Within the Resolution Cell: Super-resolution in Tomographic SAR Imaging

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Very high resolution acquisition (1.1×0.6m²)
TerraSAR-X
Very high resolution (VHR) opens up for the first time the opportunity to use SAR for urban infrastructure monitoring from space ...
Problems arise ...
SAR Side-looking Imaging Geometry

elevation angle $\theta$

range

elevation

azimuth

$x$

$r$
SAR Geometry in Range-Elevation Plane

3-D reflectivity distribution
\[ \gamma(x, r, s) \]
Bellagio hotel, Las Vegas
interpretation difficult ⇒ real 3-D imaging required ⇒ TomoSAR
TomoSAR, a **Spectral Estimation** Problem

Complex pixel value in acquisition $n$ (after some phase corrections):

$$g_n = \int_{\Delta s} \gamma(s) \exp(-j2\pi \xi_n s) ds$$

$$= FT[\gamma(s)]|_{\xi_n}, \ n = 1, \ldots, N$$

$$\xi_n = \frac{2b_n}{\lambda r}$$

**TomoSAR = spectral estimation**

- irregular sampling
- small $N$
- motion must be considered

3-D reflectivity distribution

$$\gamma(x, r, s)$$

elevation aperture

$\Delta b$

$\Delta s$

reference surface $s = 0$
TomoSAR System Model

- Discrete system model:

\[
\begin{bmatrix}
\mathbf{g}
\end{bmatrix}_{N \times 1} = \begin{bmatrix}
\mathbf{R}
\end{bmatrix}_{N \times L} \begin{bmatrix}
\mathbf{\gamma}
\end{bmatrix}_{L \times 1} + \mathbf{\varepsilon}
\]

Measurements: Irregular FT, Elevation profile, Noise

- SVD-Wiener, a MAP estimator with white noise and prior (reference)

\[
\hat{\mathbf{\gamma}} = \left( \mathbf{R}^T \mathbf{C}_{\varepsilon \varepsilon}^{-1} \mathbf{R} + \mathbf{C}_{\gamma \gamma}^{-1} \right)^{-1} \mathbf{R}^T \mathbf{C}_{\varepsilon \varepsilon}^{-1} \mathbf{g}
\]

noise covariance, prior covariance
Elevation resolution ca. 50 times worse than in azimuth and range

- Super-resolution is crucial
- Signal sparse in elevation
The SL1MMER Algorithm

Scale-down by Scale-down by
L1 norm L1 norm
Minimization – Minimization –
Model selection – Model selection –
Estimation Estimation
Reconstruction Reconstruction

Compressive sensing theory
Candès 2006, Donoho 2006, Baraniuk 2007,
Candès & Wakin 2008

Proposed by Zhu et al. in IGARSS 2010, offers an aesthetic non-parametric realization of NLS


Scale-down by $L_1$ Norm Minimization

\[
\begin{bmatrix}
g \\
\end{bmatrix}_{N \times 1} = \begin{bmatrix}
\end{bmatrix}_{N \times L} \begin{bmatrix}
\gamma \\
\end{bmatrix}_{L \times 1}
\]

under-determined system \(\Rightarrow\) infinitely many solutions

Make use of special prior — sparsity

- Nonlinear least squares (NLS) — maximum likelihood estimator (MLE)
  (Theoretically the best, but NP-hard)
  \[
  \hat{\gamma} = \arg \min_{\gamma} \left\{ \| g - R\gamma \|_2^2 + \lambda_{MS} \| \gamma \|_0 \right\}
  \]

