ANALYSIS OF RHYTHMS OF EEG SIGNALS USING ORTHOGONAL POLYNOMIAL APPROXIMATION

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ABSTRACT

We present a new method for analysis of the rhythms of the electroencephalogram (EEG) signal. The method is based on the multiresolution analysis using the orthogonal polynomial approximation (OPA). It is demonstrated that the proposed analysis technique employing the OPA gives similar performance as the well known method based on the wavelet packet decomposition. However, the proposed technique is simpler in principle and can be implemented as a one-step process.

1. INTRODUCTION

The electroencephalogram (EEG) signals are the electrical potentials sensed on the surface of the scalp caused by physiological activities of the brain. The EEG signals are transient time-varying signals. The classification of the changes of these waves is critical for understanding of the functions of the brain [1,2]. There are various modern techniques such as computerized tomography (CT), magnetic resonance imaging (MRI), etc. used for investigating the brain, but the analysis of the EEG signal plays a major role in the diagnosis of the brain [3].

The EEG signals are described in terms of amplitude and frequency. The amplitude in microvolts (µV) rises as consciousness falls from alert wakefulness through drowsiness to deep sleep [4]. The frequency bands in Hertz (Hz) corresponding to the four rhythms of the EEG signals are transient time-varying signals. The classification of the rhythms in the EEG signals plays a major role in the diagnosis of the brain [4].

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The Wavelet Packet (WP) decomposition is a commonly used method for classification of the transient rhythms of the EEG signals [5,6]. The WP decomposition can be interpreted as a signal decomposition in a set of independent frequency channels with orientation in time (or space). The wavelet orthonormal bases are generated on translation and dilation of the special functions called wavelets.

In this paper, we propose an alternative method based on the orthogonal polynomial approximation (OPA) for analysis of the rhythms in the EEG signals [7,8]. The analysis technique presented here is conceptually simpler and can be implemented as a one-step process.

2. WAVELET TRANSFORM

The wavelet transform (WT) is a useful method for analyzing a non-stationary signal. A scaled and shifted wavelet function is a rapidly decreasing oscillation function given by

\[ \psi_{ab}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t - b}{a} \right) \]  

for \( a, b \in \mathbb{R}, a \neq 0 \) where \( a \) is the scale parameter, and \( \psi(t) \) is the mother wavelet and satisfies the following admissibility condition,

\[ \int_{-\infty}^{\infty} \psi(t) dt = 0 \]  

Given a function \( f(t) \), its wavelet transform is defined as

\[ W_f(a,b) = \int_{-\infty}^{\infty} f(t) \psi^*_{ab}(t) dt \]

\[ = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{a}} \psi^* \left( \frac{t - b}{a} \right) dt \]  

for \( f(t), \psi(t) \in L^2(\mathbb{R}) \). The function \( f(t) \) can be reconstructed by the expression

\[ f(t) = \frac{1}{c_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a^{-2} W_f(a,b) \psi_{ab}(t) da db \]  

where

\[ c_\psi = \int_{-\infty}^{\infty} |\omega|^{-1} |\Psi(\omega)|^2 d\omega \]

is a non-zero finite number, and \( \Psi(\omega) \) is the Fourier transform (FT) of the wavelet function [5,9].

The multiresolution analysis (MRA) refers to approximating the signal at various resolutions in a sequence of nested linear vector subspaces. The WT provides simultaneous information on frequency and time localization of the signal characteristics in terms of the representation of the signal at various resolutions on different time-frequency (or time-scale) points.

The scaling function \( \phi(t) \) satisfies the refinement equation,

\[ \phi(t) = \sum_{k} h(k)\phi(2t - k) \]  

where
and the wavelet function $\psi(t)$ is given by

$$\psi(t) = \sum_k g(k)\varphi(2t - k) \quad (6)$$

where \{h(k)\} and \{g(k)\} are the coefficients [9]. Let $H(\omega)$ and $G(\omega)$ be the F.T.s of the coefficients \{h(k)\} and \{g(k)\} respectively.

