An Analysis of Wireless Network Coding for Unicast Sessions: The Case for Coding-Aware Routing[1]

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IEEE INFOCOM 2007

Presented by Xiucai Ye
2010.11.8

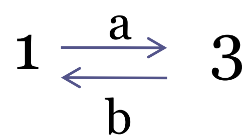
[1] S. Sengupta, S. Rayanchu, and S. Banerjee, “An Analysis of Wireless Network Coding for Unicast Sessions: The Case for Coding -Aware Routing,” *IEEE INFOCOM, 2007*.

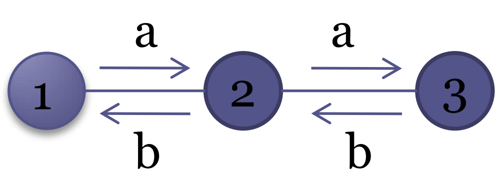
Outline

- Introduction
- Notation and Modeling Assumption
- Scheduling Broadcast Transmissions
- Coding Aware Routing
- Conclusion

Introduction

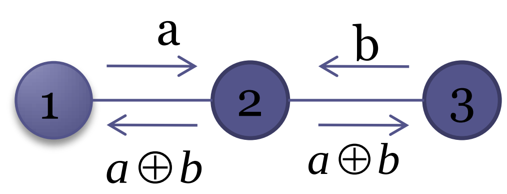
- Network coding is gaining popularity as a mechanism to increase the utilization of both wired and wireless networks.
- Example: 3 wireless nodes,





With general method

4 transmissions



With COPE[2] (network coding)

3 transmissions

Lead to a throughput improvement of 33%

[2] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard and J. Crowcroft, “XORs in the Air: Practical Wireless Network Coding”, ACM SIGCOMM 2006.

Introduction

- COPE: an opportunistic network coding scheme

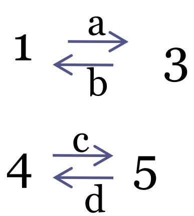
-- Uses XOR operation to perform coding.

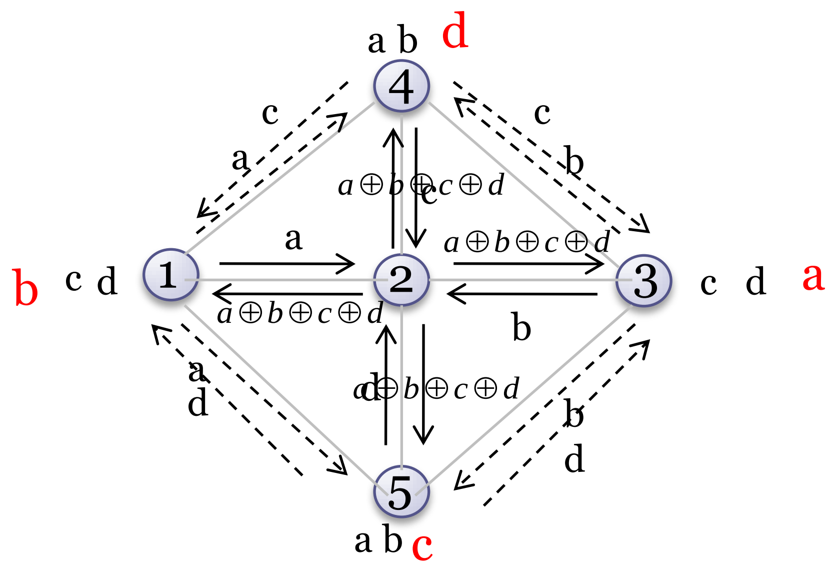
-- Two properties:

1) *Opportunistic Coding: Each wireless node uses only* packets in its local queues for coding.

2) *Opportunistic Listening: Exploiting the broadcast nature* of the wireless medium, it sets each node to snoop on all packets communicated by its neighbors. The snooped packets are used in coding decisions.