- Compressive sensing (CS) based sparse reconstruction algorithms
  (Convex optimization, solved by linear programming)
  \[
  \hat{\gamma} = \arg \min_{\gamma} \left\{ \| g - R\gamma \|_2^2 + \lambda_{C} \| \gamma \|_1 \right\}
  \]
Scale-down by $L_1$ Norm Minimization

$$\begin{bmatrix} g \end{bmatrix}_{N \times 1} = \begin{bmatrix} R \end{bmatrix}_{N \times L} \otimes \begin{bmatrix} \gamma \end{bmatrix}_{L \times 1}$$
Model Selection

\[
\begin{bmatrix}
g \\
\end{bmatrix}
= 
\begin{bmatrix}
R \\
\end{bmatrix}
\begin{bmatrix}
\gamma \\
\end{bmatrix}
\Rightarrow R(\hat{s}) = 
\begin{bmatrix}
\end{bmatrix}
\]
Estimation (De-biasing)

\[
\begin{bmatrix}
g
\end{bmatrix}_{N \times 1} =
\begin{bmatrix}
R
\end{bmatrix}_{N \times L} \gamma
\rightarrow R(\hat{s}) =
\begin{bmatrix}
gR
\end{bmatrix}_{N \times K}
\]

\[
\begin{bmatrix}
g
\end{bmatrix}_{N \times 1} =
\begin{bmatrix}
R(\hat{s})
\end{bmatrix}_{N \times \hat{K}} \begin{bmatrix}
\gamma(\hat{s})
\end{bmatrix}_{\hat{K} \times 1}
\rightarrow \hat{\gamma}(\hat{s}) =
\begin{bmatrix}
\end{bmatrix}_{\hat{K} \times 1}
\]
Two scatterers inside one SAR pixel

$\delta s$: Distance between two scatterers

$\delta s = 80m \approx 2 \rho_s$

$\Delta z = 40m$

$\Delta y = 69m$
Super-resolution of SL1MMER — Simulation

- $\delta s = 0.8 \rho_s$
Super-resolution of SL1MMER — Simulation
1) Can we separate two close scatterers?

2) If yes: How accurately can we estimate their
   - positions, ✔️
   - amplitudes, ✔️ ⇒ SL1MMER is an **efficient** estimator
   - phases? ✔️

   ✔️ I.e. its estimation accuracy approaches the Cramér–Rao lower bound

... as a function of SNR, N, distance, phase difference, ...

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   - amplitudes, ✓ ⇒ SL1MMER is an efficient estimator
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   … as a function of SNR, N, distance, phase difference, …
Super-Resolution, a Detection Problem

\[ \alpha := \frac{\delta s}{\rho_s} \]

\[ P_D \left( SNR \cdot N, \alpha, \frac{a_1}{a_2}, \Delta \varphi \right) \]

- \( H_0 \): single scatterer (not resolved)
  - \( SNR = 0 \ldots 10 \text{ dB} \)
  - \( N = 10 \ldots 100 \)
- \( H_1 \): double scatterers (resolved)
The definition of **resolution**:

The **minimum distance** between two scatterers that are separable at a **prespecified probability of detection** $P_D$.

$PD = 50\%$
Super-resolution Factor

\[ \kappa_{50\%} = \frac{\rho_s}{\rho_{50\%}^{\alpha_{50\%}}} = \frac{1}{\alpha_{50\%}} \]

\[ \alpha_{50\%} \approx 0.4 \quad \Rightarrow \quad \kappa_{50\%} \approx 2.5 \]
$\Delta \phi$ uniformly distributed in $[-\pi, \pi]$

\[
\int (.) \, d\Delta \phi
\]

Super-resolution factor: $\kappa_{50\%} = \frac{\rho_s}{\rho_{50\%}}$

SR factors: 1.5-25
Super-resolution of SL1MMER — Real Data

Bellagio hotel, Las Vegas

Optical image, © Google Earth

TerraSAR-X spotlight mode
Number of Scatterers

- **SVD - Wiener** 13% double
- **SL1MMER** 30% double

**Blue:** Null scatterers per pixel;  **Green:** Single;  **Red:** Double
1) SL1MMER detects much more double scatterers

2) Mainly contributed by the SR power
Conclusions

- VHR tomographic SAR inversion is able to reconstruct the shape and motion of individual buildings and city areas.

- Super-resolution is *crucial* and *possible*.

- The achievable super-resolution factors with the newly proposed **SL1MMER algorithm** in the typical parameter range of tomographic SAR are found to be promising and are on the order 1.5~25.

- SL1MMER algorithm is an *efficient* estimator, and offers an aesthetic non-parametric realization of NLS.
City Tour

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