

The Classification of all Single Traveling Wave Solutions to Fornberg-Whitham Equation

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Abstract: Under the traveling wave transformation, Fornberg-Whitham equation is reduced to an ordinary differential equation whose general solution can be obtained using the factorization technique. Furthermore, we apply the change of the variable and complete discrimination system for polynomial to solve the corresponding integrals and obtain the classification of all single traveling wave solutions to the Fornberg-Whitham equation.

Keywords: traveling wave solutions, Fornberg-Whitham equation, factorization technique

1. Introduction

Degasperis and Procesi (see [1]) studied the following family of third order dispersive partial differential equation (PDE) conservation laws:

$$u_t + c_0 u_x + \gamma u_{xxx} - \alpha^2 u_{xxt} = (c_1 u^2 + c_2 u_x^2 + c_3 u_{xx})_x, \quad (1)$$

where $\alpha, \gamma, c_i, i = 1, 2, 3, 4$ are real constants. They found that there are only three equations that satisfy the asymptotic integrability condition within this family: the Korteweg-de Vries equation, the Camassa-Holm equation, and the Degasperis-Procesi equation.

For $c_1 = -3c_3/2\alpha^2$ and $c_2 = c_3/2$, it became the Camassa-Holm equation modeling the unidirectional propagation of shallow water waves over a flat bottom (see [2]-[5]), $u_t(t, x)$ standing for the fluid velocity at time t in the spatial x direction. The Camassa-Holm equation was also a model for the propagation of axially symmetric waves in hyperelastic rods (see [6], [7]). It had a bi-Hamiltonian structure and was completely integrable (see [8]-[11]). The orbital stability of the peaked solitons was proved (see [12]), and that of the smooth solitons (see [13]). The explicit interaction of the peaked solitons was given (see [4]). Through discussing the dynamical behavior of the regular system, the explicit periodic blow-up solutions and solitary wave solutions of the generalized Camassa-Holm equation were obtained by Yiqing Li, Lixin Tian, Yuhai Wu (see [14]). Lixin Tian, Yunxia Wang proved the existence of global conservative solutions of the Cauchy problem for the generalized Camassa-Holm equation (see [15]).

With $c_1 = -2c_3/\alpha^2, c_2 = c_3$ in Eq.(1), by rescaling, shifting the dependent variable and applying a Galilean boost, we find the Degasperis-Procesi equation of the form

$$u_t - u_{ttx} + 4uu_x = 3u_x u_{xx} + uu_{xxx}, t > 0, x \in R, \quad (2)$$

Degasperis, Holm and Hone proved the formal integrability of Eq.(2) by constructing a Lax pair (see [16]). Lundmark and Szmigielski presented an inverse scattering approach for computing n -peakon solutions to Eq.(2) (see [17]). Vakhnenko and Parkes investigated traveling wave solutions of Eq.(2) (see [18]).

For $c_1 = -c_3/2\alpha^2, c_2 = c_3$, Eq.(1) becomes Fornberg-Whitham equation. In 1967, in order to discuss wave-breaking's Qualitative behavior, B. Fornberg and G.B Whitham gave the Fornberg-Whitham equation

$$u_t - u_{xxt} + u_x = uu_{xxx} - uu_x + 3u_x u_{xx}, \quad (3)$$

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In comparison with the Camassa-Holm equation and the Degasperis-Procesi equation, the Fornberg-Whitham equation is not integrable. Because its complexity and not integrability, research on the solution of the Fornberg-Whitham equation is very difficult, but it is important for studying wave-breaking and analysis property. Jiangbo Zhou, Lixin Tian got a new type of bounded traveling wave solutions for the Fornberg-Whitham equation (see [19]) and those bounded solutions were defined on some semifinal bounded domains and possess properties of kink waves and anti-kink waves. By using the bifurcation method, they also obtained the analytic expressions for solitons, peakons and periodic wave solutions for the Fornberg-Whitham equation and then they showed the relationships among the solitons, peakons and periodic cusp wave solutions.

