Mixing Micro and Macro Representations of Traffic Flow: a Hybrid Model Based on the LWR Theory

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ABSTRACT

Hybrid traffic flow models, coupling a microscopic (vehicle based) and a macroscopic (flow based) representations of traffic flow may be a useful tool to better understand the relationships between the various types of representation. They can also be a basis for implementing various model extensions, which may be easier using one type of representation or the other. The hybrid model presented here combines a flow and a vehicular representations of the same model, which is the classical Lighthill-Whitham-Richards model. The use of a simple and unique model makes it possible to focus on the specific problems raised by translating boundary conditions from vehicular to flow formulation and conversely. This translation is made in order to ensure the conservation of flow, a proper transmission of the information both downstream and upstream, and to minimize the perturbations induced by the transitions between a continuous and a discrete representations of flow. The resulting model is shown to have good properties, particularly concerning congestion propagation and flow smoothing at the interfaces between the two models.
INTRODUCTION

Historically, car-following and continuum traffic flow models have been developed on different theoretical bases. This has given birth to two main kinds of representation of traffic dynamics: microscopic representations, based on the description of the individual behavior of each vehicle, and macroscopic representations describing traffic as a continuous flow obeying global rules. However, the consistency between both types of models has early been studied, showing that some car-following models include a flow-density relationship under steady-state conditions (1), or pointing out similar non-linear effects of both types of models, such as shock waves (2). There have also been some attempts to develop continuum models from car-following theory, as made by Payne (3), or, more recently, to develop car-following models consistent with the continuum theory [e.g Del Castillo (4)].

Several motivations exist for developing hybrid models combining microscopic and macroscopic aspects. The first one is to adapt the traffic flow model to the scale of the phenomena to be considered. When modeling some specific elements of a network, like weaving sections, representation of local phenomena is very important since they can have global consequences. In that case, a microscopic representation is more accurate than a macroscopic one. Furthermore, there are some places where stochastic aspects are fundamental: toll gates, roundabouts… A correct description of these elements requires a microscopic approach. On the other hand, homogeneous road sections are better described, and with fewer parameters, through macroscopic models. The use of hybrid models can make it possible to combine the advantages of both types.

Some attempts have been made to develop such hybrid models, mostly without much consideration of consistency between the two representations (5; 6). A more comprehensive approach has been adopted in (7), but using complex models whose properties are difficult to fully control. These attempts have not covered the whole range of problems raised by trying to connect two different models, as will be examined in the next sections.

The approach adopted here is progressive. In a first step, a coupling is made between two representations of a unique and simple model (the continuum Lighthill-Whitham-Richards (LWR) model and a LWR-based car-following model) in order to determine the main theoretical difficulties in the development of hybrid models. The objective is to analyze precisely the effects of changing the representation when the two models used have quite similar phenomenological rules, to ensure that information propagates correctly and to reduce the introduction of undesirable effects. Using as a basis the mere LWR model, for which analytical solutions are easily derived, is also a way to have a better understanding of any side effect caused by the multiple representations, particularly at interfaces. A second step will consist of adding extensions to the microscopic model to make possible specific representations and to generalize the hybrid structure. This will primarily concern inclusion of stochastic aspects.

This paper presents the first step of the approach, the second one still being under development. The first part of this paper presents a classification of traffic models, making a distinction between the vehicle or flow representation of traffic, and the underlying flow or vehicular behavior model. The paper then presents an overview of the hybridization problem, pointing out the basic question of the flow/vehicular formulation of boundary conditions. The developed model is then described and some preliminary results presented.

CLASSIFICATION OF TRAFFIC FLOW MODELS

Though the basic theoretical premises are clear, no precise definition exists of what is a microscopic or a macroscopic model, and some models cannot be classified correctly. For example, the model developed by Del Castillo in (4) has both microscopic and macroscopic characteristics: it represents vehicles individually, which is a microscopic characteristic, but those vehicles obey a flow rule, which is a macroscopic characteristic. It can thus be considered as a particle discretisation of a continuum model rather than a car-following model. Therefore, one has to distinguish:

- The representation scale of the model, which can be vehicle or flow,
- The behavioral rule, which can be individual or collective.

