Combining Gray Relational Analysis with Cumulative Prospect Theory for Multi-sensor Target Recognition

1 Qi Wang, 2 Haiping Ren

1 Zhongshan Institute, University of Electronic Science and Technology of China, Zhongshan 528402, China
2 School of Software, Jiangxi University of Science and Technology, Nanchang 330013, P. R. China
Tel.: 13590788079
E-mail: wangqirhp@163.com

Received: 16 April 2014 /Accepted: 30 May 2014 /Published: 30 June 2014

Abstract: The aim of this paper is to propose a new multi-sensor target recognition method for solving the problem which the discriminated object has multiple characteristics indexes. The method combines the concept of gray relational analysis (GRA) method and cumulative prospect theory. This method, the characteristic vector matrix was firstly transformed to standardize membership degree decision making matrix by using a minimum and maximum membership function model. Then the positive and negative ideal points are defined. Further, according to the cumulative prospect theory and GRA, the prospect value function is defined, and an optimization model is built to solve the optimum weight vector. Finally, the rule of target recognition is given. The method can avoid the subjectivity of the weight of characteristic indexes and improve the objectivity and accuracy of target recognition. Finally, numerical simulation illustrates the effectiveness and feasibility of the proposed method.

Keywords: Multi-sensor, Target recognition, Cumulative prospect theory, Gray relational.

1. Introduction

Multi-sensor data fusion has been extensively studied recent years. It is defined as the process of integrating information from multiple sources to produce the most specific and comprehensive unified data about an entity, activity or event [1]. It has been applied many fields, such as pattern recognition, fuzzy control, robotics and medical. Multi-sensor object recognition is one of the important technologies of multi-sensor data fusion, and it has attracted many scholars’ attention and research in recent years. Many target recognition methods are put forward. For example, methods based on Dempster-Shafer evidence theory [2-5], Vague method [6, 7], variable fuzzy set method [8], extension method [9], and extension interval deviation degree method [10], VIKOR method [11]. These fusion methods are all work well, but methods based on Dempster-Shafer evidence extensively depend on the selection of basic probability assignment and the approach needs to know the distribution type and the prior probability. However, the determination of prior probability is greatly empirical in practical operation. The variable fuzzy sets method and extension method are both artificial determined the weight of characteristics. Thus they are too subjective and absolute objectivity. To improve target identification results, Wan [12] proposed entropy weight method, and Ren and Yang [11] proposed weight coefficient of variation method to determine the weight of characteristic indexes.
These methods are objectively in determining weights, but objectively weight methods are still few, therefore this paper presents a new method to determine the objective characteristics of index weight, and the multi-sensor object recognition can be treated as a multi-index decision making problem [8-12]. The prospect theory can objectively determine the weight of index, it is firstly put forward Kahneman and Tversky [13], and further developed by Kahneman and Tversky [14]. In recent years, prospect theory has extensively studied and the method is also used to multi-index decision making problems [15-17]. Thus this paper will use the prospect theory to determine weights of characteristic indexes.

This paper will propose a new multi-sensor target recognition method, which combining GRA with cumulative prospect theory. The rest of this paper is organized as follows. Section 2 constructs the multi-sensor target recognition model. The introduction of prospect theory is given in Section 3. The new multi-sensor target recognition method is proposed in Section 4. Numerical example is given in Section 5. Finally, a conclusion is given in Section 6.

2. Multi-sensor Target Recognition Model

A target recognition database contains \( n \) different target recognition category, noted as, \( \pi = \{\pi_1, \pi_2, \ldots, \pi_m\} \), and each target has a set of \( m \) characteristic indexes \( o = \{o_1, o_2, \ldots, o_n\} \). Set \( \mu_{ij} \) and \( \sigma_{ij}^2 \) are respectively the characteristic (attribute) value and variance of category \( \pi_i \) with respect to the character \( o_j \). The system has a characteristic vector matrix \( X = (\mu_{ij})_{m \times n} \).

In the target recognition problem, through the identified target of each characteristic parameters, and the observation and target database known target characteristic parameters matching to determine the identified target category. We use \( n \) different sensors to measure an unknown target object, thus we can obtained \( m \) characteristic values. That is to say, the first \( f_{th} \) sensor to measure the unknown object and gets the observed value \( \mu_{ij} (j = 1, 2, \ldots, n) \) with respect to the \( f_{th} \) characteristic index. The task of data fusion is, according to the value of state \( \mu_{ij} (j = 1, 2, \ldots, n) \), to ascertain which category will be the unknown object belonging to.