Because of the difference, a particular traveling wave solution $4/3e^{-1/2|x-ct|}$ of the Fornberg-Whitham equation refers to as a peakon with $c \rightarrow 4/3$, its changes depend on variations in c , while the Camassa-Holm equation is $ce^{|x-ct|}$. The periodic Camassa-Holm equation has periodic peakons as $u_c(t, x) = \eta \cosh[x - ct - (x - ct) - 1/2]$, where $(x - ct)$ denotes the integer part of $x - ct$.

Based on the above researches, we study the classification of all solutions of Fornberg-Whitham equation.

In the past decade, a lot of expansion methods have been proposed to seek for the traveling wave solutions to nonlinear partial differential equations. These methods are only indirect methods based on the some assumptions about the forms of the solutions of the equations considered. Applying those indirect methods, we can't give all single traveling wave solutions to the equations considered. On the other hand, some superficially different solutions are essentially the same solution. So it is worthwhile to give the classifications of all single traveling wave solutions to those equations. However, direct integral method is a rather simple and powerful methods, if a nonlinear equation can be directly reduced to the integral form as follows:

$$\pm(\xi - \xi_0) = \int \frac{du}{\sqrt{p_n(u)}}, \quad (4)$$

where $p_n(u)$ is an n -th order polynomial. We can give the classification of all solutions to the right integral in the Eq.(4) using complete discrimination system for the n -th order polynomial. But there are some nonlinear differential equations whose reduced ODES are more complex equations. Therefore we need more technical methods to get the corresponding reduced ODE and its solutions. In the present paper, using factorization technique to obtain the general solution of Fornberg-Whitham equation and furthermore using the change of variable and complete discrimination system for polynomial to solve the general solution, we obtain the classification of all single traveling wave solutions to Fornberg-Whitham equation. In Section 2, we first transform Fornberg-Whitham equation into ODE, then factorize the final ODE into the following form

$$[(U - \frac{4}{3})\partial_\xi + 3U'](\partial_{\xi\xi} - \frac{1}{4})U = 0. \quad (5)$$

Finally, the generation form of solution of this equation is derived from the factorization expression. In Section 3, using the complete discrimination system for polynomial, we drive the classification of traveling wave solutions to Fornberg-Whitham equation. A short conclusion is given in Section 4.

2. The generation form of solution of Fornberg-Whitham equation

Lemma 1 Given a nonlinear ordinary differential equation third-order

$$f(U)U''' + g(U, U')U'' + h(U)U' + k(U) = 0, \quad (6)$$

Eq.(6) owns the following factorization

$$[f(U)\partial_\xi - \phi_1(U)U' - \phi_2(U)][\partial_{\xi\xi} - \phi_3(U)\partial_\xi - \phi_4(U)]U = 0 \quad (7)$$

if and only if the following expression holds

$$\begin{cases} g(U, U') = -f(U)\phi_3(U) - \phi_1(U)U' - \phi_2(U), \\ \phi_1(U)\phi_3(U) - f(U)\frac{d\phi_3(U)}{dU} = 0, & k(U) = \phi_2(U)\phi_4(U)U, \\ h(U) = \phi_2(U)\phi_3(U) - f(U)\frac{d\phi_4(U)}{dU}U - f(U)\phi_4(U) + \phi_1(U)\phi_4(U)U. \end{cases} \quad (8)$$

Proof. Expanding Eq.(7), according to the product of differential operators, and identifying the corresponding terms of Eq.(6), Eq.(8) will be obtained. ■

Taking the traveling wave transformation

$$\xi = x + ct, u(x, t) = U(\xi) \tag{9}$$

and substituting Eq.(9) into Eq.(3) yields the following third order ODE

$$UU'' + 3U'U'' - UU' + cU' - U' + cU'' = 0. \tag{10}$$

Applying the factorization technique introduced in Lemma to Eq.(10) yields

$$[(U - c)\partial_\xi + 3U'](\partial_{\xi\xi} - \frac{1}{4})U = [(U - c)\partial_\xi + 3U'](U'' - \frac{1}{4}) = 0, \tag{11}$$

where $c = -\frac{4}{3}$. Introducing undetermined function $F(U(\xi))$, Eq.(11) can be transformed into

$$\begin{cases} U'' - \frac{1}{4}U = F(U(\xi)), \\ (U - \frac{4}{3})\frac{dF(U(\xi))}{d\xi} + 3UF(U(\xi)) = 0. \end{cases} \tag{12}$$

The generalized form solution of the second equation of Eq.(12) is

$$F(U(\omega)) = \frac{d}{(U - \frac{4}{3})}, \tag{13}$$

where d is an integration constant.