We define a new classification of traffic flow models based on these two criteria. The first one differentiates the vehicle represented (VR) models and the flow represented (FR) models. The second one makes a distinction between the models that try to reproduce the trajectory of a single vehicle reacting to its environment (individual behavioral or phenomenological rule), and the models that are based on the analysis of the propagation of concentrations or flows (collective behavioral or phenomenological rule). This makes possible a comprehensive classification of traffic flow models. For example, classical follow-the-leader models
are VR model with an individual behavioral rule, the LWR model is a FR model with a collective behavioral rule and the model proposed by Del Castillo is a VR model with a collective behavioral rule.

This classification does not refer to the deterministic or stochastic characteristics of the models. It is commonplace to associate microscopic models with stochastic aspects, and macroscopic models with deterministic ones. It is our opinion that these aspects must be considered apart from the models themselves: one can imagine deterministic microscopic models and stochastic macroscopic ones as well. We have thus to distinguish the coupling of FR and VR models (a flow and a vehicle based representations) from the coupling of stochastic and deterministic models. This will be discussed later on.

This new classification has the advantage to clarify the relationships between traffic flow models. Indeed, some are based on the same behavioral rule, and only change by their representation scale. For example, Payne’s model (3) is initially derived from a car-following model. This model is thus theoretically a FR one using an individual behavioral rule. These two models should represent the same phenomena but with two different representation scales (it actually happens that the approximations made in the derivation of the Payne’s model from the car-following equation makes it quite different from the initial vehicular model). In an hybridization process, coupling different representations or coupling models with different phenomenological rules results in different issues. This is the justification of the progressive approach adopted here.

A GENERAL VIEW OF HYBRID MODELS

Definition and Properties

The classification presented above provides us with a possible definition of a hybrid model, as the coupling of a VR model and a FR one, whatever the behavioral rules of both models are. In this paper, we will restrict our study to a special case of hybrid models: the introduction of a VR segment into a flow representation, as shown in figure 1. The two limits between flow and vehicle representations are called interfaces.

When analyzing this kind of models, some general properties can be defined. On the first hand, hybrid models must satisfy two constraints in order to be consistent:

- The conservation of vehicles at interfaces must be ensured (that is no vehicle must be lost or created),
- Information must propagate correctly at interfaces both in the upstream and downstream directions under free flow and saturated conditions (that is no information is lost and each model has to receive valid boundary conditions).

If the first constraint is not satisfied, there is an error in the propagation of traffic, and the model is not valid. If the second one is not satisfied, the two traffic flow models will give non-valid results.

On the other hand, hybrid models can be classified depending on the behavioral rules of the two traffic flow models. A hybrid model will be:

- homogeneous if the same behavioral rule is used in both models (that is the FR and the VR models must give similar results, at least in stationary conditions),
- heterogeneous otherwise.

Heterogeneous hybrid models will give two different traffic states for a single traffic condition depending on the representation used. In this case, there is no reason for the propagation speed of information to be the same for the FR and the VR models. So, a requirement for this class of hybrid model is to translate the information from the FR model to the VR one (and conversely) fulfilling the constraints of both models in terms of boundary conditions.

Those definitions and properties have already been presented in (8), together with the classification proposed in the previous section.

