THE PERFORMANCE OF DUAL-HOP DECODE-AND-FORWARD UNDERLAY COGNITIVE RELAY NETWORKS WITH INTERFERENCE POWER CONSTRAINTS OVER WEIBULL FADING CHANNELS

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To my parents Dharamasiri and Chandrakanthi,  
and sister Gayani
Abstract

With the rapid development and the increasing use of wireless devices, spectrum scarcity has become a problem. The higher frequencies have bad propagation characteristics and the lower frequencies have low data rates, therefore the radio spectrum that is available for efficient wireless transmission is a limited resource. One of the proposed solutions for this problem is cognitive relay networks (CRNs), where cognitive radio is combined with a cooperative spectrum sharing system to increase the spectrum utilization.

In this thesis, the outage probability performances of underlay CRNs with interference power constraints from the primary network over Weibull fading channels have been investigated for three different scenarios. The maximum transmit power of the secondary network is governed by the maximum interference power that the primary network’s receiver can tolerate. The first scenario is a cognitive dual-hop decode-and-forward (DF) relay network over independent non-identically distributed (i.n.i.d.) Weibull fading channels. In the second scenario, the CRN consists of a DF relay plus the direct link transmission with a selection combining receiver at the destination over i.n.i.d. Weibull fading channels. The third CRN considered has multiple DF relays where the best relay selection scheme is employed over independent identically distributed (i.i.d.) Weibull fading channels. The analytical results have been derived using the statistical characteristics of end-to-end signal-to-noise ratios, and have been verified by Monte-Carlo simulations.
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Abbreviation

AF   Amplify-and-Forward
BER  Bit Error Rate
BRS  Best Relay Selection
CCI  Co-Channel Interference
CDF  Cumulative Distribution Function
CR   Cognitive Radio
CRN  Cognitive Relay Network
CSI  Channel State Information
CSSS Cooperative Spectrum Sharing System
DF   Decode-and-Forward
EF   Estimate-and-Forward
i.i.d. independent identically distributed
i.n.i.d. independent non-identically distributed
MIMO Multiple Input Multiple Output
ML   Maximum Likelihood
MRC  Maximum Ratio Combining
OP   Outage Probability
PDF  Probability Density Function
PF   Piecewise-and-Forward
PU   Primary User
SEP  Symbol Error Probability
SNR  Signal-to-Noise Ratio
SU   Secondary User
Chapter 1

Introduction

1.1 Motivation

With the ever increasing demand on wireless devices and applications, radio spectrum scarcity is becoming a problem [1]. The higher frequencies of the electromagnetic spectrum have bad propagation characteristics and the lower frequencies have low data rates. Therefore, the spectrum that is available for efficient radio transmission is a limited resource. It has been found out that the current way of spectrum allocation has a lot of spectrum wastage in it [2], [3]. One of the proposed solutions for this is cognitive relay networks [3], where cognitive radios are combined with cooperative spectrum sharing systems to increase the spectrum utilization. Some of the parameters that can be used to measure the performance of cognitive relay networks are outage probability, symbol error probability, and capacity.

The outage probability (OP) and symbol error probability (SEP) performance of decode-and-forward (DF) relaying in a cognitive relay network (CRN) over Rayleigh fading channels have been investigated in [4]. Outage probability of CRNs under interference constraints have been discussed in [5], and cognitive transmission with multiple relays over Rayleigh fading channels are studied in [6]. The impact of multiple primary transmitters and receivers on cognitive DF relay networks over Rayleigh fading channels is investigated in [7]. In [8], closed-form expressions for OP and capacity are derived for CRNs with interference power constraints over Rayleigh fading channels. The OP of underlay cognitive cooperative networks over Rayleigh fading channels are discussed in [9].

In [10], channel estimation and optimal training design for DF relay networks under individual and total power constraints are studied. Performance of SEP, bit error rate (BER) and achievable spectral efficiency of DF relay networks over independent non-identically distributed (i.n.i.d.) Rayleigh fading channels are studied in [11]. Amplify-and-Forward (AF) relay networks under receiver power constraints are investigated in [12]. Closed-form expression for BER in a multiple input multiple output (MIMO) relay network with DF relaying scheme with maximum likelihood
(ML) detection is derived and cooperative diversity is obtained and studied in [13]. A piecewise-and-forward (PF) relaying protocol for wireless networks is proposed and compared to AF, DF and estimate-and-forward (EF) protocols in [14]. The power allocation of a dual-hop DF cooperative relay network over Rayleigh fading channels is derived based on the average SEP and investigated in [15]. Exact BER for coherent and non-coherent DF cooperative networks up to three relays is derived in [16], where a piecewise linear combiner is employed at the receiver. A new signal processing scheme for relay networks is proposed in [17], where the signal is decoded-compressed-and-forwarded with selective-cooperation.

Level crossing rate and average fade duration of the $N^{th}$ best proactive and reactive DF relaying schemes over Rayleigh fading channels are derived in [18]. An expression for the probability mass function of a relay’s transmit power in a variable gain AF opportunistic relay network is derived and examined in [19]. The OP, BER and the approximate closed-form expression of the ergodic capacity for a dual-hop DF relay network in a spectrum sharing environment under interference constraints over Rayleigh fading channels are examined in [20]. The OP and SEP of DF relay networks over Nakagami-m fading channels, have been derived in [21]. OP of AF cognitive relay networks over Nakagami-m fading channels in a spectrum sharing environment is examined in [22]. The ergodic capacity of AF dual-hop relaying systems over composite Nakagami-m/inverse Gaussian fading channels is investigated in [23]. Performance of proactive opportunistic relaying with AF protocol over generalized Gamma fading channels, have been discussed in [24].

The OP and average SEP for AF cooperative relay networks over independent identically distributed (i.i.d.) Weibull fading channels are derived in [25]. Closed-form expressions for average SEP and Shannon capacity over Weibull fading channels for dual-hop non-regenerative relaying is derived in [26]. In [27], the OP performance over i.i.d. Weibull fading channels for interference limited AF relaying systems are discussed. Power allocation and relay location for dual-hop DF relay systems are studied in [28], to maximize the ergodic capacity over i.i.d. Weibull fading channals and OP over i.n.i.d. Weibull fading channels. Average SEP, OP and average channel capacity of DF cooperative diversity networks with selection combining over i.i.d. Weibull fading channels, have been investigated in [29]. Optimization of dual-hop AF relay with multiple antennas at the destination node over Weibull fading channels is studied in [30]. In [31], the OP of AF cooperative diversity networks with single relay and multiple relays over i.n.i.d. Weibull and Weibull-lognormal fading channels have been analyzed. The performance of cooperative AF fixed-gain relays over Nakagami-m and Weibull fading channels are analyzed in [32]. In [33], the OP, average SEP and average capacity of AF relaying with cooperative diversity over i.i.d. Weibull fading channels are investigated.

This thesis is motivated by the above presented works, the objectives of this thesis are to derive and investigate, 1) OP performance of underlay CRNs with a single DF relay with interference power constraints from the primary user over i.n.i.d. Weibull fading channels, 2) OP performance of underlay cognitive DF relaying network with
cooperative diversity over i.n.i.d. Weibull fading channels with selection combining under interference power constraints, and 3) OP performance of underlay CRNs with multiple DF relays with best relay selection scheme under interference power constraints over i.i.d. Weibull fading channels. 

The organization of this thesis is as follows: In Section 1.2, a brief background on cognitive relay networks, cognitive radio and cooperative communication is provided. In Chapter 2, OP performance of underlay CRNs with a single DF relay with interference power constraints over i.n.i.d. Weibull fading channels is derived and investigated. In Chapter 3, OP performance of underlay CRNs with cooperative diversity with selection combining receiver at the destination under interference power constraints over i.n.i.d. Weibull fading channels is derived and investigated, and in Chapter 4, the OP performance of underlay CRNs with multiple DF relays with best relay selection scheme under interference power constraints over i.i.d. Weibull fading channels is derived and analyzed. Finally in Chapter 5, the thesis is concluded.
Chapter 1. Introduction

1.2 Background

1.2.1 Cognitive Relay Networks

For wireless communication, the higher frequencies have bad propagation characteristics and lower frequencies have low data rates. Therefore, the frequency spectrum that is available for efficient radio transmission is a limited resource. In the research community, cognitive radio has received a great deal of attention in the recent years with its ability to improve the efficiency of spectrum utilization [34], [35], [36], where the secondary user is able to use the primary users spectrum without interfering the primary user. This tends to severely reduce the transmission power of the secondary user. Therefore, for the secondary user to get the necessary transmission range, it is advantageous to have cooperative communication / relay networks. This cognitive radio and cooperative communication combined system can be categorized as 1) overlay cognitive relay networks, 2) underlay cognitive relay networks, or 3) interweave cognitive relay networks [37], [38].

In overlay CRNs, PU and SU use the spectrum at the same time using dirty paper coding, knowing the CSI to alleviate the interference to the PU [39]. In an underlay CRN, the SU occupy the spectrum at the same time as the PU, but the transmission power of the SU is governed by the maximum interference power of the PU’s transmitters/receivers, and in an interweave CRN, the SU use the spectrum only when the PU is not occupying the spectrum. The signal to noise ratio (SNR) of the signal received at the destination can be effected by 1) SUs transmit power limitation, 2) Interference from the PU [38].