As the variation coefficient weight method easy to use is proposed in this paper, we put the original model was transformed to a minimum and maximum membership function model firstly, the feature vector matrix into the index membership degree matrix \( R = (r_{ij})_{m \times n} \), where

\[
r_{ij} = \frac{\min\{\mu_{ij}, \mu_{ij}'\}}{\max\{\mu_{ij}, \mu_{ij}'\}}
\]

Eq. (1) shows that \( r_{ij} \) is the relative membership degree between measured value and the characteristic values. The target recognition task is to find the closest to the target class and each sensor measurements.

3. Prospect Theory

The prospect theory selected the course of action based on the prospect value. The prospect value \( V \) is defined as follows [13],

\[
V = \sum_{j=1}^{n} \pi(p_j)\nu(x_j),
\]

where \( \pi(p_j) \) is the probability weight function which is the monotone increasing function with respect to \( p_j \), and \( \nu(x) \) is the value function coming from the subjective feeling of the decision maker. The functional formula \( \nu(x) \) is proposed by Kahneman and Tversky [13], and given as follows:

\[
\nu(x) = \begin{cases} 
  x^\alpha, & x > 0 \\
  -\theta(-x)^\beta, & x < 0,
\end{cases}
\]

where \( x \) is the gains or the losses of the surface value, and the gains are the positive values and the losses are the negative values; \( \alpha \) and \( \beta \) are the concave-convex degree of the region value power function of the gains and the losses, respectively, \( 0 < \alpha, \beta < 1 \); when the values of \( \alpha \) and \( \beta \) are larger, then the decision maker is tend to risk; \( \theta \) shows that the region value power function is more steeper for the losses than for the gains, and \( \theta > 1 \) shows the losses aversion. Kahneman and Tversky [13] experimentally determined the values of \( \alpha = \beta = 0.88 \) and \( \theta = 2.25 \), which are consistent with empirical data.

The characters of the prospect theory's value function are given as follows:

i) The gains and the losses are relative in terms of decision making reference points;

ii) People are tending to risk aversion when they face the gains, but tend to risk seeking when they face the losses;

iii) People are more sensitive to the losses than to the gains.

Kahneman and Tversky [14] considered that the probability weight is the subjective judgment of the
decision maker based on the probability \( p \) of the event outcome, and it is neither the probability nor the linear function of the probability. It is the corresponding weight on the probability. The probability weight function is shown as follows \([14]\):

\[
\pi(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{1/\gamma}} \tag{4}
\]

4. Target Recognition Method
Combining GRA with Prospect Theory

Gray relational analysis (GRA) method was originally developed by Deng \([18]\), and has been successfully applied in solving a variety of multi-attribute decision making (MADM) problems \([19-21]\). In this section, we will give the calculation steps of the gray relational analysis (GRA) method based on prospect theory for the multi-sensor target recognition as follows:

Step 1. Characteristic matrix into the membership matrix indicators \( R = (r_{jm})_{m \times n} \):

- The positive ideal point:
  \[
  R^+ = (r_{i1}^+, r_{i2}^+, \ldots, r_{in}^+) = (\max_i \{r_{i1}\}, \max_i \{r_{i2}\}, \ldots, \min_i \{r_{in}\}) \tag{5}
  \]
- The negative ideal point:
  \[
  R^- = (r_{i1}^-, r_{i2}^-, \ldots, r_{in}^-) = (\min_i \{r_{i1}\}, \min_i \{r_{i2}\}, \ldots, \min_i \{r_{in}\}) \tag{6}
  \]

Step 2. Define the positive and negative ideal point, as follows:

\[
\pi^+(w_j) = \frac{w_j^\gamma}{[w_j^\gamma + (1 - w_j)^\gamma]^{1/\gamma}} \tag{11}
\]

\[
\pi^-(w_j) = \frac{w_j^\gamma}{[w_j^\gamma + (1 - w_j)^\gamma]^{1/\gamma}} \tag{12}
\]

According to the Ref. \([17]\), this article will adopt the parameter values in function utility value prospects and prospects weighting function as follows:

\[
\alpha = \beta = 0.88, \theta = 2.25, \gamma^+ = 0.61, \gamma^- = 0.69 \tag{13}
\]
Step 6. Determine the optimal characteristic index weights.