As the idea of hybridization is quite recent, few realizations of hybrid models exist. To the authors knowledge, only three correspond to our definition. The first one (6) is called MICMAC and mixes two distinct models: SIMRES (a discretized version of the Payne model) and SITRA-B+ (a car-following model). As those models are quite different, compatibility between them is studied, but only in stationary conditions. This hybrid model seems to verify the two constraints defined above. However, the transition from a continuous flow to a
vehicle based representation, and conversely, is not treated in a comprehensive way. Results are provided on examples using a very large time step, avoiding most possible undesirable effects to appear. The second model (7) is a coupling between a car-following model named Intelligent Driver Model (IDM), and a continuum model directly derived from the IDM. The consistency between the two models is studied in detail, but little information is given concerning the communication process at interfaces. It is thus quite difficult to fully understand the behavior of the model. The third and most recent example (5) is a coupling between a simplified higher order continuum traffic flow model and the IDM. Here, the transmission of information between the models is explained in detail, but consistency is not treated. This model has some limitations: it is valid only under free flow conditions and flows at interfaces have an oscillatory behavior (because of the calculus of flows in the vehicle representation).

Examining those papers makes clear two main difficulties to be encountered when coupling two different traffic flow models:

- The consistency or compatibility of both models,
- The translation of boundary conditions at interfaces.

The first one raises the question of the comparison of different models in order to estimate the impact of the difference of behavior of the VR and FR models on the resulting hybrid model. The second one is directly related to the problem of changing a continuous flow information into a discrete one and conversely.

The hybrid model presented here is based on the first order LWR traffic flow theory (9; 10) and uses vehicular and flow representations of this unique model. This approach is quite different from those present in the literature since the three models listed above are based on higher order models, but the issues of the coupling are similar. The objective is to focus on the issues raised by the coupling of two different representations. A further step will be to generalize the approach to other VR or FR models.

A single lane with uniform characteristics and a unique type of vehicles are considered. Considering several lanes would not change fundamentally the hybridization problem but would add complexity. Taking account of several vehicle classes could be a further extension.

An Idealized Approach of Hybridization

A first hybrid model based on the LWR theory has already been presented in (8). It combines two new event-based calculations of the analytical solutions of the LWR model: the FR model uses a discretization of the fans of the entropic solution in “acceleration waves”, and the VR model is derived from the FR one. As the VR model is ideally consistent with the FR one and both models use a common type of information, the authors developed a hybrid model where information propagates through interfaces without any distortion. However, this “ideal” model has few practical applications. When adding extensions such as stochastic aspects or heterogeneity, it is very difficult (if not impossible) to satisfy the conservation constraint at interfaces. A conclusion of this paper is that, when adding extensions to a homogeneous deterministic hybrid model, we need to define time horizons to evaluate the flow conservation, i.e to differentiate possible flow oscillations or viscosity effects from non conservation.

Since it appears quite impossible to introduce such a constraint in this first model, a more pragmatic approach is adopted, using the now classical space-time discretization of the LWR model based on the Godunov scheme (11; 12). Indeed, time steps present in the discretization will be used to define time horizons for imposing the conservation constraint when extending the model. The resulting hybrid model will be less “ideal” from a theoretical point of view, but will have extensions abilities, and consequently will be more useful for practical applications.

THE HYSutra MODEL

In connecting vehicular and flow representations of the same model, the translation of boundary conditions at the interfaces remains as the main possible cause of errors, when moving from a continuous to a discrete representation of traffic and conversely. Our objective is to minimize those errors in order to limit undesirable effects.
**General Presentation of the two Traffic Flow Models**

The two models to be coupled are derived from the LWR model \((9; 10)\). It is based on three variables (density \(K\), flow \(Q\) and average speed \(V\)) and three equations:

\[
\frac{\partial K}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad \text{(conservation equation)} \tag{1}
\]

\[
Q = KV \quad \text{(definition of the average speed)} \tag{2}
\]

\[
Q = Q_e(K) \quad \text{(equilibrium relation between flow and density)} \tag{3}
\]

