Near-far Resistant MIMO Iterative Receiver for Uplink LTE

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Abstract—Equalization for uplink MIMO SC-FDMA LTE multi-stream transmission is considered utilizing linear filtering methods and redundancy in the form of channel coding. The equalizer jointly removes inter-symbol (ISI) as well as spatial (MIMO) interference. It is observed that in an unequal received power situation, gains over traditional MMSE frequency equalization can be obtained via group-based cancellation with turbo equalization. It is shown that signal co-existence is possible with this methodology and therefore QoS for cell-edge users can be maintained at considerably higher noise levels as compared to a baseline receiver.

Index Terms—SC-FDMA, MMSE, MIMO, Turbo equalization, Interference cancellation.

I. INTRODUCTION

Cellular system users operating close to the cell edge can expect much lower QoS than those located closer to the center. In those scenarios received power differences between signals from the strongest and weakest terminals can be considerable, up to tens of decibels. Traditionally, different groups of users occupy different resources like time and frequency additionally to spatial separation. However with advances in signal processing, especially with the development of joint detection methods this orthogonal allocation may not be necessary anymore and mix of different traffic can occupy the same resources and still be successfully resolved at the receiver.

Interference combating detectors were first applied in the code-division multiple access (CDMA) systems. It was shown in [1], [2] that the optimal detector has prohibitively high complexity for any practical implementation and many years of research in the field were devoted to the development of suboptimal methods [3], [4]. Linear methods are primary example of these and because of their simplicity and relatively straightforward mathematical treatment gained a great deal of attention in the community. To this group belong simple correlation receiver, sometimes called matched filter receiver, decorrelator and the widely celebrated minimum-mean-square-error (MMSE) filter, which is of most interest nowadays. Essentially, this method, adopted from control theory, depends on inverting the covariance matrix of the input signal to achieve interference suppression [5]. As shown in [6] a serial cascade of MMSE filters achieves the vertices of the capacity region of the multiple access channel. This is a direct application of the chain rule of mutual information [7] and is usually called successive interference cancellation (SIC), sometimes also onion peeling decoding. However, its main drawback is a long decoding delay increasing proportionally with the number of signals in the channel. Very often those high latencies may not be tolerable in practical high-speed wireless communication systems.

The remedy for that is parallel interference cancellation (PIC), where the detector aims at estimating the signals of all streams at once, running an algorithm in an iterative manner. Usually the number of iterations is much smaller than the number of signals in the channel and therefore much lower delays are possible with PIC comparing to SIC.

The literature mostly concentrates on the so-called power controlled situation, where the powers of signals arriving at the receiver are very similar. This requirement originates from traditional processing for CDMA via simple correlation, which does not take into account the structure of the interference and therefore favors equal powers situation. However modern iterative cancellation receivers actually prefer to deal with unequal power signals and as shown in [8] the equal power situation presents the worst case for this type of receivers. In [9], [10] it was demonstrated that parallel interference cancellation with the additional use of redundancy in the coded bits with a simple LLR-combiner achieves excellent performance with a very low complexity two-stage processing. Finally, in [11] it was shown that two-stage decoding with unequal power distributions can achieve the capacity of the linear multi-user channel.

As mentioned before coding may, in many situations, improve the performance of SIC and PIC methods. The application of channel codes to improve the performance of joint receivers was initially considered in [12], [13] where per-user MMSE filter accepting the a priori information from the outer channel code, acronymed as IC-MMSE, was introduced. This scheme exhibits visible performance advantage over non-iterative MMSE, but its drawback is a fairly high complexity, since for each turbo iteration a separate MMSE filter has to be computed for each user. For large systems this complexity barrier may be considerable and hard to overcome.

It is also known that linear processors, particularly the
MMSE filter of interest here, perform best in the situations when the number of transmitted waveforms is smaller than the number of available orthogonal dimensions (under-loaded system) otherwise they suffer from the problem of rank deficiency (over-loaded system). For any MIMO system measure of loading can be defined as ratio of number transmit to number receive antennas $\frac{N_t}{N_r}$. Based on these observations we propose an intermediate solution. It combines the low complexity of the parallel interference cancellation with the excellent performance of successive cancellation with coding utilizing the feedback from the outer error control code. We consider the case of the LTE standard rate 1/3 turbo code [14]. The ordering in successive IC is a very crucial issue and most often depends on the strength of the signals received from users belonging to different traffic classes in a way that strong signals are detected/decoded first and subsequently fed back to the cancellation block. To assure optimal behavior of the linear MMSE filter, in this paper the equalization process is split into a number of subprocesses. Therefore partial (intermediate) MMSE filters operate on the channel with substantially reduced loading. The grouping is performed solely depending on the received power of each group and by group we mean a number of $N_t$ independent data streams transmitted by a certain user $k$. The proposed processing is particularly suitable for systems with many streams and large receive arrays, since then the complexity advantage of the proposed group detection is most visible.