Assume that the weights of characteristic index are satisfy the following set $H$, where

$$H = \{0 \leq a_j \leq w_j \leq b_j \leq 1, j = 1, 2, \ldots, n\}$$  \hspace{1cm} (14)

For each alternative, its comprehensive prospect value is always the big the better, so we can construct the optimization model with the follow objective function:

$$\max V = (V_1, V_2, \ldots, V_m)$$ \hspace{1cm} (15)

Since the alternatives are fair competition, thus we can construct the following optimization model:

$$\max V = \sum_{j=1}^{n} \sum_{i=1}^{n} v_{ij} \pi^j (w_j) + \sum_{j=1}^{n} \sum_{i=1}^{n} v_{ij} \pi^j (w_j)$$

$$\text{s.t.} \sum_{j=1}^{n} w_j = 1$$

$$w_j \geq 0, j = 1, 2, \ldots, n$$  \hspace{1cm} (16)

The above model can be solved by Matlab software, i.e. one can use the Matlab genetic algorithm toolbox to solve. The obtained optimal solution $w^* = (w_1^*, w_2^*, \ldots, w_n^*)$ is the optimal characteristic index weights vector.

Step 7. Calculate the optimal comprehensive prospect value.

The optimal comprehensive prospect value of $A_i$ is

$$V_i^* = \sum_{j=1}^{n} v_{ij} \pi^j (w_j) + \sum_{j=1}^{n} v_{ij} \pi^j (w_j)$$  \hspace{1cm} (17)

Step 8. Recognition rule

From the above analysis, according to the optimal comprehensive prospect value of candidate objects $A_i (i = 1, 2, \ldots, m)$, we give the target recognition rule:

If

$$k_0 = \arg \max_{1 \leq i \leq m} \{V_i^*\}$$  \hspace{1cm} (18)

Then the unknown object belonging to the target $\pi_{k_0}$.

5. Example Study

To illustrate the effectiveness of the new multisensor target recognition method, an example adopted from the paper [22] is given. In order to realize the automatic recognition and classification for intelligent robots, several sensors are fixed in the robot system. The system by SCARA robot, the robot control and drive, sensor system, the main computer, etc., in the multi-sensor system equipped with six force sleep, and close to sleep, and contact sleep and sliding sleep, and array touch, heat sensation for the sensor and the corresponding signal processor. In the experiment determined the four independent characteristic indexes to show the work piece, they were shape factor $\theta_1$, the section center moment $\theta_2$, surface reflection ability $\theta_3$, surface roughness $\theta_4$ of the work piece (part). The weights of characteristic indexes are partly known, and they satisfy

$$H : 0.16 \leq w_1 \leq 0.20, 0.14 \leq w_2 \leq 0.16, 0.15 \leq w_3 \leq 0.18, 0.13 \leq w_4 \leq 0.17$$  \hspace{1cm} (19)

There are four standard parts used to test in the experiment, and the characteristic index value and variance of the four parts are reported in Table 1.

<table>
<thead>
<tr>
<th>Part</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.30</td>
<td>1.86</td>
<td>3.07</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>2</td>
<td>2.43</td>
<td>3.71</td>
<td>2.28</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(0.17)</td>
<td>(0.37)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>3</td>
<td>2.18</td>
<td>1.93</td>
<td>1.37</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.11)</td>
<td>(0.13)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>4</td>
<td>1.85</td>
<td>2.52</td>
<td>2.97</td>
<td>1.93</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.23)</td>
<td>(0.25)</td>
<td>(0.19)</td>
</tr>
</tbody>
</table>

The sensor signal through the data collect and input to the computer and through the information analysis and characteristic level data fusion for some unknown characteristic indexes values are reported in Table 2.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Measurement Value</th>
<th>Standard Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.15</td>
<td>(1.30)</td>
</tr>
<tr>
<td>2</td>
<td>1.30</td>
<td>(0.32)</td>
</tr>
<tr>
<td>3</td>
<td>2.15</td>
<td>(0.17)</td>
</tr>
<tr>
<td>4</td>
<td>2.12</td>
<td>(0.21)</td>
</tr>
</tbody>
</table>

This paper presents the use of the proposed method to recognize the unknown work piece.