The FR model used here is the Strada model\((13)\), a discretized version of the LWR model close to Daganzo’s Cell transmission model \((12)\). Each link is divided into cells of length \(\delta x\), and we define a time step \(\delta t_f\). For each cell \(i\), we define \(K^i_t\), the average density at time \(t\), and \(Q^i_t\), the average exit flow from \(t\) to \(t + \delta t_f\). At each time \(t\), \(K^i_t\) is known for all \(i\), and \(K^{i+\delta x}_t\) is computed applying the conservation of vehicles, which leads to the following formula:

\[
K^{i+\delta x}_t = K^i_t + \frac{\delta t}{\delta x} (Q^{i+1}_t - Q^i_t) \tag{4}
\]

To compute \(Q^i_t\), two functions are introduced: the traffic supply \(\Omega_s(K^i_t)\) and the traffic demand \(\Delta_s(K^i_t)\). A consequence of those definitions is that boundary conditions for a link are the upstream demand at the entry point and the downstream supply at the exit point.

This model is valid if the two discretization steps \(\delta x\) and \(\delta t_f\) satisfy the following inequality (Courant-Friedrich-Lewy condition):

\[
\frac{\delta x}{\delta t_f} \geq V_f \tag{5}
\]

\(V_f\) being the free-flow speed.

In order to minimize numerical viscosity, it is convenient to take: \(\frac{\delta x}{\delta t_f} = V_f\). More details on that model can be found in \((13)\).

This will be the FR part of the hybrid model. The VR part must also be based on the LWR theory, but few models correspond to that definition. To the authors’ knowledge, only three models can be used: the model developed by Del Castillo \((4)\), the optimal velocity model initially proposed by Newell \((2)\) and presented by Helbing in \((14)\) and the model developed in \((8)\).

Del Castillo first presents a VR model named the propagation model. This model supposes that the speed of a vehicle \(i\) is equal to the speed of the vehicle \(i-1\) just in front with a time lag \(T_r\):

\[
V_i(t + T_r) = \lambda V_{i-1}(t) \tag{6}
\]

with \(\lambda\) a gain factor. Del Castillo shows that this model is consistent with the LWR theory if \(\lambda = 1\) and \(T_r\) is equal to “the time it takes for a disturbance to propagate between two consecutive vehicles” in the LWR theory. However, time \(T_r\) is quite difficult to obtain in the general case, so Del Castillo computes it only in the case of small disturbances. The VR model developed in \((8)\) is an alternative solution to compute time \(T_r\) in the general case. Unfortunately, this results in the “idealized” hybrid model described above, which is unusable in practice.

In the same paper, Del Castillo uses the propagation model as a basis to develop a new VR model consistent with the LWR theory. The resulting model is similar to the propagation model in the case of small
disturbances. However, this model includes some approximations resulting in instabilities in the vehicles trajectories when a platoon decelerates.

The optimal velocity model studied by Newell in (2) considers the speed of a vehicle i as a function $G$ of the spacing between the vehicles with a certain delay $\delta t$:

$$V_i(t) = G\left(x_{i-1}(t-\delta t) - x_i(t-\delta t)\right)$$

$x_i(t)$ being the position of vehicle i at time $t$.

Since spacing is the inverse of concentration, $G$ is similar to an equilibrium speed-density relation. Thus, with a delay $\delta t$ small enough, this model gives results very similar to those of the LWR theory (as the appearance of shock waves).

Analyzing those three models led us to use the optimal velocity model with the same equilibrium relation between flow and density as for the FR model. This model is a VR model, has a behavioral rule derived from the LWR theory and it does not suffer from undesirable effects. Actually, the choice of this model is not crucial for our hybrid model, and another one could be chosen: the only constraint is that it be a VR model with a behavior based on the LWR theory.

In order to derive numerical solutions, this model must be discretized in time. The time step will be chosen equal to $\delta t$ for simplicity, and will be noted $\delta t_f$. To make possible a proper representation of vehicles trajectories, this time has to be much shorter than $\delta t_f$.