### 2.1 Wavelet Packet Decomposition

Given a sequence \(f(n), n = 0, \pm 1, \pm 2, \ldots\), a binary tree structure can be expressed as

$$f_0(n) = \sum_k f(k)h^*(2n - k) \quad (7)$$

$$f_1(n) = \sum_k f(k)g^*(2n - k) \quad (8)$$

The analysis filters $H(\omega)$ and $G(\omega)$ satisfy the quadrature mirror filter conditions, and the filters divide the frequency range of the discrete-time signal into two halves. The outputs of the filters can be combined through the synthesis filters to reconstruct the signal. This method is known as the tree structured perfect reconstruction filter banks (PRFB). The sequences $f_1(n)$ and $f_0(n)$ can be further decomposed by using (7) and (8). The frequency resolution of the sub-band filters can be adjusted by choosing a desired tree structure, and the components of $f(n)$ decomposed at different levels are obtained by choosing the specified tree structure corresponding to the different frequency bands. The four rhythms of the EEG signal, which occupy four different frequency bands can be separated in the process [6].

### 3. ORTHOGONAL POLYNOMIAL APPROXIMATION

Let \(\hat{f}(n), n = 0, 1, \ldots, N - 1\) be a sequence of sampled points and $f(n)$ is the true value of the discrete-time signal. The orthogonal polynomial approximation of order $m$ for the function is given by

$$f^m(n) = \sum_{j=0}^{m} c_j p_j(n) \quad (9)$$

where the set of polynomials $p_j(n)$ is orthogonal over sampled points [7,10]:

$$\sum_{n=0}^{N-1} p_l(n)p_k(n) = 0, \ l \neq k \quad (10)$$

and, the polynomials are expressed by the recurrence relation,

$$p_{j+1}(n) = (n - a_{j+1})p_j(n) - b_jp_{j-1}(n), j \geq 0 \quad (11)$$

with $p_0(n) = 1, p_{-1}(n) = 0,$ and

$$a_{j+1} = \frac{\sum_{n=0}^{N-1} n|p_j(n)|^2}{\sum_{n=0}^{N-1} |p_j(n)|^2} \quad (12)$$

The coefficients of the polynomial series are computed in the least squares sense as

$$b_j = \frac{\sum_{n=0}^{N-1} |p_j(n)|^2}{\sum_{n=0}^{N-1} |p_{j-1}(n)|^2} \quad (13)$$

Moreover, when the order of approximation $m$ is chosen, by the criterion of minimum error-variance, the reconstructed signal becomes free from uncorrelated noise [7,8].

Figure 1 shows the plots of the polynomials $p_0, p_1, p_2, p_3, p_4, p_5$ over time $n$. It is apparent that higher the degree $j$ of the polynomial $p_j$, the polynomial becomes more oscillatory and covers a range of higher frequency. Therefore, the inclusion of higher degree polynomials in the approximation will extend the high frequency range of the approximating signal. Moreover, since it can be shown that there is a direct relationship between the degree of polynomial and the frequency range that it covers, the representation of a signal by the polynomial series, in effect, provides a decomposition of the signal on various frequency ranges.
3.1 Multiresolution Signal Decomposition using OPA

The principle of obtaining the MRA using the OPA is explained in the sequel. Let \( P_j \) be the operator which approximates a signal with a combination of first \( j \) orthogonal polynomials. Then it is easy to verify that \( P_j \) is a linear projection operator on a vector space \( W_j \) spanned by these polynomials, and among all the approximated functions in \( W_j \), \( P_j f(n) \) is the function which is most similar to \( f(n) \) (with least square approximation) [8,10]. Moreover the approximation of a signal by first \( j \) polynomials contains all necessary information to compute the same signal with first \( k \) polynomials for all \( k \leq j \).