Example





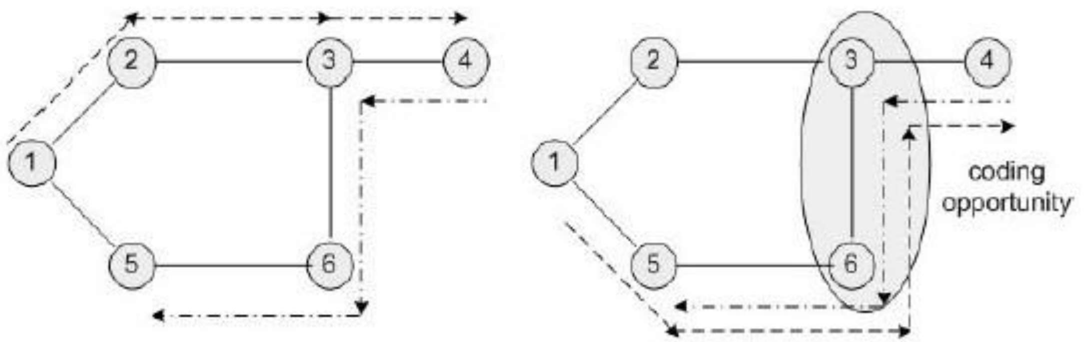
General method: 8 transmissions
Coding without opportunistic listening: 6 transmissions
COPE: 5 transmissions

Introduction

- ***Contributions of this paper***
- Analyze throughput improvements obtained by COPE-type network coding in wireless networks from a theoretical perspective.
- Provide a theoretical framework for investigating the potential interactions between coding opportunities and routing decisions (coding aware routing).
- Introduce the notion of joint coding-aware and interference-aware routing, illustrate the tradeoffs between needs of increased coding and decreased interference.

Coding aware routing

- Example: two flows: 1→ 4, 4→ 5.



Coding-oblivious routing
(a)

Coding (and interference) aware routing
(b)

In absence of network coding, Figure (a) shows the shortest and minimum interference paths for the flows.
The throughput of the flows can be improved by choosing paths to get the opportunity to perform network coding.

Notation and Modeling Assumption

Notation

- The wireless network topology is modeled as a graph G = (N,E) with node set N and (directed) edge set E.

- The sets of incoming and outgoing edges at node *i are denoted by E−(i) and E+(i)* respectively.

- *e = (i, j) represent a directed link in the* network from node *i to node j*. The transmitting node for link *e will be denoted by t(e) and its receiving node by r(e)*. Denote the reverse of link *e = (i, j) by* $\bar{e}$ *= (j, i)*.

- The rate of transmission on link *e* is denoted by *Re* and its delivery probability by p_e*, the effective rate of transmission on* link *e is* $u_e = p_e R_e$.

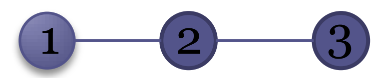
Notation and Modeling Assumption

- *D is the set of demands. A demand k* $\in$ *D has source* node *s(k), destination node d(k), and traffic value t(k).*
- *For* a given routing/coding scheme, the *throughput is defined as* the maximum multiplier λ *such that all demands with their* traffic values multiplied by λ *can be feasibly routed by the* network. (Objective: maximize λ *for coding-aware network routing).*
- For a path *P and links e, e1, e2, use e* $\in$ *P to denote* that link *e is on path P and e1e2* $\in$ *P to denote that path P* contains link *e1 followed by link e2 (r(e1) = t(e2)). For a path P and node i, use i* $\in$ *P to denote that node i is on path P.*

Notation and Modeling Assumption

Coding Rules and Modeling Assumptions

- *k packets* $p_1, p_2, \ldots, p_k$ *at a node that have distinct* next-hop nodes $n_1, n_2, \ldots, n_k$ *respectively. Suppose these are* coded as $p = p_1 \oplus p_2 \oplus \ldots \oplus p_k$, and it is broadcast to all the above next-hop nodes. This is valid if the next-hop node n_i *for each* p_i *already has all other* p_j , $i \neq j$.