1.2.2 Cognitive Radio

The radio spectrum available for wireless transmission is a limited resource. Currently the radio spectrum is allocated in a fixed-based manner. Through studies, it has come to light that the spectrum utilization is low and inefficient [2]. The cognitive radio invented by Mitola [34], has been suggested as a solution for this problem. Cognitive radio enables us to access the radio spectrum in a dynamic fashion. Some of the essential components for a cognitive radio system are spectrum sensing, cognitive medium access control and cognitive networking capabilities [1].

1.2.3 Cooperative Communication

Cooperative communication / relay networks is a new communication paradigm, different from the conventional point-to-point communication. In cooperative communication networks, users and nodes share resources [40]. Cooperative communication was first introduced by E.C. van der Meulen in his Ph.D dissertation ”Transmission of information in a T-terminal discrete memoryless channel,” at the Department of Statistics, University of California, Berkeley, California, and in [41].
Depending on the relaying operation, cooperative communication networks can be categorized as 1) decode-and-forward, 2) amplify-and-forward, 3) estimate-and-forward, and 4) Piecewise-and-forward. Each of these relaying operations has their advantages and disadvantages.

When a message is received at the relay node in a relay network with decode-and-forward relaying protocol, it is decoded and is retransmitted from the relay to the destination only if the message transmitted from the source gets decoded properly at the relay. In a relaying network with amplify-and-forward relaying protocol, the message gets simply amplified at the relay and retransmitted to the destination. The piecewise-and-forward relaying scheme proposed in [42], where the signal received at the relay is compared to an adaptive threshold and if the amplitude of the signal meet the threshold, then the relay will decode the signal if not the relay will forward the received signal after linear processing, and in [43], estimate-and-forward relaying protocol is explained in detail.

Cooperative communication relay networks can be dual-hop, multiple hops, multiple dual-hop relays, or multiple relays with multiple hops depending on the requirements of the system. Systems with multiple relays, require a relay combining strategy to achieve full diversity. Some of the strategies used are best relay selection, partial relay selection, etc. [38].
Chapter 2

Performance Analysis of Underlay Cognitive Relay Networks with a Single DF Relay Under Interference Power Constraints

In this chapter, the performance of dual-hop DF cognitive relay networks over independent non-identically distributed (i.n.i.d.) Weibull fading channels are evaluated, in a spectrum sharing environment. Here, the transmit power of the secondary network is governed by the maximum interference power that the primary networks’ receiver can tolerate. Specifically, a closed-form OP expression under interference power constraints from the primary network is derived using statistical characteristics of the SNR. The analytical results are verified by Monte-Carlo simulation.

2.1 Introduction

CRNs have gained a considerable amount of attention in the research community during the past few years [35], [36]. By combining cooperative spectrum sharing systems (CSSSs) with cognitive radio (CR) the efficiency of the spectrum utilization can be improved.

The performance of CSSS over Rayleigh fading channels and Nakagami-m fading channels have been investigated for AF relay networks in [35], [44], [45] and for DF relay networks in [46]. In [47], the SEP of DF relay networks over Rayleigh fading channels has been investigated. In [48] and [49], the capacities for relay networks over Nakagami-m fading channels are discussed. OP of AF relaying networks over i.n.i.d. Rayleigh fading channels with interference power constraints from the primary network is investigated in [50].

OP of AF relaying networks over i.i.d. Weibull fading channels have been investigated in [27]. In [29], the OP, SEP and capacity for i.i.d. Weibull fading channels in
a DF relay network with selection combining have been discussed, and in [33], for an AF relaying network. SEP and Shannon capacity over i.n.i.d. Weibull fading channel with non-regenerative relaying have been investigated in [26].

While all of the aforementioned works provide a good understanding of CRNs, most of them have been done over Rayleigh fading channels and Nakagami-m fading channels. As such, the objective is to further extend the above presented works for Weibull fading channels. Exact close-form expression for OP in a DF relay network over i.n.i.d. Weibull fading channels under interference power constraints from the primary network has been derived.

The organization of this chapter is as follows: In Section 2.2, the system and channel model is described. In Section 2.3, the analytical calculations for OP is performed, and in Section 2.4 the analytical results are verified by Monte-Carlo simulations. Finally, the chapter is concluded in Section 2.5.
2.2 System and Channel Model

Consider a CRN as in Fig. 2.1, where the CRN spectrum co-exist with the primary user (PU). For the secondary user (SU) to exist in the same spectrum as the PU, a spectrum sharing strategy is employed by having a limit on the SU’s transmit power where it cannot exceed a certain threshold power, which is the maximum peak interference power ($I_p$) that the PU’s receiver can tolerate.

The communication from source to destination takes place in two phases. In Phase I, the signal is transmitted from the source to the relay and if the message gets decoded correctly at the relay then in Phase II, it is transmitted to the final destination. In Phase I, the source transmits with a power of $P_s = \frac{I_p}{|h_{s,p}|^2}$, where $h_{s,p}$ denotes the channel coefficient of the link SU-Tx $\rightarrow$ PU-Rx. In Phase II, the relay retransmits the signal with a power of $P_r = \frac{I_p}{|h_{r,p}|^2}$, where $h_{r,p}$ denotes the channel coefficient of the link Relay $\rightarrow$ PU-Rx. The instantaneous end-to-end SNR for the DF relay can be written as

$$\gamma_d = \min \left( \frac{\alpha_1 |h_{s,r}|^2}{|h_{s,p}|^2}, \frac{\alpha_2 |h_{r,d}|^2}{|h_{r,p}|^2} \right) \gamma_{sr} \gamma_{rd},$$

where $\alpha_i = \frac{I_p}{N_0}$, with $N_0$ representing the noise variance, $\gamma_{sr}$ representing the SNR

Figure 2.1: System model for the cognitive relay network with a single DF relay under interference power constraints. SU-Tx: Secondary user transmitter, SU-Rx: Secondary user receiver, PU-Rx: Primary user receiver.
for the SU-Tx → Relay link, $\gamma_{rd}$ representing SNR for Relay → SU-Rx link and $\gamma_d$ representing the end-to-end SNR for SU-Tx → Relay → SU-Rx relay. While $h_{s,r}$, $h_{s,p}$, $h_{r,d}$, $h_{r,p}$, are the channel coefficients of the particular links. We have assumed Weibull fading channels, with fading parameters $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, and $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$, where $\omega_i = \sqrt{r_i^2/\Gamma(1+\frac{2}{\beta_i})}$ for Weibull parameter $\beta_i \geq 0$. $r_i^2$ is the average signal fading power and $\Gamma(\bullet)$ is the Gamma function [51].
2.3 Performance Analysis

2.3.1 Outage Probability

Since $\gamma_{sr}$ and $\gamma_{rd}$ are independent Weibull distributed random variables, the cumulative distribution function (CDF) of $\gamma_{sr}$ ($F_{\gamma_{sr}}(\gamma)$) and the CDF of $\gamma_{rd}$ ($F_{\gamma_{rd}}(\gamma)$) can be written as

$$F_{\gamma_{sr}}(\gamma) = \int_0^\infty F_{x_1} \left( \frac{\gamma y_1}{\alpha_1} \right) f_{y_1}(y_1) dy_1,$$

and

$$F_{\gamma_{rd}}(\gamma) = \int_0^\infty F_{x_2} \left( \frac{\gamma y_2}{\alpha_2} \right) f_{y_2}(y_2) dy_2,$$

where $x_1 = |h_{s,r}|^2$, $x_2 = |h_{r,d}|^2$, $y_1 = |h_{s,p}|^2$, and $y_2 = |h_{r,p}|^2$. After some mathematical manipulations the CDFs can be written as follows:

$$F_{\gamma_{sr}}(\gamma) = \frac{\beta_2}{\omega_2^{\beta_2}} \int_0^\infty (y_1)^{\beta_2-1} \exp\left(-\left(\frac{y_1}{\omega_2}\right)^{\beta_2}\right) dy_1,$$

and

$$F_{\gamma_{rd}}(\gamma) = \frac{\beta_4}{\omega_4^{\beta_4}} \int_0^\infty (y_2)^{\beta_4-1} \exp\left(-\left(\frac{y_2}{\omega_4}\right)^{\beta_4}\right) dy_2.$$

For the sake of simplicity, we will write the above integrals as follows

$$I_1 = \int_0^\infty (y_1)^{\beta_2-1} \exp\left(-\left(\frac{y_1}{\omega_2}\right)^{\beta_2}\right) dy_1,$$

$$I_2 = \int_0^\infty (y_1)^{\beta_2-1} \exp\left(-\left(\frac{y_1}{\omega_2}\right)^{\beta_2}\right) \exp\left(-\left(\frac{\gamma y_1}{\alpha_1}\right)^{\beta_1}\right) dy_1,$$

$$I_3 = \int_0^\infty (y_2)^{\beta_4-1} \exp\left(-\left(\frac{y_2}{\omega_4}\right)^{\beta_4}\right) dy_2,$$

$$I_4 = \int_0^\infty (y_2)^{\beta_4-1} \exp\left(-\left(\frac{y_2}{\omega_4}\right)^{\beta_4}\right) \exp\left(-\left(\frac{\gamma y_2}{\alpha_3}\right)^{\beta_3}\right) dy_2.$$

The integrals of $I_1$ and $I_3$ have been evaluated according to [52, Eq. (3.383)] and have been determined to be

$$I_1 = \frac{\omega_2^{\beta_2}}{\beta_2},$$

$$I_3 = \frac{\omega_4^{\beta_4}}{\beta_4}.$$
Therefore, by substituting the values of $F$ with interference power constraints from the primary network can be expressed as