Utilizing the concept of the MRA, the different rhythms of the EEG signal can be separated by taking the difference of the signal approximations at two assigned orders of approximations. This is so because the rhythms of the EEG signal occupy different frequency ranges, as such the rhythms will be represented by the orthogonal polynomials at different ranges of polynomial orders.

Figure 2: The EEG signal under test

4. SIMULATION STUDY AND DISCUSSION

The real EEG signals are sampled and stored as data files for further analysis. The EEG signals are converted to digital format through A/D converter at a sampling frequency of 100 Hz. The signals are prefiltered by the 1 – 50 Hz band-pass filter, and the artifact rejection has been performed off-line.

The EEG signal under test includes the signals under different brain functions. We show the plot of 100 samples of the EEG signal in Figure 2. The MRA of the signal is obtained in turn by the WP decomposition and the OPA. For the WP decomposition, a six-level decomposition of Daubechies wavelet is applied to the EEG signal and the four rhythms of the EEG are obtained: Delta rhythm (0.78 – 3.91 Hz), alpha rhythm (7.81 – 13.28 Hz) and beta rhythm (13.28 – 30.47 Hz). For the polynomial approximation the optimal degree of polynomial is found to be 50. The error-variance is minimal for this choice of order of approximation [7,10]. Table 1 shows the ranges of the polynomial orders for reconstruction of the four rhythms of the EEG signal.

The correspondence between the chosen ranges of the polynomial order and the frequency intervals that the chosen ranges cover are shown in Figures 3 - 7. The template signal is formed by adding the polynomials of successive orders in the specified range, and the power spectral density (PSD) of the template signal is computed. The template signals and the corresponding PSD plots used for decomposition of the EEG signal are shown in Figures 3 - 7. The signal component above 30 Hz is not used for analysis.

Figure 8 shows the four rhythms of the EEG signal separated by the WP decomposition, and Figure 9 shows the same result when the method based on OPA is employed. It should be observed that the EEG signal under test is taken while the brain activity has been going through transitional states. It is for this reason that we are getting all the four rhythms present in the signal. When the brain activity settles in one state, the EEG signal reveals usually two and rarely three or more rhythms.

Table 1: Polynomial orders for four rhythms of EEG signal

<table>
<thead>
<tr>
<th>No.</th>
<th>Rhythm</th>
<th>Ranges of polynomial order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Delta</td>
<td>0 – 4</td>
</tr>
<tr>
<td>2</td>
<td>Theta</td>
<td>5 – 9</td>
</tr>
<tr>
<td>3</td>
<td>Alpha</td>
<td>10 – 14</td>
</tr>
<tr>
<td>4</td>
<td>Beta</td>
<td>15 – 32</td>
</tr>
</tbody>
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Figure 3: Polynomial order range (0-4): (a) Template signal (left) (b) PSD plot (right)
5. CONCLUSION

An important application of the MRA is to decompose a signal into various sub-bands of frequency. Conventionally, the WP decomposition is used for the MRA. The four rhythms of the EEG signal can be separated by the WP decomposition.

In this paper, we present an alternative method based on the OPA for analysis of the four rhythms of the EEG signal. The method provides the approximations of the signal at various resolutions. We can decompose the signal into various sub-band of frequency by taking the difference of approximations at two different levels. The method is applied for analysis of the rhythms of the EEG signal. The results are similar to the results obtained by the WP decomposition.

There are some advantages of the MRA using the OPA over the well known technique by the WP decomposition. In the wavelet-based approach, any reduction of the step size of resolution will result in increased computation. However, the polynomial approximations can be obtained with smaller step size of resolution without additional computation. Moreover, since the decomposition and reconstruction of a signal by polynomial approximation are one-step processes, the implementation of the method based on the OPA can be made on-line. The OPA can process nonuniformly sampled data because the orthogonal polynomials are generated with arbitrarily sampled points [7]. Finally, the OPA has inherent noise rejection capability when the optimal order for the polynomial approximation is chosen by the minimum error-variance criterion [7,10].
REFERENCES