When

(i) node n_i *is the previous-hop node of packet* p_j*, or*

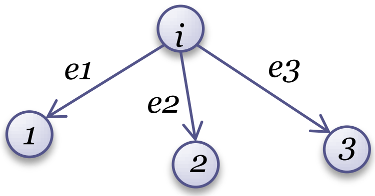
(ii) node n_i *overheard packet* p_j *from the transmission*

of its previous-hop node (opportunistic listening).

Scheduling Broadcast Transmissions

Broadcast Transmission Rates

- Let *B be a subset of outgoing links at some node.*



B={e1,e2,e3}

- Assume that the transmission rate for broadcast on *B is the minimum* rate of its *component* links, $R(B) = \min_{e \in B} Re$.
- Losses on individual links are independent, the delivery probability of the broadcast *B* is at least $\prod_{e \in B} p_e$

.

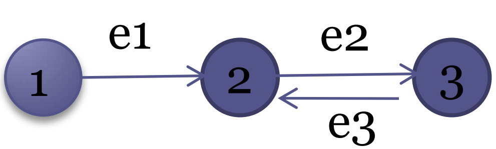
- The effective rate of transmission for broadcast on *B is given by*

$$u(B) = (\prod_{e \in B} p_e) \min_{e \in B} R_e$$

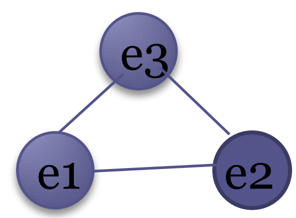
Scheduling Broadcast Transmissions

Conflict graph[3] :

- The nodes in conflict graph correspond to links in the topology graph. Two nodes are connected by an (undirected) edge in the conflict graph if the corresponding links cannot be scheduled simultaneously.
- Scheduling link transmissions for link interference are then modeled using constraints corresponding to *cliques in the conflict* graph.



Topology graph

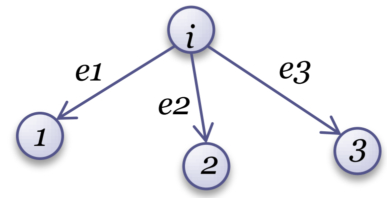


Conflict graph

[3]K. Jain., J. Padhye, V. N. Padmanabhan, L. Qiu, "Impact of Interference on Multi-hop Wireless Network Performance", ACM MOBICOM 2003,

Scheduling Broadcast Transmissions

Broadcast Conflict Graph F



B={e1,e2,e3}

- A broadcast transmission at node *i on a subset B of its* outgoing links will represented as (*i,B) and the associated* broadcast traffic as y_i^B .
- Each node in topology graph represents a broadcast transmission (*i,B*). *r(B)* denote the set of receiver nodes for the links in broadcast set *B*.
- *Two broadcasts (i1,B1) and (i2,B2) interfere and hence* have an edge between them in the broadcast conflict graph **F** if either
  - *Some node j* $\in$ *r(B1) is within interference range of* node *i2, or*
  - *Some node j* $\in$ *r(B2) is within interference range of* node *i1*

Scheduling Broadcast Transmissions

Clique Constraints for Broadcast Transmission Scheduling

- Consider a *clique in the broadcast conflict graph*. *Let C* be the set of broadcast nodes (*i*,*B*) *that correspond to nodes of* this clique. The fraction of time that broadcast (*i*,*B*) *is active* is $y_i^B / u(B)$.

- *Since the broadcasts in C mutually conflict with* each other, at most one of them can be active at any given time. This can be modeled by the constraint

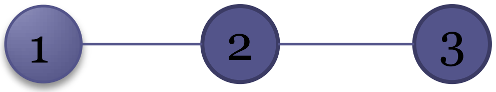
$$\sum_{(i,B)\in C} \frac{y_i^B}{u(B)} \leq 1 \quad \forall \text{ cliques } C \text{ in } F$$

Coding Aware Routing

Two cases:

- Without opportunistic listening

The coding opportunity at a node involves XOR-ing exactly two packets – these packets enter and leave the node using the same links but in opposite directions.



- With opportunistic listening

The coding opportunity at a node involves XOR-ing at least two packets.