The OP of underlay CRN with a single DF relay over i.n.i.d. Weibull fading channel can be written as

Charles Fox. Finally, substituting $H^m.n$ denotes the FoxH function as introduced by

where $H^m.n_{p,q} \left[ z \begin{pmatrix} (a_1, a_1), ..., (a_p, a_p) \\ (b_1, b_1), ..., (b_q, b_q) \end{pmatrix} \right]$, denotes the FoxH function as introduced by

Chapter 2. Performance Analysis of Underlay Cognitive Relay Networks with a
Single DF Relay Under Interference Power Constraints

and the integrals of $I_2$ and $I_4$ have been evaluated according to [53, Eq. (2.25.1)] and have been determined as

$$I_2 = \left( \frac{\omega_2^{\beta_2}}{\beta_2} \right) \times H_{1,1}^{1,1} \left[ \left( \frac{\gamma}{\alpha_1 \omega_1} \right)^{\beta_1} \left( \frac{1}{\omega_2} \right)^{\beta_1} \left( 1 - \beta_2, \beta_1 \right) \left( 0, \frac{1}{\beta_2} \right) \right] , \quad (2.12)$$

$$I_4 = \left( \frac{\omega_4^{\beta_4}}{\beta_4} \right) \times H_{1,1}^{1,1} \left[ \left( \frac{\gamma}{\alpha_2 \omega_2} \right)^{\beta_3} \left( \frac{1}{\omega_4} \right)^{\beta_4} \left( 1 - \beta_4, \beta_3 \right) \left( 0, \frac{1}{\beta_4} \right) \right] , \quad (2.13)$$

The end-to-end CDF of the relay ($F_{\gamma_d} (\gamma)$) can be written as

$$F_{\gamma_d} (\gamma) = [1 - (1 - F_{\gamma_{sr}} (\gamma))(1 - F_{\gamma_{rd}} (\gamma))]. \quad (2.16)$$

Therefore, by substituting the values of $F_{\gamma_{sr}} (\gamma)$ and $F_{\gamma_{rd}} (\gamma)$ in (2.16), the CDF of $\gamma_d$ can be written as

$$F_{\gamma_d} (\gamma) = 1 - H_{1,1}^{1,1} \left[ \left( \frac{\gamma}{\alpha_1 \omega_1} \right)^{\beta_1} \left( \frac{1}{\omega_2} \right)^{\beta_1} \left( 1 - \beta_2, \beta_1 \right) \left( 0, \frac{1}{\beta_2} \right) \right] H_{1,1}^{1,1} \left[ \left( \frac{\gamma}{\alpha_2 \omega_2} \right)^{\beta_3} \left( \frac{1}{\omega_4} \right)^{\beta_4} \left( 1 - \beta_4, \beta_3 \right) \left( 0, \frac{1}{\beta_4} \right) \right] . \quad (2.17)$$

The OP of underlay CRN with a single DF relay over i.n.i.d. Weibull fading channel with interference power constraints from the primary network can be expressed as

$$P_{out} = 1 - H_{1,1}^{1,1} \left[ \left( \frac{\gamma}{\alpha_1 \omega_1} \right)^{\beta_1} \left( \frac{1}{\omega_2} \right)^{\beta_1} \left( 1 - \beta_2, \beta_1 \right) \left( 0, \frac{1}{\beta_2} \right) \right] H_{1,1}^{1,1} \left[ \left( \frac{\gamma}{\alpha_2 \omega_2} \right)^{\beta_3} \left( \frac{1}{\omega_4} \right)^{\beta_4} \left( 1 - \beta_4, \beta_3 \right) \left( 0, \frac{1}{\beta_4} \right) \right] . \quad (2.18)$$
2.4 Numerical Results and Discussion

The considered CRN is a linear dual-hop DF relay network where all the SUs are in a straight line, where the coordinates of the SU-Tx is (0,0) and SU-Rx (1,0), the distances have been normalized. The relay node is placed between the SU-Tx and SU-Rx at (0.5,0) coordinates. The pathloss follows an exponential-decay model, where the pathloss exponent $\epsilon$ is set to 4 for a typical non line of sight propagation model as described in [54]. The effect of the PU’s location on the secondary CRN is evaluated for the following three scenarios (0.44, 0.44), (0.55, 0.55), and (0.66, 0.66). Also, the OP performances of direct transmission from SU-Tx to SU-Rx under the same interference power constraints from the PU have been compared to the CRN’s OP performance.

2.4.1 Outage Probability

The OP for the direct transmission from SU-Tx to SU-Rx under interference power constraints from the PU can be written as follows, after performing some mathematical calculations (detailed calculations are provided in the Appendix):

$$P_{out}^{DT} = 1 - H_{1,1} \left( \frac{\gamma_{SP}}{\alpha_{SD}} \right)^{\beta_{SD}} \left| \frac{(1 - \beta_{SP}, \beta_{SD})}{(0, \frac{1}{\beta_{SP}})} \right|. \quad (2.19)$$

In this section, the analytical results have been verified by Monte-Carlo simulation of

![Figure 2.2](image-url)

Figure 2.2: Outage probability of spectrum sharing DF relay network over Weibull fading channels.

the OP performance for the DF relay network over i.n.i.d. Weibull fading channels.
From Fig.2.2 it can be clearly seen that the simulation curves closely match the analytical curves. Furthermore, as expected, the greater the interference power that the PU receiver can tolerate, better the OP performance of the SU network. Also, from Fig.2.2 it is evident that the DF relay performance is better than the direct transmission performance. Furthermore, it can be seen that the best performance for the DF relay is achieved when the PU is located at (0.66, 0.66).
2.5 Conclusions

In this chapter, the OP of an underlay cognitive DF relay network over an independent non-identically distributed Weibull fading channel with interference power constraints from the primary user has been derived and has been verified by Monte-Carlo simulation. Furthermore, it is evident from the results that the cognitive DF relay performs better than the conventional cognitive direct transmission. Also, it is shown that the location of the PU makes a significant influence on the SU cognitive DF relay networks’ performance.

2.6 Appendix

The CDF of $\gamma_{sd}$ ($F_{\gamma_{sd}}(\gamma)$) can be written as

$$F_{\gamma_{sd}}(\gamma) = \int_{0}^{\infty} F_{x}\left(\frac{\gamma y}{\alpha}\right) f_{y}(y) \, dy,$$  \hspace{1cm} (2.20)

where $x = |h_{s,d}|^2$ and $y = |h_{s,p}|^2$. After some algebraic manipulations, the CDF of $\gamma_{sd}$ can be expressed as

$$F_{\gamma_{sd}}(\gamma) = \frac{\beta_{sp}}{\omega_{sp}} \int_{0}^{\infty} (y)^{\beta_{sp} - 1} e^{-\left(\frac{y}{\omega_{sp}}\right)^{\beta_{sp}}} \, dy$$

$$- \frac{\beta_{sp}}{\omega_{sp}} \int_{0}^{\infty} (y)^{\beta_{sp} - 1} e^{-\left(\frac{y}{\omega_{sp}}\right)^{\beta_{sp}}} e^{-\frac{\gamma y}{\alpha \omega_{sd}}} \, dy.$$  \hspace{1cm} (2.21)

The above integrals have been evaluated by using [52, Eq. (3.383)], [53, Eq. (2.25.1)] and can be written as

$$F_{\gamma_{sd}}(\gamma) = 1 - H_{1,1}^{1,1}\left[\begin{align*}
\left(\frac{\gamma \omega_{sp}}{\alpha \omega_{sd}}\right)^{\beta_{sd}} & | (1 - \beta_{sp}, \beta_{sd}) \\
(0, \frac{1}{\beta_{sp}})
\end{align*}\right].$$  \hspace{1cm} (2.22)

The expression for the OP, for direct transmission from SU-Tx to SU-Rx over i.n.i.d. Weibull fading channel with interference power constraints from the PU can be expressed as

$$P_{\text{out}}^{\text{DT}} = 1 - H_{1,1}^{1,1}\left[\begin{align*}
\left(\frac{\gamma \omega_{sp}}{\alpha \omega_{sd}}\right)^{\beta_{sd}} & | (1 - \beta_{sp}, \beta_{sd}) \\
(0, \frac{1}{\beta_{sp}})
\end{align*}\right].$$  \hspace{1cm} (2.23)
Chapter 3

Performance Analysis of Underlay Cognitive Relay Networks with DF Relay Plus Direct Link Transmission Under Interference Power Constraints

In this chapter, the performance of cognitive relay networks with dual-hop DF relay plus direct link transmission over i.n.i.d. Weibull fading channels are evaluated. Specifically the OP, where the maximum transmit power of the secondary network is governed by the maximum interference power that the primary networks’ receiver can tolerate. A selection combining (SC) receiver is employed at the destination to combine the signals. The OP performance of DF relay only scenario and direct transmission only scenario are compared with the OP of DF relay plus direct link transmission with SC receiver scenario. The results are derived using the statistical characteristics of end-to-end SNRs. The analytical results are verified by Monte-Carlo simulation.

3.1 Introduction

The OP performance of cooperative relay networks over Nakagami-m fading channels have been investigated in [55], and in [56] the OP performance of dual-hop DF L-relay plus direct link over Nakagami-m fading channels have been investigated. The SEP and OP for fixed DF cooperative relay networks over Nakagami-m fading channels are discussed in [21]. The performance of OP in dual-hop AF relay networks over i.n.i.d. Nakagami-m fading channel in [22]. In [57], the OP and SEP of dual-hop channel state information (CSI)-assisted AF cooperative networks are evaluated.