The paper is organized as follows. In Section II the LTE channel model using the matrix notation is outlined. Section III briefly explains the baseline processing algorithm and describes the proposed near-far group receiver. Section IV is devoted to a graphical semi-analytical method of capturing the convergence behavior of the proposed receiver using variance transfer curves [15]–[17]. Section V presents simulation results for specific MIMO configurations and confirms the validity of the semi-analytical approach discussed before. Finally, Section VI concludes the paper and sketches future work directions.

II. UPLINK LTE CHANNEL MODEL

LTE uplink communication utilizes a so-called single-carrier FDMA (SC-FDMA) modulation format, where a block of $M$ time-domain symbols is transformed via an $M$-size DFT into the frequency domain and transmitted as $M$ out of $N$ OFDM tones [18]. The resulting channel can be modeled as follows

$$Y = HAX + N = HAFx + N,$$  

where $H$ is a matrix composed of diagonal matrices in a form solely dependent on the MIMO configuration. For instance for a $2 \times 2$ MIMO case

$$H = \begin{bmatrix} H_{1,1} & H_{1,2} \\ H_{2,1} & H_{2,2} \end{bmatrix},$$  

and each matrix $H_{n_r,n_t}$ is a diagonal matrix of frequency gains between the $n_r$-th receive and $n_t$-th transmit antenna. The matrix $A$ is a diagonal matrix of stream (user) amplitudes which implies that $W = AA^H$ is matrix of powers. $F$ is a size-$M$ Fourier transform matrix. $x$ is the vector of transmitted QAM modulation symbols. The noise $N$ is complex Gaussian with variance $\sigma^2$. In this paper we concentrate on a specific cellular situation where two different power MIMO users are communicating to the base station. They use the same resources at the same time. The strong user is close to the base station and the weak one is far, possibly on the cell edge. Usually, in such situations the weak user experiences very low QoS and in many cases its transmitted information cannot be retrieved from the signal that is dominated by presence of the strong interferer. The proposed approach aims at separating both users, so both of them can deliver good rates.

III. RECEIVER ALGORITHMS

Two algorithms are compared. The baseline frequency-domain MMSE equalization and the proposed group detection using turbo equalization.

A. Baseline Linear MMSE MIMO Equalizer

A key advantage of adopting OFDM for LTE is that it transforms the channel from a finite-state time-domain channel into a frequency-domain multiplicative channel, greatly simplifying basic signal processing functions. The generic MMSE equalizer for the channel model (1) is given by

$$\hat{X}_{MMSE} = A^H H^H M^{-1} Y = A^H H^H (H W H^H + \sigma^2 I)^{-1} Y,$$

which using the matrix inversion lemma [4] can be transformed into

$$\hat{X}_{MMSE} = A^{-1} M^{-1} H^H Y = A^{-1} (H^H H + \sigma^2 W^{-1})^{-1} H^H Y.$$

Inversion of the matrix $M$ is simple, since it consists of $N_t$ blocks of diagonal matrices and can be accomplished by $M$ separate inversions of $N_t \times N_t$ size matrices. Subsequently to obtain modulation symbol estimates the resulting signal in (3) or (4) is passed to the inverse DFT block

$$\hat{x}_{MMSE} = F^H \hat{X}_{MMSE}.$$

Subsequently, these signals are passed to the corresponding error control decoders.

B. Group Turbo MMSE MIMO Equalizer

The key idea of the proposed receiver relies on the use of the following splitting of the received signal depending on the received powers. Rewriting (1)

$$Y = H_1 FA_1 x_1 + H_2 FA_2 x_2 + N$$

it can be seen that signals belonging to each user (group of $N_t$ streams) are separated and we applied the trivial equality $AF = FA$. $H_i$ is the $i$-th group of columns of the MIMO matrix, see matrix in (2).