Coding Aware Routing

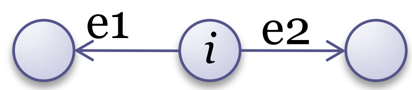
Without opportunistic listening

Formulate the problem

- p^k *denote the set of available paths from source s(k)* to destination *d(k) for routing demand k.*
- Routing variable $f^k(P)$ *denotes the amount* of traffic on path *P for routing demand k.*
- y_i^B denote the traffic that is broadcast at node *i on link set B* $\subseteq$ *E+(i),*
- $|B| \leq 2$. *If* $|B| = 1$, *the corresponding transmission* is a unicast (of a native packet) on the single link in *B*.
- *λ denote the throughput for routing all demands in D.*

Coding Aware Routing

Without opportunistic listening



- linear program (LP):

$$\text{maximize} \quad \lambda$$

subject to

$$\sum_{P \in \mathcal{P}^k} f^k(P) = t(k)\lambda \quad \forall\, k \in D \qquad (4)$$

$$y_i^{\{e_1,e_2\}} \le \sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni \bar{e}_1 e_2} f^k(P) \quad \forall\, e_1, e_2 \in E^+(i), i \in N \qquad (5)$$

$$y_i^{\{e_1,e_2\}} \le \sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni \bar{e}_2 e_1} f^k(P) \quad \forall\, e_1, e_2 \in E^+(i), i \in N \qquad (6)$$

$$y_i^{\{e\}} = \sum_{k \in D, s(k)=i} \sum_{P \in \mathcal{P}^k, P \ni e} f^k(P) + \sum_{e_1 \in E^-(i)} \Big[\sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni e_1 e} f^k(P) - y_i^{\{e, \bar{e}_1\}}\Big] \quad \forall\, e \in E^+(i), i \in N \qquad (7)$$

$$\sum_{(i,B) \in C, |B| \le 2} \frac{y_i^B}{u(B)} \le 1 \quad \forall \text{ cliques } C \text{ in } F \qquad (8)$$

(4) : the total traffic routed on the available paths for a demand must equal the demand value multiplied by its throughput.

(5), (6): $y_i^{\{e1,e2\}}$ is the maximum amount of coded traffic *that can be* broadcast on outgoing links *{e1, e2} at node i, it* is at most the total traffic traversing node *i along link sequence* $\bar{e}_1 e_2$ *or* $\bar{e}_2 e_1$.

(7): $y_i^{\{e\}}$ is the total amount of traffic unicast on outgoing link *e at node i.*
Two parts: 1) the traffic that originates at node *i and is sent on link e;* 2) the amount of transit traffic at node *i with next-hope could not be coded with other flows.*

(8) : broadcast transmission scheduling constraints in the broadcast conflict graph

Coding Aware Routing

With opportunistic listening

- The coding opportunity for a packet at a node is determined by two factors:
- *The combination of its incoming and outgoing links at* that node,
- *Whether the packet was received at that node as a coded* or native packet.

-- *useful listening opportunities* that involve transmission of a native packet only.

- Use a structure *S to specify* a coding opportunity at node *i*, $s = (e1e2, v)$, *e1 is the incoming link of the packet, e2* is the outgoing link of the packet, and $v = c, n$ *depending on* whether the packet was received as coded (*c*) *or native* (*n*), $s \in S$.

Coding Aware Routing

With opportunistic listening

- Formulate the problem
- Let *xi(S)* denote the traffic associated with coding structure *S at node i*– this is the traffic amount associated with each *e1e2 link-pair* participating in the structure.

- $z_i^k(P)$ *is the portion* of the traffic on path *P for demand k that is transmitted as* native from node *i*.

- The broadcast set *B in variables* y_i^B can include all the outgoing links at node *i*.