Dual-hop AF and DF relaying systems with multiple interferences over Rayleigh fading channels are discussed in [58]. In [59], the OP of DF cognitive relay networks with interference from primary user over Rayleigh fading channels are investigated. In [60], closed-form expressions for OP and bit error probability (BEP) for threshold-
based opportunistic relaying with selection cooperation over Rayleigh fading channels are derived. Exact closed-form OP of DF relaying networks over a mixed Rayleigh and generalized Gamma fading channels are examined in [61].

OP performance of AF relay networks with co-channel interference (CCI) over i.i.d. Weibull fading channels are discussed in [27]. Average symbol error probability and Shannon capacity in dual-hop non-regenerative relaying over i.n.i.d. Weibull fading channels are investigated in [26]. The OP of single AF relay networks and multiple AF relay networks over i.n.i.d. Weibull fading channels and Weibull-lognormal fading channels are investigated in [62]. In [28], the optimal power allocation and relay location of DF relay networks over Weibull fading channels are discussed. In [33], the performance of OP, SEP and average channel capacity of AF cooperative diversity networks over i.i.d. Weibull fading channels with selection combining, and in [29] for DF cooperative diversity networks, have been investigated.

As such, in this chapter the objective is to further extend the above presented work and what is presented in Chapter 2 of this thesis for i.n.i.d. Weibull fading channels under interference power constraints from the primary network for the scenario of single DF relay plus direct link transmission with a selection combining receiver at the destination (SU-Rx). Furthermore, the OP results of single DF relay plus direct link networks’ performance is compared to the performance of single DF relay only scenario and direct transmission scenario.

The organization of this chapter is as follows: In Section 3.2, the system and channel model are described. In Section 3.3, the OP calculations for the single DF relay plus direct transmission with SC receiver is performed. In Section 3.4, the analytical results are verified by Monte-Carlo simulation. Finally, the chapter is concluded in Section 3.5.
3.2 System and Channel Model

For the cognitive DF relay network in Fig. 3.1, it is considered that there are two paths that the signal could take when it is transmitted from the source (SU-Tx). The communication from source to destination through the relay takes place in two phases. In Phase I, the signal is transmitted from the source to the relay and in Phase II, the message is retransmitted to the final destination (SU-Rx). In Phase I, the source transmits with a power of $P_s = \frac{I_p}{|h_{s,p}|^2}$, where $h_{s,p}$ denotes the channel coefficient of the link SU-Tx → PU-Rx. In Phase II, the signal is transmitted with a transmit power of $P_r = \frac{I_p}{|h_{r,p}|^2}$, where $h_{r,p}$ denotes the channel coefficient of the link SU-Relay → PU-Rx.

For the direct link, the transmission takes place in one phase, where the signal is transmitted with the same amount of power as the relay $P_s = \frac{I_p}{|h_{s,p}|^2}$ from the SU-Tx. The two signals are combined by the SC receiver at the destination. The instantaneous SNRs received at the destination from the DF relaying link and the direct link transmission can be written as follows,

![System Model for cognitive decode-and-forward relay network with a selection combining receiver](image)

Figure 3.1: System Model for cognitive decode-and-forward relay network with a selection combining receiver
\[ \gamma_{\text{relay}} = \min \left( \frac{\alpha_1 |h_{s,r}|^2}{|h_{s,p}|^2}, \frac{\alpha_2 |h_{r,d}|^2}{|h_{r,p}|^2} \right), \]

\[ \gamma_{\text{direct}} = \left( \frac{\alpha_3 |h_{s,d}|^2}{|h_{s,p}|^2} \right), \quad (3.1) \]

where \( \alpha_i = \frac{I_i}{N_0} \), with \( N_0 \) representing the noise variance. \( \gamma_{\text{relay}} \) represents the SNR for SU-Tx \( \rightarrow \) SU-Relay \( \rightarrow \) SU-Rx DF relay, where \( \gamma_{sr} \) is the SNR for the SU-Tx \( \rightarrow \) SU-Relay link and \( \gamma_{rd} \) is the SNR for the SU-Relay \( \rightarrow \) SU-Tx link. \( \gamma_{\text{direct}} \) represents the SNR for the SU-Tx \( \rightarrow \) SU-Rx link. Furthermore, \( h_{s,p}, h_{r,p}, h_{s,r}, h_{r,d}, h_{s,d} \) are the channel coefficients of the particular links. We have assumed i.n.i.d. Weibull fading channels, with fading parameters \( \beta_{sr}, \beta_{sp}, \beta_{rd}, \beta_{rp}, \beta_{sd} \) and \( \omega_{sr}, \omega_{sp}, \omega_{rd}, \omega_{rp}, \omega_{sd} \), where \( \omega_i = \sqrt{\frac{r_i^2}{\Gamma(1+\frac{2}{\beta_i})}} \) for Weibull parameter \( \beta_i > 0 \), \( r_i^2 \) is the average signal fading power and \( \Gamma(\bullet) \) is the Gamma function [51]. The instantaneous end-to-end SNR for the combined system \( \gamma_d \) can be written as

\[ \gamma_d = \max (\gamma_{\text{relay}}, \gamma_{\text{direct}}) \quad (3.2) \]
3.3 Performance Analysis

3.3.1 Outage Probability

From (3.1), it can be seen that $\gamma_{\text{direct}}$ and $\gamma_{\text{relay}}$ are statistically dependent due to the presence of the common random variable $h_{s,p}$ which makes the analysis of OP complicated. An analytical approach where the non-independent variables have been taken into consideration has been proposed in [63], where the CDF of $\gamma_d (F_{\gamma_d}(\gamma))$ conditioned on $h_{s,p}$ has been written as

$$F_{\gamma_d}(\gamma|h_{s,p}) = F_{\gamma_{\text{relay}}}(\gamma|h_{s,p}) \times F_{\gamma_{\text{direct}}}(\gamma|h_{s,p}).$$  \hfill (3.3)

The CDF of $\gamma_{\text{relay}} (F_{\gamma_{\text{relay}}}(\gamma))$ conditioned on $h_{s,p}$ can be written as

$$F_{\gamma_{\text{relay}}}(\gamma|y_{sp}) = [1 - (1 - F_{\gamma_{sr}}(\gamma|y_{sp})) (1 - F_{\gamma_{rd}}(\gamma|y_{sp}))],$$  \hfill (3.4)

where $y_{sp} = |h_{s,p}|^2$, $y_{rp} = |h_{r,p}|^2$, $x_{sr} = |h_{s,r}|^2$, $x_{rd} = |h_{r,d}|^2$, $x_{sd} = |h_{s,d}|^2$, $\gamma_{sr} = \frac{\alpha_1 x_{sr} y_{sp}}{y_{sp}}$, $\gamma_{rd} = \frac{\alpha_2 x_{rd} y_{rp}}{y_{rp}}$ and $\gamma_{direct} = \frac{\alpha_3 x_{sd} y_{sp}}{y_{sp}}$. Since $\gamma_{rd}$ is statistically independent of $y_{sp}$, its CDF conditioned on $y_{sp}$ ($F_{\gamma_{rd}}(\gamma|y_{sp})$) can be written as

$$F_{\gamma_{rd}}(\gamma|y_{sp}) = \int_0^{\infty} F_{x_{rd}}\left(\frac{\gamma y_{rp}}{\alpha_2}\right) f_{y_{rp}}(y_{rp}) dy_{rp}.$$  \hfill (3.5)

After some mathematical manipulations, the CDF of $\gamma_{rd}$ can be written as

$$F_{\gamma_{rd}}(\gamma|y_{sp}) = \frac{\beta_{rp}}{\omega_{rp}} \int_0^{\infty} (y_{rp})^{\beta_{rp}-1} e^{-\frac{y_{rp}}{\omega_{rp}}} d y_{rp}$$

$$- \frac{\beta_{rp}}{\omega_{rp}} \int_0^{\infty} (y_{rp})^{\beta_{rp}-1} e^{-\frac{\gamma_{rd} y_{rp}}{\omega_{rd}}} d y_{rp}.$$  \hfill (3.6)

For simplicity, the above integrals can be written as

$$I_1 = \frac{\beta_{rp}}{\omega_{rp}} \int_0^{\infty} (y_{rp})^{\beta_{rp}-1} e^{-\frac{\gamma y_{rp}}{\omega_{rp}}} d y_{rp},$$  \hfill (3.7)

$$I_2 = \frac{\beta_{rp}}{\omega_{rp}} \int_0^{\infty} (y_{rp})^{\beta_{rp}-1} e^{-\frac{\gamma_{rd} y_{rp}}{\omega_{rd}}} d y_{rp}.$$  \hfill (3.8)

The integral $I_1$ can be evaluated according to [52, Eq. (3.326)] and can be written as

$$I_1 = 1.$$  \hfill (3.9)

Integral $I_2$ can be evaluated according to [53, Eq. (2.25.1)] and has been determined as

$$I_2 = H^{1,1}_{1,1} \left[ \left( \frac{\gamma \omega_{rp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \left| \begin{array}{c} \left( \frac{\beta_{rd}}{\beta_{rp}} \right) \\ (0, 1) \end{array} \right. \right].$$  \hfill (3.10)
By substituting $I_1$ and $I_2$ in (3.6), the CDF of $\gamma_{rd}$ can be written as

$$F_{\gamma_{rd}}(\gamma \mid y_{sp}) = 1 - H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha \omega_{rd}} \right]^{\beta_{rd}} \left[ \begin{array}{c} \left( 0, \frac{\beta_{rd}}{\beta_{rp}} \right) \\ \left( 0, 1 \right) \end{array} \right].$$