Step 1. Table 1, 2 of the data and (1), (2) type get feature matrix and confidence distance matrix is as

$$X = \begin{bmatrix}
1.30 & 1.86 & 3.07 & 2.75 \\
2.43 & 3.71 & 2.28 & 2.34 \\
2.18 & 1.93 & 1.37 & 1.52 \\
1.85 & 2.52 & 2.97 & 1.93
\end{bmatrix}$$  \hspace{1cm} (20)
Step 2. Determine the ideal point and negative ideal vectors respectively are:

\[ R^* = (r_1^*, r_2^*, r_3^*, r_4^*) = (0.9862, 0.9127, 0.9428, 0.9107) \]  

\[ R^- = (r_1^-, r_2^-, r_3^-, r_4^-) = (0.6047, 0.6199, 0.4983, 0.7170) \]  

Step 3. Calculating the distance measures of alternative object with positive and negative ideal point as follows:

The distance set of the alternative \( A_i \) with the positive ideal point is

\[
D^+ = (d(r_{ij}, r_{ij}^*))_{nn} = \begin{bmatrix}
    d(r_{11}, r_{1j}^*) & \ldots & d(r_{1n}, r_{1j}^*) \\
    \vdots & \ddots & \vdots \\
    d(r_{nj}, r_{nj}^*) & \ldots & d(r_{nn}, r_{nj}^*) 
\end{bmatrix}
\]  

The distance set of the alternative \( A_i \) with the negative ideal point is

\[
D^- = (d(r_{ij}, r_{ij}^-))_{nn} = \begin{bmatrix}
    d(r_{11}, r_{1j}^-) & \ldots & d(r_{1n}, r_{1j}^-) \\
    \vdots & \ddots & \vdots \\
    d(r_{nj}, r_{nj}^-) & \ldots & d(r_{nn}, r_{nj}^-) 
\end{bmatrix}
\]  

Step 4. Calculate \( v^-(\tilde{r}_{ij}) = -\theta(d(\tilde{r}_{ij}, r_{ij}^-))^\beta \) and \( v^+(r_{ij}) = (d(\tilde{r}_{ij}, r_{ij}^+))^\alpha \), where \( \alpha = \beta = 0.8, \ \theta = 2.5 \).

Then the positive prospect matrix is:

\[
V^+ = (v_{ij}^+)_nn = \begin{bmatrix}
    0.3815 & 0.1040 & 0.0307 & 0.1398 \\
    0.1040 & 0.2928 & 0.1285 & 0.0047 \\
    0 & 0.0736 & 0.4335 & 0.1397 \\
    0.1257 & 0 & 0 & 0 
\end{bmatrix}
\]

The negative prospect matrix is:

\[
V^- = (v_{ij}^-)_nn = \begin{bmatrix}
    0 & 0.1888 & 0.4228 & 0.0539 \\
    0.2801 & 0 & 0.3250 & 0.1890 \\
    0.3815 & 0.2192 & 0 & 0 \\
    0.2558 & 0.2928 & 0.4535 & 0.1937 
\end{bmatrix}
\]

Step 5. Set \( \gamma^+ = 0.61, \gamma^- = 0.69 \), according to the equations, we can get the optimal characteristic indexes weight vector by using the Matlab software, and given as follows:

\[ w^* = (0.5, 0.2, 0.1, 0.2) \]  

Step 6. Calculate the optimal comprehensive prospect value of each part \( A_i \).

\[ V_i = -0.3934, V_j = -0.0980, V_k = -0.0376, V_l = 0.0818 \]  

Step 7. Due to the maximum \( V_l = 0.0818 \), so the time to check the unknown work piece as the fourth kind of work piece. The recognition results are consistent with [22].

6. Conclusions

To fully consider the characteristics of multi-sensor indicators of the degree of importance for target recognition, we use the prospect theory to determine the weights of characteristic indexes. It not only reflects the objective reality, but also can avoid the subjective arbitrariness, reducing the interference of artificial subjective factors. The proposed method, which combing the GRA with prospect theory, can work well with the multi-sensor target recognition. This algorithm provides a new target recognition approach, which is simpler and easy to use Matlab and other software to solve. The method can also be applied other multi-attribute decision making problems.

Acknowledgements

This work is partially supported by Natural Science Foundation of Jiangxi Province of China (No. 20132BAB211015), and Natural Science Foundation of Jxust (No. NSFJ2014-G38).

References

[5]. K. J. Cao, Z. G. Zhao, H. Jiang, Target identification based on D-S theory and rule of conditioning,