**Hybridization Scheme**

We have two traffic flow models chosen in order to be consistent: the discretized model Strada and the vehicular optimal velocity model. The hybridization scheme of both models finally consists of properly translating boundary conditions at interfaces between the two models: information must propagate simultaneously in both directions (from the FR model to the VR model and conversely), and also be able to propagate upstream and downstream depending on the fluid or congested traffic conditions (to satisfy the second constraint). The resulting hybrid model has to be conservative (to satisfy the first constraint). In this section, a hybrid scheme meeting these requirements will be presented, and we will see that the main difficulty in the translation of boundary conditions is due to the difference of representation of the traffic flow models.

A first study of this hybridization scheme has shown that it was very difficult (if not impossible) to realize a direct coupling while respecting the behavior of both traffic flow models. We thus introduce a “transition cell” at each interface in order to split up the transition process. The upstream transition cell will ensure transition of traffic from the FR model to the VR one, and conversely for the downstream transition cell (figure 2). The specificity of the transition cell is that both traffic flow models coexist there: both vehicle characteristics (position, speed of vehicles) and flow characteristics (concentration, flow), computed by aggregation of the vehicle ones, are determined. This makes it possible to determine correct and consistent boundary conditions for both models, i.e a supply or a demand for the FR model and vehicle trajectories for the VR model. It is clear that only the global flow characteristics will be valid to describe traffic states on that cell since trajectories of vehicles may be modified for the needs of hybridization. The VR cell and the two transition cells will be called the vehicular zone.

Supposing that the state of the system (concentration of the FR cells, position and speed of the vehicles in the vehicular zone) is known at time $t$, we want to determine the state of the system at the next time step. Since we have two different models that use two different time steps, the first problem to solve is to synchronize those models. Each model knows the state of a part of the system at discrete times, but we need to define times when both models will know the state of the system to exchange this information. As $\delta t_v < \delta t_f$, we impose $\delta t_f$ to be a multiple of $\delta t_v$, that is: $\delta t_f = N\delta t_v$, $N$ being an integer. A direct consequence is that at time $t + \delta t_f$, both models will know the state of the system, which allows the transmission of boundary conditions.

The problem is then to determine the state of the system at time $t + \delta t_f$. Firstly, we calculate the upstream demand and the downstream supply given by the FR model for the transition cells for the current time
step, that is between \( t \) and \( t + \delta t_F \). The upstream demand represents the number of vehicles that wish to enter the upstream transition cell, and the downstream supply the number of vehicles which could exit the downstream transition cell during the time step \( \delta t_F \). They are maximum values, which means that the flows that will be realized must be less or equal (otherwise, the FR model behavior would not be respected). Those values are imposed as boundary conditions for the vehicular zone during \( \delta t_F \). The VR model can thus compute the evolution of the vehicular zone with the time step \( \delta t_v \) until \( t + \delta t_F \). As the state of the vehicular zone will be known at \( t + \delta t_F \), flows effectively realized at the interfaces of the transition cells from \( t \) to \( t + \delta t_F \) can be deduced. The entry flow of the upstream transition cell and the exit flow of the downstream one are imposed to the FR model, allowing it to compute the state of the system at time \( t + \delta t_F \).

However, some points in that procedure must be solved: how is it possible to impose a demand and a supply to the vehicular zone? How can we derive flows at interfaces from the entry and exit times of vehicles?

Translation of Flow Boundary Conditions and Vehicle Progression

The problem here is to fix flow boundary conditions for the vehicular zone: the flow demand must be taken into account to create entering vehicles, and the flow supply when making vehicles to exit.

Upstream Transition Cell

Considering the upstream transition cell, the question is how to take into account the demand in the vehicles generation process at the entry of the cell. At time \( t \), this demand \( \Delta_F \) is imposed to the vehicular zone during \( \delta t_F \). It means that \( \Delta_F \cdot \delta t_F \) vehicles wish to enter during that period. At time \( t \), we define tentative generation (or creation) times for all vehicles during \( \delta t_F \); those times are uniformly distributed by defining a generation gap (time interval between generating two consecutive vehicle).