(3.11)

The CDF of $\gamma_{sr}$ conditioned on $y_{sp}$ can be written as

$$F_{\gamma_{sr}}(\gamma \mid y_{sp}) = 1 - e^{-\left( \frac{y_{sp} \alpha_{1}}{\omega_{sr}} \right)^{\beta_{sr}}}.$$  (3.12)

Furthermore by substituting the expressions of $F_{\gamma_{sr}}(\gamma \mid y_{sp})$ and $F_{\gamma_{rd}}(\gamma \mid y_{sp})$ in (3.4), the CDF of $\gamma_{relay}$ conditioned on $y_{sp}$ can be written as

$$F_{\gamma_{relay}}(\gamma \mid y_{sp}) = 1 - e^{-\left( \frac{y_{sp} \alpha_{1}}{\omega_{sr}} \right)^{\beta_{sr}}} H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha \omega_{rd}} \right]^{\beta_{rd}} \left[ \begin{array}{c} \left( 0, \frac{\beta_{rd}}{\beta_{rp}} \right) \\ \left( 0, 1 \right) \end{array} \right].$$

(3.13)

The CDF of the direct link ($F_{\gamma_{direct}}(\gamma)$) conditioned on $y_{sp}$ can be written as

$$F_{\gamma_{direct}}(\gamma \mid y_{sp}) = 1 - e^{-\left( \frac{y_{sp} \alpha_{3}}{\omega_{sd}} \right)^{\beta_{sd}}}.$$  (3.14)

The CDF of the end-to-end SNR for the DF relay plus direct link ($F_{\gamma_{d}}(\gamma)$) can be written as

$$F_{\gamma_{d}}(\gamma) = \int_{0}^{\infty} F_{\gamma_{relay}}(\gamma \mid y_{sp}) F_{\gamma_{direct}}(\gamma \mid y_{sp}) f_{y_{sp}}(y_{sp}) dy_{sp}. $$

(3.15)

Therefore, by substituting (3.13) and (3.14) in (3.15), the CDF of $\gamma_{d}$ can be written as

$$F_{\gamma_{d}}(\gamma) = \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_{0}^{\infty} (y_{sp})^{\beta_{sp}-1} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sp}}} dy_{sp}$$

$$- \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_{0}^{\infty} (y_{sp})^{\beta_{sp}-1} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sp}}} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sr}}} dy_{sp}$$

$$- H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha \omega_{rd}} \right]^{\beta_{rd}} \left[ \begin{array}{c} \left( 0, \frac{\beta_{rd}}{\beta_{rp}} \right) \\ \left( 0, 1 \right) \end{array} \right]$$

$$\times \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_{0}^{\infty} (y_{sp})^{\beta_{sp}-1} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sp}}} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sr}}} dy_{sp}$$

$$+ H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha \omega_{rd}} \right]^{\beta_{rd}} \left[ \begin{array}{c} \left( 0, \frac{\beta_{rd}}{\beta_{rp}} \right) \\ \left( 0, 1 \right) \end{array} \right]$$

$$\times \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_{0}^{\infty} (y_{sp})^{\beta_{sp}-1} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sp}}} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sr}}} e^{-\left( \frac{y_{sp}}{\omega_{sp}} \right)^{\beta_{sd}}} dy_{sp}.$$  (3.16)
3.3. Performance Analysis

For the sake of simplicity, the above integrals shall be written as follows:

\[ I_3 = \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_0^\infty (y_{sp})^{\beta_{sp}-1} e^{-\left(\frac{y_{sp}}{\omega_{sp}}\right)^{\beta_{sp}}} dy_{sp}, \tag{3.17} \]

\[ I_4 = \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_0^\infty (y_{sp})^{\beta_{sp}-1} e^{-\left(\frac{y_{sp}}{\omega_{sp}}\right)^{\beta_{sp}}} e^{-\left(\frac{y_{sp}}{\omega_{sd}^{\beta_{sd}}}\right)^{\beta_{sd}}} dy_{sp}, \tag{3.18} \]

\[ I_5 = H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha_{2\omega_{rd}}} \right]^{\beta_{rd}} \left[ 0, \frac{\beta_{rd}}{\beta_{rp}} \right] (0, 1) \times \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_0^\infty (y_{sp})^{\beta_{sp}-1} e^{-\left(\frac{y_{sp}}{\omega_{sp}}\right)^{\beta_{sp}}} e^{-\left(\frac{y_{sp}}{\omega_{sr}^{\beta_{sr}}}\right)^{\beta_{sr}}} dy_{sp}, \tag{3.19} \]

\[ I_6 = H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha_{2\omega_{rd}}} \right]^{\beta_{rd}} \left[ 0, \frac{\beta_{rd}}{\beta_{rp}} \right] (0, 1) \times \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_0^\infty (y_{sp})^{\beta_{sp}-1} e^{-\left(\frac{y_{sp}}{\omega_{sp}}\right)^{\beta_{sp}}} e^{-\left(\frac{y_{sp}}{\omega_{sr}^{\beta_{sr}}}\right)^{\beta_{sr}}} dy_{sp}. \tag{3.20} \]

Integral \( I_3 \) can be evaluated according to [52, Eq. (3.326)] and has been determined as

\[ I_3 = 1. \tag{3.21} \]

The integrals \( I_4 \) and \( I_5 \) have been determined according to [53, Eq. (2.25.1)] and have been determined to be

\[ I_4 = H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{sp}}{\alpha_{3\omega_{sd}}} \right]^{\beta_{sd}} \left[ 0, \frac{\beta_{sd}}{\beta_{sp}} \right] (0, 1), \tag{3.22} \]

and

\[ I_5 = H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{sp}}{\alpha_{3\omega_{sd}}} \right]^{\beta_{sd}} \left[ 0, \frac{\beta_{sd}}{\beta_{sp}} \right] (0, 1) \times H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{sr}}{\alpha_{1\omega_{sr}}} \right]^{\beta_{sr}} \left[ 0, \frac{\beta_{sr}}{\beta_{sp}} \right] (0, 1). \tag{3.23} \]

After some mathematical manipulations integral \( I_6 \) can be written as

\[ I_6 = \left( \frac{\beta_{sp}^2}{\beta_{sr} \beta_{sd}} \right) \times H_{1,1}^{1,1} \left[ \frac{\gamma \omega_{rp}}{\alpha_{2\omega_{rd}}} \right]^{\beta_{rd}} \left[ 0, \frac{\beta_{rd}}{\beta_{rp}} \right] (0, 1) \times \int_0^\infty (t)^0 e^{-t} H_{0,1}^{1,0} \left[ \frac{\gamma \omega_{sr}}{\alpha_{1\omega_{sr}}} \right]^{\beta_{sr}} \left[ t, 0, \frac{\beta_{sr}}{\beta_{sp}} \right] (0, 1) \times H_{0,1}^{1,0} \left[ \frac{\gamma \omega_{sp}}{\alpha_{3\omega_{sd}}} \right]^{\beta_{sd}} \left[ t, 0, \frac{\beta_{sd}}{\beta_{sp}} \right] dt. \tag{3.24} \]

Integral $I_6$ can be evaluated according to [64, Eq. (2.6.2)] and has been determined as

\[
I_6 = \left( \frac{\beta_{sp}^2}{\beta_{sr} \beta_{sd}} \right) \times H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
\times H_{1,0,0,1,1}^{1,1,0,0,1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_1 \omega_{sr}} \right)^{\beta_{sr}} \mid \begin{bmatrix} 1, 1 \\ (1, 1), (1, 1) \end{bmatrix} \right] 
\times \left( \frac{\beta_{sp}^2}{\beta_{sr} \beta_{sd}} \right) \times H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
\times H_{1,0,0,1,1}^{1,1,0,0,1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_1 \omega_{sr}} \right)^{\beta_{sr}} \mid \begin{bmatrix} 1, 1 \\ (1, 1), (1, 1) \end{bmatrix} \right]. \tag{3.25}
\]

Finally, substituting $I_3$, $I_4$, $I_5$ and $I_6$ in (3.16), the end-to-end CDF of $\gamma_d$ can be written as

\[
F_{\gamma_d}(\gamma) = 1 - H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha_3 \omega_{sd}} \right)^{\beta_{sd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
- H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha_1 \omega_{sr}} \right)^{\beta_{sr}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
+ \left( \frac{\beta_{sp}^2}{\beta_{sr} \beta_{sd}} \right) \times H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
\times H_{1,0,0,1,1}^{1,1,0,0,1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha_3 \omega_{sd}} \right)^{\beta_{sr}} \mid \begin{bmatrix} 1, 1 \\ (1, 1), (1, 1) \end{bmatrix} \right]. \tag{3.26}
\]

Therefore, the OP of the underlay CRN with DF relay plus the direct link with SC over i.n.i.d. Weibull fading channels under interference power constraints from the PU can be expressed as

\[
P_{\text{out}} = 1 - H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha_3 \omega_{sd}} \right)^{\beta_{sd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
- H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha_1 \omega_{sr}} \right)^{\beta_{sr}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
+ \left( \frac{\beta_{sp}^2}{\beta_{sr} \beta_{sd}} \right) \times H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{tp}}{\alpha_2 \omega_{rd}} \right)^{\beta_{rd}} \mid \begin{bmatrix} 0, \beta_{sd} \\ 0, 1 \end{bmatrix} \right] 
\times H_{1,0,0,1,1}^{1,1,0,0,1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha_3 \omega_{sd}} \right)^{\beta_{sr}} \mid \begin{bmatrix} 1, 1 \\ (1, 1), (1, 1) \end{bmatrix} \right]. \tag{3.27}
\]
3.4 Numerical Results and Discussion

From Fig. 3.1, it is possible to see that all the nodes considered have a co-linear topology, and that the relay node is placed between SU-Tx and SU-Rx. All distances have been normalized. The coordinates at SU-Tx are (0,0), SU-Rx are (0,0) and the coordinates where the relay node is located are (0.5,0). The pathloss follows an exponential decay model. Pathloss exponent $\epsilon$ is set to 4 for a typical non line of sight propagation model [54].