Substituting Eq.(13) into the first equation of Eq.(12) and integrating once, yields

$$U'^2 = \frac{1}{4}U^2 - d(U - \frac{4}{3})^{-2} + 2l, \tag{14}$$

where l is an integration constant and ξ_0 is an arbitrary constant, which is the general form of solution of ODE (3). If we can derive the solutions of Eq.(14), the traveling wave solutions of Eq.(3) are obtained. In section 3 we will study this problem well by means of complete discrimination system for polynomial and direct integral method. The key steps are to change the given Fornberg-Whitham equation into the integral form like Eq.(14) and to analyze the solutions of Eq.(14). Eq.(3) can be transformed into the following form by the convolution :

$$u_t + uu_x + \partial_x(1 - \partial_x^2)^{-1}(u) = 0 \tag{15}$$

Hence, if the solution has a continuous first derivative, it must be the global solution of Eq.(15).

3. Classification of traveling wave solutions to Fornberg-Whitham equation

The general solution of Eq.(3) is as follows:

$$\pm(\xi - \xi_0) = \int \frac{dU}{\sqrt{\frac{1}{4}U^2 - d(U - \frac{4}{3})^{-2} + 2l}}. \tag{16}$$

In order to solve the integral (16), we take a variable transformation $v = U - \frac{4}{3}$, then the corresponding integral becomes

$$\pm\frac{1}{2}(\xi - \xi_0) = \int \frac{v dv}{\sqrt{a_4 v^4 + a_3 v^3 + a_2 v^2 + a_1 v + a_0}}, \tag{17}$$

where $a_4 = 1, a_3 = \frac{8}{3}, a_2 = \frac{16}{9} + 8l, a_1 = 0, a_0 = -4d$. We take the change of variable as follows:

$$\omega = (v + \frac{2}{3}), \tag{18}$$

Then Eq.(17) becomes

$$\pm \frac{1}{2}(\xi - \xi_0) = \int \frac{\omega d\omega}{\sqrt{\omega^4 + p\omega^2 + q\omega + r}}. \quad (19)$$

where

$$p = \frac{a_2}{\sqrt{a_4}}, q = a_4^{-\frac{1}{4}} \left(\frac{a_3^3}{8a_4^2} - \frac{a_2 a_3}{2a_4} + a_1 \right),$$

$$r = a_0 - \frac{a_1 a_3}{4a_4} + \frac{a_2 a_3^2}{16a_4^2} - \frac{3a_3^4}{256a_4^3}.$$

According to the classification of the solutions to the ODE

$$(\omega'(\xi))^2 = \omega^4 + p\omega^2 + q\omega + r. \quad (20)$$

We can give the classification of all single traveling wave solutions to Fornberg-Whitham equation. We denote

$$f(\omega) = \omega^4 + p\omega^2 + q\omega + r. \quad (21)$$

We first write the complete discrimination system for the fourth order polynomial as follows:

$$D_1 = 4, D_2 = -p, D_3 = 8rp - 2p^3 - 9q^2,$$

$$D_4 = 4p^4 r - p^3 p^2 + 36prq^2 - 32r^2 - \frac{27}{4}q^4 + 64r^3,$$

$$E_2 = 9p^2 - 32pr. \quad (22)$$

According to the complete discrimination system (21), we give the classification of all single wave solutions to Fornberg-Whitham equation. We obtain the following result.