However, generating vehicles at those times would require the local supply on the transition cell to be high enough to allow these vehicles to enter. We define the local supply as follows. When a generation time is reached, the vehicle is thus created only if the space between the entry interface and the previously created vehicle is higher than the minimum space \( s_{min} = 1/K_{max} \) (\( K_{max} \) being the maximum concentration defined in the equilibrium relationship). In that case, the vehicle is created with the equilibrium speed given by the equilibrium relation for the current spacing. That speed is bounded by the equilibrium speed corresponding to the demand. If the spacing is inferior to \( s_{min} \), generation is delayed, and the vehicle will be generated as soon as the previous condition is satisfied. This procedure thus gives a local definition of the transition cell supply; if this supply is always sufficient, the flow will be equal to the demand. If the supply is not sufficient, vehicles generation will be delayed, and the flow will be inferior to the demand. This is not an explicit supply calculation, but this limitation of the flow corresponds to an actual supply constraint enforcement and a proper congestion propagation is ensured. The drawback of this method is that, under congested conditions, all vehicles are created with a null initial speed. This has no consequences on the macroscopic behavior of the transition cell and particularly on its global flow and density if the transition cell is long enough, but is one of the reasons why vehicles trajectories in this cell are not completely realistic. It also makes it necessary to use long enough transition cells.

Two elements must be mentioned to complete that description. First, vehicles generation times are continuous, whereas the VR model is discrete in time. As a delay in creation would reduce flows, we have introduced a procedure of spatiotemporal correction: the continuous generation time is computed, and the position of the vehicle at the discrete generation time is corrected by the distance the vehicle has covered since its generation. Second, the number of vehicles to be created is not necessarily an integer. A transition stack has thus been introduced in order to store and group the portions of vehicles generated and to ensure the conservation of flow. The notion of portion of vehicles will be defined later on.

Downstream Transition Cell

In the downstream transition cell, the first problem is to define the trajectory of the first vehicle present at the downstream end of the cell. The VR model cannot be applied since there is no leading vehicle before it. However, this vehicle is not free: a flow does exist on the downstream FR cell and should determine the behavior of this first vehicle. The solution here is to use a ghost vehicle fictively located on the FR cell, which actually corresponds to the last vehicle having left the transition cell, and whose speed is the equilibrium flow speed corresponding to the concentration of the FR cell. This ghost vehicle transmits downstream traffic flow conditions to the first vehicle of the vehicular zone. Such a procedure is used in (6) to ensure the entire
transmission of boundary conditions. However, though satisfactory under fluid traffic conditions, it is not sufficient to ensure the limitation of flow up to the downstream supply provided by downstream flow conditions.

This supply represents the maximum number of vehicles that can exit from the transition cell. If the number of vehicles wishing to exit the transition cell, which constitute the demand, is lower than this supply, then the flow will be equal to this demand. Otherwise, the exit of vehicles should be delayed in order to have a flow equal to the supply. The solution proposed is symmetric to the one of the upstream transition cell. We define an exit gap equal to the inverse of the supply. This gap corresponds to the time interval between two consecutive exits that would ensure the realization of a flow equal to the supply. We then define at time $t$ the minimum exit times between $t$ and $t + \delta t$: no vehicle may leave the cell before its exit minimum time is elapsed. Then, as soon as a vehicle exits the cell, a tentative exit time for the next vehicle is computed using the procedure explained above (with the ghost vehicle). If that exit time is lower than the minimum defined by the exit gap, the trajectory of the vehicle is modified to ensure an exit time equal to the minimum. This limitation guarantees that congestion will properly propagate through the interface.

As for the upstream transition cell, the number of vehicles exiting the cell is not necessarily an integer. However, it does not affect the proposed procedure. The only modification is when considering the change of flow time step: at the end of a time step, we estimate the portion of the first vehicle that has left the transition cell, and only the complement will have to leave at the next time step. This implies to define a way to estimate portions of vehicles: that definition will be presented in the next section.