Coding Aware Routing

With opportunistic listening LP

$$\text{maximize} \quad \lambda$$

subject to

$$\sum_{P \in \mathcal{P}^k} f^k(P) = t(k)\lambda \ \ \forall\, k \in D \quad (9)$$

$$\sum_{S \ni (e_1 e_2, n)} x_i(S) \leq \sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni e_1 e_2} z^k_{t(e_1)}(P) \quad \forall\, e_1 \in E^-(i), e_2 \in E^+(i), i \in N \quad (10)$$

$$\sum_{S \ni (e_1 e_2, c)} x_i(S) \leq \sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni e_1 e_2} [f^k(P) - z^k_{t(e_1)}(P)] \quad \forall\, e_1 \in E^-(i), e_2 \in E^+(i), i \in N \quad (11)$$

$$\sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni e_1 e_2} f^k(P) = \sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni e_1 e_2} z^k_i(P) + \sum_{S \ni (e_1 e_2, n)} x_i(S) + \sum_{S \ni (e_1 e_2, c)} x_i(S) \quad \forall\, e_1 \in E^-(i), e_2 \in E^+(i), i \in N \quad (12)$$

Coding Aware Routing

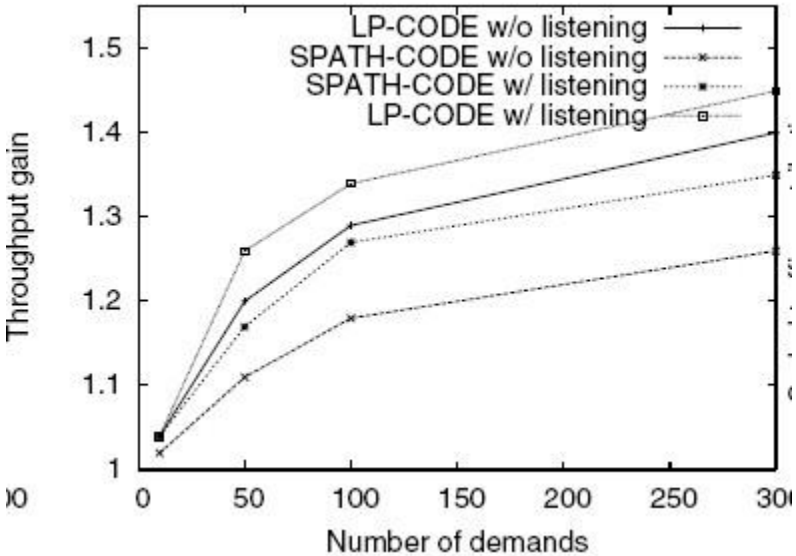
With opportunistic listening

$$
\begin{aligned}
z^k_{s(k)}(P) &= f^k(P) \;\; \forall\, P \in \mathcal{P}^k, k \in D && (13) \\
z^k_i(P) &\leq f^k(P) \;\; \forall\, i \in P - \{s(k), d(k)\}, \\
& \quad P \in \mathcal{P}^k, k \in D && (14) \\
y^{\{e\}}_i &= \sum_{k \in D} \sum_{P \in \mathcal{P}^k, P \ni e} z^k_i(P) \\
& \quad \forall\, e \in E^+(i), i \in N && (15) \\
y^B_i &= \sum_{S \in \Gamma_i, b(S) = B} x_i(S) \\
& \quad \forall\, i \in N, |B| \geq 2 && (16) \\
\sum_{(i,B) \in C} \frac{y^B_i}{u(B)} &\leq 1 \;\; \forall \text{ cliques } C \text{ in } F && (17)
\end{aligned}
$$

Coding Aware Routing

With opportunistic listening

- Evaluation



SPATH-CODE: shortest path routing with network coding
LP-CODE: network coding aware multi-path routing
w/o listening： without opportunistic listening
w/ listening: with opportunistic listening

Conclusion

- Provide a theoretical framework for investigating the coding aware routing.
- The formulations provide a systematic method to quantify the benefits of using network coding.
- The formulation results are valid both in presence and absence of opportunistic listening mechanisms.
- Introduced the notion of joint coding-aware and interference aware routing for choosing routes that optimize the tradeoffs between the coding opportunities and wireless interference.

Thank you very much!