3.4.1 Outage Probability

OP for the direct transmission from SU-Tx to SU-Rx under interference power constraints from the primary network can be written as follows (detailed calculations are provided in Chapter 2.6):

$$P_{out}^{DT} = 1 - H_{1,1}^{1,1} \left[ \frac{(\gamma \omega_{sp})^{\beta_{sd}}}{(\alpha \omega_{ad})} \right] \left[ (1 - \beta_{sp}, \beta_{sd}) \right] \left[ (0, \frac{1}{\beta_{sp}}) \right]. \quad (3.28)$$

OP for the single DF relay under interference power constraints from the primary network can be written as (detailed calculations are provided in Chapter 2.3):

$$P_{out}^{DF Relay} = 1 - H_{1,1}^{1,1} \left[ \frac{(\gamma \omega_{sp})^{\beta_{1}}}{(\alpha \omega_{1})^{\beta_{2}}} \right] \left[ (1 - \beta_{2}, \beta_{1}) \right] H_{1,1}^{1,1} \left[ \frac{(\gamma \omega_{sp})^{\beta_{3}}}{(\alpha \omega_{3})^{\beta_{4}}} \right] \left[ (1 - \beta_{4}, \beta_{3}) \right] \left[ (0, \frac{1}{\beta_{2}}) \right] \left[ (0, \frac{1}{\beta_{4}}) \right]. \quad (3.29)$$

From Fig. 3.2, it can be seen that the OP performance of DF relay plus the direct link CRN is better than both the DF relay only scenario and direct transmission scenario. From Fig. 3.3, it is evident that when PU is located at coordinates (0.66,0.66), the best OP performance is achieved for the CRN. The diversity order of the relay only link and direct transmission only is one while the diversity order of the direct link plus the DF relay with a SC receiver is two. Furthermore, from Fig. 3.2 and Fig. 3.3, it can be seen that the simulated curves closely match with the analytical curves.

Figure 3.2: Outage probability of spectrum sharing DF relay network over Weibull fading channels.

Figure 3.3: Outage probability of spectrum sharing DF relay plus direct transmission with a SC receiver over Weibull fading channels: Varying the location of PU.
3.5 Conclusions

In this chapter, a closed-form expression for OP of a CRN with DF relay plus direct link with a SC receiver under interference power constraints from the primary network over i.n.i.d. Weibull fading channels has been derived. The analytical results have been verified by Monte-Carlo simulations. The OP performance of DF relay plus direct transmission scenario is better than single DF relay only scenario and direct transmission only scenario. Furthermore, it can be seen that the best performance for the CRN is achieved when the PU is located further away from the SU-Tx.
Chapter 4

Performance Analysis of Underlay Cognitive Relay Networks with Multiple DF Relays with Best Relay Selection under Interference Power Constraints

In this chapter, the performance of underlay cognitive dual-hop decode-and-forward multi-relay networks with best relay selection (BRS) over i.i.d. Weibull fading channels, are evaluated. Here, the maximum transmit power of the secondary network is governed by the maximum interference power that the primary networks’ receiver can tolerate. Specifically, a closed-form OP expression with interference power constraints from the primary network is derived, using the statistical characteristics of end-to-end SNR. The analytical results have been verified by Monte-Carlo simulations.

4.1 Introduction

Spectrum sharing relay networks have gained a considerable amount of attention in the recent years as a promising technology that can improve the spectrum utilization. Cooperative diversity is used to improve the performance of wireless network systems, and BRS scheme is where only the best relay participates in the relaying operation [65].

The OP performance of relay selection criterion, relay number, channel fading parameters and availability of direct link over Nakagami-m fading channels are discussed in [66]. In [21], the OP and SEP for fixed DF cooperative networks with relay selection over i.n.i.d. Nakagami-m fading channels are derived. In [67], the OP for a DF cognitive dual-hop system with interference temperature limit over Nakagami-m fading channels is investigated.

The OP performance of cognitive cooperative networks over Rayleigh fading chan-
Chapter 4. Performance Analysis of Underlay Cognitive Relay Networks with Multiple DF Relays with Best Relay Selection under Interference Power Constraints

Channels with interference power and maximum transmit power constraints are investigated in [68]. In [58], OP of dual-hop DF and AF relaying systems with multiple interferences over Rayleigh fading channels have been investigated. OP performance of partial relay selection in dual-hop AF relaying systems over Rayleigh fading channels are discussed in [69]. In [70], Nth best opportunistic AF cooperative diversity systems are evaluated over i.n.i.d. Rayleigh fading channels. OP and SEP of AF cooperative communication system with cooperative diversity over Rayleigh fading channels are discussed in [71]. In [72], the outage performance of DF cooperative multicast systems with BRs over Rayleigh fading channels have been investigated.

Error probability distribution and density functions for Weibull fading channels with diversity combining and without diversity combining are discussed in [73]. In [62], the OP for cooperative diversity wireless networks using AF relays over i.n.i.d. Weibull and Weibull-lognormal fading channels for single and multiple relays are discussed. Second-order statistics and channel capacity of Weibull fading channels are investigated in [74]. The performance of dual selection combining receiver over correlated Weibull fading channels with arbitrary parameters are studied in [75]. Performance of cooperative dual-hop AF relaying mechanisms with fixed gain relays over Nakagami-m and Weibull fading channels are discussed in [32]. In [28], power allocation and relay positioning for dual-hop DF relaying networks over Weibull fading channels, and in [76], power allocation and relay positioning in AF cooperative networks over Weibull fading environments are investigated. In [77], the OP of selection combining receivers over exponentially correlated composite Weibull-gamma fading channels is analyzed. Outage performance of interference-limited AF relaying systems over i.i.d. Weibull fading channels, is evaluated in [27].

In this chapter, the performance of underlay CRNs with multiple DF relays with BRs scheme under interference power constraints over i.i.d. Weibull fading channels are investigated. Specifically, the OP of multiple DF relay networks where the secondary users’ transmit power is governed by the maximum interference power that the primary networks’ receiver can tolerate. The chapter is organized as follows: In Section 4.2, the system and channel model is described. In Section 4.3, the analytical calculations are performed. In Section 4.4, the analytical results are verified by Monte-Carlo simulations and the results are discussed. Finally, in Section 4.5, the chapter is concluded.
4.2 System and Channel Model

Consider a dual-hop CRN as in Fig. 4.1 with one SU-Tx with multiple relays \((R_k)\), where \((k = 1, 2, \ldots, K)\) and one SU-Rx at the destination. In the first hop of the transmission, the SU source transmits the signal to \(K\) relays under interference power constraints, to prevent interference to the PU.

The transmit power at the source \(P_s = I_p / |h_{s,p}|^2\), where \(h_{s,p}\) denote the channel coefficient of the SU-Tx → PU-Rx link and \(I_p\) is the maximum interference power that the PU-Rx can tolerate. At the relay, the received signal is decoded and if the message is correctly decoded then it is transmitted from the relay towards the destination. The signal is transmitted from the relay with a power of \(P_r = I_p / |h_{r_k,p}|^2\), where \(h_{r_k,p}\) denotes the channel coefficient of the link \(R_k \rightarrow \text{PU-Rx}\). Knowing that BRS is employed for the selection of the SU communication, the relay which has the best SNR \((\gamma_k)\) is selected. Therefore the end-to-end SNR \((\gamma_D)\) at the destination (SU-Rx) can be given by

\[
\gamma_D = \max_{k=1,2,\ldots,K} (\gamma_k) \tag{4.1}
\]

![Figure 4.1: System model for cognitive relay network with best relay selection.](image-url)
Chapter 4. Performance Analysis of Underlay Cognitive Relay Networks with Multiple DF Relays with Best Relay Selection under Interference Power Constraints

\[ \gamma_k = \min_{k = 1, 2, ..., K} \left( \frac{\alpha_1 |h_{s,r_k}|^2}{|h_{s,p}|^2}, \frac{\alpha_2 |h_{r_k,d}|^2}{|h_{r_k,p}|^2} \right), \]  \tag{4.2}

where \( \alpha_i = \frac{I_P}{N_0} \), with \( N_0 \) representing the noise variance, and \( \gamma_{1k} \) represents the SNR for SU-Tx \( \rightarrow R_k \) and \( \gamma_{2k} \) represents the SNR for \( R_k \rightarrow SU-Rx \). Furthermore, \( h_{s,p}, h_{r_k,p}, h_{s,r_k}, h_{r_k,d} \), are the channel coefficients of the particular relay links. We have assumed Weibull fading channels, with fading parameters \( \beta_{s,p}, \beta_{s,r_k}, \beta_{r_k,d}, \beta_{r_k,p}, \) and \( \omega_{s,p}, \omega_{s,r_k}, \omega_{r_k,p}, \omega_{r_k,d} \), where \( \omega_i = \sqrt{\frac{r_i^2}{\Gamma(1+\beta_i)}} \) for Weibull parameter \( \beta_i \geq 0 \). \( \sqrt{r_i^2} \) is the average signal fading power and \( \Gamma(*) \) is the Gamma function [51].
4.3 Performance Analysis