**Vehicle Size and Flow Calculus**

Our problem is to convert a discrete information on traffic (the trajectory of the vehicles) into a continuous one. If we consider punctual vehicles, the aggregation of individual variables can produce oscillations into the resulting aggregated variables. For example, if we consider a platoon of vehicles at equilibrium (with the same speed and the same equilibrium spacing) and estimate the corresponding concentration on a cell at two different times, it is possible to find situations where concentration will be different although traffic is in the same situation, as illustrated in figure 3. In that case, concentration varies from $4/L$ to $3/L$ depending on the position of the vehicles. The same problem exists when calculating flows. Those discrete values of flow and concentration would have bad effects on the hybrid model: in a stationary situation, it would introduce oscillations around the mean values, as those observed in (5).

We thus define a spatial extension of vehicles in order to smooth the aggregation of individual variables. The presence of a vehicle $i$ at position $x_i$ is interpreted as the presence of a constant density $K = 1/(x_{i-1} - x_i)$ between $x_i$ and $x_{i-1}$ ($x_{i-1}$ being the position of the leading vehicle). It is then possible to estimate portions of vehicles. For example, if we suppose that the vehicles $i-1$ and $i$ are situated on two different cells, we can estimate the portion of the vehicle $i$ being on the first cell, and the other portion being on the second cell by integrating the concentration. Notating $X$ the position of the limit between the two cells, we split the vehicle $i$ in two portions whose sizes are $(X - x_i)/(x_{i-1} - x_i)$ for the first one, $(x_{i-1} - X)/(x_{i-1} - x_i)$ for the other one.

This definition is exact under stationary situations, but the hypothesis of constant density in the space interval is no longer valid in transitory phases. Further developments on that point can be found in (15).

This definition of a spatial extension of vehicles makes possible the computation of a continuous flow based on the trajectories of vehicles. Rather than counting the number of vehicles that went through one point during a period of time $\delta t$, which leads to discrete values of flows, we propose to compute flows between time $t$ and time $t + \delta t$ using the notion of portion of vehicles, so the number of vehicles will be a real value.

**Conclusion**

The hybrid model presented here has been designed in a way to minimize undesirable effects, but some hypothesis like the estimation of portions of vehicles can be sources of errors. The possible incidence of these has been investigated numerically by analyzing the results of the model in some typical situations.
RESULTS

In order to analyze the hybrid model and to test the influence of the hybridization, we study the results of the model in three typical cases: a stationary situation, a single state change and an incident. Our objective is to validate our model in stationary situations and study the propagation of perturbations through the transition cells. The length of the studied road is of 11 km. We chose the Greenshields fundamental diagram but an other diagram could be used. The parameters of this diagram are fixed as follows: \( K_{\text{max}} = 0.1 \, \text{veh/m} \) and \( V_f = 25 \, \text{m/s} \). We fix \( \delta t_r \) to 0.1 sec, \( \delta x \) to 1000 m and \( \delta t_r \) to 40 sec. So, we define 11 cells: cells 1 to 4 and 8 to 11 are FR cells, cell 6 is the VR cell, cells 5 and 7 are transition cells.

Stationary Situation

The simplest situation is a stationary one. We suppose that the road is empty at \( t = 0 \), and a constant demand is applied at the entry of the road. Figure 4a shows the results of the model for a demand of 0.3 veh/s: it represents the flows at the entry of the upstream transition cell and at the exit of the downstream transition cell. Those cells have been chosen because they are the more illustrative of the effect of hybridization. We observe that after a loading period, the flow is exactly the same for both cells and equal to the demand. We also compare those flows to the flows calculated by Strada in the same situation. A direct conclusion is that hybridization does not change flow results in stationary situations, and that the calculus of flow based on portion of vehicles is correct. Trajectories of vehicles on the VR cell can be traced, and we observe vehicles in a stationary state: they all have the same speed and headway, corresponding to the equilibrium state of the demand. As those trajectories are not very illustrative, they have not been plotted in that paper.