Case 1: If $D_4 = 0, D_3 = 0, D_2 < 0$, then we have

$$f(\omega) = \left((\omega - l_1)^2 + s_1^2 \right)^2, \quad (23)$$

where l_1, s_1 are real numbers, $s_1 > 0$. We have

$$(\xi - \xi_0) = \frac{1}{2} \ln[s_1^2 + (U - \frac{2}{3} - l_1)^2] + \frac{\arctan(\frac{U - \frac{2}{3} - l_1}{s_1}) l_1}{s_1}. \quad (24)$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 2: If $D_4 = 0, D_3 = 0, D_2 = 0$, then we have

$$f(\omega) = \omega^4, \quad (25)$$

We have

$$U = \exp \left[-\frac{1}{2} |\xi - \xi_0| \right] + \frac{2}{3}. \quad (26)$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 3: If $D_4 = 0, D_3 = 0, D_2 > 0, E_2 > 0$, then we have

$$f(\omega) = (\omega - \alpha)^2 (\omega - \beta)^2, \quad (27)$$

where α, β are real numbers $\alpha > \beta$. We have

$$\pm \frac{1}{2}(\xi - \xi_0) = \frac{\alpha \ln |U - \frac{2}{3} - \alpha| - \beta \ln |U - \frac{2}{3} - \beta|}{\alpha - \beta}. \quad (28)$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 4: If $D_4 = 0, D_3 > 0, D_2 > 0$, then we have

$$f(\omega) = (\omega - \alpha)^2 (\omega - \beta) (\omega - \gamma), \quad (29)$$

where α, β, γ are real numbers, $\beta > \gamma$. We have

$$\begin{aligned} \pm \frac{1}{2}(\xi - \xi_0) = & \frac{\sqrt{-\beta + (U - 2/3)}\sqrt{-\gamma + (U - 2/3)} \ln(-\beta - \gamma + 2(U - \frac{2}{3}))}{2\sqrt{-\beta + (U - 2/3)}\sqrt{-\gamma + (U - 2/3)}} \\ & - \frac{a}{\lambda} \ln[2\sqrt{-\beta + (U - 2/3)}\sqrt{-\gamma + (U - 2/3)} + \frac{2\beta\gamma - \alpha\beta - \alpha\gamma + (2\alpha - \beta - \gamma)(U - \frac{2}{3})}{\alpha\lambda(\alpha - U + \frac{2}{3})}] \end{aligned} \quad (30)$$

where

$$\lambda = \sqrt{\alpha - \beta}\sqrt{\alpha - \gamma}.$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 5: If $D_4 = 0, D_3 = 0, D_2 > 0, E_2 = 0$, then we have

$$f(\omega) = (\omega - \alpha)^3(\omega - \beta), \quad (31)$$

where α, β, γ are real numbers. When $\omega > \alpha, \omega > \beta$ or $\omega < \alpha, \omega < \beta$ we have

$$\pm \frac{1}{4}(\xi - \xi_0) = \frac{\frac{\alpha(\alpha - U + \frac{2}{3})(U - \frac{2}{3} - \beta)}{\alpha - \beta} + (U - \frac{2}{3} - \alpha)^{3/2}\sqrt{U - \frac{2}{3} - \beta} \ln(U - \frac{2}{3}\sqrt{\alpha} + U - \frac{2}{3}\sqrt{\beta})}{\sqrt{(-\alpha + U - \frac{2}{3})^3(-\beta + U - \frac{2}{3})}}. \quad (32)$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 6: If $D_4 = 0, D_2D_3 < 0$, then we have

$$f(\omega) = (\omega - \alpha)^2((\omega - l_1)^2 + s_1^2), \quad (33)$$

where α, l_1 and s_1 are real numbers. We have

$$\begin{aligned} \pm \frac{1}{2}(\xi - \xi_0) = & \left\{ -(\alpha - U + \frac{2}{3})\sqrt{l_1^2 - 2l_1(U - \frac{2}{3}) + (U - \frac{2}{3})^2 + s_1^2} \right. \\ & \left. (\sqrt{l_1^2 - 2l_1\alpha + \alpha^2 + s_1^2} \ln(U - \frac{2}{3} - 1 + \sqrt{l_1^2 - 2l_1(U - \frac{2}{3}) + (U - \frac{2}{3})^2 + s_1^2}) \right. \\ & \left. - \alpha \ln[\frac{\sqrt{l_1^2 - 2l_1(U - \frac{2}{3}) + (U - \frac{2}{3})^2 + s_1^2}}{\alpha(\alpha - U + \frac{2}{3})} + \frac{l_1^2 + s_1^2 - l_1(U - \frac{2}{3}) + \alpha(U - \frac{2}{3} - l_1)}{\alpha(\alpha - U + \frac{2}{3})\sqrt{h(\alpha)}}] \right\} \\ & / (\sqrt{l_1^2 - 2l_1\alpha + \alpha^2 + s_1^2}\sqrt{s_1^2 + (l_1 - \omega)^2(\alpha - U + \frac{2}{3})^2}) \end{aligned} \quad (34)$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 7: If $D_4 > 0, D_3 > 0, D_2 > 0$, then we have