4.3.1 Outage Probability

The OP is the probability that the instantaneous SNR at the receiver falls below a threshold SNR \( \gamma_{th} \). From (4.2), it is evident that \( \gamma_{1k} \) has a common random variable \( h_{s,p} \) for all relays. Therefore, \( \gamma_{1k} \) is statistically dependent upon the random variables \( h_{s,p} \). The CDF of the end-to-end SNR \( (F_{\gamma_{D}}(\gamma)) \) for multiple DF relays with BRS conditioned on \( h_{s,p} \) can be expressed as follows: \[78\], \[63\],

\[
F_{\gamma_{D}}(\gamma|h_{s,p}) = \prod_{k=1}^{K} F_{\gamma_{k}}(\gamma | h_{s,p}), \tag{4.3}
\]

where

\[
F_{\gamma_{k}}(\gamma|h_{s,p}) = [1 - (1 - F_{\gamma_{1k}}(\gamma | h_{s,p})) (1 - F_{\gamma_{2k}}(\gamma | h_{s,p}))]. \tag{4.4}
\]

The CDF of \( \gamma_{1k} \) \( (F_{\gamma_{1k}}(\gamma)) \) conditioned on \( h_{s,p} \) can be written as follows

\[
F_{\gamma_{1k}}(\gamma|h_{s,p}) = 1 - e^{-\left(\frac{\gamma_{y1}}{\alpha_{1k} \omega_{srk}}\right)^{\beta_{srk}}}. \tag{4.5}
\]

Since \( \gamma_{2k} \) is independent of \( h_{s,p} \) the CDF \( \gamma_{2k} \) \( (F_{\gamma_{2k}}(\gamma)) \) conditioned on \( h_{s,p} \) can be written as

\[
F_{\gamma_{2k}}(\gamma|h_{s,p}) = \int_{0}^{\infty} F_{x_{2k}}\left(\frac{\gamma y_{2k}}{\alpha_{2k}}\right) f_{y_{2k}}(y_{2k}) \, dy_{2k}, \tag{4.6}
\]

where \( y_{1} = |h_{s,p}|^2, y_{2k} = |h_{r_{kp}}|^2, x_{1k} = |h_{srk}|^2 \) and \( x_{2k} = |h_{r_{kd}}|^2 \). After some mathematical manipulations, (4.6) can be written as

\[
F_{\gamma_{2k}}(\gamma|h_{s,p}) = \frac{\beta_{r_{kp}}}{\omega_{r_{kp}}} \int_{0}^{\infty} (y_{2k}) e^{-\left(\frac{y_{2k}}{\omega_{r_{kp}}}\right)^{\beta_{r_{kp}}}} dy_{2k} - \frac{\beta_{r_{kp}}}{\omega_{r_{kp}}} \int_{0}^{\infty} (y_{2k}) e^{-\left(\frac{\gamma y_{2k}}{\alpha_{2k} \omega_{r_{kp}}^{2}}\right)^{\beta_{r_{kd}}}} dy_{2k}. \tag{4.7}
\]

For the sake of simplicity, the above integrals have been written as

\[
I_{1} = \frac{\beta_{r_{kp}}}{\omega_{r_{kp}}} \int_{0}^{\infty} (y_{2k}) e^{-\left(\frac{y_{2k}}{\omega_{r_{kp}}^{2}}\right)^{\beta_{r_{kp}}}} dy_{2k}, \tag{4.8}
\]

\[
I_{2} = \frac{\beta_{r_{kp}}}{\omega_{r_{kp}}} \int_{0}^{\infty} (y_{2k}) e^{-\left(\frac{\gamma y_{2k}}{\alpha_{2k} \omega_{r_{kp}}^{2}}\right)^{\beta_{r_{kd}}}} dy_{2k}. \tag{4.9}
\]

Integral \( I_{1} \) can be evaluated according to \[52, \text{Eq. (3.383)}\] and has been determined as

\[
I_{1} = 1. \tag{4.10}
\]
Integral $I_2$, after some mathematical manipulations, has been evaluated according to [53, Eq. (2.25.1)] and has been determined to be

$$I_2 = \int_1 \left[ \frac{\frac{\gamma}{\alpha_{2k} \omega_{rkd}}}{\frac{1}{\omega_{rkd}}^{\beta_{rkd}}} \right]^{\beta_{rkd}} \left[ 0, \frac{\beta_{rkd}}{\beta_{rkp}} \right] \left[ 0, 1 \right] .$$

(4.11)

Therefore, by substituting $I_1$ and $I_2$ in (4.7), we have

$$F_{\gamma_{2k}} (\gamma | h_{s,p}) = 1 - H_{1,1}^{1,1} \left[ \frac{\frac{\gamma}{\alpha_{2k} \omega_{rkd}}}{\frac{1}{\omega_{rkd}}^{\beta_{rkd}}} \right]^{\beta_{rkd}} \left[ 0, \frac{\beta_{rkd}}{\beta_{rkp}} \right] \left[ 0, 1 \right] .$$

(4.12)

By substituting (4.5) and (4.12) in (4.4), we obtain

$$F_{\gamma_k} (\gamma | h_{s,p}) = 1 - e^{-\left( \frac{\gamma}{\alpha_{1k} \omega_{rkd}} \right)^{\beta_{srk}}} H_{1,1}^{1,1} \left[ \frac{\frac{\gamma}{\alpha_{2k} \omega_{rkd}}}{\frac{1}{\omega_{rkd}}^{\beta_{rkd}}} \right]^{\beta_{rkd}} \left[ 0, \frac{\beta_{rkd}}{\beta_{rkp}} \right] \left[ 0, 1 \right] .$$

(4.13)

Furthermore, by substituting (4.13) in (4.3), we get

$$F_{\gamma_D} (\gamma | h_{s,p}) = \prod_{k=1}^{K} \left( 1 - e^{-\left( \frac{\gamma}{\alpha_{1k} \omega_{rkd}} \right)^{\beta_{srk}}} H_{1,1}^{1,1} \left[ \frac{\frac{\gamma}{\alpha_{2k} \omega_{rkd}}}{\frac{1}{\omega_{rkd}}^{\beta_{rkd}}} \right]^{\beta_{rkd}} \left[ 0, \frac{\beta_{rkd}}{\beta_{rkp}} \right] \left[ 0, 1 \right] \right) .$$

(4.14)

For i.i.d. Weibull fading channels, (4.14) can be further simplified to

$$F_{\gamma_D} (\gamma | h_{s,p}) = \left( 1 - e^{-\left( \frac{\gamma}{\alpha_{1k} \omega_{rkd}} \right)^{\beta_{srk}}} H_{1,1}^{1,1} \left[ \frac{\frac{\gamma}{\alpha_{2k} \omega_{rkd}}}{\frac{1}{\omega_{rkd}}^{\beta_{rkd}}} \right]^{\beta_{rkd}} \left[ 0, \frac{\beta_{rkd}}{\beta_{rkp}} \right] \left[ 0, 1 \right] \right)^K .$$

(4.15)

Applying the following mathematical identity

$$(1 + x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k ,$$

(4.16)

in (4.15), $F_{\gamma_D} (\gamma | h_{s,p})$ can be rewritten as

$$F_{\gamma_D} (\gamma | h_{s,p}) = \sum_{k=0}^{n} \binom{\text{NoR}_k}{k} (-1)^k e^{-k \left( \frac{\gamma}{\alpha_{1k} \omega_{rkd}} \right)^{\beta_{srk}}} H_{1,1}^{1,1} \left[ \frac{\frac{\gamma}{\alpha_{2k} \omega_{rkd}}}{\frac{1}{\omega_{rkd}}^{\beta_{rkd}}} \right]^{\beta_{rkd}} \left[ 0, \frac{\beta_{rkd}}{\beta_{rkp}} \right] \left[ 0, 1 \right] \right)^k .$$

(4.17)
where \( NoR \) denotes the number of relays in the CRN. Finally, applying the concepts of probability theory, the CDF of \( \gamma_D \) can be expressed as

\[
F_{\gamma_D}(\gamma) = \int_0^\infty F_{\gamma_D}(\gamma | y_1) f_{y_1}(y_1) \, dy_1.
\]  

(4.18)

By substituting (4.17) and the PDF of Weibull fading in (4.18), we obtain

\[
F_{\gamma_D}(\gamma) = \sum_{k=0}^n \left( \frac{NoR}{k} \right) (-1)^k \left( H_{1,1}^{1,1} \left[ \frac{\gamma}{\alpha_{2k} \omega r_k d} \frac{\beta_{r_k d}}{1 / \omega_{r_k p}} \left| \begin{array}{c} 0, \frac{\beta_{r_k d}}{\beta_{r_k p}} \\ (0, 1) \end{array} \right. \right] \right)^k
\times \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_0^{\infty} (y_1)^{\beta_{sp} - 1} e^{-\left( \frac{y_1}{\omega_{sp}} \right)^{\beta_{sp}}} e^{-k \left( \frac{\gamma y_1}{\alpha_{1k} \omega r_k} \right)^{\beta_{sr_k}}} \, dy_1.
\]  