A Single State Change

We study the propagation of a single perturbation through the transition interfaces. The case studied is a demand drop, going from 0.5 veh/s to 0.2 veh/s: this creates a perturbation that propagates downward. Results are plotted in figure 4b. As in the stationary situation, hybridization does not introduce unexpected effects.

Multiple State Changes

This is a more complex situation, with perturbations going in both directions. We suppose that an incident occurs at 8000m, leading to a complete road closure. This results in a very strong perturbation that propagates through the transition cells. Results are presented in figure 4c and 4d for two different lengths and time steps of the flow cells: (1000m, 40s) and (200m, 8s). They are compared to those given by Strada in the same case. This shows the influence of the size of the cells of the FR model on hybridization. The trajectories of all the vehicles affected by the perturbation on the VR cell are represented on figure 4e, which clearly shows the propagation of the various waves corresponding to the formation and dissipation of the queue caused by the incident. It is to notice that the duration of the incident is different for each of those three illustrations, in order to improve their legibility. When the time step is small (fig 4d), instabilities appear in the model at the level of the transition cells. This is due to the estimation of the portions of vehicles: the hypothesis of constant concentration between consecutive vehicles is false in transitory phases, so there is an error in the calculus of flow. This error is less than one vehicle, so it is more visible when the number of vehicles exiting during a flow time step is small.

Conclusion

Globally, the numerical results confirm that our homogeneous hybrid model correctly translates boundary conditions from a model to the other, both under fluid and congested conditions. In transitory phases, limited undesirable effects are still observable. Their impact is directly related to the flow time step, and thus may be controlled by tuning this time step.

CONCLUSION - EXTENSIONS

The model we have developed in this paper is a first approach of a deterministic homogeneous hybrid model in the case of a discretized FR model. The proposed model satisfies the two constraints that we defined for hybrid models, and the results obtained in the different situations studied above are satisfactory since hybridization introduces few undesirable effects. This model overcomes the main issues of mixing different representations of traffic.

Consequently, a second step is the development of a heterogeneous hybrid model, introducing extensions to the previous model such as stochastic aspects or heterogeneity. For example, we can introduce headway and speed distributions when creating vehicles. As we have a time period with a defined demand, it is possible to impose the constraint of conservation of vehicles, which was impossible in the model proposed in (8).
We also have the possibility to introduce heterogeneity in our model using a VR model based on a different
behavioral rule. Both extensions are under development and will enlarge the possible practical applications of
the hybrid model.

REFERENCES

Research* 7 (4), 1959, pp. 499-505.
209-229.
by G. A. Bekey (Simulation Council, La Jolla, CA), Vol. 1, 1971, pp. 51-61.
4. Del Castillo, J. M. A car following model based on the Lighthill-Whitham theory. Transportation and
Traffic Theory: Proceedings of the 13th International Symposium on Transportation and Traffic
5. Poschinger, A., R. Kates, and H. Keller. Coupling of concurrent macroscopic and microscopic traffic
flow models using hybrid stochastic and deterministic disaggregation. Transportation and Traffic
Theory for the 21st century: Proceedings of the 15th International Symposium on Transportation and Traffic
INFORMS spring 2000 meeting, Salt Lake City, Utah, U.S.A. 7 - 10 May 2000.
step. Proceedings of the 9th Meeting of the Euro Working Group on Transportation, Polytechnic of
Bari, Italy, 2002.
Transportation and Traffic Theory: proceedings of the 13th International Symposium on Transportation and Traffic
12. Daganzo, C. F. The CELL transmission model: a dynamic representation of highway traffic consistent
13. Buisson C., J.-P. Lebacque and J.-B. Lesort. STRADA, a discretized macroscopic model of vehicular
traffic flow in complex networks based on the Godunov scheme. Proceedings CESA'96 IMACS
Multiconference. Computational Engineering in Systems Applications, Lille, France, 9-12 July 1996,
pp 976-981.
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