$$f(\omega) = (\omega - \alpha_1)(\omega - \alpha_2)(\omega - \alpha_3)(\omega - \alpha_4), \quad (35)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are real numbers, and $\alpha_1 > \alpha_2, \alpha_3 > \alpha_4$. We have

$$\begin{aligned} \pm \frac{1}{2}(\xi - \xi_0) = & [2\alpha_3 + \text{EllipticF}(\sqrt{\frac{(\alpha_1 - \alpha_3)(U - \frac{2}{3} - \alpha_4)}{(\alpha_1 - \alpha_4)(U - \frac{2}{3} - \alpha_4)}}, \sqrt{\frac{(\alpha_3 - \alpha_2)(\alpha_4 - \alpha_1)}{(\alpha_4 - \alpha_2)(\alpha_3 - \alpha_1)})} \\ & + 2(\alpha_4 - \alpha_3)(\alpha_4 - \alpha_1)(U - 2/3 - \alpha_3)\sqrt{\frac{(\alpha_1 - \alpha_3)(U - 2/3 - \alpha_4)}{(\alpha_1 - \alpha_4)(U - 2/3 - \alpha_3)}}\sqrt{\frac{(\alpha_3 - \alpha_2)(\alpha_4 - \alpha_1)}{(\alpha_4 - \alpha_2)(\alpha_3 - \alpha_1)}} \\ & \sqrt{\frac{(\alpha_3 - \alpha_4)(U - 2/3 - \alpha_2)}{(\alpha_2 - \alpha_4)(U - 2/3 - \alpha_3)}}\sqrt{\frac{(\alpha_1 - \alpha_3)(U - 2/3 - \alpha_4)}{(\alpha_1 - \alpha_4)(U - 2/3 - \alpha_3)}}\sqrt{\frac{(\alpha_3 - \alpha_4)(U - 2/3 - \alpha_1)}{(\alpha_1 - \alpha_4)(U - 2/3 - \alpha_3)}})] / \\ & [(\alpha_1 - \alpha_3)(\alpha_3 - \alpha_4)\sqrt{(U - 2/3 - \alpha_1)(U - 2/3 - \alpha_2)(U - 2/3 - \alpha_3)(U - 2/3 - \alpha_4)}] \end{aligned} \quad (36)$$

This solution has a continuous first derivative, so it is the overall solution of Eq. (3).

Case 8: If $D_4 < 0, D_2D_3 \geq 0$, then we have

$$f(\omega) = (\omega - \alpha)(\omega - \beta)((\omega - l_1)^2 + s_1^2), \quad (37)$$