The proposed group iterative equalizer is shown in Figure 1. The receiver operates as follows. First the received signal $Y$
is passed to the MMSE filter tuned to the strong user. The resulting signal is
\[ \hat{x}_1 = M_1 Y \] (7)
where
\[ M_1 = A_1 H_1 F H_1^H (H_1 W_1 H_1^H + H_2 W_2 H_2^H + \sigma^2 I)^{-1} \] (8)
is the is scaled covariance matrix of \( Y \) which is the input signal to the filter at the initial iteration. The estimate \( \hat{x}_1 \) is then passed to the soft-demodulator and subsequently to the turbo code decoder, which produces estimates of all (systematic and parity) bits \( \hat{x}_1 \). These estimates can be erroneous and the influence of these errors can be captured using the soft-symbol variance parameter
\[ \sigma^2_{s,1} = \mathbb{E} |x_1 - \hat{x}_1|^2 \] (9)
which is related to the power of the residual interference that the weak user will experience from imperfect cancellation of the strong user. The estimates \( \hat{x}_1 \) are QAM remodulated to obtain \( Y_1 \) and cancelled from the composite signal \( Y \) to produce the interference-reduced signal for the weak group detector/decoder
\[ Y_2 = Y - Y_1 \] (10)
which can be further developed into
\[ Y_2 = H_1 F A_1 (x_1 - \hat{x}_1) + H_2 F A_2 x_2 + N \] (11)
where interstream interference and intersymbol interference is present. The covariance matrix of (11), used in the subsequent MMSE filtering, is
\[ \mathbb{E} [Y_2 Y_2^H] = H_1 W_1 \sigma^2_{s,1} H_1^H + H_2 W_2 H_2^H + \sigma^2 I. \] (12)
Similarly, the process of MMSE filtering is repeated for the weak user to obtain
\[ \hat{x}_2 = M_2 Y_2 \] (13)
where
\[ M_2 = A_2^H F H_2 H_2^H (H_1 W_1 \sigma^2_{s,1} H_1^H + H_2 W_2 H_2^H + \sigma^2 I)^{-1} \] (14)
The cancellation and demodulation/decoding is performed and this completes global turbo iteration number 1. The number of global iterations can be arbitrary, however avoiding high complexity it must be aimed at a moderate number of \( 2 - 3 \) in order to facilitate practical implementation.

The soft symbols are generated by first generating LLR values for each component bit of the constellation, then (after decoding) rebuilding the soft symbols using the decoded soft bits \( \bar{b}_i \). We assume that all quadrature constellations are bit-mapped in the binary superposition form, where in each dimension, a PAM symbol is created as
\[ x = \sum_{i=0}^{B-1} \bar{b}_i 2^i \] (15)
and \( B \) is the order of the PAM constellation.

IV. ANALYSIS

The method of analyzing the convergence behavior of the proposed receiver is based on variance transfer charts, which are related to the popular EXIT analysis [19]. With this tool the error free performance of the detector can be predicted as a function of the SNR. The only important knowledge required to sketch those graphs is the SNR transfer of each of the blocks involved in the processing chain.

A. Variance transfer method

The procedure of obtaining variance transfer curves will be briefly outlined. To comply with the LTE standards [14], [20], the choice of channel coding and modulation is fixed therefore we do not consider other alternatives. For the case considered two transfer functions have to be evaluated. One for the MMSE detector and one for the demodulation/decoding block as shown in Figure 2. The MMSE filter for user \( k \) accepts as its input the cancelled signal with residual interference power of group \( k' \), \( \sigma^2_{s,k'} \), and outputs signal plus disturbance of power \( \sigma^2_{\text{norm},k} \) (VT analysis assumes that the interference is modeled as Gaussian process \( \mathcal{CN}(0, \sigma^2_{\text{norm},k}) \)), which is subsequently passed to the demodulator/decoder and re-modulator to produce soft-symbol values that will be used in subsequent MMSE filtering of group \( k' \). The variance \( \sigma^2_k \) can be calculated using equations (13) and (14). Usually the signal power is alternated through the MMSE filtering and in general \( P_k \neq 1 \). Therefore the normalization w.r.t to unit signal power is necessary and new normalized noise plus interference power is obtained as
\[ \sigma^2_{\text{norm},k} = \frac{\sigma^2_k}{P_k}. \] (16)

