A8)

Considering boundary conditions of pt 1 \((z = 0)\) and pt 2 \((z = L_1)\), Eqs. (A7) and (A8) can be rearranged as

\[
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix} =
\begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2
\end{bmatrix}
\]  

(A9)

\[
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix} =
\begin{bmatrix}
e^{-k_+ L_1} & e^{k_+ L_1} \\
e^{-k_- L_1} & e^{k_- L_1}
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2
\end{bmatrix}
\]  

(A10a)

Where

\[
k_+ = \frac{k_0}{1 + M_k} \quad k_- = \frac{k_0}{1 - M_k}
\]  

(A10b)

Combination of Eqs. (A9) and (A10) carries out that
Where

\[
\begin{align*}
  b1' &= \cos\left(\frac{kL_2}{1-M_i^2}\right); \quad b12' = j\sin\left(\frac{kL_2}{1-M_i^2}\right); \\
  b21' &= j\sin\left(\frac{kL_2}{1-M_i^2}\right); \quad b22' = \cos\left(\frac{kL_2}{1-M_i^2}\right)
\end{align*}
\]

(A11b)

As the derivation in Eq. (A11), the four-pole matrix between pt 4 and pt 5 with mean flow is expressed in equation (A12).

\[
\begin{bmatrix}
  p_1 \\ p_2 \\
  \rho V_c U_4 \\ \rho V_c U_5
\end{bmatrix}
= e^{-jM_i^2} \begin{bmatrix}
  \cos \left(\frac{kL_4}{1-M_i^2}\right) & j\sin \left(\frac{kL_4}{1-M_i^2}\right) \\
  j\sin \left(\frac{kL_4}{1-M_i^2}\right) & \cos \left(\frac{kL_4}{1-M_i^2}\right)
\end{bmatrix}
\begin{bmatrix}
  p_1 \\ p_2 \\
  \rho V_c U_4 \\ \rho V_c U_5
\end{bmatrix}
\]

(A12a)

Where

\[
\begin{align*}
  c1' &= \cos\left(\frac{kL_5}{1-M_i^2}\right); \quad c12' = j\sin\left(\frac{kL_5}{1-M_i^2}\right); \\
  c21' &= j\sin\left(\frac{kL_5}{1-M_i^2}\right); \quad c22' = \cos\left(\frac{kL_5}{1-M_i^2}\right)
\end{align*}
\]

(A12b)

Similarly, the four-pole matrix between pt 7 and pt 8 with mean flow is expressed in equation (A13).

\[
\begin{bmatrix}
  p_4 \\ p_5 \\
  \rho V_c U_7 \\ \rho V_c U_7
\end{bmatrix}
= e^{-jM_i^2} \begin{bmatrix}
  \cos \left(\frac{kL_8}{1-M_i^2}\right) & j\sin \left(\frac{kL_8}{1-M_i^2}\right) \\
  j\sin \left(\frac{kL_8}{1-M_i^2}\right) & \cos \left(\frac{kL_8}{1-M_i^2}\right)
\end{bmatrix}
\begin{bmatrix}
  p_4 \\ p_5 \\
  \rho V_c U_7 \\ \rho V_c U_7
\end{bmatrix}
\]

(A13a)

Where

\[
\begin{align*}
  d1' &= \cos\left(\frac{kL_9}{1-M_i^2}\right); \quad d12' = j\sin\left(\frac{kL_9}{1-M_i^2}\right); \\
  d21' &= j\sin\left(\frac{kL_9}{1-M_i^2}\right); \quad d22' = \cos\left(\frac{kL_9}{1-M_i^2}\right)
\end{align*}
\]

(A13b)

**Appendix B**

As derived by Munjal (1997), the four-pole matrix representing flow relations between pt 2 and pt 4 is expressed as equation (B1) [6].

\[
\begin{bmatrix}
  p_2 \\ v_2 \\
  \rho V_c U_2 \\ \rho V_c U_2
\end{bmatrix}
= e^{-jM_i^2} \begin{bmatrix}
  \cos \left(\frac{kL_6}{1-M_i^2}\right) & j\sin \left(\frac{kL_6}{1-M_i^2}\right) \\
  j\sin \left(\frac{kL_6}{1-M_i^2}\right) & \cos \left(\frac{kL_6}{1-M_i^2}\right)
\end{bmatrix}
\begin{bmatrix}
  p_2 \\ v_2 \\
  \rho V_c U_2 \\ \rho V_c U_2
\end{bmatrix}
\]

(B1a)

Where

\[
\begin{align*}
  Z_i &= -j \frac{c_c}{S_i} \cot(kL_i); \quad Y_i = Y_c = \frac{c_c}{S_i}; \\
  Y_7 &= \frac{c_c}{S_7}; \quad k_i = \left(\frac{S_i}{S_i}ight)^2 - 1
\end{align*}
\]

(B1b)

Similarly, the four-pole matrix representing flow relations between pt 5 and pt 7 is expressed as equation (B2).

\[
\begin{bmatrix}
  p_7 \\ v_7 \\
  \rho V_c U_7 \\ \rho V_c U_7
\end{bmatrix}
= e^{-jM_i^2} \begin{bmatrix}
  \cos \left(\frac{kL_7}{1-M_i^2}\right) & j\sin \left(\frac{kL_7}{1-M_i^2}\right) \\
  j\sin \left(\frac{kL_7}{1-M_i^2}\right) & \cos \left(\frac{kL_7}{1-M_i^2}\right)
\end{bmatrix}
\begin{bmatrix}
  p_7 \\ v_7 \\
  \rho V_c U_7 \\ \rho V_c U_7
\end{bmatrix}
\]

(B2a)

Where

\[
\begin{align*}
  Z_i &= -j \frac{c_c}{S_i} \cot(kL_i); \quad Y_i = Y_c = \frac{c_c}{S_i}; \quad Y_i = \frac{c_c}{S_i}; \\
  k_i &= \left(1 - \frac{S_i}{S_i}ight) / 2; \quad S_i = S_4 = S_7 = S_8
\end{align*}
\]

(B2b)

The transformation formula of \( V_i \) (the acoustic mass velocity) and \( u_i \) (acoustic particle velocity) with respect to point \( i \) is

\[
V_i = \rho_i V_c U_i
\]

(B3)

By substituting Eq. (B3) into Eqs. (B1) and (B2) respectively, the equivalent forms are derived in Eqs. (B4) and (B5).
The mathematical gradient formulation in maximizing the STL is

\[
\text{Minimize } F(X) = -\text{STL}(X) \text{ Objective function}
\]

Subject to \( g_j(X) \leq 0 \) \( j = 1, 4 \) inequality constraints

For exterior penalty function method, \( \Phi \) is defined as

\[
\Phi(X,r_p) = F(X) + r_p P(X)
\]

\[
= F(X) + r_p \sum_{j=1}^{4} \{ \max[0, g_j(X)] \}^2
\]

For interior penalty function method, \( \Phi \) is defined as

\[
\Phi(X,r_p) = F(X) + r_p \sum_{j=1}^{4} \frac{-1}{g_j(X)}
\]

References


[12] Jong, D., An Analysis of the Behavior of a Class of


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