where α, β, l_1 and s_1 are real numbers, and $\alpha > \beta, s_1 > 0$, we have

$$\begin{aligned} \pm \frac{1}{4}(\xi - \xi_0) &= (\beta - l_1 + s_1 I)(U - 2/3 - \alpha)(U - 2/3 - \alpha)^2 \sqrt{\frac{(l_1 - s_1 I - \alpha)(U - 2/3 - l_1)}{(l_1 - s_1 I - \beta)(U - 2/3 - \alpha)}} \\ &\sqrt{\frac{(\alpha - \beta)(U - 2/3 - l_1 - s_1 I)}{(U - 2/3 - \beta)(l_1 - s_1 I - \beta)}} \sqrt{\frac{(\alpha - \beta)(U - 2/3 - l_1 + s_1 I)}{(U - 2/3 - \alpha)(l_1 - s_1 I - \beta)}} \\ &(\alpha \text{Elliptic}F(\sqrt{\frac{(U - 2/3 - \beta)(l_1 - s_1 I - \alpha)}{(U - 2/3 - \alpha)(l_1 - s_1 I - \beta)}}, \sqrt{\frac{(\alpha - l_1 - s_1 I)(\beta - l_1 + s_1 I)}{(\beta - l_1 - s_1 I)(\alpha - l_1 + s_1 I)}}) + (\beta - \alpha)) \\ &(\text{Elliptic}Pi(\sqrt{\frac{(l_1 - s_1 I - \alpha)(U - 2/3 - \beta)}{(l_1 - s_1 I - \beta)(U - 2/3 - \alpha)}}, \frac{(l_1 - s_1 I - \beta)}{(l_1 - s_1 I - \alpha)}, \sqrt{\frac{(\alpha - l_1 - s_1 I)(\beta - l_1 + s_1 I)}{(\beta - l_1 - s_1 I)(\alpha - l_1 + s_1 I)}})/(\alpha - \beta) \\ &((l_1 - s_1 I - \alpha)\sqrt{(U - 2/3 - \beta)(U - 2/3 - \alpha)(U - 2/3 - l_1 - s_1 I)(U - 2/3 - l_1 + s_1 I)})) \end{aligned} \quad (38)$$

This solution has a continuous first derivative, so it is the overall solution of Eq.(3).

Case 9: If $D_4 > 0, D_2 D_3 \leq 0$, then we have

$$f(\omega) = ((\omega - l_1)^2 + s_1^2)((\omega - l_2)^2 + s_2^2), \quad (39)$$

where l_1, l_2, s_1 and s_2 are real numbers, and $s_1 > s_2 > 0$. We have

$$\begin{aligned} \pm \frac{1}{2}(\xi - \xi_0) &= 2I(U - 2/3 - l_1 + s_1 I)^2 \frac{B}{C} \sqrt{\frac{-Is_1(U - 2/3 - l_2 - s_2 I)}{(U - 2/3 - l_1 + s_1 I)}} \sqrt{\frac{-Is_1(U - 2/3 - l_2 + s_2 I)}{(U - 2/3 - l_1 - s_1 I)}} [(l_1 - s_1 I) \\ &\text{Elliptic}F(\frac{B}{-A} \sqrt{\frac{(U - 2/3 - l_1 - s_1 I)}{(U - 2/3 - l_1 + s_1 I)}}, \frac{DA}{BC}) + 2Is_1 \text{Elliptic}Pi(\frac{B}{-A} \sqrt{\frac{(U - 2/3 - l_1 - s_1 I)}{(U - 2/3 - l_1 + s_1 I)}}, (\frac{B}{A})^2, -\frac{DA}{BC})] \\ &/[B^2 s_1 \sqrt{(U - 2/3 - l_1 - s_1 I)(U - 2/3 - l_1 + s_1 I)(U - 2/3 - l_2 - s_2 I)(U - 2/3 - l_2 + s_2 I)}] \end{aligned} \quad (40)$$

where

$$\begin{aligned} A &= (l_1 + s_1 I - l_2 + s_2 I), B = (l_2 - s_2 I - l_1 + s_1 I), \\ C &= (l_2 + s_2 I - l_1 - s_1 I), D = (l_1 - s_1 I - l_2 - s_2 I). \end{aligned}$$

This solution has a continuous first derivative, so it is the overall solution of Eq.(3).

These are all possible cases, so we obtain the complete classification of all single traveling wave solutions to Fornberg-Whitham equation.

4. Conclusions

In summary, the classifications of all single traveling wave solutions to nonlinear partial differential equations were a rather difficult problem. But there were a lot of nonlinear differential equations whose all traveling wave solutions could be obtained using direct integral method and complete discrimination system for polynomial. On the other hand, if a nonlinear differential equation whose reduced ODE could't be obtained by simple integral method, then we needed to find more powerful tricks and methods to do this thing. We used symmetry group to reduce the order of ODE, and furthermore used factorization technique to reduce the equation to an integral ODE. Using complete discrimination system for polynomial, we obtained the classification of all single traveling wave solutions to some nonlinear partial differential equations. The methods and tricks could be expected to solve more complex and more extensive equations.

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