Having the VT plots one can precisely predict the number of turbo equalization iterations needed for the system to achieve error-free performance for both types of traffic. This is particularly interesting for mixed traffic, where one user affects others depending on the parameters like modulation format, coding applied. Although the analysis given below is carried only for a two user case, it is straightforward to extend to other cases. The system delivers vanishing error-rates when the decoding trajectory has its fixed point at \( \sigma^2 = 0 \).
The following system is analyzed. A 6-path channel model with impulse response

$$E[|h|^2] = [0.6429 0.2553 0.0671 0.0222 0.0084 0.0042]$$

is assumed. These values are averages over a number of runs and were obtained from an SCM channel simulator [21] for the urban macro scenario case. Each of the channel impulse response taps has distribution $h_i \sim \mathcal{CN}(0, E[|h_i|^2])$ implying Rayleigh fading. It is also assumed that the entire block of $M$ modulation symbols experiences the same fading and since $M$ is usually smaller than number of symbols in a data frame therefore the whole turbo frame experiences a number of independent fading channels. At the receiver front-end complex Gaussian noise $\mathcal{CN}(0, \sigma^2)$ is added to every receive antenna. The average signal-to-noise ratio per bit for both systems with the assumption that both terminals have the same number of antennas is obtained as

$$\frac{E_b}{N_0} = \frac{1}{2} \left( \frac{E_{s,1}}{R_{t,1}R_{m,1}} \sigma^2 + \frac{E_{s,2}}{R_{t,2}R_{m,2}} \sigma^2 \right)$$

and used throughout the paper. $R_{t,k}$ and, $R_{m,k}$ are the turbo code rate (here $R_{t,1} = 1/3$) and the modulation rate for user $k$. In the simulations localized subcarrier mapping was used [18]. Both figures below present system behavior with the parameters listed in Table I. Figure 3 presents the detection/decoding process for an 4x4 MIMO system at average $E_b/N_0 = 7.0$ dB. Detection/decoding trajectory is outlined - dashed lines indicate infinite loop.

![Fig. 3. Variance transfer chart for 4 x 4 MIMO system at average $E_b/N_0 = 7.0$ dB. Detection/decoding trajectory is outlined - dashed lines indicate infinite loop.](image)

The maximum gain is ca. 2.5 dB (at $\sigma^2_s = 0$) point. After increasing the SNR to 8dB we obtain system behavior presented in Figure 4. In this case everyone converges, since the detection/decoding trajectory arrives at $\sigma^2_s = 0$ for all users, however after a number of turbo iterations. This number is not well captured by the VT plot in Figure 4, but it is certain that it is larger than one and at the same time relatively small, since line (4) is already very close to $\sigma^2_s = 0$ point.

It is difficult to exactly point the moment when the system achieves almost error-free performance using the outlined variance transfer analysis. The main reason is that it is not expected for this system to exhibit a sharp cliff behavior due to the application of finite length codes as is presented in the simulations section below.

V. NUMERICAL RESULTS

To prove the validity of the above proposed semi-analytical approach numerical experiments were performed. All the important simulation parameters are given in Table I. It can be seen that the proposed receiver outperforms the baseline processor. From the results presented in Figure 5 we notice that 3 global turbo iterations are necessary for the system to achieve almost error-free performance, which agrees with the predictions obtained via the variance transfer analysis in Figure 4. The maximum gain is ca. 2.5dB (at $P_b = 10^{-5}$) after three global turbo iterations and no visible gains were observed with performing more iterations. The performance of the strong user in the initial iteration is the same as for the baseline algorithm, since no cancellation benefits (and...
Variance transfer analysis gives insights into the iterative process and closely matches the behavior of the system obtained using numerical experiments. As possible future extension the reduction of the complexity of the MMSE inversion, especially for large MIMO arrays will have to be addressed, preferably via the use of iterative, multistage filters [16], [22].

### VI. Conclusions and Future Work

We have studied the use of iterative, turbo demodulation to separate data streams in LTE uplink scenarios utilizing SC-FDMA. The proposed scheme is a hybrid of successive (SIC) and parallel interference cancellation (PIC) and outperforms baseline non-iterative processing. The coexistence of mixed traffic is achieved and better QoS levels can be maintained. The