(4.19)

For the sake of simplicity, the above integral shall be written as

\[
I_3 = \frac{\beta_{sp}}{\omega_{sp}^{\beta_{sp}}} \int_0^{\infty} (y_1)^{\beta_{sp} - 1} e^{-\left( \frac{y_1}{\omega_{sp}} \right)^{\beta_{sp}}} e^{-k \left( \frac{\gamma y_1}{\alpha_{1k} \omega r_k} \right)^{\beta_{sr_k}}} \, dy_1.
\]  

(4.20)

After some mathematical manipulations, the above integral can be determined according to [53, Eq. (2.25.1)] and can be written as

\[
I_3 = H_{1,1}^{1,1} \left[ k \left( \frac{\gamma}{\alpha_{1k} \omega r_k} \right)^{\beta_{sr_k}} \frac{1 / \omega_{sp}^{\beta_{sr_k}}} {1 / \omega_{sp}^{\beta_{sr_k}}} \left| \begin{array}{c} 0, \frac{\beta_{sr_k}}{\beta_{sp}} \\ (0, 1) \end{array} \right. \right].
\]  

(4.21)

Finally, by substituting (4.21) in (4.19) the CDF of \( \gamma_D \) can be written as

\[
F_{\gamma_D}(\gamma) = \sum_{k=0}^n \left( \frac{NoR}{k} \right) (-1)^k \left( H_{1,1}^{1,1} \left[ \frac{\gamma}{\alpha_{2k} \omega r_k d} \frac{\beta_{r_k d}}{1 / \omega_{r_k p}} \left| \begin{array}{c} 0, \frac{\beta_{r_k d}}{\beta_{r_k p}} \\ (0, 1) \end{array} \right. \right] \right)^k
\times H_{1,1}^{1,1} \left[ k \left( \frac{\gamma}{\alpha_{1k} \omega r_k} \right)^{\beta_{sr_k}} \frac{1 / \omega_{sp}^{\beta_{sr_k}}} {1 / \omega_{sp}^{\beta_{sr_k}}} \left| \begin{array}{c} 0, \frac{\beta_{sr_k}}{\beta_{sp}} \\ (0, 1) \end{array} \right. \right].
\]  

(4.22)

The OP for an underlay CRN with multiple DF relays with BRS under interference power constraints from the PU network over i.i.d. Weibull fading channels can be expressed as

\[
P_{out} = \sum_{k=0}^n \left( \frac{NoR}{k} \right) (-1)^k \left( H_{1,1}^{1,1} \left[ \frac{\gamma}{\alpha_{2k} \omega r_k d} \frac{\beta_{r_k d}}{1 / \omega_{r_k p}} \left| \begin{array}{c} 0, \frac{\beta_{r_k d}}{\beta_{r_k p}} \\ (0, 1) \end{array} \right. \right] \right)^k
\times H_{1,1}^{1,1} \left[ k \left( \frac{\gamma}{\alpha_{1k} \omega r_k} \right)^{\beta_{sr_k}} \frac{1 / \omega_{sp}^{\beta_{sr_k}}} {1 / \omega_{sp}^{\beta_{sr_k}}} \left| \begin{array}{c} 0, \frac{\beta_{sr_k}}{\beta_{sp}} \\ (0, 1) \end{array} \right. \right].
\]  

(4.23)
4.4 Numerical Results and Discussion

The above presented analytical results are verified by Monte-Carlo simulation in this section. The parameters of the i.i.d. Weibull fading channels are for $S \rightarrow R_k$ link $\beta_{sr_k} = 3$, $R_k \rightarrow D$ link $\beta_{rd_k} = 4$, $R_k \rightarrow P$ link $\beta_{rp_k} = 6$, and $S \rightarrow P$ link $\beta_{sp} = 5$, for the scenarios of $K = \{2, 3, 5\}$. The OP performance of multiple DF relays are compared to the OP performance of direct transmission from SU-Tx to SU-Rx without a relay scenario in Fig. 4.2. Furthermore the impact of channel fading parameters on the OP performance of the CRN is illustrated in Fig. 4.3, and the OP performance of the CRN relative to the PU-Rx location in Fig. 4.4.

The CRN considered has a co-linear network topology, where all distances are normalized. The coordinates of the source SU-Tx, destination SU-Rx, and the relays are respectively $[0,0]$, $[0,1]$, and $[0,0.5]$. An exponentially decaying pathloss model is assumed for the channels, and $\epsilon$ is set to 4 for a typical non-line of sight propagation model [54], [79]. The impact of the PU-Rx location on the CRN’s OP performance has been investigated for the following three scenarios $(0.44, 0.44)$, $(0.55, 0.55)$ and $(0.66, 0.66)$.

4.4.1 Outage Probability

The OP for direct transmission from SU-Tx to SU-Rx without a relay node under interference power constraints from the primary network can be written as follows after performing some mathematical calculations (detailed calculations are provided in Chapter 2.6):

$$P_{out}^{DT} = 1 - H_{1,1}^{1,1} \left[ \left( \frac{\gamma \omega_{sp}}{\alpha \omega_{sd}} \right)^{\beta_{sd}} \left( 1 - \frac{(1 - \beta_{sp}, \beta_{sd})}{(0, \frac{1}{\beta_{sp}})} \right) \right]. \quad (4.24)$$

From Fig. 4.2, it is evident that when the number of relays increases, the OP performance of the CRN improves. Furthermore, from Fig. 4.2 it is possible to see that the OP performance of multiple DF relays with BRS scheme performs better than the OP performance of direct transmission.

The impact of channel fading parameters on the secondary CRN is illustrated in Fig. 4.3. It is evident from Fig. 4.3, that the worst OP performance for the secondary CRN is achieved when the fading parameter is minimum.

PU-Rx location has a significant impact on the OP performance of the secondary CRN, this can be clearly seen from Fig. 4.4. The best OP performance for the secondary CRN is achieved when the PU-Rx is located at $(0.66, 0.66)$.

As expected, the greater interference power that the PU’s receiver can tolerate, the better the OP performance of the secondary CRN. From the curves in Fig. 4.2, Fig. 4.3, and Fig. 4.4, it can be seen that the analytical results closely match with the Monte-Carlo simulations.
### 4.4. Numerical Results and Discussion

Figure 4.2: Outage Probability of underlay cognitive relay network with multiple DF relays over i.i.d. Weibull fading channels with best relay selection scheme under interference power constraints: Varying the number of relays $K$.

Figure 4.3: Outage Probability of underlay cognitive relay network with multiple DF relays over i.i.d. Weibull fading channels with best relay selection scheme under interference power constraints: Varying the fading channel parameters $\beta_i$. 

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Figure 4.4: Outage Probability of underlay cognitive relay network with multiple DF relays over i.i.d. Weibull fading channels with best relay selection scheme under interference power constraints: Varying the location of PU.
4.5 Conclusions

In this chapter, a closed-form expression for OP of an underlay CRN with multiple DF relays with best relay selection over i.i.d. Weibull fading channels with interference power constraints from the PU has been derived. The analytical results have been verified by Monte-Carlo simulation. It is shown that the OP of underlay CRN with multiple DF relays with best relay selection scheme performs better than the OP of direct transmission scenario over i.i.d. Weibull fading channels, and that with higher diversity orders better OP performance can be obtained. Furthermore, it is shown that the location of the PU-Rx impacts the OP performance of the CRN significantly, and also that the OP performance depends on the fading parameters of the relay.
Chapter 5

Conclusions

In this thesis, a closed-form expression for outage probability has been derived for underlay cognitive relay networks with DF relaying over Weibull fading channels under interference power constraints.

The OP of CRNs with a single DF relay over i.n.i.d. Weibull fading channels is derived and the performance is investigated in Chapter 2, where the SU maximum transmit power is governed by the PU receiver maximum interference power. It is evident from Fig. 2.2 that the OP performance of the CRN is better than the OP performance of direct transmission. Furthermore, from Fig. 2.2 it is possible to infer that the location of the PU makes a significant impact on the OP performance of the CRN. The best OP performance for the CRN is achieved when the PU is located further away from the SU-Tx.

In Chapter 3, the OP of CRNs with a single DF relay plus direct link transmission with a selection combining receiver at the destination over i.n.i.d. Weibull fading channels is derived and the performance is investigated. From Fig. 3.2 it is possible to see that the OP performance of CRN with single DF relay plus direct link transmission with SC receiver is better than both CRN with single DF relay transmission and direct transmission (without relay). The best OP performance is achieved when the PU is located further away from the SU-Tx, this is evident from Fig. 3.3.

The OP of underlay CRNs with multiple DF relays with best relay selection over i.i.d. Weibull fading channels under interference power constraints from the PU is derived and investigated in Chapter 4. It is evident from Fig. 4.2, that the OP performance of CRNs can be improved with cooperative diversity. From Fig. 4.3, it is possible to infer that the OP performance depends on the fading parameters of the DF relay. Furthermore, the PU-Rx location has a significant impact on the CRN’s OP performance, and from Fig. 4.4, it is evident that the best OP performance for the CRN is achieved when the PU-Rx is located further away from the SU-Tx.

It is also evident from Fig. 2.2, Fig. 3.2, Fig. 3.3, Fig. 4.2, Fig. 4.3, and Fig. 4.4 that the analytical results closely match with the Monte-Carlo simulations.
Bibliography


